

ULTIMATE KCET



CRASH COURSE 2026

Mathematics

Lecture : 01

Probability, Permutation & Combination

By – Guru sir



Recap *of previous lecture*

1 *Probability*

2

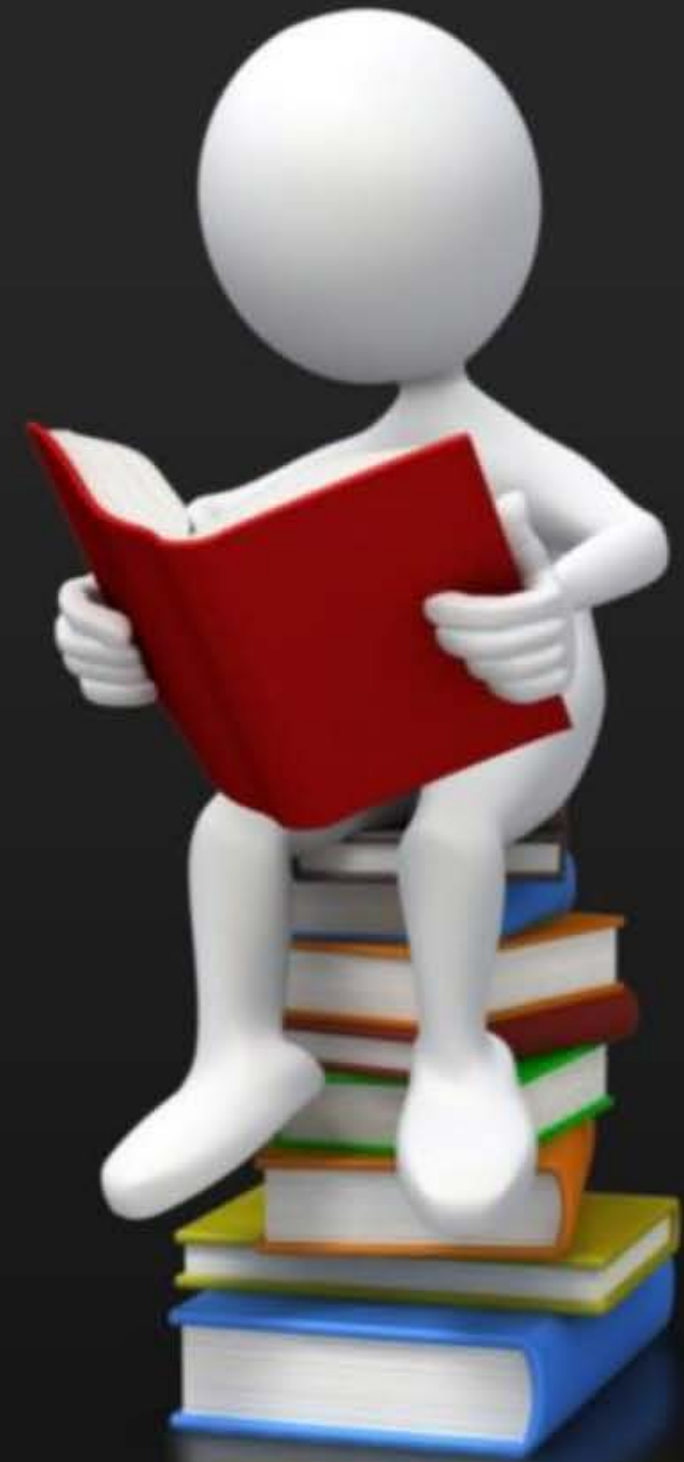
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Topics *to be covered*

- 1 Probability — Continue
- 2 P & C
- 3
- 4



QUESTION



#Q. Two cards are drawn at random from a pack of 52 cards. The probability of these two being "Aces" is

A $1/26$

B $1/221$

C $1/2$

D $1/13$

$$\frac{4}{52} \times \frac{3}{51} = \frac{1}{13} \times \frac{1}{17}$$

(B)

$$\frac{{}^4C_2}{{}^{52}C_2} = \frac{\frac{4}{2} \times 3}{\frac{52}{2} \times 51} = \frac{1}{221}$$

$$= \frac{1}{13 \times 17}$$

$$= \frac{1}{221}$$

QUESTION



#Q. Two letters are chosen from the letters of the word 'EQUATIONS'. The probability that one is **vowel** and the other is **consonant** is

A 8/9

B 3/9

C 4/9

D 5/9

E Q
 V T
 A N
 I S
 O
 ↙ ↘
 5C₁ 4C₁

$$\frac{{}^5C_1 \times {}^4C_1}{{}^9C_2} = \frac{5 \times 4}{\frac{9 \times 8}{2}} = \frac{5}{9}$$

Out of 9 objects
 You are picking 2

QUESTION



#Q. Two letters are chosen from the letters of the word 'EQUATIONS'. The probability that one is vowel and the other is consonant is

A 8/9

B 3/9

C 4/9

D 5/9

without Replacement

order of picking the letter is not given

$$P(\text{vowel}) \cdot P(\text{consonant}) \text{ (or) } P(\text{consonant}) \cdot P(\text{vowel})$$

$$= \frac{5}{9} \cdot \frac{4}{8} + \frac{4}{9} \cdot \frac{5}{8}$$

$$= \frac{5}{9} \cdot \frac{4}{8} + \frac{4}{9} \cdot \frac{5}{8} = \frac{5}{9}$$

Permutations: -

① If we are provided
 n distinct objects



$${}^n P_r = \frac{n!}{(n-r)!}$$

② If we are provided with
 repeated objects (same objects)

