ULTIMATE KCET

CRASH COURSE 2026

Mathematics | Lecture – 01

Application of Derivatives & Statistics

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Recap *of previous lecture*

1. DE
2. AOD
3.
4.

Topics *to be covered*

1. AOD → continue
2. Statistics
3.
4.

QUESTION

#Q. The function $f(x) = x^3 - 3x^2 + 3x - 100, x \in R$ is

[✓ A] increasing

B decreasing

C not increasing in (1, 2)

D neither increasing nor decreasing

$$f'(x) = 3x^2 - 6x + 3$$
$$= 3[x^2 - 2x + 1]$$
$$= 3(x-1)^2$$

WKT

$$3(x-1)^2 \geq 0 \quad \forall x \in R$$
$$f'(x) \geq 0 \quad \forall x \in R$$

QUESTION

if $a \cdot b \leq 0$

& if $a > 0$

$\Rightarrow b < 0$

$\oplus\ \underline{\ominus} = \ominus$

#Q. Find the values of x such that $f(x) = x^4 - 2x^3 + 1$ is a decreasing function.

$$f'(x) = 4x^3 - 6x^2$$
$$= 4x^2\left[x - \frac{3}{2}\right]$$

$$\Downarrow$$
$$f'(x) \leq 0$$
$$4x^2\left(x - \frac{3}{2}\right) \leq 0$$
$$\underset{>0}{\downarrow} \qquad \Downarrow$$
$$x - \frac{3}{2} \leq 0$$
$$x \leq \frac{3}{2}$$

A. $x > -1$

B. $x < 3/2$

C. $x > 3/2$

D. $x < -1$

QUESTION

$$\frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2} \quad \Big| \quad \int \frac{1}{x}\,dx = \log|x| + c$$

#Q. The function $f(x) = \frac{3}{x} + 7$ where $x \neq 0$ is

$\hookrightarrow f'(x) = -\frac{3}{x^2}$

- **A** [✓ decreasing for all $x \in R - \{0\}$]
- **B** increasing for all $x \in R - \{0\}$
- **C** neither increasing nor decreasing for all $x \in R - \{0\}$
- **D** None of these

WKT

$x^2 \geq 0 \quad \forall x \in R$

$\frac{1}{x^2} > 0 \quad \forall x \in R - \{0\}$

$-\frac{3}{x^2} < 0 \quad \forall x \in R - \{0\}$

$f'(x) < 0 \quad \forall x \in R - \{0\}$

QUESTION

$e^{1-x} \rightarrow$ exponential func

$\Downarrow$

Range $= (0, \infty)$

$\downarrow$

+ve

#Q. The function $f(x) = xe^{1-x}$

$$f'(x) = xe^{1-x}(-1) + e^{1-x}(1)$$

$$= e^{1-x}(-x+1)$$

A ✗ strictly increases in the interval (1/2, 2)

B ✗ increases in the interval $(0, \infty)$

C decreases in the interval (0, 2)

D ✓ strictly decreases in the interval $(1, \infty)$

Case 1:- if $f(x)$ is increasing

$$f'(x) \geq 0$$

$$e^{1-x}(-x+1) \geq 0$$

$\downarrow$

>0

$$\Rightarrow -x+1 \geq 0$$

$$1 \geq x$$

$$x \leq 1$$

Case 2:- if $f(x)$ is strictly decreasing

$$f'(x) < 0$$

$$e^{1-x}(-x+1) < 0$$

$\downarrow$

>0

$$\Rightarrow -x+1 < 0$$

$$1 < x$$

$$x > 1$$

$$x \in (1, \infty)$$

QUESTION

#Q. The function $f(x) = e^{ax} + e^{-ax}$, $a > 0$ is monotonically increasing for

$f'(x) = a[e^{ax} - e^{-ax}]$

A $-1 < x < 1$

B $x < -1$

C $x > -1$

[✓ D] $x > 0$

$f'(x) \geq 0$

$a[e^{ax} - e^{-ax}] \geq 0$

$e^{ax} \geq e^{-ax}$

Take log

$ax \geq -ax$

$2ax \geq 0$

$\boxed{x \geq 0}$

QUESTION

#Q. The function $\tan x - 4x$ for $-\frac{\pi}{3} < x < 0$ is

$$f(x) = \tan x - 4x$$

$$f'(x) = \sec^2 x - 4$$

- A [✓ decreasing]
- B increasing
- C neither increasing nor decreasing
- D None of these

given

$$-\frac{\pi}{3} < x < 0$$

$$\sec\left(-\frac{\pi}{3}\right) > \sec x > \sec 0$$

$$2 > \sec x > 1$$

$$\Downarrow$$

$$1 < \sec x < 2$$

$$1 < \sec^2 x < 4$$

$$-3 < \sec^2 x - 4 < 0$$

$$-3 < f'(x) < 0$$

$$\Rightarrow \underline{f'(x)} = -ve$$

QUESTION

#Q. The function $f(x) = x + \cos x$ is

$f'(x) = 1 - \sin x$

WKT

$-1 \le -\sin x \le 1 \quad \forall x \in R$

$0 \le 1 - \sin x \le 2$

$\Downarrow$

$0 \le f'(x) \le 2$

$\Downarrow$

$f'(x) \geq 0 \quad \forall x \in R$

[✓ **A**] always increasing

B always decreasing

C increasing for certain range of x

D None of these

QUESTION

#Q. The function $f(x) = \tan x - x$

$$f'(x) = \sec^2 x - 1$$

A [✓ always increases]

B always decreases

C never increases

D sometimes increases and sometimes decreases

WKT

$$\sec x \in (-\infty, -1] \cup [1, \infty)$$

$$\sec^2 x \in [1, \infty)$$

$$\sec^2 x - 1 \in [0, \infty)$$

$$f'(x) \in [0, \infty)$$

$$\Downarrow$$

$$\underline{f'(x) \geq 0}$$

QUESTION

#Q. What is the value of b for which $f(x) = \sin x - bx + c$ is decreasing in the interval $(-\infty, \infty)$?

A $b < 1$

[✓ B] $b \geq 1$

C $b > 1$

D $b \leq 1$

$$f'(x) = \cos x - b$$

$$f'(x) \leq 0$$

$$\cos x - b \leq 0$$

$$\cos x \leq b \Rightarrow \text{max of } \cos x \leq b$$

$$\text{since } \cos x \in [-1, 1]$$

$$1 \leq b$$

$$\boxed{b \geq 1}$$

$$\underset{\text{min}}{\underset{\downarrow}{A}} \leq f(x) \leq \underset{\text{max}}{\underset{\downarrow}{B}}$$

QUESTION

#Q. What is the value of b for which $f(x) = \sin x - bx + c$ is increasing in the interval $(-\infty, \infty)$?

