ULTIMATE KCET

CRASH COURSE 2026

Mathematics | Lecture - 01

Methods of Differentiation

By - Guru sir

Recap of previous lecture

1. Continuity
2. Differentiability
3.
4.

Topics *to be covered*

1. Differentiability
2. Differentiation of ITF
3.
4.

QUESTION

#Q. The correct statement among the following is

A $f(x) = x - [x]$ is continuous for all integral values of x

B $f(x) = x - [x]$ is continuous for all non integral values of x

C $f(x) = \frac{|x|}{x}$ is continuous at $x = 0$

D $f(x) = \frac{x^2}{x}$ is continuous at $x = 0$

if

$$f(x) = \begin{cases} x^2+2x+3 & \text{if } x \geq 1 \\ x^2-4x+1 & \text{if } x < 1 \end{cases}$$

1. If $f(x)$ differentiable at $x=1$
2. Find $f'(5)$, $f'(-6)$

Soln:-

$$f'(x) = \begin{cases} 2x+2 = 2(1)+2 = 4 & x \geq 1 \quad (RHD) \\ 2x-4 = 2(1)-4 = -2 & x < 1 \quad (LHD) \end{cases}$$

$LHD \neq RHD$

$\therefore f(x)$ is not differentiable at $x=1$

$x = 5 \geq 1$

$f'(x) = 2x+2$

$f'(5) = 2(5)+2$
$= 12$

$x = -6 < 1$

$f'(x) = 2x-4$

$f'(-6) = -12-4$
$= -16$

QUESTION

$(x-2)(x-1) = x^2 - 3x + 2$

#Q. The function $f(x) = \begin{cases} 1-x & \text{if } x < 1 \\ (1-x)(2-x) & \text{if } 1 \le x \le 2 \\ 3-x & \text{if } x > 2 \end{cases}$

(A) is differentiable at $x = 1$

(B) is differentiable at $x = 2$

(C) is differentiable at $x = 1$ and $x = 2$

(D) is not differentialbe either at $x = 1$ or at $x = 2$

$$f(x) = \begin{cases} 1-x & x<1 \\ x^2-3x+2 & x \geq 1 \\ x^2-3x+2 & x \le 2 \\ 3-x & x>2 \end{cases}$$

$$f'(x) = \begin{cases} -1 & \text{LHD at } x=1 \\ 2x-3 = 2(1)-3 = -1 & \text{RHD at } x=1 \\ 2x-3 = 2(2)-3 = 1 & \text{LHD at } x=2 \\ -1 & \text{RHD at } x=2 \end{cases}$$

QUESTION

#Q. The function is $f(x) = \begin{cases} x^2 & x \leq 0 \\ x & x > 0 \end{cases}$

(A) differentiable at $x = 0$

(B) not differentialbe at $x = 0$

(C) differentiable for all $x \in R$

(D) not differentiable for any $x \in R$

$f'(x) = \begin{cases} 2x = 2(0) = 0 & x \leq 0 \quad \text{LHD at } x = 0 \\ 1 & x > 0 \quad \text{RHD at } x = 0 \end{cases}$