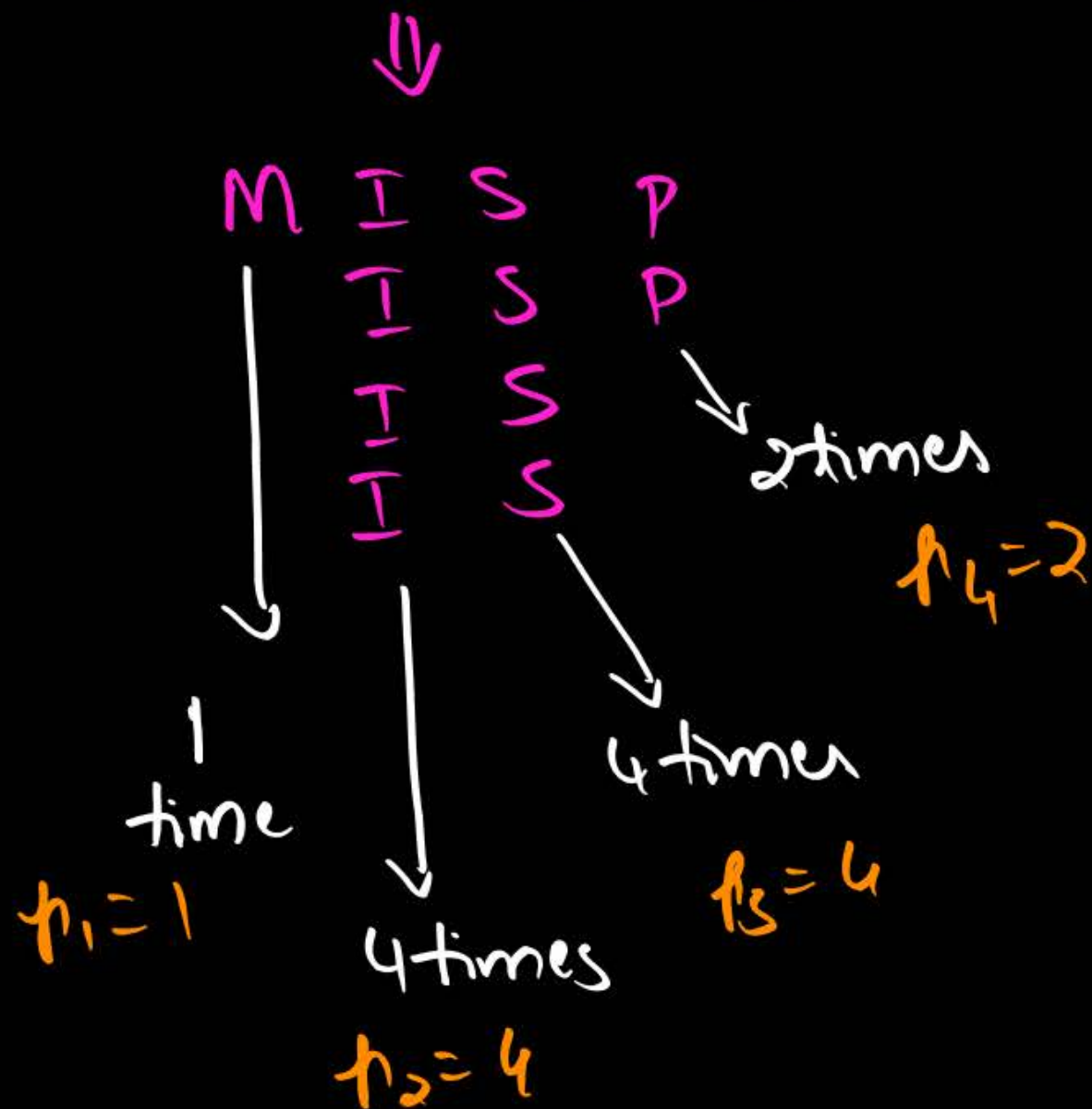
Then

$${}^n P_r = \frac{n!}{r_1! r_2! r_3! \dots r_k!}$$

$r_i \rightarrow$ object repeated r_i times

MISSISSIPPI

⇒ Total objects = 11



No of Permutations

$$= \frac{11!}{1! 4! 4! 2!}$$

Given the word BENCH Find the permutations of



① No of ways in which the word BENCH can be arranged = ${}^5P_5 = 5!$
 $= 120$

② No of ways in which —)) —)) —)) —
such that E N C stays at first, last of the word.

③ No of ways in which —)) —)) —)) —
such that N E C always stays together

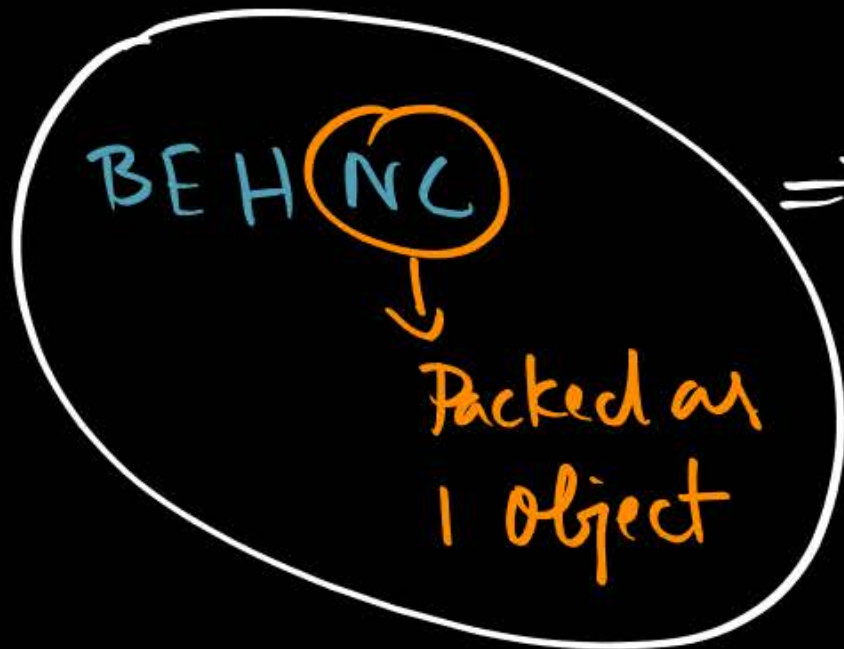
(2) \underline{E} _____ \underline{C}
 (5) ${}^3P_3 = 3! = 6$

\underline{C} _____ \underline{E}
 ${}^3P_3 = 3! = 6$

$6 + 6$

$= 12$

(3)