(A) $b < 1$

(B) $b \geq 1$

(C) $b > 1$

(D) $b \leq -1$

$f'(x) = \cos x - b$

$f'(x) \geq 0$

$\cos x - b \geq 0$

$\cos x \geq b \Rightarrow$ min value of $\cos x \geq b$

WKT $-1 \leq \cos x \leq 1$

$\Rightarrow -1 \geq b$

$\Rightarrow \underline{b \leq -1}$

QUESTION

#Q. If the function $f(x) = kx^3 - 9x^2 + 9x + 3$ is monotonically increasing in every interval, then

(A) $k < 3$

(B) $k \leq 3$

[✓ (C) $k > 3$]

(D) $k \geq 3$

$f'(x) = 3kx^2 - 18x + 9$

then $f'(x) \geq 0$

$3kx^2 - 18x + 9 \geq 0$

$a = 3k \mid b = -18 \mid c = 9$

then $D < 0$

$(-18)^2 - 4(3k)(9) < 0$

$324 < 108k$

if $f(x) = ax^2 + bx + c$

such that

$f(x) \geq 0$

$\Rightarrow a > 0$ & $D < 0$

$108k > 324$

$\boxed{k > 3}$

QUESTION

$x \in [1,2]$

$\Uparrow$

#Q. If the function $f(x) = 2x^2 - kx + 5$ is increasing on [1,2], then k lies in the interval

$f'(x) = 4x - k$

A [✓ $(-\infty, 4)$]

Here $f'(x) \geq 0$

B $(4, \infty)$

$4x - k \geq 0$

$4x \geq k$

C $(-\infty, 8)$

$x \geq \frac{k}{4} \Rightarrow$ $\underline{\text{min}}$ value of $x \geq \frac{k}{4}$

D $(8, \infty)$

Here $x \in [1, 2]$

$\Rightarrow 1 \geq \frac{k}{4}$ | $k \leq 4$

QUESTION

#Q. The function $f(x) = x^3 + 6x^2 + (9 + 2k)x + 1$ is strictly increasing for all x, if

- **A** [✓ $k > \frac{3}{2}$]
- **B** $k \geq \frac{3}{2}$
- **C** $k < \frac{3}{2}$
- **D** $k \leq \frac{3}{2}$

$f'(x) = 3x^2 + 12x + (9+2k)$

$\Rightarrow f'(x) > 0$

$3x^2 + 12x + (9+2k) > 0$

$a = 3 \mid b = 12 \mid c = 9 + 2k$

Here $D < 0$

$144 - 4(3)(9+2k) < 0$

$12 < 9 + 2k$

$2k > 3$

$k > \frac{3}{2}$

QUESTION

#Q. The function $f(x) = \frac{x}{\log x}$ increases in the interval

A $(0, \infty)$

B $(0, e)$

C [✓ (e, ∞)]

D None of these

$$f'(x) = \frac{\log x(x) - 1}{(\log x)^2}$$

$$f'(x) \geq 0$$

$$\log x - 1 \geq 0$$

$$\log x \geq 1$$

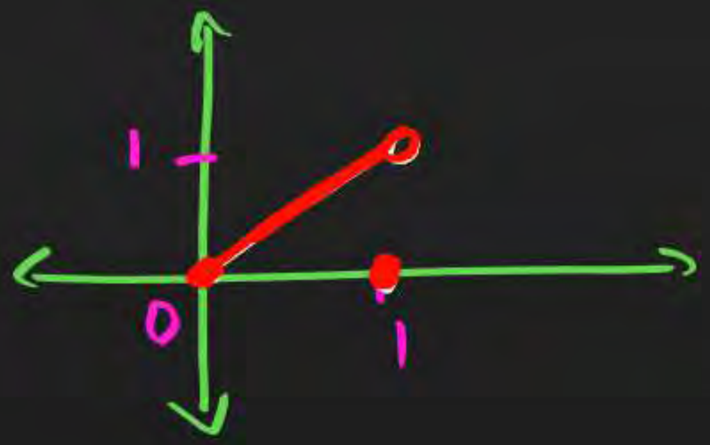
$$\boxed{x \geq e}$$

QUESTION

#Q. $f(x) = x - [x]$ in the interval $[0, 1]$ is

$x \in [0,1) \Rightarrow f(x) = (0,1)$

$x = 1 \Rightarrow f(x) = 1 - [1] = 0$



A Increasing

B Decreasing

C [✓ Neither increasing nor decreasing]

D None of these

$f(x) = x - [x]$

$x \in (0,1)$

$f(0.5) = 0.5 - [0.5]$

$= 0.5 - 0 = 0.5$

$f(0.6) = 0.6 - [0.6]$

$= 0.6 - 0$

$= 0.6$

$x = 1$

$f(1) = 1 - [1] = 0$

↓

is on x-axis

↓

Always increases

↓

neither increasing
nor decreasing

QUESTION

#Q. $f(x) = x - [x]$ in the interval $[0, 1)$ is [2023]

A [✓ increasing]