LHD $\neq$ RHD

$$f(x) = \begin{cases} x^2 & x \le 0 \\ x & x > 0 \end{cases}$$

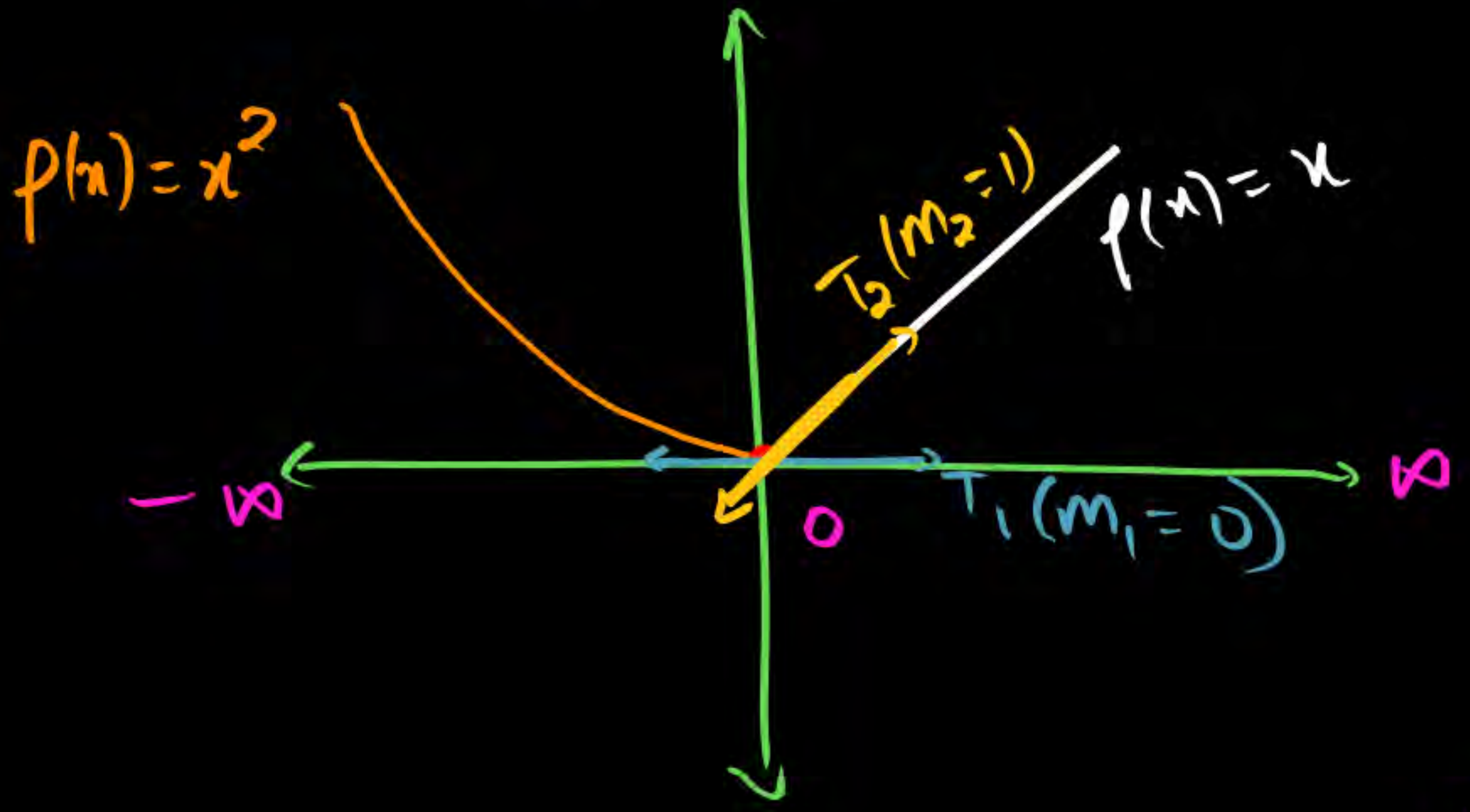
$x > 0$

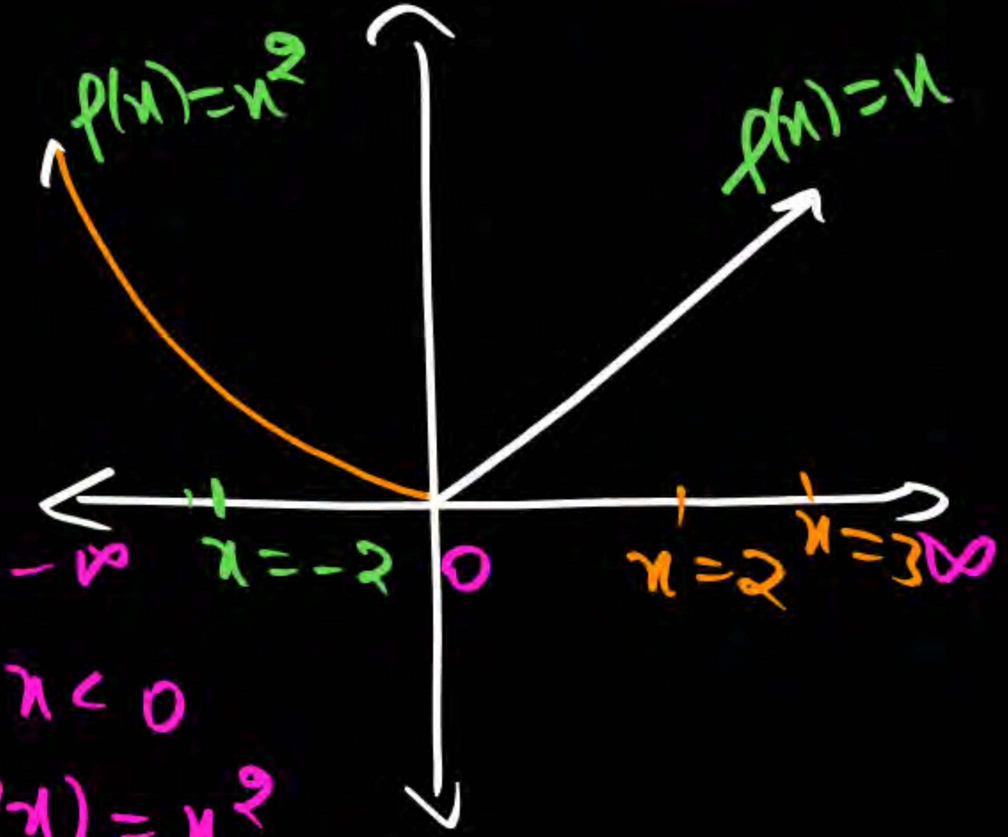
$f(x) = x$

$f'(x) = 1$

$f'(2) = 1$

$f'(3) = 1$





$x < 0$

$f(x) = x^2$

$f'(x) = 2x$

$f'(-2) = 2(-2) = -4$

QUESTION

#Q. If $f(x) = \begin{cases} 1+3x & x \geq 0 \\ 1-2x & x < 0 \end{cases}$ then RHD of $f(x)$ at $x = 0$ is

RHD

(A) 3

(B) –2

(C) 1

(D) –1

$f'(x) = \begin{cases} 0+3 = 3 \end{cases}$ RHD at $x = 0$

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

RHD	LHD
Put $x = a + h$	Put $x = a - h$
$\therefore$ As $x \to a$	$\therefore$ As $x \to a$
$h \to 0$	$h \to 0$

$$Rf'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$Lf'(a) = \lim_{h \to 0} \frac{f(a-h) - f(a)}{h}$$

QUESTION

#Q. If $f(x) = \begin{cases} x^2 \sin\frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$ then RHD is

A) 1

B) –1

C) 0

D) not defined

$f(x) = x^2 \sin\frac{1}{x}$

$f(a+h) = f(0+h) = h^2 \sin\frac{1}{h}$

&

$f(a) = f(0) = 0 \sin\frac{1}{0}$

$= 0 \times [-1, 1] = 0$

$f(x) = x^2 \sin\frac{1}{x}$ | $a = 0$

$$Rf'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$= \lim_{h \to 0} \frac{h^2 \sin 1/h - 0}{h}$$

$$= \lim_{h \to 0} h \sin\frac{1}{h} = 0 \times [-1, 1] = 0$$

QUESTION

#Q. If $y = |x|$ then $\frac{dy}{dx}$ for $x \neq 0$ is

$x > 0$ or $x < 0$

(A) 1

(B) 0

(C) -1

(D) $\frac{|x|}{x}$

$$y = |x| = \begin{cases} x & x > 0 \\ 0 & x = 0 \\ -x & x < 0 \end{cases}$$

$$\frac{dy}{dx} = \begin{cases} 1 & x > 0 \\ -1 & x < 0 \end{cases} = \text{similar } \cancel{\text{to}} \text{ signum func} = \frac{|x|}{x} \text{ for } x \neq 0$$

$$\frac{|x|}{x} = \begin{cases} \frac{x}{x} = 1 & \text{if } x > 0 \\ \frac{-x}{x} = -1 & \text{if } x < 0 \end{cases}$$

QUESTION

#Q. If $f(x) = |x|$ then $f(x)$ is

(A) continuous and differentialbe $\forall\, x \in R$

(B) continuous $\forall\, x \in R$ and differentiable $\forall\, x \in R - \{0\}$