⇒ Together 4 objects = ${}^4P_4 = 4! = 24$

Now C & N can be arranged in **2 ways** among themselves

$24 \times 2 = 48$

BEHLN

BEHNC

BE CN H

BENCH

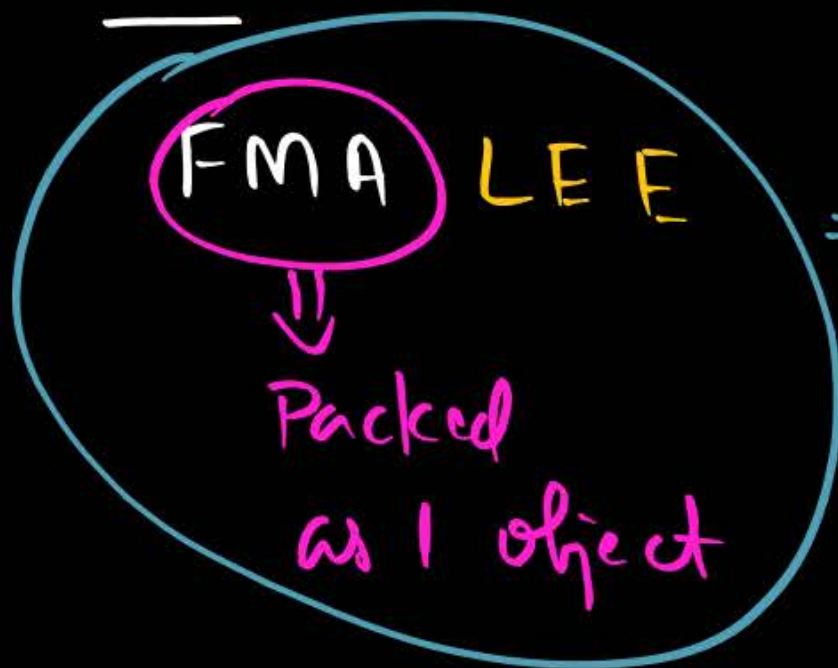


① Find the permutations of the word

FEMALE in which FMA are always together

Solu:

- FMA
- FAM
- MAF
- MFA
- AMF
- AFM



⇒ Total 4 objects = $\frac{4!}{2!} = 12$ (since E is repeated twice)

Here FMA can be arranged in $3! = 6$ ways among themselves

$12 \times 6 = 72$

FMA
FAM
AMF
AFM
MAF
MFA

← (FMA) LEE

() ELE

() EEL

L()EE

E()LE

E()EL

LE()E

EL()E

EE()L

LEE()

ELE()

EEL()





(2) Find the Permutations of the word

FEMALE in which FMA never come together.

FEMALE

E
↓
2

Soln:

Total Permutations — NO of Permutations
in which PMA are
always together

$$= \frac{6!}{2!} - \frac{4!}{2!} (3!)$$

$$= \frac{720}{2} - (12 \times 6) = 360 - 72 = \underline{288}$$

QUESTION



#Q. The probability that in a random arrangement of the letters of the word "FAVOURABLE", the two 'A' do not come together is

- A** 1/5
- B** 1/10
- C** 9/10
- D** ✓ 4/5

F A V O U R B L E
 (A
 A)

Total Permutation = $\frac{10!}{2!}$

A's always come together = 9!

$$\begin{aligned}
 P(\bar{A}) &= \frac{10! - 9!}{\frac{10!}{2!}} = 1 - \frac{9!}{\frac{10!}{2!}} \\
 &= 1 - \frac{9! \times 2}{10 \times 9!} \\
 &= 1 - \frac{2}{10} = 1 - \frac{1}{5} = \frac{4}{5}
 \end{aligned}$$

QUESTION



#Q. The letters of the word 'UNIVERSITY' are randomly arranged, then the probability of the arrangement that no two 'I' come together is

Same as previous one

A ✓ $4/5$

B $1/5$

C $1/10$

D $7/10$

QUESTION



Total = ${}^6P_6 = 6!$

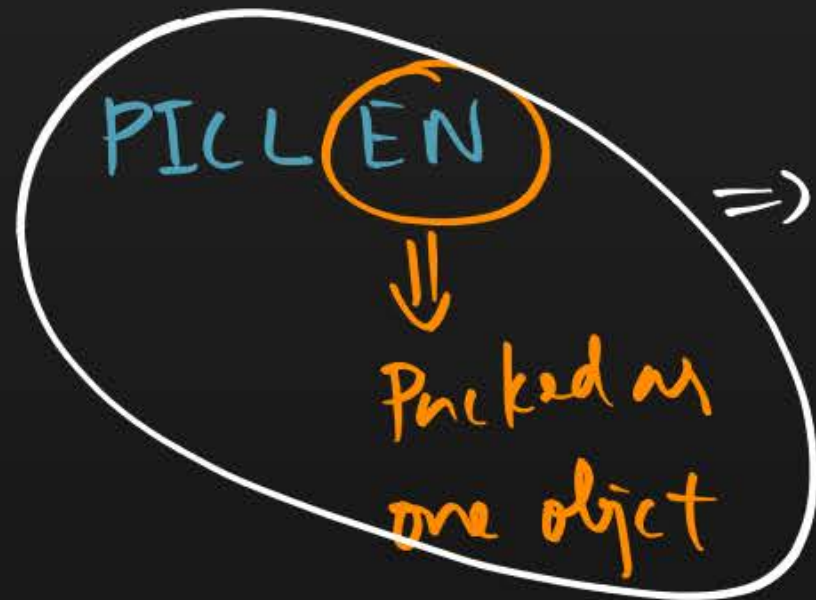
→ All objects are distinct



#Q. If letters of the word 'PENCIL' are arranged in random order, then the probability that N is always next to E is

NE stays together

- A** 1/6
- B** 1/2
- C** 4/3
- D** 2/6



⇒ 5 objects in total = 5!

EN can be arranged in 2 ways among themselves

$120 \times 2 = 240$

$P(A) = \frac{240}{720}$

$= \frac{24}{72}$

$= \frac{1}{3} = \frac{2}{6}$

QUESTION



#Q. Words with or without meaning are to be formed using all the letters of the word EXAMINATION. The probability that the letter M appears at the fourth position in any such word is

A 2/11

B 1/11

C 1/9

D 1/66

EXAMINATION
A I N

$$n(S) = \frac{11!}{2!2!2!}$$

$$n(A) = \frac{10!}{2!2!2!} \quad (\text{Since the position of M is fixed, we need to arrange remaining 10 letters})$$

$$P(A) = \frac{10!}{11!} = \frac{10!}{11 \times 10!} = \frac{1}{11}$$



19th of April (Sunday) → Morning (6 to 10)

↓
3 to 4 hrs

- ↓
- ① Bayes
 - ② PNL

QUESTION



#Q. Two dice of different colours are thrown at a time. The probability that the sum is either 7 or 11 is