B decreasing

C neither increasing nor decreasing

D none of these

$[-0.6] = -1$

⊛ Range of $f(x) = x - [x]$

Soln:-

① $x \in Z \Rightarrow f(x) = 0$

$f(1) = 1 - [1] = 1 - 1 = 0$

$f(2) = 2 - [2] = 2 - 2 = 0$

② $x \in R - Z \Rightarrow f(x) \in (0, 1)$

$f(0.5) = 0.5 - [0.5] = 0.5 - 0 = 0.5$

$f(3.5) = 3.5 - [3.5] = 3.5 - 3 = 0.5$

$f(-0.6) = -0.6 - [-0.6] = -0.6 - (-1) = 0.4$

$f(-2.3) = -2.3 - [-2.3] = -2.3 + 3 = 0.7$

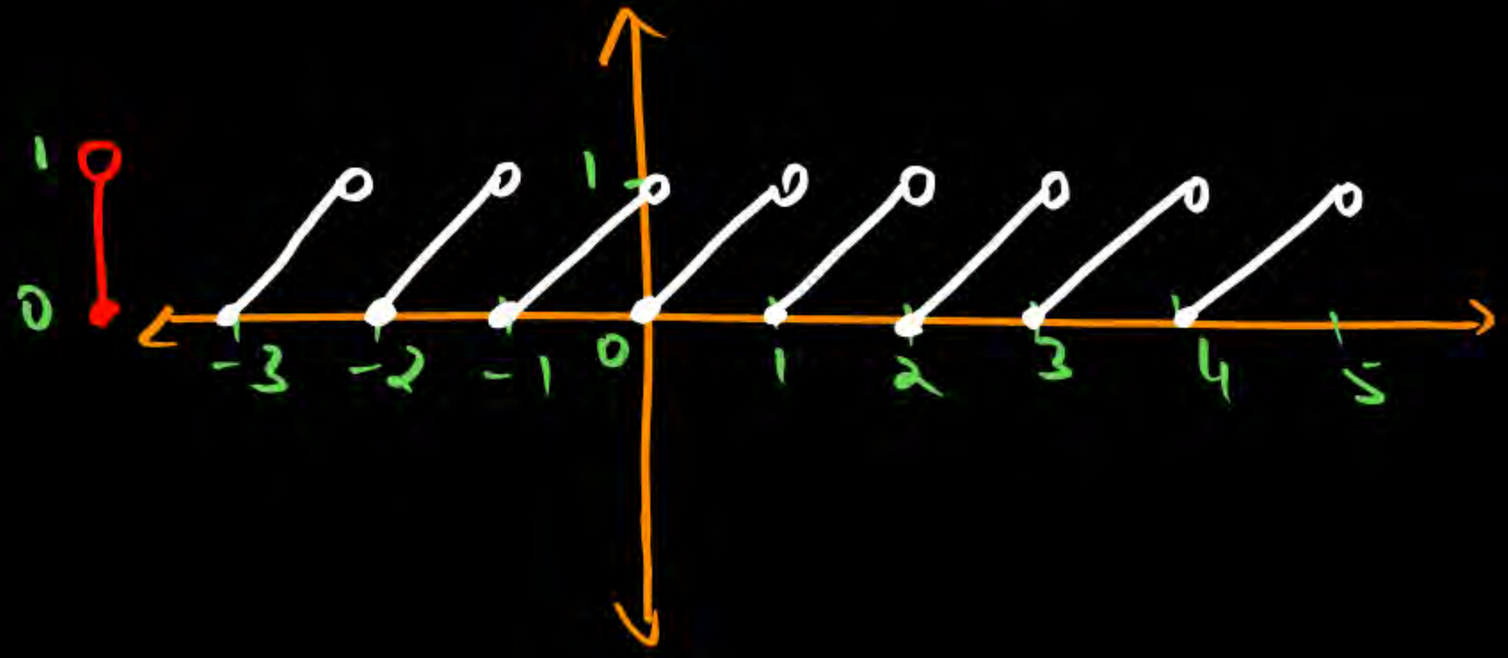
$\therefore$ if $x \in R$

$f(x) \in [0, 1)$

$f(x) = x - [x]$

Domain = R

Range = $[0, 1)$



QUESTION

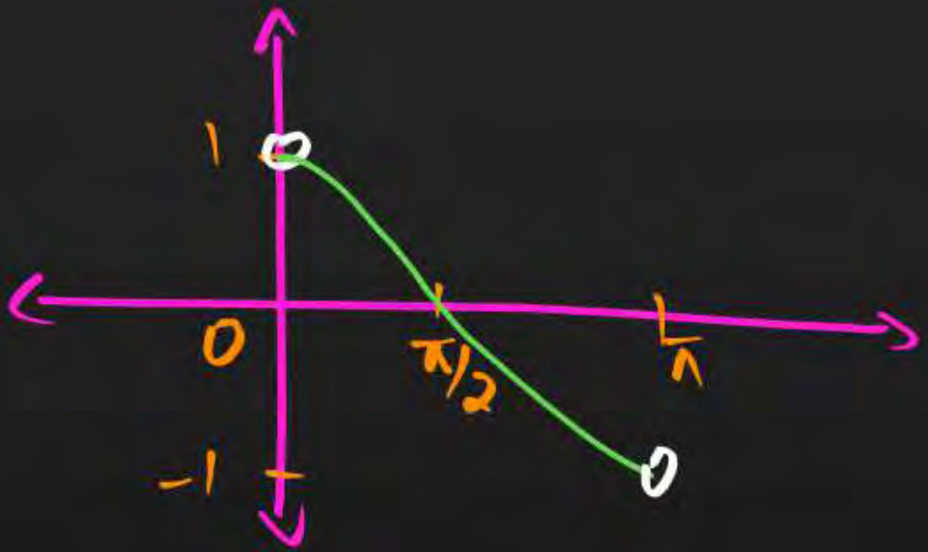
#Q. The function $f(x) = \cos x, 0 < x < \pi$ is [2024]

Ⓐ increasing

Ⓑ [✓ decreasing]

Ⓒ neither increasing nor decreasing

Ⓓ none of these



$f'(x) = -\sin x$

in $(0, \pi) \Rightarrow \sin x > 0$

$\Downarrow$

$-\sin x < 0$

$\underline{f'(x) < 0}$

QUESTION

#Q. The function $f(x) = x^3 - 6x^2 + 12x - 16, x \in R$ is [2019]

$f'(x) = 3x^2 - 12x + 12$

$= 3[x^2 - 4x + 4]$

[✓ A] increasing for all $x \in R$

$f'(x) = 3(x-2)^2$

B decreasing for all $x \in R$

Here $(x-2)^2 \geq 0$

$f'(x) \geq 0$

C increasing for all $x \in (-1, \infty)$

D decreasing for all $x \in (2, \infty)$

QUESTION

$\sqrt{x^2} = |x|$

#Q. Find the values of x if $f(x) = \frac{x}{x^2+1}$ is a decreasing function.