(C) continuous but not differentialbe for $x \in R$

(D) neither continuous nor differentialbe $\forall\, x \in R$

QUESTION

#Q. The function $f(x) = |\sin x|$ is

A differentialbe at $x = 0$

B not differentialbe at $x = \pi/2$

C not differentialbe at $x = 0$

D differentiable at $x = 0$ and $x = \pi/2$

QUESTION

#Q. The function $f(x) = |\cos x|$ is

(A) differentialbe at $x = 0$

(B) not differentialbe at $x = \pi/2$

(C) not differentialbe at $x = 0$

(D) differentiable at $x = 0$ and $x = \pi/2$

QUESTION

#Q. The function $f(x) = \tan x$ is

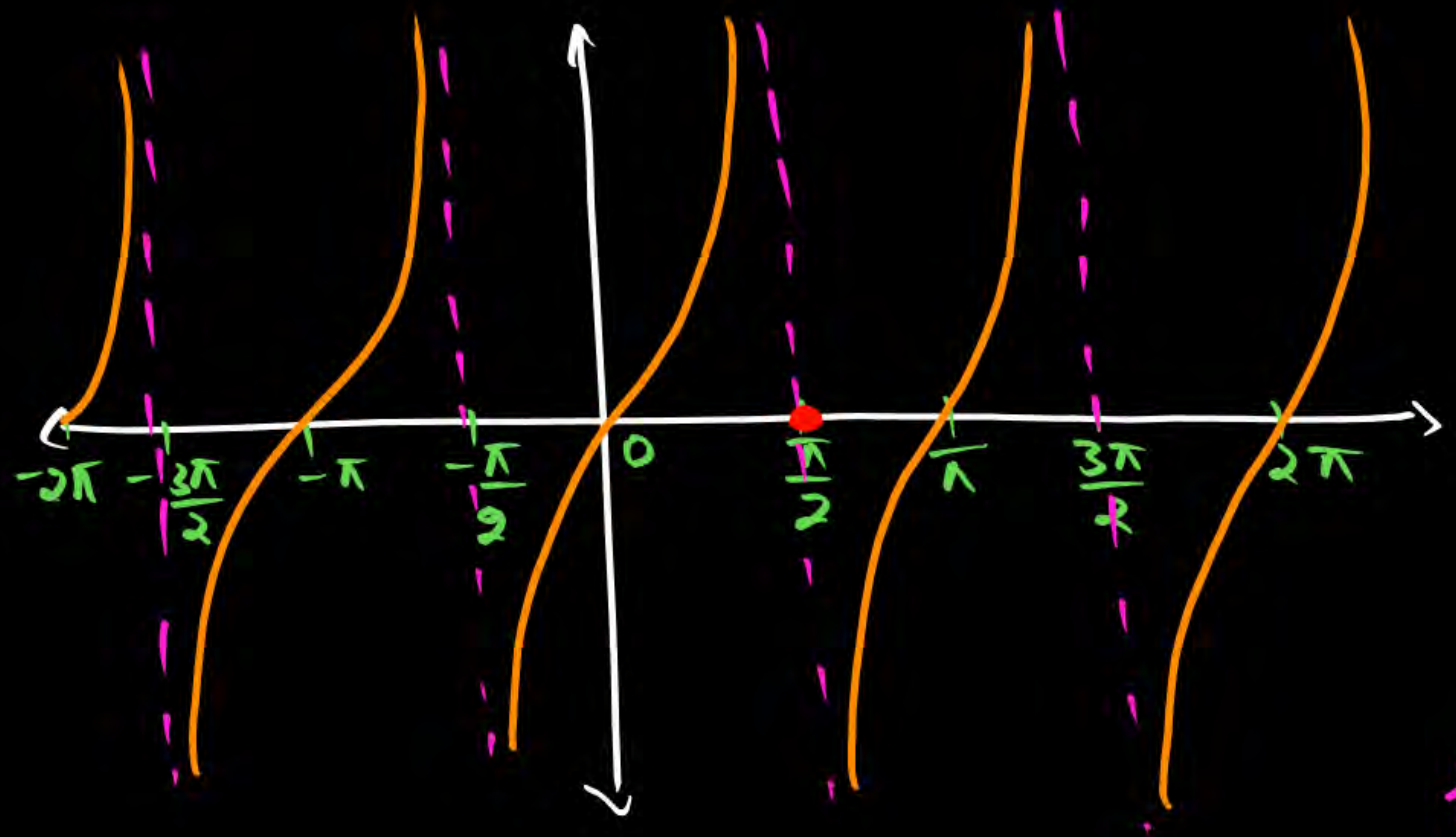
(A) continuous at $x = 0$

(B) continuous at $x = \pi/2$

(C) differentiable at $x = \pi/2$

(D) not continuous and differentiable at $x = \pi/2$

Discontinuous & not Differentiable at $x = \frac{\pi}{2}$



$f(x) = \tan x$

is discontinuous

at $x = \frac{\pi}{2}$

$\Downarrow$

Every Discontinuous

func is not Differentiable

$\Downarrow$

$f(x) = \tan x$ is not

Differentiable at $x = \frac{\pi}{2}$

QUESTION

#Q. The function $f(x) = 1 + |\sin x|$ is

↪ Differentiable at $R - \{n\pi, n \in Z\}$

(A) continuous no where

(B) continuous every where

(C) differentiable no where

(D) differentiable at $x = 0$ False

$$\lim_{\theta \to 0} \frac{\tan^{-1}\theta}{\theta} = 1$$

$$\lim_{\theta \to 0} \frac{\theta}{\tan^{-1}\theta} = 1$$

$$\lim_{\theta \to 0} \frac{\sin^{-1}\theta}{\theta} = 1$$

$$\lim_{\theta \to 0} \frac{\theta}{\sin^{-1}\theta} = 1$$

QUESTION

#Q. If $f(x)=\begin{cases}\dfrac{|x+2|}{\tan^{-1}(x+2)} & \text{for } x\neq -2\\ 2 & \text{for } x=-2\end{cases}$ then

(A) f is continuous at $x=-2$

(B) f is neither continuous nor derivable at $x=-2$

(C) f is derivable at $x=-2$

(D) f is continuous at $x=-2$ but not derivable

Check for continuity:-

$$f(x)=\begin{cases}\dfrac{x+2}{\tan^{-1}(x+2)} & \text{if } x\geq -2 \text{ RHL}\\ \dfrac{-(x+2)}{\tan^{-1}(x+2)} & \text{if } x<-2 \text{ LHL}\\ 2 & \text{if } x=-2\end{cases}$$

RHL = 1

LHL = −1

⇓

Discontinuous

⇓

Not Diff

$$\lim_{x \to -2} \frac{x+2}{\tan^{-1}(x+2)}$$

Put $x+2=t$ | As $x \to -2$

$t \to 0$

$$\lim_{t \to 0} \frac{t}{\tan^{-1} t} = 1$$

QUESTION

In such type of problems, it is enough to check the differentiable at only split points

#Q. The set of all points where the function $f(x) = x\,|x|$ is differentialbe is