↓ ↓
6 2

$$\frac{6+2}{36} = \frac{8}{36} = \frac{2}{9}$$

- A** 7/36
- B** 2/9
- C** 2/3
- D** 5/9

Sum	NO of outcomes
2	1
3	2
4	3
5	4
6	5
7	6
8	5
9	4
10	3
11	2
12	1

QUESTION

#Q. A coin is tossed and a die is rolled. The probability that the coin shows head and the die shows 3 is

$$S = \{ (H, 1), \dots, (H, 6) \\ (T, 1), \dots, (T, 6) \}$$

A $1/6$

B $1/12$

C $1/9$

D $11/12$

$$A = \{ (H, 3) \}$$

$$P(A) = \frac{1}{12}$$

QUESTION



Total = 4 + 2 + 4 = 10
 ↓ Red ↓ white ↓ Black

$$n(S) = {}^{10}C_4 = \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} = 30 \times 7 = 210$$

#Q. From 4 red balls, 2 white balls and 4 black balls, four balls are selected. The probability of getting 2 red balls is

- A** 7/21
- B** 8/21
- C** 9/21
- D** 10/21

$$n(A) = {}^4C_2 \times {}^6C_2$$

↓ ↓
 2 from 2 from
 4 Red Remaining

$$= 6 \times 15$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{6 \times 15}{10 \times 3 \times 7} = \frac{3}{7}$$

$$= \frac{3}{7} \times \frac{3}{3} = \frac{9}{21}$$

QUESTION



$$n(S) = {}^{15}C_{11} = {}^{15}C_4 = \frac{15 \times 14 \times 13 \times 12}{4 \times 3 \times 2 \times 1} \quad \begin{matrix} 5B \\ 10G \end{matrix}$$

#Q. In a class of 15 students, 5 of them are boys and 10 students are girls. A team of 11 members has to be formed at random. The probability that the team has at least 4 boys is

A 37/91

B 54/95

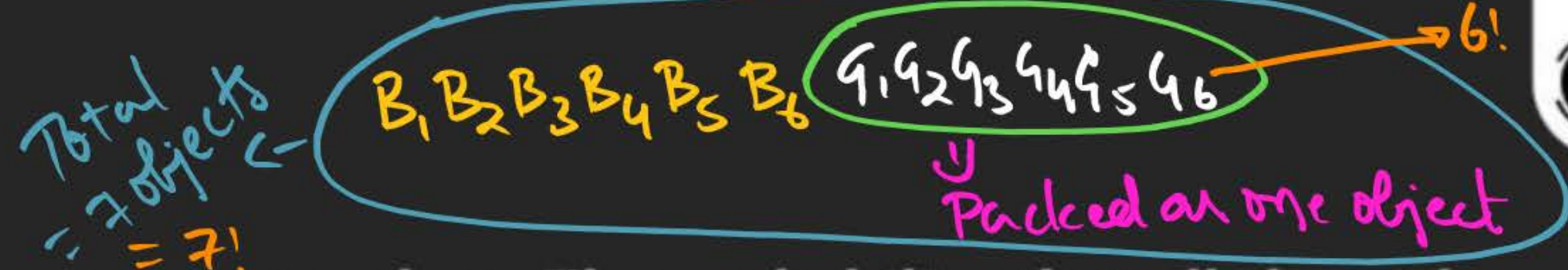
C 54/91

D 51/91

$$\begin{aligned} n(A) &= {}^5C_4 \times {}^{10}C_7 + {}^5C_5 \times {}^{10}C_6 = ({}^5C_1 \times {}^{10}C_3) + (1 \times {}^{10}C_4) \\ &= \left(5 \times \frac{10 \times 9 \times 8}{3 \times 2 \times 1} \right) + \left(\frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} \right) \\ &= (25 \times 24) + (30 \times 7) \\ &= 600 + 210 \\ &= 810 \end{aligned}$$

$$\begin{aligned} P(A) &= \frac{810}{15 \times 14 \times 13} \\ &= \frac{54}{91} \end{aligned}$$

QUESTION



#Q. 6 boys and 6 girls sit in a row at random. The probability that all the girls sit together is

- A** 1/432
- B** 12/431
- C** 1/132
- D** None of these

$n(S) = {}^{12}P_{12} = 12!$

$n(A) = 7! \cdot 6!$

Handwritten calculation for the probability:

$$P(A) = \frac{7! \cdot 6!}{12!}$$

$$= \frac{7! \cdot 6!}{12 \times 11 \times 10 \times 9 \times 8 \times 7!}$$

$$= \frac{6}{11 \times 9 \times 8 \times 4}$$

$$= \frac{1}{11 \times 3 \times 4} = \frac{1}{11 \times 12}$$

$$= \frac{1}{132}$$

— $B_1 B_2 B_3 B_4 B_5 B_6$
 girls

ABC

B_1 — $B_2 B_3 B_4 B_5 B_6$
 | girls

$B_1 B_2$ — $B_3 \dots B_6$

$B_1 B_2 B_3$ — $B_4 B_5 B_6$

B_4 — $B_5 B_6$

B_5 — B_6

$B_5 B_6$ —

QUESTION



Total Permutations = ${}^8P_8 = 8! = n(s)$

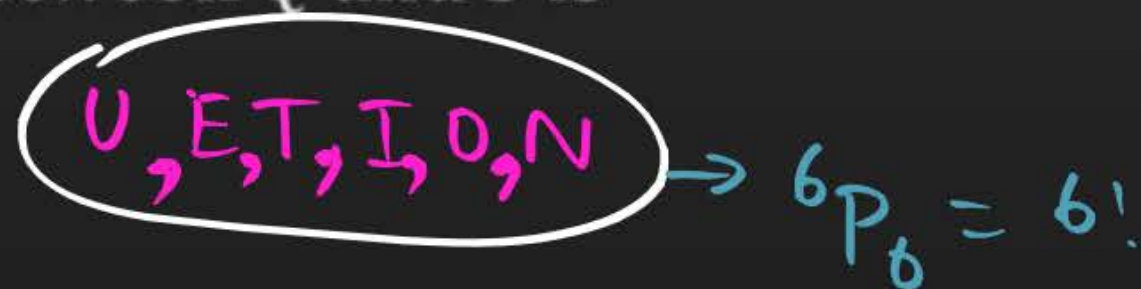
#Q. The letters of the word "QUESTION" are arranged in a row at random. The probability that there are exactly two letters between Q and S is