$x=0$

A $x < 1$

$x=0$

B $x > -1$ or $x < 1$

✓ **C** $x < -1$ or $x > 1$

D $x > -1$ $x=0$

$$f'(x) = \frac{(x^2+1)(1) - x(2x)}{(x^2+1)^2}$$

$$= \frac{1-x^2}{(x^2+1)^2}$$

$f'(x) \le 0$

$1 - x^2 \le 0$

$1 \le x^2$

$x^2 \ge 1$

$|x| \ge 1$

$x \in (-\infty, -1] \cup [1, \infty)$

$x \le -1$ or $x \ge 1$

$|x| \geq 1$

↓

we need to find input values for which the above statement is true

$x = 1, 2, 3, 4 \ldots$

$x = -1, -2, -3, -4 \ldots$

$|-2| = 1$

$|-1| = 1$

QUESTION

#Q. The set of values of ‘x’ for which $f(x) = \cos x - x$ is decreasing in

(A) $(-\infty, 0)$

(B) $(0, \infty)$

(C) $(-\infty, \infty)$

(D) $\emptyset$

$f'(x) = -\sin x - 1$

WKT

$-1 \le -\sin x \le 1 \quad \forall x \in R$

$-2 \le -\sin x - 1 \le 0$

$-2 \le f'(x) \le 0$

$\Downarrow$

$f'(x) = -ve \quad \forall x \in R$

QUESTION

$$\frac{-ve}{+ve} = -ve$$

$$f(x) = \frac{1}{\sqrt{g(x)}}$$

① $g(x) > 0$

② $f(x) > 0$

#Q. $f(x) = \sqrt{x^2 - 4}$ is decreasing in

- **A** $(-2, 2)$
- **B** $(2, \infty)$
- [✓ **C** $(-\infty, -2)$]
- **D** $(-\infty, \infty)$

$$f'(x) = \frac{x}{\sqrt{x^2-4}}$$

Here

$$f'(x) \le 0$$

$$\frac{x}{\sqrt{x^2-4}} \le 0$$

Here

$$\sqrt{x^2-4} = +ve$$

$x \le 0 \to$ ①

but for $f(x) = \sqrt{x^2-4}$

$$x^2 - 4 \geq 0$$

$$x^2 \geq 4$$

$$|x| \geq 2$$

$x \in (-\infty, -2] \cup [2, \infty) \to$ ①

① ∩ ②

$$x \in (-\infty, -2]$$

QUESTION

#Q. $y=\sqrt{x-x^2}$ strictly increases in the interval

(A) ✓ $\left(0,\frac{1}{2}\right)$

(B) $\left(-\frac{1}{2},0\right)$

(C) ✗ $\left[0,\frac{1}{2}\right]$

(D) $\left[0,\frac{1}{2}\right)$

$$f'(x)=\frac{1-2x}{2\sqrt{x-x^2}}$$

$$\Downarrow$$

$$f'(x)>0$$

$$\frac{1-2x}{\underbrace{2\sqrt{x-x^2}}_{+ve}}>0 \Rightarrow x\neq 0 \text{ in the soln}$$