A $(-\infty, \infty)$

B $(-\infty, 0) \cup (0, \infty)$

C $(0, \infty)$

D $[0, \infty)$

$$f(x) = x|x| = \begin{cases} x(x) = x^2 & \text{if } x \geq 0 \\ x(-x) = -x^2 & \text{if } x < 0 \end{cases}$$

split point $x = 0$

$$f'(x) = \begin{cases} 2x = 2(0) = 0 & \text{RHD } (x \geq 0) \\ -2x = -2(0) = 0 & \text{LHD } (x < 0) \end{cases}$$

LHD = RHD at $x = 0$

QUESTION

#Q. The function 'f' defined by $f(x)=\begin{cases} x^2+3x+a, & x\leq 1 \\ bx+2, & x>1 \end{cases}$ is given to be derivable at every x. Then the values of a and b are,

$f'(x)=\begin{cases} 2x+3 = 2(1)+3 = 5 & \text{LHD } (x\leq 1) \\ b & \text{RHD } (x>1) \end{cases}$

$\rightarrow$ LHD = RHD

$b=5$

A. × $a=5, b=3$

B. ✓ $a=3, b=5$

C. $a=2, b=5$

D. × $a=1, b=2$

$f(x)$ is differentiable at $x=1$

$\Downarrow$

$f(x)$ is continuous at $x=1$

WKT Every Differentiable func is continuous

LHL at $x=1$

$= 1+3+a$

$= 4+a$

RHL at $x=1$

$= b(1)+2$

$= b+2$

LHL = RHL

$4+a = b+2$

Here $b=5$

$4+a = 5+2$

$a=3$

QUESTION

#Q. If $f(x) = |\cos x|$ then $f'\left(\frac{3\pi}{4}\right)$ equals

(A) $1/\sqrt{2}$

(B) $1/2$

(C) $\sqrt{3}/2$

(D) $\sqrt{3}$

At $x = \frac{3\pi}{4} \in 2^{nd}$ Quad

$\Downarrow$

$\cos x = -ve$

$|\cos x| = -\cos x$

$f(x) = -\cos x$

$f'(x) = \sin x$

$f'\left(\frac{3\pi}{4}\right) = \sin\frac{3\pi}{4} = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

if $f(x) = |\sin x|$ find $f'\left(\frac{7\pi}{6}\right)$

Soln:

$$x = \frac{7\pi}{6} = 210^\circ \in 3^{rd}\ \text{Quad}$$

$$\Downarrow$$

$$\sin x = -ve$$

$$|\sin x| = -\sin x$$

$$f(x) = -\sin x$$

$$f'(x) = -\cos x$$

$$f'\left(\frac{7\pi}{6}\right) = -\cos\left(\pi + \frac{\pi}{6}\right) = +\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

if $f(x) = |x^2 - 3x|$ find $f'(2)$

<u>Soln</u>:

Here $2 \in (0,3)$

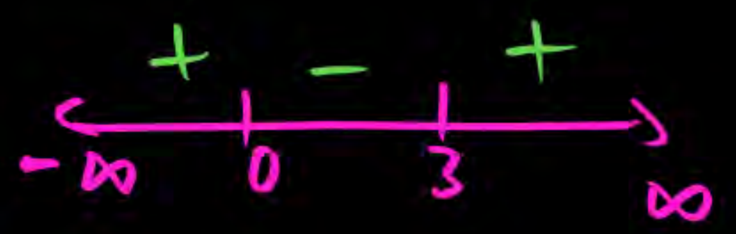
$$f(x) = -(x^2 - 3x)$$
$$= 3x - x^2$$

$$f'(x) = 3 - 2x$$

$$f'(2) = 3 - 2(2) = 3 - 4 = \underline{-1}$$

$$f(x) = |x^2 - 3x| = |x(x-3)|$$

$$= \begin{cases} x^2 - 3x & \text{if } x(x-3) \geq 0 \\ & x \in (-\infty, 0) \cup (3, \infty) \\ -(x^2 - 3x) = 3x - x^2 & \text{if } x(x-3) < 0 \\ & x \in (0,3) \end{cases}$$



if $f(x) = |4x - x^3|$ find $f'(4)$

Soln:-

Here $4 \in (-2, 0) \cup (2, \infty)$

$$f(x) = x^3 - 4x$$

$$f'(x) = 3x^2 - 4$$

$$f'(4) = 3(16) - 4$$

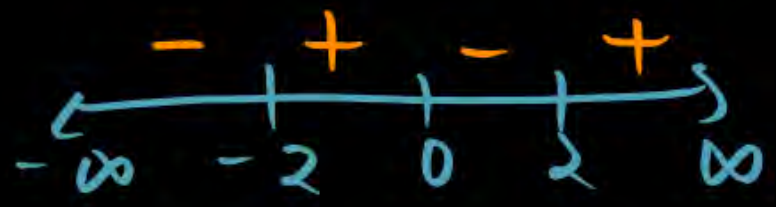
$$= 48 - 4$$

$$= \underline{44}$$

$$|4x - x^3| = |x(4 - x^2)|$$

$$= |x(2 - x)(2 + x)|$$

$$= \begin{cases} 4x - x^3 & \text{if } x(2-x)(2+x) \geq 0 \\ & x(x-2)(x+2) \leq 0 \\ & x \in (-\infty, -2) \cup (0, 2) \\ -(4x - x^3) & \text{if } x(2-x)(2+x) < 0 \\ = x^3 - 4x & x(x-2)(x+2) > 0 \\ & x \in (-2, 0) \cup (2, \infty) \end{cases}$$



if $f(x) = |x+3| + |4-x|$ find $f'(1)$

Soln:

$$x = 1 \begin{cases} > -3 \\ < 4 \end{cases}$$

$$|4-x| = \begin{cases} 4-x & \text{if } 4-x > 0 \\ & x < 4 \\ -(4-x) = x-4 & \text{if } 4-x < 0 \\ & x > 4 \end{cases}$$