A 1/14

B 5/7

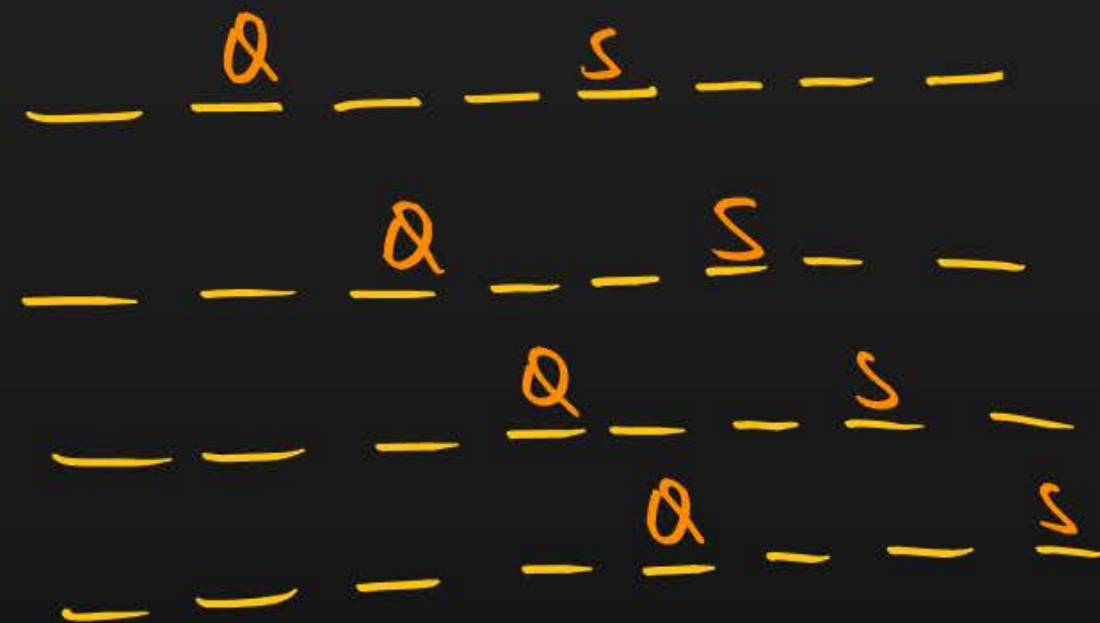
C 1/7

D 5/28



→ Q & S can be arranged in 2 ways among themselves = 2

→ The positions of Q & S can be shifted = 5 ways



$$n(A) = 6! \times 2 \times 5$$

$$\begin{aligned} \therefore P(A) &= \frac{6! \times 2 \times 5}{8!} \\ &= \frac{\cancel{6!} \times \cancel{2} \times 5}{\cancel{4} \times 8 \times 7 \times \cancel{6!}} \\ &= \frac{5}{28} \end{aligned}$$

QUESTION



$$X = | \text{No of head} - \text{No of Tails} |$$

#Q. A coin is tossed three times. If X denotes the absolute difference between the number of heads and the number of tails, then $P(X = 1) =$

$S = \{ \text{HHH}, \text{HHT}, \text{HTH}, \text{THH}, \text{TTT}, \text{TTH}, \text{THT}, \text{HTT} \}$
 ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓
 $|3-0| = 3$ $|2-1| = 1$ $|2-1| = 1$ $|2-1| = 1$ $|0-3| = 3$ $|1-2| = 1$ $|1-2| = 1$ $|1-2| = 1$

$$\{X=1\} = \{ \text{HHT}, \text{HTH}, \text{THH}, \text{TTH}, \text{THT}, \text{HTT} \}$$

$$P(X=1) = \frac{6}{8} = \frac{3}{4}$$

- A** 1/2
- B** 2/3
- C** 1/6
- D** 3/4

QUESTION

Diamond cards are Red in colour.



#Q. From a deck of cards, all the diamonds are removed. From remaining cards, a card is chosen randomly. What is the probability that it will be a black card?

A $1/2$

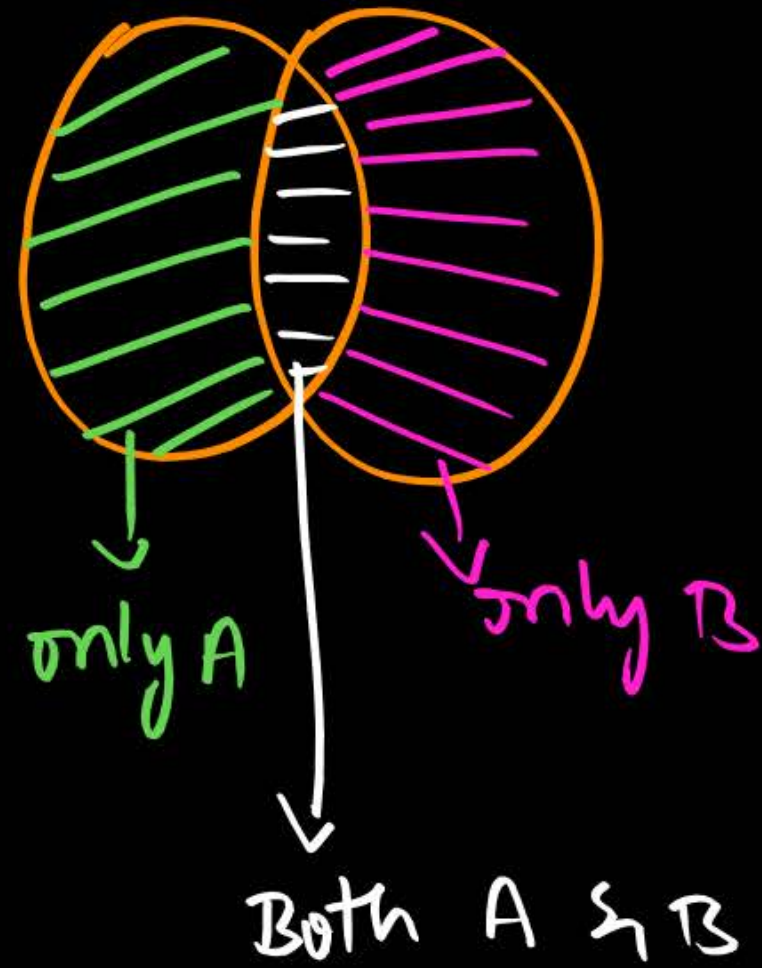
B $2/3$

C $1/3$

D $3/4$

$$\begin{aligned}n(S) &= \text{Total} - \text{Diamond} \\ &= 52 - 13 \\ &= 39\end{aligned}$$

$$P(A) = \frac{26}{39} = \frac{2}{3}$$



$A \cup B$



either A



B

At least one of A & B

QUESTION



$$\begin{array}{l}
 P(A) = 0.4 \mid P(B) = 0.3 \mid P(C) = 0.2 \mid P(A') = 0.6 \\
 P(A \text{ hit}) \mid \mid \mid P(A \text{ Missed the target})
 \end{array}$$