$$\therefore 1-2x>0$$

$$1>2x$$

$$2x<1$$

$$x<\frac{1}{2}\rightarrow ①$$

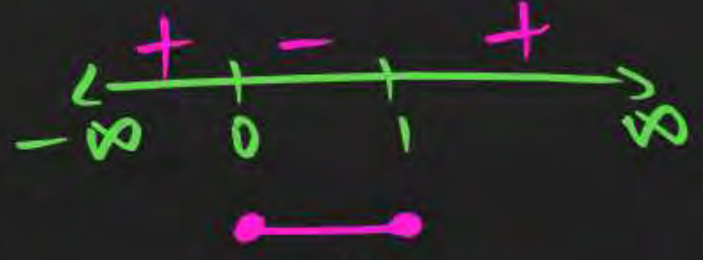
Now in $f(x)=\sqrt{x-x^2}$

$$x-x^2\geq 0$$

$$x(1-x)\geq 0$$

$$-x(x-1)\geq 0$$

$$x(x-1)\leq 0$$



$x\in[0,1]\rightarrow ②$

① ∩ ② − {0}

$$x\in\left(0,\tfrac{1}{2}\right)$$

$$\frac{1-2x}{\sqrt{x-x^2}} > 0$$

$$\frac{-2\left(x-1/2\right)}{\sqrt{x(1-x)}} > 0$$

$$\frac{x-1/2}{\sqrt{x(1-x)}} < 0$$

$\longrightarrow D^r > 0$

$\Downarrow$

$x \neq 0$ & $x \neq 1$

in the soln

$$x - 1/2 < 0$$

$$x < 1/2 \longrightarrow ①$$

$$x(1-x) > 0$$

$$-x(x-1) > 0$$

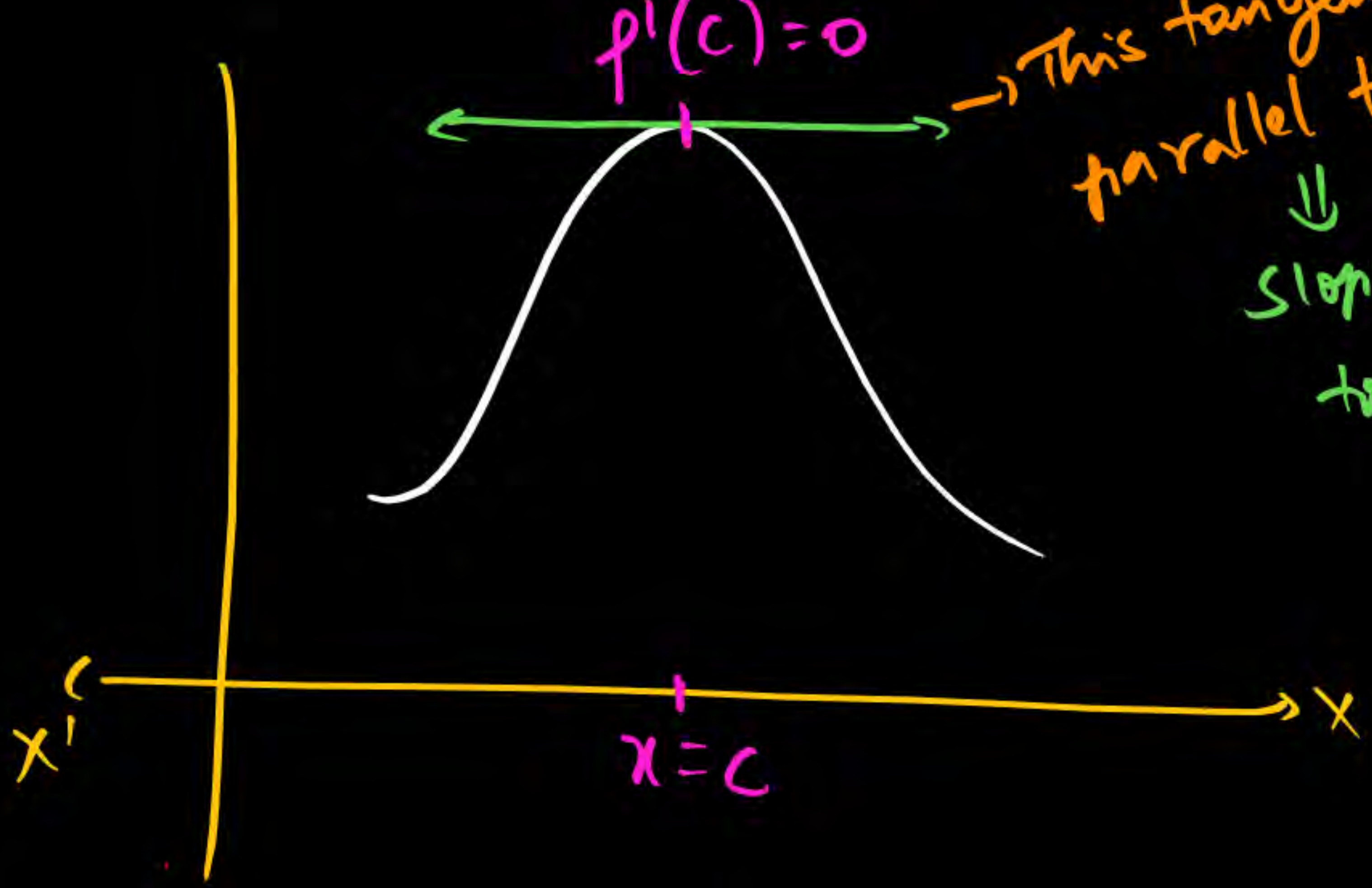
$$x(x-1) < 0$$

$$x \in (0,1) \longrightarrow ②$$

$$① \cap ②$$

$$x \in \underline{(0, 1/2)}$$

⊛ Stationary Point :-



→ This tangent is parallel to x-axis

$\Downarrow$

slope of tangent to the curve $= 0$

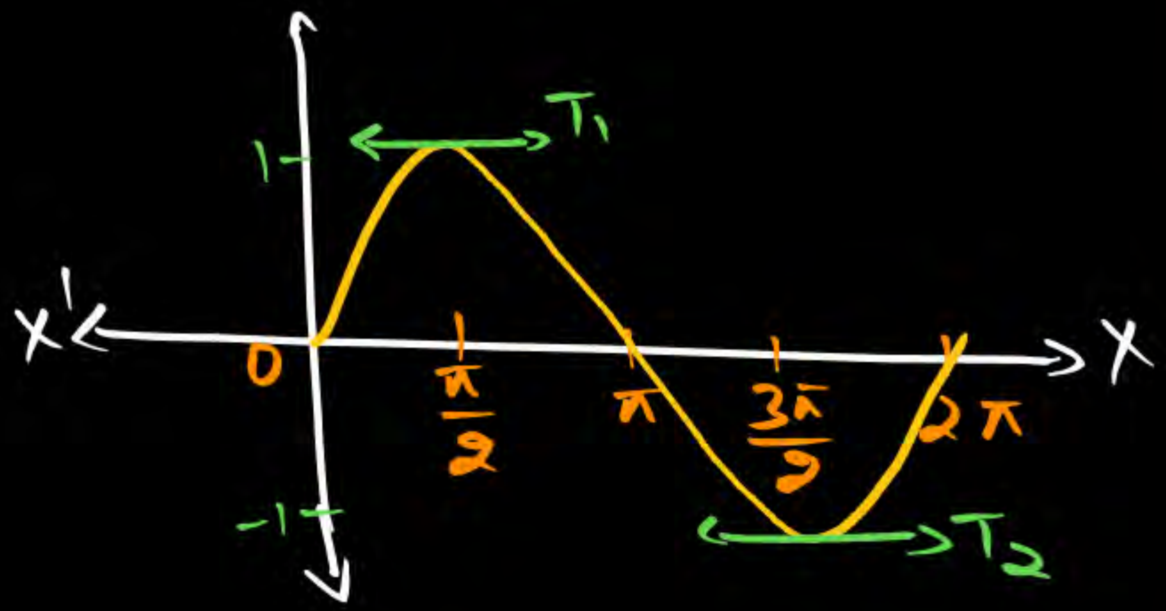
$\Downarrow$

$f'(c) = 0$

$\therefore x = c$ is the stationary Point

⊛ if $f(x) = \sin x$, $x \in [0, 2\pi]$

How many stationary Points exist



No of stationary Points = 2

i.e, At $x = \frac{\pi}{2}$ &

$x = \frac{3\pi}{2}$

⊛ $f(x) = \sin x$

$f'(x) = \cos x$

Consider

$f'(x) = 0$

$\cos x = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}.$

QUESTION

#Q. The values of 'x' at which $f(x) = \sin x$ is stationary are given by

A $n\pi, \forall n \in Z$

B [✓ $(2n+1)\frac{\pi}{2}, \forall n \in Z$]

C $\frac{n\pi}{4}, \forall n \in Z$

D $\frac{n\pi}{2}, \forall n \in Z$

$f'(x) = \cos x$

Consider $f'(x) = 0$

$\cos x = 0$

$x = \frac{\pi}{2}, -\frac{\pi}{2}, \frac{3\pi}{2}, -\frac{3\pi}{2}$ etc.

$x = (2n+1)\frac{\pi}{2}, n \in Z$

QUESTION

#Q. The values of 'x' at which $f(x) = \cos x$ is stationary are given by

$f'(x) = -\sin x$

Consider $f'(x) = 0$

$-\sin x = 0$

$\sin x = 0$

$x = n\pi, n \in Z$

[✓ A] $n\pi, \forall n \in Z$

B $(2n+1)\frac{\pi}{2}, \forall n \in Z$

C $\frac{n\pi}{4}, \forall n \in Z$

D $\frac{n\pi}{2}, \forall n \in Z$

QUESTION

#Q. Stationary point of $y = \frac{\log x}{x} (x > 0)$ is

A $(\overset{x}{1}, \overset{y}{0})$

B [✓ $\left(e, \frac{1}{e}\right)$]