$$f(x) = \cancel{x} + 3 + (4 - \cancel{x})$$

$$f(x) = 7$$

$$f'(x) = 0$$

$$f'(1) = 0$$

in $\left(0, \frac{\pi}{4}\right)$

$\cos x > \sin x$

in $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$

$\cos x < \sin x$

in $\left(\frac{5\pi}{4}, 2\pi\right)$

$\cos x > \sin x$

$\frac{7\pi}{4} \in \left(\frac{5\pi}{4}, 2\pi\right)$

$0 > \sin x - \cos x$

$\Downarrow$

$\sin x - \cos x < 0$

$\therefore |\sin x - \cos x| = -(\sin x - \cos x)$

$= \cos x - \sin x$

if $f(x) = |\sin x - \cos x|$ find $f'\left(\frac{7\pi}{4}\right)$

Soln:-

Here $x = \frac{7\pi}{4} \in \left(\frac{5\pi}{4}, 2\pi\right)$

$\Downarrow$

$\cos x > \sin x$

$0 > \sin x - \cos x$

$\sin x - \cos x < 0$

$\therefore f(x) = |\sin x - \cos x| = -(\sin x - \cos x)$

$f(x) = \cos x - \sin x$

$f'(x) = -\sin x - \cos x$

$f'\left(\frac{7\pi}{4}\right) = -\sin\frac{7\pi}{4} - \cos\frac{7\pi}{4}$

$= +\sin\frac{\pi}{4} - \cos\frac{\pi}{4}$

$= \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}$

$= \underline{0}$

$$-\sin\frac{7\pi}{4}$$

$$= -\sin\left(2\pi - \frac{\pi}{4}\right)$$

$$= -\left[-\sin\frac{\pi}{4}\right]$$

$$= +\sin\frac{\pi}{4}$$

$$= \frac{1}{\sqrt{2}}$$

$$\frac{7\pi}{4} = \frac{8\pi - \pi}{4} = 2\pi - \frac{\pi}{4}$$

$\downarrow$

4^{th} Quad

$\Downarrow$

$\sin = -ve$

QUESTION

$$\tan^{-1}A - \tan^{-1}B = \tan^{-1}\left[\frac{A-B}{1+AB}\right]$$

$$\tan^{-1}A + \tan^{-1}B = \tan^{-1}\left[\frac{A+B}{1-AB}\right]$$

#Q. $\frac{d}{dx}\left[\tan^{-1}\left(\frac{a-x}{1+ax}\right)\right]$ is

constant

$y = \tan^{-1}a - \tan^{-1}x$

(A) $-\frac{1}{1+x^2}$

(B) $\frac{1}{1+\left(\frac{a-x}{1-ax}\right)^2}\frac{d}{dx}\left(\frac{a-x}{1+ax}\right)$

$$\frac{dy}{dx} = 0 - \frac{1}{1+x^2}$$

$$= \frac{-1}{1+x^2}$$

(C) $\frac{-1}{\sqrt{1-\left(\frac{a-x}{1+ax}\right)^2}}$

(D) $\frac{1}{1+a^2} - \frac{1}{1+x^2}$

(2) if $y = \tan^{-1}\left[\frac{7x}{1-12x^2}\right]$ Then $\frac{dy}{dx}$

observe :- D^r, 1^{st} term is 1

$D^r \to$ In b/w 1 & $12x^2$, There is $-ve$ sign

Soln :-

$$y = \tan^{-1}\left[\frac{\overset{A+B}{7x}}{1-\underset{AB}{12x^2}}\right]$$

$$\tan^{-1}\left[\frac{A+B}{1-AB}\right] = \tan^{-1}A + \tan^{-1}B$$

$$12x^2 = 3x(4x)$$

$$7x = 3x + 4x$$

$$y = \tan^{-1}\left(\frac{3x+4x}{1-12x^2}\right)$$

$$y = \tan^{-1}3x + \tan^{-1}4x$$

$$\frac{dy}{dx} = \frac{3}{1+9x^2} + \frac{4}{1+16x^2}$$

$$\frac{d}{dx}\left(\tan^{-1}3x\right)$$

$$= \frac{1}{1+9x^2}\frac{d}{dx}(3x)$$

$$= \frac{3}{1+9x^2}$$

QUESTION

#Q. If $y=\frac{x}{2}\sqrt{a^2+x^2}+\frac{a^2}{2}log\left(x+\sqrt{x^2+a^2}\right)$ then $\frac{dy}{dx}$ is

A $\sqrt{x^2-a^2}$

B $\sqrt{a^2+x^2}$

C $\sqrt{a^2-x^2}$

D none of these

WKT

$$\int\sqrt{a^2+x^2}\,dx=\frac{x}{2}\sqrt{a^2+x^2}+\frac{a^2}{2}\log\left(x+\sqrt{x^2+a^2}\right)+c$$

$\Downarrow$

Diff of this

$\Downarrow$

$\sqrt{a^2+x^2}$

QUESTION

#Q. The derivative of $tan^{-1} x^m + \tan^m x - \tan^{-1}\left(\frac{a+x^m}{1-ax^m}\right)$ is

(A) $\frac{1}{1+(x^{2m})} m x^{m-1}$

(B) $m\tan^{m-1} x \cdot \sec^2 x$

(C) 0

(D) $\frac{1}{1+x^{2m}} + \frac{1}{1+x^m}$

$y = \cancel{\tan^{-1} x^m} + \tan^m x - \left[\tan^{-1} a + \cancel{\tan^{-1} x^m}\right]$

$y = \tan^m x - \underbrace{\tan^{-1} a}_{\text{const}}$

$\frac{dy}{dx} = m \tan^{m-1} x \sec^2 x$

QUESTION

$\cos^{-1}A + \sin^{-1}A = \frac{\pi}{2}$

#Q. If $y = cos^{-1}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right) + \sin^{-1}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right)$ then $\frac{dy}{dx}$ is

A $\frac{1}{\sqrt{1-x^2}}$

$y = \frac{\pi}{2}$

B $\frac{1}{x\sqrt{1-x^2}}$

$\frac{dy}{dx} = 0$

C 0

D -1

QUESTION

$$2\tan^{-1}\theta = \begin{cases} ① \ \sin^{-1}\left(\frac{2\theta}{1+\theta^2}\right) \\ ② \ \cos^{-1}\left(\frac{1-\theta^2}{1+\theta^2}\right) \\ ③ \ \tan^{-1}\left(\frac{2\theta}{1-\theta^2}\right) \end{cases}$$

#Q. If $y=\cos^{-1}\left(\frac{x-x^{-1}}{x+x^{-1}}\right)$ then y' is

(A) $\frac{1}{1+x^2}$

(B) $-\frac{1}{1+x^2}$

(C) $\frac{2}{1+x^2}$

(D) $-\frac{2}{1+x^2}$

$$y = \cos^{-1}\left[\frac{x\,[1-x^{-2}]}{x\,[1+x^{-2}]}\right]$$

$$= \cos^{-1}\left[\frac{1-(x^{-1})^2}{1+(x^{-1})^2}\right]$$

$$\rightarrow \cos^{-1}\left(\frac{1-t^2}{1+t^2}\right) = 2\tan^{-1}t$$

$$= 2\tan^{-1}(x^{-1})$$

$$= \frac{2}{1+(x^{-1})^2}\frac{d}{dx}(x^{-1})$$

$$= \frac{2}{1+\frac{1}{x^2}}\left(-\frac{1}{x^2}\right)$$

$$= \frac{2x^2}{1+x^2}\left(-\frac{1}{x^2}\right) = \frac{-2}{1+x^2}$$

QUESTION

$a^{\log_a f(x)} = f(x)$

#Q. If $y = 10^{\log_{10}\left[\sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right]} + e^{\log_e(\sin^{-1}x+\cos^{-1}x)}$ then $y' =$

