Indefinite Integration

If f & g are functions of x such that $g'(x) = f(x)$ then,

$\int f(x)dx = g(x) + c \Leftrightarrow \frac{d}{dx}\{g(x)+c\} = f(x)$, where c is called the constant of integration.

Standard formula:

(i) $\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c, n \neq -1$

(ii) $\int \frac{dx}{ax+b} = \frac{1}{a}\ln|ax+b| + c$

(iii) $\int e^{ax+b} dx = \frac{1}{a} e^{ax+b} + c$

(iv) $\int a^{px+q} dx = \frac{1}{p}\frac{a^{px+q}}{\ln a} + c; a > 0$

(v) $\int \sin(ax+b)dx = -\frac{1}{a}\cos(ax+b) + c$

(vi) $\int \cos(ax+b)dx = \frac{1}{a}\sin(ax+b) + c$

(vii) $\int \tan(ax+b)dx = \frac{1}{a}\ln\sec|(ax+b)| + c$

(viii) $\int \cot(ax+b)dx = \frac{1}{a}\ln|\sin(ax+b)| + c$

(ix) $\int \sec^2(ax+b)dx = \frac{1}{a}\tan(ax+b) + c$

(x) $\int cosec^2(ax+b)dx = -\frac{1}{a}\cot(ax+b) + c$

(xi) $\int \sec x dx = \ln|(\sec x + \tan x)| + c$

Or $\ln\left|\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right| + c$

(xii) $\int cosec\, x\, dx = \ln|(cosec\, x - \cot x)| + c$

Or $\ln\left|\tan\frac{x}{2}\right| + c$

Or $-\ln|(\text{cosec}\, x + \cot x)| + c$

(xiii) $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + c$

(xiv) $\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

(xv) $\int \frac{dx}{|x|\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\frac{x}{a} + c$

(xvi) $\int \frac{dx}{\sqrt{x^2+a^2}} = \ln\left|x+\sqrt{x^2+a^2}\right| + c$

(xvii) $\int \frac{dx}{\sqrt{x^2-a^2}} = \ln\left|x+\sqrt{x^2-a^2}\right| + c$

(xviii) $\int \frac{dx}{\left(a^2-x^2\right)} = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right| + c$

(xix) $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + c$

(xx) $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + c$

(xxi) $\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\ln\left|\frac{x+\sqrt{x^2+a^2}}{a}\right| + c$

(xxii) $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\ln\left|\frac{x+\sqrt{x^2-a^2}}{a}\right| + c$

Integration by substitution:

If we substitute $f(x) = t$, then $f'(x)\,dx = dt$

Integration by part:

$$\int (f(x)g(x)dx = f(x))\int (g(x))dx$$

$$-\int\left(\frac{d}{dx}(f(x))\int (g(x))dx\right)dx$$

Integration of type:

$$\int \frac{dx}{ax^2+bx+c}, \int \frac{dx}{\sqrt{ax^2+bx+c}}, \int \sqrt{ax^2+bx+c}bx$$

Make the substitution $x+\frac{b}{2a}=t$

Integration of type:

$$\int \frac{px+q}{ax^2+bx+c}dx, \int \frac{px+q}{\sqrt{ax^2+bx+c}}dx,$$

$$(px+q)\sqrt{ax^2+bx+c}dx$$

Make the substitution $x+\frac{b}{2a}=t$, then split the integral as sum of two integrals one containing the linear term and the other containing constant term.

Integration of trigonometric functions:

(i) $\int \frac{dx}{a+b\sin^2 x}$ $Or, \int \frac{dx}{a+b\cos^2 x}$ $Or,$

$\int \frac{dx}{a\sin^2 x + b\sin x\cos x + c\cos^2 x}$, put $\tan x = t$

(ii) $\int \frac{dx}{a+b\sin x}$ Or, $\int \frac{dx}{a+b\cos x}$ Or

$\int \frac{dx}{a+b\sin x + c\cos x}$, put $tan\frac{x}{2} = t.$

(iii) $\int \frac{a\cdot\cos x + b\cdot\sin x + c}{l\cdot\cos x + m\cdot\sin x + n}dx$ Express

$N^r \equiv A\left(D^r\right) + B\frac{d}{dx}\left(D^r\right) + c$ & proceed.

Integration of Type:

$\int \frac{x^2 \pm 1}{x^4 + Kx^2 + 1}dx$ where K is any constant.

Divide N^r & Dr by x^2 & put $x \mp \frac{1}{x} = t$

Integration of type:

$\int \frac{dx}{(ax+b)\sqrt{px+q}}$ $Or \int \frac{dx}{\left(ax^2+bx+c\right)\sqrt{px+q}}$

put $px + q = t^2$.

Integration of type:

$\int \frac{dx}{(ax+b)\sqrt{px^2+qx+r}}$, put $ax + b = \frac{1}{t}$;

$\int \frac{dx}{\left(ax^2+b\right)\sqrt{px^2+q}}$ put $x = \frac{1}{t}$

Some Standard Substitution

1. $\int f(x)^n f'(x)dx$ Or $\int \frac{f'(x)}{[f(x)]^n}dx$ put $f(x) = t$ & proceed.

2. $\int \frac{dx}{ax^2+bx+c}, \int \frac{dx}{\sqrt{ax^2+bx+c}}, \int \sqrt{ax^2+bx+c}\,dx$

   Express $ax^2 + bx + c$ in the form of perfect square & then apply the standard results.

3. $\int \frac{(px+q)dx}{ax^2+bx+c}, \int \frac{(px+q)}{\sqrt{ax^2+bx+c}}dx$

   Express $px + q = A$ (differential coefficient of denominator) + B.

4. $\int e^x\left[f(x) + f'(x)\right]dx = e^x \cdot f(x) + c$

5. $\int\left[f(x)+xf'(x)\right]dx = xf(x)+c$

6. $\int\frac{dx}{x\left(x^n+1\right)}, n\in N$, take x^n common & put $1 + x^{-1} = t$.

7. $\int\frac{dx}{x^2\left(x^n+1\right)^{(n-1)/n}}, n\in N$, take x^n common & put $1 + x^{-n} = t^n$.

8. $\int\frac{dx}{x^n\left(1 \quad x^n\right)^{1/n}}$, *take* x^n take x^n common & put $1 + x^{-n} = t$.

9. $\int\sqrt{\frac{x-\alpha}{\beta-x}}dx$ Or $\int\sqrt{(x-\alpha)(\beta-x)}$ put $x = \alpha\cos^2\theta + \beta\sin^2\theta$

$\int\sqrt{\frac{x-\alpha}{x-\beta}}dx$ Or $\int\sqrt{(x-\alpha)(x-\beta)}$ put $x = \alpha\sec^2\theta + \beta\tan^2\theta$

$\int\frac{dx}{\sqrt{(x-\alpha)(x-\beta)}}$; put $x-\alpha = t^2$ put $x - \alpha\ t^2$ or $x - \beta = t^2$

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