#Q. Three persons A, B and C, fire at a target in turn, starting with A. Their probability of hitting the target are 0.4, 0.3 and 0.2, respectively. The probability of **at least one hits the target**

$$P(\text{At least one of A, B \& C hits target})$$

A 0.024

B 0.188

C 0.336

D ~~None of these~~

$$= P(A \cup B \cup C) = 1 - P(A') P(B') P(C')$$

$$= 1 - \frac{6}{10} \frac{7}{10} \frac{8}{10} \mid 1 - 0.336$$

$$= 1 - \frac{42 \times 8}{1000} \mid = \underline{\underline{0.664}}$$

$$= 1 - \frac{336}{1000}$$

QUESTION



#Q. The probability that a certain person will buy a shirt is 0.2 , the probability that he will buy a trouser is 0.3 and the probability that he will buy a shirt given that he buys a trouser is 0.4 . Find the probability that he will buy both a shirt and a trouser.

- A** 0.2
- B** ✓ 0.12
- C** 0.3
- D** 0.06

$$\begin{aligned}P(\text{shirt}) &= P(A) = 0.2 \\P(\text{trouser}) &= P(B) = 0.3 \\P(A|B) &= 0.4\end{aligned}$$

$$\begin{aligned}P(A \cap B) &= P(B) P(A|B) \\&= (0.3)(0.4) \\&= \underline{0.12}\end{aligned}$$

QUESTION



#Q. For the married couple living in Jammu, the probability that a husband will vote in an election is 0.5 and the probability that his wife will vote is 0.4 . The probability that the husband votes, given that his wife also votes is 0.7 . Then the probability that husband and wife both will vote is

conditional probability

A 0.28

B 0.20

C 0.35

D 0.15

$$P(\text{Husband vote}) = P(A) = 0.5$$

$$P(\text{wife vote}) = P(B) = 0.4$$

$$P(A|B) = 0.7$$

$$P(A \cap B) = ?$$

$$P(A \cap B) = P(B) P(A|B)$$

$$= (0.4)(0.7)$$

$$= \underline{0.28}$$

QUESTION



Let $P(A) = p$
 $P(B) = q$

#Q. A and B are two **independent** events. The probability that both A and B occur is $\frac{1}{6}$ and the probability that neither of them occurs is $\frac{1}{3}$. Find the probability of the occurrence of A.

- A** 4/5
- B** 2/3
- C** 1/4
- D** 1/3

$P(A \cap B) = \frac{1}{6}$
 $P(A)P(B) = \frac{1}{6}$
 $p \cdot q = \frac{1}{6}$

$P(A' \cap B') = \frac{1}{3}$
 \Downarrow
 $P(A')P(B') = \frac{1}{3}$
 $(1-p)(1-q) = \frac{1}{3}$
 $1 - p - q + pq = \frac{1}{3}$
 $1 - (p+q) + \frac{1}{6} = \frac{1}{3}$

$p + q = 1 + \frac{1}{6} - \frac{1}{3} = \frac{6+1-2}{6} = \frac{5}{6}$
 $q = \frac{5}{6} - p$

 $p \cdot q = \frac{1}{6}$
 $p(\frac{5}{6} - p) = \frac{1}{6}$
 $\frac{5p}{6} - p^2 = \frac{1}{6}$

$$\frac{5p}{6} - p^2 = \frac{1}{6}$$

$$6p^2 - 5p + 1 = 0$$

$$6p^2 - 3p - 2p + 1 = 0$$

$$3p(2p-1) - 1(2p-1) = 0$$

$$p = \frac{1}{3} \quad | \quad p = \frac{1}{3}$$

$$+6$$

$$\swarrow \searrow$$

$$-3 \quad -2$$

QUESTION



Fails if Atleast one of them Fails.

#Q. A machine operates if all of its three components function. The probability that the first component fails during the year is 0.14, the second component fails is 0.10 and the third component fails is 0.05. What is the probability that the machine will fail during the year?

A 0.1542

B 0.2647

C 0.3642

D 0.4231

$$P(\text{1st Fail}) = P(A) = 0.14$$

$$P(B) = 0.10$$

$$P(C) = 0.05$$

$$P(\text{Machine Fails}) = P(\text{Atleast one of } A, B \text{ \& } C \text{ Fails})$$

$$= P(A \cup B \cup C)$$

$$= 1 - P(A') P(B') P(C')$$

$$= 1 - (0.86)(0.9)(0.95)$$



$$\begin{aligned} & 1 - (0.86)(0.9)(0.95) \\ &= 1 - (0.774)(0.95) \\ &= 1 - 0.7353 \\ &= \underline{0.2647} \end{aligned}$$

QUESTION



#Q. Given that, the events A and B are such that $P(A) = \frac{1}{2}$, $P(A \cup B) = \frac{3}{5}$ and $P(B) = p$. Then probabilities of B if A and B are mutually exclusive and independent respectively are

- A** $1/2, 1/3$
- B** $1/5, 1/3$
- C** $2/3, 1/3$
- D** $1/10, 1/5$

① A & $B \rightarrow$ mutually exclusive



$$P(A \cap B) = 0$$

$$P(A \cup B) = P(A) + P(B)$$

$$\frac{3}{5} = \frac{1}{2} + p$$

$$p = \frac{3}{5} - \frac{1}{2} = \frac{1}{10}$$

② A & B are independent

$$P(A \cup B) = 1 - P(A') P(B')$$

$$\frac{3}{5} = 1 - \frac{1}{2} (1 - p)$$

$$\frac{3}{5} = 1 - \frac{1}{2} + \frac{p}{2}$$

$$\frac{3}{5} - \frac{1}{2} = \frac{p}{2}$$

$$\frac{p}{2} = \frac{1}{10}$$

$p = \frac{1}{5}$

QUESTION



Let $P(F) = p$

#Q. Two events E and F are independent. If $P(E) = 0.3$ $P(E \cup F) = 0.5$, then $P(E | F) - P(F | E)$ equals to