A) $\frac{2}{1+x^2}$

B) $-\frac{2}{1+x^2}$

C) $\frac{2}{\sqrt{1-x^2}}$

D) $-\frac{2}{\sqrt{1-x^2}}$

$y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right) + (\sin x + \cos^{-1}x)$

$y = \frac{\pi}{2} - \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) + \frac{\pi}{2}$

$y = \pi - 2\tan^{-1}x$

$\frac{dy}{dx} = 0 - \frac{2}{1+x^2} = \frac{-2}{1+x^2}$

$\sin^{-1}A = \frac{\pi}{2} - \cos^{-1}A$

$\cos^{-1}\left(\frac{1-t^2}{1+t^2}\right) = 2\tan^{-1}t$

if $y = \tan^{-1}\left[\frac{4-8x}{4+8x}\right]$ find $\frac{dy}{dx}$

Soln:

$\div$ both N^r & D^r by 4

$$y = \tan^{-1}\left[\frac{1-2x}{1+2x}\right]$$

$$y = \frac{\pi}{4} - \tan^{-1}2x$$

$$\frac{dy}{dx} = 0 - \frac{2}{1+4x^2}$$

$$= \frac{-2}{1+4x^2}$$

$$\tan^{-1}\left(\frac{1-A}{1+A}\right) = \frac{\pi}{4} - \tan^{-1}A$$

$$\tan^{-1}\left(\frac{1+A}{1-A}\right) = \frac{\pi}{4} + \tan^{-1}A$$

$$\frac{d}{dx}(\tan^{-1}2x) = \frac{1}{1+(2x)^2}\frac{d}{dx}(2x)$$

$$= \frac{2}{1+4x^2}$$

QUESTION

$1-2\sin^2\theta$

#Q. The value of $\frac{d}{dx}\cos^{-1}(1-2x^2)$ is

$1-2\sin^2\theta = \cos 2\theta$

(A) $\frac{-1}{\sqrt{1-4x^4}}$

(B) $\frac{2}{\sqrt{1-x^2}}$ ✓

(C) $\sin^{-1}(1-2x^2)$

(D) $\frac{-2}{\sqrt{1-x^2}}$

Put $x = \sin\theta$

$\theta = \sin^{-1}x$

$y = \cos^{-1}(1-2\sin^2\theta)$

$y = \cos^{-1}(\cos 2\theta)$

$y = 2\theta$

$y = 2\sin^{-1}x$

$\frac{dy}{dx} = \frac{2}{\sqrt{1-x^2}}$

if $y = \cos^{-1}\left(\frac{\sqrt{1+x}}{\sqrt{2}}\right)$ find $\frac{dy}{dx}$.

Soln:-

Put $x = \cos 2\theta$

$2\theta = \cos^{-1} x$

$\theta = \frac{1}{2}\cos^{-1} x$

$\sqrt{1+x} = \sqrt{1+\cos 2\theta}$

$= \sqrt{2\cos^2\theta}$

$= \sqrt{2}\cos\theta$

$y = \cos^{-1}\left(\frac{\sqrt{2}\cos\theta}{\sqrt{2}}\right)$

$y = \cos^{-1}(\cos\theta) = \theta$

$y = \frac{1}{2}\cos^{-1} x$

$\frac{dy}{dx} = \frac{-1}{2\sqrt{1-x^2}}$

if $y = \cos^{-1}\frac{1}{\sqrt{1+x}}$

Soln:-

Put $x = \tan^2\theta$

$\tan\theta = \sqrt{x}$

$\theta = \tan^{-1}\sqrt{x}$

$y = \cos^{-1}\left(\frac{1}{\sqrt{1+\tan^2\theta}}\right)$

$y = \cos^{-1}\left(\frac{1}{\sqrt{\sec^2\theta}}\right)$

$y = \cos^{-1}\left(\frac{1}{\sec\theta}\right)$

$y = \cos^{-1}(\cos\theta)$

$y = \theta$

$y = \tan^{-1}\sqrt{x}$

$\frac{dy}{dx} = \frac{1}{1+x}\frac{d}{dx}\sqrt{x}$

$= \frac{1}{2\sqrt{x}(1+x)}$

$1+x$

① $\cos^{-1}\left[\left(\frac{1+x}{2}\right)\right]$ ② $\sin^{-1}\left[\left(\frac{1+x}{2}\right)\right]$

$1+x$

$1+\cos 2\theta$	$1+\cos\theta$	$1+\tan^2\theta$
$\Downarrow$	$\Downarrow$	$\Downarrow$
$2\cos^2\theta$	$2\cos^2\frac{\theta}{2}$	$\sec^2\theta$
$\Downarrow$	$\Downarrow$	$\Downarrow$
Put $x=\cos 2\theta$	Put $x=\cos\theta$	Put $x=\tan\theta$

$1-x$

$1-\cos 2\theta = 2\sin^2\theta \Downarrow$ Put $x=\cos 2\theta$

$1-\cos\theta = 2\sin^2\theta \Downarrow$ Put $x=\cos\theta$

$1-\sin^2\theta = \cos^2\theta \Downarrow$ Put $x=\sin^2\theta$

$$\frac{1-x}{1+x}$$

$$\frac{1-\cos 2\theta}{1+\cos 2\theta} = \frac{2\sin^2\theta}{2\cos^2\theta} = \tan^2\theta$$

$$\frac{1-\cos\theta}{1+\cos\theta} = \tan^2\frac{\theta}{2}$$

$$\frac{1-\tan\theta}{1+\tan\theta} = \tan\left(\frac{\pi}{4}-4\right)$$

$$\frac{1-\tan^2\theta}{1+\tan^2\theta}$$

$\cos 2\theta$

Ex:-

$$y = \tan^{-1}\sqrt{\frac{1-x}{1+x}}$$

Ex:-

$$y = \tan^{-1}\sqrt{\frac{1-x}{1+x}}$$

Ex.