C $\left(\frac{1}{e}, -e\right)$

D $\left(\frac{1}{e}, \frac{1}{e}\right)$

$f'(x) = y_1 = \frac{1-\log x}{x^2}$

consider $f'(x) = 0$

$\frac{1-\log x}{x^2} = 0$

$\Rightarrow 1-\log x = 0$

$\log x = 1$

$\boxed{x = e}$

$\therefore f(e) = \frac{\log e}{e} = \frac{1}{e} = y$

$\therefore (x, y) = \underline{\left(e, \frac{1}{e}\right)}$

QUESTION

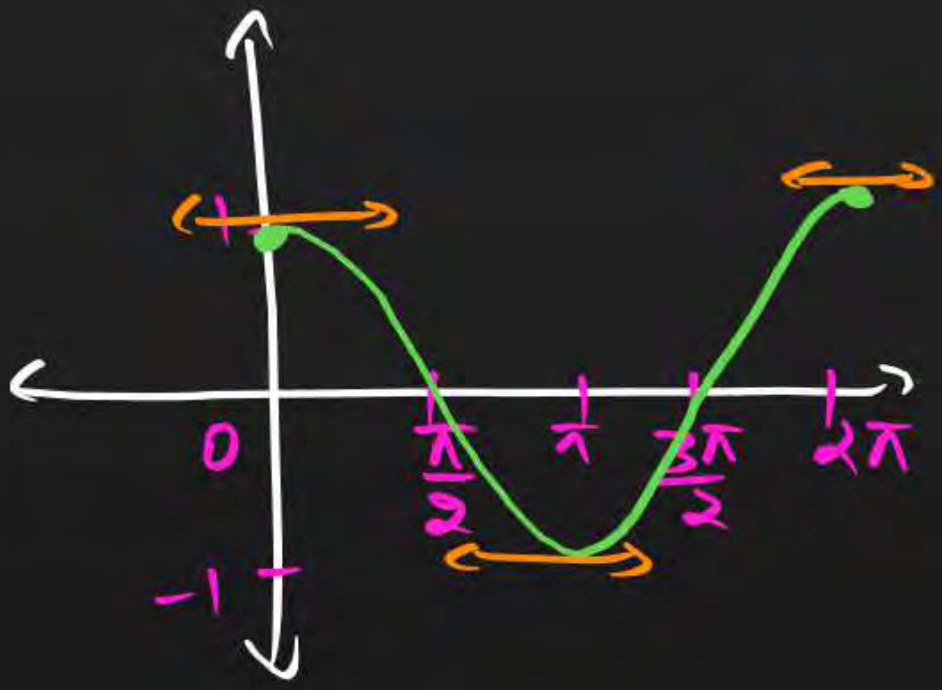
#Q. The number of stationary points of $f(x) = \cos x$ in $[0, 2\pi]$ are

$$f'(x) = -\sin x$$

$$f'(\underline{0}) = 0$$

$$f(\underline{\pi}) = -\sin \underline{\pi} = 0$$

$$f'(\underline{2\pi}) = -\sin 2\underline{\pi} = 0$$



- **A** 1
- **B** 2
- [✓ **C** 3]
- **D** 4

QUESTION

#Q. If $a > b$ maximum value of $a\sin^2x + b\cos^2x$ is

[✓ **A**] a

B b

C $a + b$

D $\sqrt{a^2 + b^2}$

$$f(x) = a\sin^2x + b(1 - \sin^2x)$$

$$= \sin^2x(a - b) + b$$

WKT

$$0 \le \sin^2x \le 1$$

$$0 \le \sin^2x(a - b) \le a - b$$

$$\underset{\substack{\downarrow \\ \text{min}}}{b} \le \sin^2x(a - b) + b \le \underset{\substack{\downarrow \\ \text{max}}}{a}$$

$a\sin x + b\cos x$

min $= -\sqrt{a^2+b^2}$

max $= \sqrt{a^2+b^2}$

QUESTION

#Q. A stationary point of $f(x) = \sqrt{16 - x^2}$ is

x y

A (4, 0)

B (−4, 0)

C ✓ (0, 4)

D (−4, 4)

$$f'(x) = \frac{-x}{\sqrt{16-x^2}}$$

consider $f'(x) = 0$

$$\boxed{x = 0}$$

$$f(0) = \sqrt{16-0} = 4 = y$$

$$\underline{(x, y) = (0, 4)}$$

QUESTION

→ finding $f(c)$

stationary point :- finding $x = c$ for which $f'(c) = 0$

#Q. A stationary value of $f(x) = x(\ln x)^2$ is

A $2e^{-2}$

[✓ **B** $4e^{-2}$]

C $2e^{2}$

D $4e^{2}$

$$f'(x) = \frac{2\cancel{x}\log x}{\cancel{x}} + (\log x)^2$$

$$= \log x[2 + \log x]$$

Consider $f'(x) = 0$

$$\log x\,(2 + \log x) = 0$$

$\log x = 0$ or $\log x = -2$

$x = e^0$ | $\boxed{x = e^{-2}}$

$\boxed{x = 1}$

$f(1) = 1(\log 1)^2 = 0$ ✗

$$f(e^{-2}) = e^{-2}(\log e^{-2})^2$$

$$= e^{-2}(-2\log e)^2$$

$$= e^{-2}(2)^2$$

$$= \underline{4e^{-2}}$$

QUESTION

#Q. $f(x) = (\sin^{-1}x)^2 + (\cos^{-1}x)^2$ is stationary at

A [✓ $x = 1/\sqrt{2}$]

B $x = \pi/4$

C $x = 1$

D $x = 0$

$$f'(x) = \frac{2\sin^{-1}x}{\sqrt{1-x^2}} - \frac{2\cos^{-1}x}{\sqrt{1-x^2}}$$

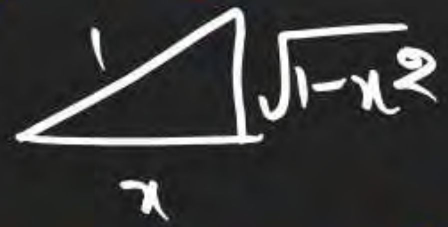
Consider $f'(x) = 0$

$$\sin^{-1}x = \cos^{-1}x$$

$$\boxed{x = \frac{1}{\sqrt{2}}}$$

$$\sin^{-1}x = \cos^{-1}x$$

$$x = \sin[\cos^{-1}x]$$



$$x = \sin[\sin^{-1}\sqrt{1-x^2}]$$

$$x = \sqrt{1-x^2}$$

$$x^2 = 1-x^2$$

$$2x^2 = 1$$

$$x^2 = \frac{1}{2}$$

$$\boxed{x = \frac{1}{\sqrt{2}}}$$

$\sin x = \cos x$

$\Downarrow$

$x = \frac{\pi}{4}$

$\sin^{-1} x = \cos^{-1} x$

$\Downarrow$

$x = \frac{1}{\sqrt{2}}$

Critical Point:-

The point $\underline{x=a}$

at which either $\underline{f'(a)=0}$

(or)