↓ ↓
 $P(E) - P(F)$

$P(E) = \frac{3}{10}$

$P(E \cup F) = 1 - P(E') P(F')$

$0.5 = 1 - (0.7)(1-p)$

$0.5 = 1 - 0.7 + 0.7p$

$0.5 - 0.3 = 0.7p$

$p = \frac{0.2}{0.7} = \frac{2}{7}$

$\frac{3}{10} - \frac{2}{7} = \frac{21 - 20}{70}$

$= \frac{1}{70}$

- A** 2/7
- B** 3/35
- C** 1/70
- D** 1/7

QUESTION



#Q. If A and B are two independent events with $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{4}$, then $P(B' | A)$ is equal to



$$1 - P(B|A)$$

$$1 - P(B)$$

$$1 - \frac{1}{4}$$

$$= \frac{3}{4}$$

A $\frac{1}{4}$

B $\frac{1}{8}$

C $\frac{3}{4}$

D 1

QUESTION



#Q. For two events $P(A \cup B) = \frac{5}{6}$, $P(A) = \frac{1}{6}$, $P(B) = \frac{2}{3}$, then A and B are

$$\begin{aligned} P(A \cap B) &= \frac{1}{6} + \frac{2}{3} - \frac{5}{6} \\ &= \frac{1+4-5}{6} = 0 \end{aligned}$$

- A** mutually exclusive events
- B** independent events
- C** dependent events
- D** none of these

QUESTION



#Q. If A and B are two independent events with $P(A) = \frac{3}{5}$ and $P(B) = \frac{4}{9}$, then $P(A' \cap B')$ equals to

A $4/15$

B $8/45$

C $1/3$

D $2/9$

$\rightarrow P(A') \cdot P(B')$

$= \frac{2}{5} \cdot \frac{5}{9}$

$= \frac{2}{9}$

QUESTION



$$\text{Let } P(B) = p$$

#Q. Suppose A and B are such events that $P(A) = 0.3$ $P(A \cup B) = 0.8$. If A and B are independent events then $P(B)$ is

A $5/7$

B $5/4$

C $1/3$

D $1/2$

$$P(A \cup B) = 1 - P(A')P(B')$$

$$0.8 = 1 - (0.7)(1-p)$$

$$0.8 = 1 - 0.7 + 0.7p$$

$$0.8 - 0.3 = 0.7p$$

$$p = \frac{0.5}{0.7} = \frac{5}{7}$$

QUESTION



#Q. The probability of A to be failed in an examination is 0.2 and that of B to be failed is 0.3 then the probability that either A or B to be failed is

$$P(A \text{ Fail}) = P(A) = 0.2$$

$$P(B \text{ Fail}) = P(B) = 0.3$$

$$P(A \cup B) = 1 - P(A')P(B')$$

$$= 1 - (0.8)(0.7)$$

$$= 1 - 0.56$$

$$= 0.44$$

A 0.3

B 0.44

C 0.8

D 0.25

QUESTION



#Q. A problem is given to three students whose probabilities of solving it are $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{6}$ respectively. If the events of solving the problem are independent, find the probability that at least one of them solves it.

A 5/12

B 7/12

C 1/2

D None of these

$$P(A) = \frac{1}{3}$$

$$P(B) = \frac{1}{4}$$

$$P(C) = \frac{1}{6}$$

$$P(A \cup B \cup C) = 1 - P(A')P(B')P(C')$$

$$= 1 - \frac{2}{3} \cdot \frac{3}{4} \cdot \frac{5}{6}$$

$$= 1 - \frac{5}{12}$$

$$= \frac{7}{12}$$

QUESTION



#Q. A problem is given to three students whose probabilities of solving it are $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{6}$ respectively. If the events of solving the problem are independent, find the probability that at least ~~Two~~ of them solves it.

A 5/12

B 7/12

C 1/2

D None of these

$$P(A) = \frac{1}{3}$$

$$P(B) = \frac{1}{4}$$

$$P(C) = \frac{1}{6}$$

$P(\text{At least 2 of } A, B \text{ \& } C)$

$$= P(A)P(B)P(C') + P(A)P(B')P(C) + P(A')P(B)P(C) + P(A)P(B)P(C)$$

$$= \frac{1}{3} \cdot \frac{1}{4} \cdot \frac{5}{6} + \frac{1}{3} \cdot \frac{3}{4} \cdot \frac{1}{6} + \frac{2}{3} \cdot \frac{1}{4} \cdot \frac{1}{6} + \frac{1}{3} \cdot \frac{1}{4} \cdot \frac{1}{6}$$

$$= \left[\frac{5+3+2+1}{3(4)(6)} \right] = \frac{11}{72}$$

PYQ of KLET

#Q. If A, B and C are three independent events such that $P(A) = P(B) = P(C) = p$, then P (at least two of A, B and C occur) is equal to

- A** $p^2 - p$
- B** $2p^2 - 2$
- C** $3p^2 - 2p^3$
- D** $3p^3 - 2p^2$

$$= \overset{A}{p} \overset{B}{p} \overset{C'}{(1-p)} + \overset{A}{p} \overset{B'}{(1-p)} \overset{C}{p} + \overset{A'}{(1-p)} \overset{B}{p} \overset{C}{p} + \overset{A}{p} \overset{B}{p} \overset{C}{p}$$

$$= 3p^2(1-p) + p^3$$

$$= 3p^2 - 3p^3 + p^3$$

$$= 3p^2 - 2p^3$$

QUESTION



$$P(A) = \frac{3}{7} \quad | \quad P(B) = \frac{5}{7}$$

#Q. The probabilities of two students A and B coming to the school in time are $\frac{3}{7}$ and $\frac{5}{7}$ respectively. Assuming that the events, 'A coming in time' and 'B coming in time' are independent, then find the probability of only one of them coming to the school in time.

A $25/48$

B $24/47$

C $26/49$

D $27/48$

$$\begin{aligned} & P(A \cap B') + P(A' \cap B) \\ &= P(A)P(B') + P(A')P(B) \\ &= \frac{3}{7} \cdot \frac{2}{7} + \frac{4}{7} \cdot \frac{5}{7} \\ &= \frac{26}{49} \end{aligned}$$

$A \cap B'$
↳ A but not B.