$$y = \tan^{-1}\left(\frac{1-x}{1+x}\right)$$

Ex:

$$y = \cos^{-1}\left(\frac{1-x}{1+x}\right)$$

if $y=\tan^{-1}\sqrt{\frac{1-x}{1+x}}$

Find $\frac{dy}{dx}$

Soln:- Put $x=\cos 2\theta$

$\theta=\frac{1}{2}\cos^{-1}x$

$$y=\tan^{-1}\sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}$$

$$y=\tan^{-1}\sqrt{\tan^2\theta} = y=\theta$$

$$y=\frac{1}{2}\cos^{-1}x$$

$$\frac{dy}{dx}=\frac{-1}{2\sqrt{1-x^2}}$$

② if $y=\cot^{-1}\left(\frac{1-x}{1+x}\right)$

Find $\frac{dy}{dx}$

Soln:- Put $x=\tan\theta$

$\theta=\tan^{-1}x$

$$y=\cot^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right)$$

$$y=\frac{\pi}{2}-\tan^{-1}\left(\tan\left(\frac{\pi}{4}-\theta\right)\right)$$

$$y=\frac{\pi}{2}-\frac{\pi}{4}+\theta=\frac{\pi}{4}+\tan^{-1}x$$

$$\frac{dy}{dx}=\frac{1}{1+x^2}$$

③ if $y=\sin^{-1}\left(\frac{1-x}{1+x}\right)$

Find $\frac{dy}{dx}$

Soln:- Put $x=\tan^2\theta$

$\theta=\tan^{-1}\sqrt{x}$

$$y=\frac{\pi}{2}-\cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$$

$$y=\frac{\pi}{2}-\cos^{-1}(\cos 2\theta)$$

$$y=\frac{\pi}{2}-2\theta=\frac{\pi}{2}-2\tan^{-1}\sqrt{x}$$

$$\frac{dy}{dx}=\frac{-2}{1+x}\left(\frac{1}{2\sqrt{x}}\right)$$

$$=\frac{-1}{\sqrt{x}(1+x)}$$

QUESTION

#Q. If $y = \sin^2 \cot^{-1}\left(\sqrt{\frac{1+x}{1-x}}\right)$ then y' is

$\frac{dy}{dx}$

Put $x = \cos 2\theta$

$\sqrt{\frac{1+\cos 2\theta}{1-\cos 2\theta}} = \sqrt{\frac{2\cos^2\theta}{2\sin^2\theta}} = \cot\theta$

(A) 0

(B) 1/2

(C) −1/2 ✓

(D) 1

$y = \sin^2 \cot^{-1}(\cot\theta)$

$y = \sin^2\theta$

$y = \frac{1-\cos 2\theta}{2}$

$y = \frac{1}{2} - \frac{x}{2}$

$\frac{dy}{dx} = 0 - \frac{1}{2} = \frac{-1}{2}$

method 2:-

From here

$y = \sin^2\theta$

$\frac{dy}{dx} = 2\sin\theta\cos\theta \cdot \frac{d\theta}{dx}$

$= \sin 2\theta\left(\frac{-1}{2\sin 2\theta}\right)$

$= \frac{-1}{2}$

$x = \cos 2\theta$

$\frac{dx}{d\theta} = -2\sin 2\theta$

$\frac{d\theta}{dx} = -\frac{1}{2\sin 2\theta}$

QUESTION

#Q. If $y = cos^{-1}\left(\frac{x^{2n}-1}{x^{2n}+1}\right)$ then $\frac{dy}{dx}$ is

(A) $\frac{2}{1+x^{2n}}$

(B) $-\frac{2nx^{n-1}}{1+x^{2n}}$

(C) $\frac{2nx^{n-1}}{1+x^{2n}}$

(D) $\frac{2}{1-x^{2n}}$

$$y = \cos^{-1}\left[\frac{-(1-(x^2)^n)}{(1+(x^2)^n}\right]$$

$$= \pi - \cos^{-1}\left(\frac{1-(x^2)^n}{1+(x^2)^n}\right)$$

$$\cos^{-1}\left(\frac{1-t^2}{1+t^2}\right) = 2\tan^{-1} t$$

$$y = \pi - 2\tan^{-1}(x^n)$$

$$\frac{dy}{dx} = 0 - \frac{2}{1+(x^n)^2}\frac{d}{dx}(x^n)$$

$$= -\frac{2nx^{n-1}}{1+x^{2n}}$$

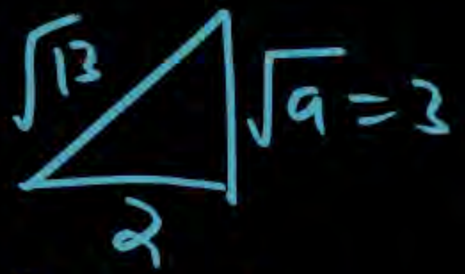
$$y = \sin^{-1}\frac{1}{\sqrt{13}}\left[2\sin x - 3\cos x\right] \qquad \text{Find } \frac{dy}{dx}$$

Soln:

$$y = \sin^{-1}\left[\underbrace{\sin x \frac{2}{\sqrt{13}} - \cos x \frac{3}{\sqrt{13}}}_{\text{similar to}}\right]$$

$\sin A \cos B - \cos A \sin B$

$\therefore$ Let $\cos B = \frac{2}{\sqrt{13}}$



$\therefore \sin B = \frac{3}{\sqrt{13}}$

$$y = \sin^{-1}\left[\sin x \cos B - \cos x \sin B\right]$$

$$y = \sin^{-1}\left[\sin(x - B)\right]$$

$$y = x - B \rightarrow \text{constant}$$

$$\frac{dy}{dx} = 1$$

QUESTION

#Q. $\frac{d}{dx}\left[\sin^{-1}\left(\frac{5x+12\sqrt{1-x^2}}{13}\right)\right]$ is equal to

$y = \sin^{-1}\left[x\left(\frac{5}{13}\right) + \sqrt{1-x^2}\,\frac{12}{13}\right]$

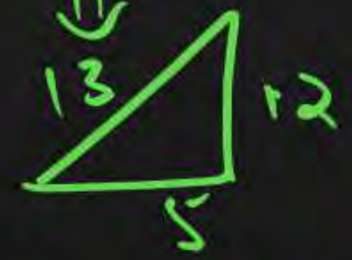
Let $x = \sin A$ & $\frac{5}{13} = \cos B$

$A = \sin^{-1} x$

$\Downarrow$

$\sqrt{1-x^2} = \cos A$

$\Downarrow$



$\frac{12}{13} = \sin B$

$y = \sin^{-1}[\sin A \cos B + \cos A \sin B]$

$y = \sin^{-1}(\sin(A+B))$

$y = A + B.$

$y = \sin^{-1} x + B \rightarrow$ constant

$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$

QUESTION

$\sin A \quad \cos B \quad \sin B \quad \cos A$

$A = \sin^{-1} x^2$

$B = \sin^{-1}\sqrt{x}$

#Q. If $y = \sin^{-1}\left[x^2\sqrt{1-x} - \sqrt{x}\sqrt{1-x^4}\right]$ then dy/dx is equal to