$\underline{f'(a)}$ $\underline{\text{doesnot exist}}$

① $f(x)=x^2$

$f'(x)=2x$

consider $f'(x)=0$

$2x=0$

$\boxed{x=0}$

is the critical point

② $f(x)=|x-1|$

Here at $x=1$, $f(x)$ is not differentiable

$\therefore x=1$ is the critical Point

QUESTION

#Q. The critical point of $f(x) = |2x + 7|$ at $x =$

$$2x+7=0$$

$$\boxed{x = -\frac{7}{2}}$$

- **A** 0
- **B** 7
- **C** [✓ $-7/2$]
- **D** -7

Relationship b/w A·M & G·M

$A \cdot M \geq G \cdot M$

if a & b are any 2 no such that

$A \cdot M = \frac{a+b}{2}$ & $G \cdot M = \sqrt{ab}$

$$\frac{a+b}{2} \geq \sqrt{ab}$$

$$a+b \geq 2\sqrt{ab}$$

⊛ if we are given sum of 2 terms, where the two terms are reciprocal to each other

Now if we are to find the minimum value of sum of these 2 terms

Then we can use concept of

$a+b \geq 2\sqrt{ab}$ where $\boxed{b = \frac{1}{a}}$

① Find the min value of $f(x) = \frac{4}{x^2} + 9x^2$

Soln:

Let $a = \frac{4}{x^2}$ & $b = 9x^2$

WKT $a + b \geq 2\sqrt{ab}$

$$\frac{4}{x^2} + 9x^2 \geq 2\sqrt{\frac{4}{x^2}(9x^2)}$$

$$\geq 2\sqrt{4(9)}$$

$$f(x) \geq 2(6)$$

$$f(x) \geq 12$$

$\therefore$ min value $= 12$.

QUESTION

#Q. For $0 < a < x$, the minimum value of the function $\log_x a + \log_a x$ is

A 1

B [✓ 2]

C -2

D $-1/2$

$$\log_x a + \log_a x \geq 2\sqrt{\log_x a \cdot \log_a x}$$

$$\geq 2\sqrt{\frac{\log a}{\log x}\cdot\frac{\log x}{\log a}}$$

$$\geq 2\sqrt{1}$$

$$\underline{\geq 2}$$

QUESTION

#Q. The minimum value of $16 \cot x + 9 \tan x$ is

$$a + b \geq 2\sqrt{ab}$$

$$16 \cot x + 9 \tan x \geq 2\sqrt{16 \cot x \, (9 \tan x)}$$

$$\geq 2\sqrt{144}$$

$$\geq 2(12)$$

$$\geq 24$$

A 12

B 6

C [✓ 24]

D 25

QUESTION

#Q. The minimum value of $f(x) = 4\sec^2 x + 9\,\mathrm{cosec}^2 x$ is

A 5

B [✓ 25]

C 13

D 13^2

$$f(x) = 4(1+\tan^2 x) + 9(1+\cot^2 x)$$

$$= 13 + \underbrace{4\tan^2 x + 9\cot^2 x}$$

$$= 13 + g(x)$$

WKT

$$4\tan^2 x + 9\cot^2 x \geq 2\sqrt{4\cancel{\tan^2 x}(9\cancel{\cot^2 x})}$$

$$\geq 2\sqrt{4(9)}$$

$$\geq 2(6)$$

$$g(x) \geq 12$$

$$13 + g(x) \geq 13 + 12$$

$$\underline{f(x) \geq 25}$$

QUESTION

#Q. The maximum height of the curve $y = 6\cos x - 8\sin x$ above the x-axis

$$y = A\cos x - B\sin x$$

$$\Downarrow$$

$$\max = \sqrt{A^2+B^2}$$

$$= \sqrt{6^2+8^2}$$

$$= \sqrt{100}$$

$$= \underline{10}$$

- **A** 6
- **B** 8
- **C** 14
- **D** [✓ 10]

QUESTION

#Q. The greatest value of the function $f(x) = \sin^2 x - 20\cos x + 1$ is

A 20

B 11

C ✓ 21

D 0

$$f(x) = 1 - \cos^2 x - 20\cos x + 1$$

$$= -[\cos^2 x + 20\cos x - 2] \quad \text{Put } \cos x = t$$

$$= -[t^2 + 20t + 10^2 - 10^2 - 2]$$

$$= -[(t+10)^2 - 102]$$

$$f(x) = 102 - (\cos x + 10)^2$$

WKT $-1 \le \cos x \le 1$

$$9 \le \cos x + 10 \le 11$$

$$81 \le (\cos x + 10)^2 \le 121$$

$$-121 \le -(\cos x + 10)^2 \le -81$$

$$-19 \le 102 - (\cos x + 10)^2 \le 21$$

$$-19 \le f(x) \le 21 \rightarrow \text{max}$$

max & min

methods :-

① 2nd Derivative test → Algebraic func

② Range method → Trigonometric func , Quadratic expressions

③ A.M & G.M → Reciprocal of 2 +ve terms

④ Absolute max & min → when closed interval is given

QUESTION

#Q. Maximum value of $1 + \overset{2(4)}{8} \sin^2(x^2) \cos^2(x^2)$ is → Range method

A [✓ 3]

B −1

C −8

D 9

$f(x) = 1+2[2\sin(x^2)\cos(x^2)]^2$

$f(x) = 1+2[\sin 2(x^2)]^2$

WKT

$0 \le \sin 2(\overset{\theta}{x^2}) \le 1$

on squaring

$0 \le [\sin 2(x^2)]^2 \le 1$

$0 \le 2[\sin 2(x^2)]^2 \le 2$

$1 \le 1+2[\sin 2(x^2)]^2 \le 3$ → max $= 3$

$2\sin\theta\cos\theta = \sin 2\theta$

$4\sin^2\theta\cos^2\theta = (2\sin\theta\cos\theta)^2$

$= (\sin 2\theta)^2$

$-1 \le \sin \textcircled{A} \le 1$

$-1 \le \sin 2x \le 1$

$0 \le \sin 2(x^2) \le 1$

but

$0 \le \sin^2 2(x^2) \le 1$

#Q. The minimum value of $x^2 - 8x + 17$ is

A 17

B −1

C [✓ 1]

D 2

$$f(x) = \underbrace{x^2 - 8x + 4^2}_{} - 4^2 + 17$$

$$= (x-4)^2 - 16 + 17$$

$$f(x) = (x-4)^2 + 1$$

WKT $(x-4)^2 \geqslant 0$

$$(x-4)^2 + 1 \geqslant 1$$

$$f(x) \geqslant 1$$

QUESTION

This should be first a critical point ⇒ At $x = \frac{5\pi}{6}$

$f'(x) = 0$

#Q. At $x = 5\pi/6$, $f(x) = 2 \sin 3x + 3 \cos 3x$ is:

$f'(x) = 6\cos 3x - 9\sin 3x$

$x = \frac{5\pi}{6}$

$3x = \frac{5\pi}{2}$

$$f'\left(\frac{5\pi}{6}\right) = 6\cos\left(\frac{5\pi}{2}\right) - 9\sin\left(\frac{5\pi}{2}\right)$$

$$= 6(0) - 9(1)$$

$$= -9 \neq 0$$

$\therefore x = \frac{5\pi}{6}$ is not a critical point

$\Downarrow$

neither a point of minima nor a point of maxima

A maximum

B minimum

C zero

D [✓ neither maximum nor minimum]

QUESTION

#Q. Find the maximum value of the function $f(x) = 3x^2 + 6x + 8, x \in R$.