QUESTION



$$P(A) = \frac{3}{7} \quad | \quad P(B) = \frac{5}{7}$$

#Q. The probabilities of two students A and B coming to the school in time are $\frac{3}{7}$ and $\frac{5}{7}$ respectively. Assuming that the events, 'A coming in time' and 'B coming in time' are independent, then find the probability of

① Both come on time

② At least one of them come on time

Soln:

$$\textcircled{1} P(A \cap B) = P(A) P(B) = \frac{15}{49}$$

$$\textcircled{2} P(A \cup B) = 1 - P(A') P(B') = 1 - \frac{4}{7} \frac{2}{7} = 1 - \frac{8}{49} = \frac{41}{49}$$

QUESTION

Equally likely means



$$P(A) = P(B)$$



#Q. Let A and B be two events such that $P(\overline{A \cup B}) = \frac{1}{6}$, $P(A \cap B) = \frac{1}{4}$ and $P(\bar{A}) = \frac{1}{4}$. Then events A and B are

$$P(A \cup B) = \frac{5}{6}$$

$$P(A) = \frac{3}{4}$$

$$P(A \cap B) = P(A) \cdot P(B)$$

A ✗ equally likely, but not independent

$$P(B) = \frac{5}{6} + \frac{1}{4} - \frac{3}{4} = \frac{10 + 3 - 9}{12} = \frac{4}{12} = \frac{1}{3}$$

B ✗ equally likely and mutually exclusive
since $P(A \cap B) \neq 0$

C mutually exclusive and independent

$$P(A)P(B) = \frac{3}{4} \cdot \frac{1}{3} = \frac{1}{4} = P(A \cap B)$$

D ✓ independent but not equally likely

$$P(A) \neq P(B)$$

QUESTION

$$\text{Let } P(B) = p$$



#Q. Let A and B be independent events with $P(A) = 1/4$ and $P(A \cup B) = 2P(B) - P(A)$. Find $P(B)$.

- A** $1/4$
- B** $3/5$
- C** $2/3$
- D** $2/5$

$$\downarrow$$

$$1 - P(A')P(B') = 2P(B) - \frac{1}{4}$$

$$1 - \frac{3}{4}(1-p) = 2p - \frac{1}{4}$$

$$1 - \frac{3}{4} + \frac{3}{4}p = 2p - \frac{1}{4}$$

$$\frac{1}{4} + \frac{1}{4} = 2p - \frac{3}{4}p$$

$$\frac{1}{2} = \frac{5p}{4}$$

$$\uparrow = \frac{2}{5}$$

#Q. Two events A and B will be independent, if

- A** A and B are mutually exclusive
- B** $P(A' \cap B') = [1 - P(A)][1 - P(B)]$
- C** $P(A) = P(B)$
- D** $P(A) + P(B) = 1$

QUESTION



#Q. If A and B are events such that $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$ and $P(A \cap B) = \frac{1}{12}$, then find $P(\text{not } A \text{ and not } B)$.

- A** $1/4$
- B** $1/2$
- C** $2/3$
- D** $1/3$

QUESTION



#Q. Fatima and John appear in an interview for two vacancies in the same post. The probability of Fatima's selection is $\frac{1}{7}$ and that of John's selection is $\frac{1}{5}$. What is the probability that both of them will be selected?

- A** 34/35
- B** 12/35
- C** 1/35
- D** None of these

$$P(\text{Fatima}) = P(A) = \frac{1}{7}$$

$$P(\text{John}) = P(B) = \frac{1}{5}$$

selection of Fatima doesn't
effect that of John's

& vice versa

\therefore events A & B are independent

$$P(A \cap B) = P(A) P(B)$$

$$= \frac{1}{35}$$

QUESTION



#Q. A salesman has a 60% chance to sell a product to **any customer**. The behavior of two different customer is independent. If two customer A and B came into the shop, then what is the probability that salesman will sell the product to customer A **or** B ?

$$P(A) = \frac{60}{100} = 0.6 = P(B)$$

A & B are independent

$$\begin{aligned} P(A \cup B) &= 1 - P(A')P(B') \\ &= 1 - (0.4)(0.4) \\ &= 1 - 0.16 = \underline{0.84} \end{aligned}$$

A 0.84

B 0.91

C 0.72

D 0.60

QUESTION

Let $P(A) = p$ & $P(B) = q$



#Q. Two independent events A and B are such that $P(A \cup B) = \frac{2}{3}$ and $P(A \cap B) = \frac{1}{6}$. If $P(B) < P(A)$, then what is $P(B)$ equal to?

- A** $1/4$
- B** $1/3$
- C** $1/2$
- D** $1/6$

$$P(A \cap B) = \frac{1}{6}$$

$$P(A)P(B) = \frac{1}{6}$$

$$pq = \frac{1}{6}$$

$$P(A \cup B) = \frac{2}{3}$$

$$1 - P(A')P(B') = \frac{2}{3}$$

$$1 - (1-p)(1-q) = \frac{2}{3}$$

$$1 - 1 + q + p - pq = \frac{2}{3}$$

$$p + q = \frac{2}{3} + \frac{1}{6}$$

$$p + q = \frac{5}{6}$$

$$p = \frac{5}{6} - q$$

$$pq = \frac{1}{6}$$

$$\left(\frac{5}{6} - q\right)q = \frac{1}{6}$$

$$\frac{5q}{6} - q^2 = \frac{1}{6}$$

$$6q^2 - 5q + 1 = 0$$

→ option verification

$$6q^2 - 5q + 1 = 0$$

$$6q^2 - 3q - 2q + 1 = 0 \quad \begin{array}{l} +b \\ -3 \quad -2 \end{array}$$

$$3q(2q-1) - 1(2q-1) = 0$$

$$q = \frac{1}{3} \text{ (or) } q = \frac{1}{2}$$

$$\Downarrow$$

$$p = \frac{5}{6} - q$$

$$p = \frac{5}{6} - \frac{1}{6}$$

$$p = \frac{1}{2}$$

$$\Downarrow$$

$$p = \frac{1}{3}$$

Here $P(B) < P(A)$

$$\Downarrow$$

Since

$$\frac{1}{3} < \frac{1}{2}$$

$$\therefore P(B) = \frac{1}{3}$$

Thank

You