(A) $\cos^{-1}\left[x^2\sqrt{1-x} - \sqrt{x}\sqrt{1-x^4}\right]$

(B) $\frac{2x}{\sqrt{1-x^4}} - \frac{1}{2\sqrt{x}\sqrt{1-x}}$

(C) $\frac{1}{x^2\sqrt{1-x}}$

(D) $\frac{1}{\sqrt{x}\sqrt{1-x^4}}$

Put $x^2 = \sin A$

$\Downarrow$

$\sqrt{1-x^4} = \cos A$

$\sqrt{x} = \sin B$

$\Downarrow$

$\sqrt{1-x} = \cos B$

$y = \sin^{-1}[\sin(A-B)]$

$y = A - B$

$y = \sin^{-1}x^2 - \sin^{-1}\sqrt{x}$

$\frac{dy}{dx} = \frac{2x}{\sqrt{1-x^4}} - \frac{1}{2\sqrt{x}\sqrt{1-x}}$

If $y=\sin^{-1}\left[x^3\sqrt{1-x^2}-x\sqrt{1-x^6}\right]$ find $\frac{dy}{dx}$

$x^3 \to \sin A$, $\sqrt{1-x^2} \to \cos B$, $x \to \sin B$, $\sqrt{1-x^6} \to \cos A$

$A=\sin^{-1}(x^3)$

$B=\sin^{-1}x$

Soln:

$$y=\sin^{-1}(\sin(A-B))$$

$$y=A-B$$

$$y=\sin^{-1}x^3-\sin^{-1}x$$

$$\frac{dy}{dx}=\frac{3x^2}{\sqrt{1-x^6}}-\frac{1}{\sqrt{1-x^2}}$$

QUESTION

#Q. If $\tan^{-1}\left(\frac{x^2-y^2}{x^2+y^2}\right) = a$ then $\frac{dy}{dx}$ is

(A) $\frac{x}{y}$

(B) $\frac{y}{x}$

(C) $\frac{x(1-\tan a)}{y(1+\tan a)}$

(D) $\frac{y(1+\tan a)}{x(1-\tan a)}$

QUESTION

#Q. $\frac{d}{dx}\tan^{-1}\left[\frac{x+1}{1-x}\right]$ is

$y = \tan^{-1}1 + \tan^{-1}x$

$\frac{dy}{dx} = +\frac{1}{1+x^2}$

A $\frac{1}{(1-x)^2}$

B $\frac{1}{1-x^2}$

C $\frac{1}{1+x^2}$

D $\frac{-1}{1+x^2}$

QUESTION

#Q. If $y=\sin^{-1}\left[\frac{3\cos x-4\sin x}{5}\right]$, then y' is

(A) 1

(B) −1

(C) 3

(D) 4

$$y=\sin^{-1}\left[\frac{3}{5}\cos x-\frac{4}{5}\sin x\right]$$

$\frac{3}{5} \downarrow \sin A$ $\qquad$ $\frac{4}{5} \downarrow \cos A$

$$y=\sin^{-1}\left[\sin(A-x)\right]$$

$$y=A-x$$

$$\frac{dy}{dx}=0-1=-1$$

QUESTION

#Q. If $y=\tan^{-1}\left(\frac{3x-2}{2x+3}\right)$ then $\frac{dy}{dx}$ is equal to

(A) $\frac{2}{1+x^2}$

(B) $\frac{3}{1+x^2}$

(C) $\frac{1}{1+x^2}$

(D) $\frac{-1}{1+x^2}$

$$y=\tan^{-1}\left[\frac{3x-2}{2x+3}\right]$$

↳ This should be +1

÷ N^r & D^r by $2x$

$$y=\tan^{-1}\left[\frac{\frac{3}{2}-\frac{1}{x}}{1+\frac{3}{2}\frac{1}{x}}\right]$$

$$y=\tan^{-1}\left(\frac{3}{2}\right)-\tan^{-1}\left(\frac{1}{x}\right)$$

$$y=\tan^{-1}\left(\frac{3}{2}\right)-\cot^{-1}x$$

$$\frac{dy}{dx}=0-\left(\frac{-1}{1+x^2}\right)=\frac{1}{1+x^2}$$

QUESTION

#Q. If $y = \sin^{-1}(3x - 4x^3)$ then $\frac{dy}{dx}$ is

(A) $3\sin^{-1} x$

$y = 3\sin^{-1} x$

(B) $\frac{3}{\sqrt{1-x^2}}$

$\frac{dy}{dx} = \frac{3}{\sqrt{1-x^2}}$

(C) 3

(D) $-\frac{3}{\sqrt{1-x^2}}$

QUESTION

$1+\sin x = \cos^2\frac{x}{2}+\sin^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2} = \left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2$

#Q. If $y=\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$ then y' is

$\rightarrow a^2-b^2=(a-b)(a+b)$

$y=\tan^{-1}\left[\frac{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}\right]$

$y=\tan^{-1}\left[\frac{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)\cancel{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)}}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^{\cancel{2}}}\right]$

$\div$ by $\cos\frac{x}{2}$ in N^r & D^r

$y=\tan^{-1}\left(\frac{1-\tan x/2}{1+\tan x/2}\right)$

$y=\tan^{-1}\left(\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right)$

$y=\frac{\pi}{4}-\frac{x}{2}$

$\frac{dy}{dx}=-\frac{1}{2}$

A. -1

B. 1

C. $\frac{1}{2}$

D. $-\frac{1}{2}$ ✓

$$\frac{\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}}$$

$\div$ by $\cos\frac{x}{2}$ in N^r & D^r

$$\frac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}$$

$$=\frac{\tan\frac{\pi}{4}-\tan\frac{x}{2}}{1+\tan\frac{\pi}{4}\tan\frac{x}{2}}=\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)$$

QUESTION

#Q. If $y = \tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$ then $\frac{d^2y}{dx^2}$ is

(A) $\frac{1}{2}\sec^2\frac{x}{2}$

(B) $\sec\frac{x}{2}$

(C) $\frac{1}{2}$

(D) 0

$$y = \tan^{-1}\left(\frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2 x/2}\right)$$

$$y = \tan^{-1}\left(\tan\frac{x}{2}\right)$$

$$y = \frac{x}{2}$$

$$\frac{dy}{dx} = \frac{1}{2}$$

$$\frac{d^2y}{dx^2} = 0$$

Thank
You