- **A** 2
- **B** −8
- **C** 5
- **D** [✓ does not exist]

$$f(x) = 3\left[x^2 + 2x + 4 - 4 + \frac{8}{3}\right]$$

$$= 3\left[(x+2)^2 - \frac{4}{3}\right]$$

$$= 3(x+2)^2 - 4$$

WKT

$$3(x+2)^2 \geq 0$$

$$3(x+2)^2 - 4 \geq -4$$

$$f(x) \geq -4$$

$$-4 \leq f(x) < ?$$

$$\downarrow \qquad\qquad \downarrow$$

min = −4

max = DNE

16th evening

① St lines

② sequence & series

③ complex nos

17th → Morning (5:30)

① Stats

② P & C.

③ Conics

2nd Derivative test

① $x=a$ is the Point of maxima

if ① $f'(a)=0$ ($x=a$ is the critical point)

② $f''(a)<0$

$f(a) \to$ max value

② $x=a$ is the Point of minima

if ① $f'(a)=0$ $\Rightarrow f(a) \to$ min value.

② $f''(a)>0$

QUESTION

$\log \frac{1}{x} = -\log x$

#Q. The maximum value of $\left(\frac{1}{x}\right)^x$ is

A e

B e^e

[✓ C] $e^{1/e}$

D $(1/e)^e$

$f(x) = \left(\frac{1}{x}\right)^x$

$\log f(x) = -x \log x$

Diff

$\frac{f'(x)}{f(x)} = -[1 + \log x]$

$f'(x) = -f(x)[1 + \log x]$

$= -\underset{\neq 0}{\left(\frac{1}{x}\right)^x}(1 + \log x)$

Conside $f'(x) = 0$

$-(1 + \log x) = 0$

$\log x = -1$

$x = e^{-1} = \frac{1}{e}$

$f\left(\frac{1}{e}\right) = \left(\frac{1}{1/e}\right)^{1/e} = \underline{e^{1/e}}$

Solve PYQ's
⇓
AOD (Must & Should)

YT
↓
on saturday
⇓
PYQ's of AOD

QUESTION

$f(x) =$

#Q. The maximum value of $\frac{\log_e x}{x}$, if $x > 0$ is

$$f'(x) = \frac{1 - \log x}{x^2}$$

Consider

$f'(x) = 0$

$\log x = 1$

$x = e$

$f(e) = \frac{1}{e}$

A 1

B ✓ $1/e$

C $-1/e$

D e

QUESTION

#Q. Maximum value of the function $f(x) = \frac{x}{8} + \frac{2}{x}$ on the interval [1, 6] is

- **A** 1
- **B** 9/8
- **C** 13/12
- **D** [✓ 17/8]

$$f'(x) = \frac{1}{8} - \frac{2}{x^2}$$

$$f'(x) = 0$$

$$\frac{1}{8} = \frac{2}{x^2}$$

$$x^2 = 16$$

$$x = \pm 4$$

$$f(1) = \frac{1}{8} + \frac{2}{1} = \frac{17}{8} = 2.1$$

$$f(4) = \frac{1}{2} + \frac{1}{2} = 1$$

$$f(-4) = -\frac{1}{2} - \frac{1}{2} = -1$$

$$f(6) = \frac{6}{8} + \frac{2}{6} = \frac{3}{4} + \frac{1}{3}$$

$$= \frac{9+4}{12} = \frac{13}{12} = \underline{1.08}$$

↳ Absolute max & Absolute min.

QUESTION

sqrt func

$\Uparrow$

#Q. Find the local maximum value of the following function $f(x) = x\sqrt{1-x}$.

A ✓ $2/3\sqrt{3}$

B $1/\sqrt{3}$

C $3/2\sqrt{3}$

D $2/\sqrt{3}$

$$f(x) = x\sqrt{1-x}$$

$$[f(x)]^2 = x^2(1-x)$$

$$z = x^2(1-x)$$

$$\frac{dz}{dx} = x^2(-1) + 2x(1-x)$$

$$= -x^2 + 2x - 2x^2$$

$$= -3x^2 + 2x$$

$$\frac{dz}{dx} = 0$$

$$-3x^2 + 2x = 0$$

$$x(-3x+2) = 0$$

$x = 0$ or $x = \frac{2}{3}$

✗

$$\Downarrow$$

$$f\left(\frac{2}{3}\right) = \frac{2}{3}\sqrt{1-\frac{2}{3}}$$

$$= \frac{2}{3}\sqrt{\frac{1}{3}} = \frac{2}{3\sqrt{3}}$$

QUESTION

$\rightarrow$ method of range

#Q. If $f(x)=\frac{x^2-1}{x^2+1}$, $x \in R$, then the minimum value of f is

- (A) 0
- (B) 4/5
- (C) 3/5
- (D) [✓ −1]

Consider

$$f(x) = y$$

$$\frac{x^2-1}{x^2+1} = y$$

$$x^2 - 1 = yx^2 + y$$

$$x^2(1-y) = 1+y$$

$$x^2 = \frac{y+1}{1-y}$$

WKT $x^2 \geq 0$

$$\frac{y+1}{1-y} \geq 0$$

$$\frac{y+1}{-(y-1)} \geq 0$$

$$\frac{y+1}{y-1} \leq 0$$

$$-1 \leq y < 1$$

$$\Uparrow$$

$$\therefore y \in [-1, 1)$$

min = −1

max = DNE

$$A \le f(x) \le B$$

↓ min $= A$

↓ max $= B$

$$A < f(x) < B$$

↓ min $=$ DNE

⇓ max $=$ DNE

Reason:- There is no equal to sign

The value is not Definite

Thank

You