Vernacular

Ultimate kcet crash course 2026

maths

DPP: 1

Differential Equations, Application of derivatives

Q1 Find the degree of the differential equation

$\sqrt{1+\left(\frac{dy}{dx}\right)^2}=\left(\frac{d^2y}{dx^2}\right)^{1/3}$

(A) 2 (B) 1
(C) 3 (D) 4

Q2 The degree of the differential equation

$y''+9(y')^{5/2}=e^x$ is

(A) 5 (B) 2
(C) 1 (D) 5/2

Q3 Order and degree of the differential equation

$\frac{d^4y}{dx^4}+\sin\left(\frac{d^3y}{dx^3}\right)=0$ are

(A) order = 4, degree = 1
(B) order = 3, degree = 1
(C) order = 4, degree = not defined
(D) order = 4, degree = 2

Q4 The order of the differential equation of the curve $y^2=4ax$ is:

(A) 2
(B) 1
(C) 3
(D) Can't be determined

Q5 The order and degree of the differential equation

$\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^{\frac{1}{3}}+x^{\frac{1}{4}}=0$ are respectively

(A) 2, 3 (B) 3, 3
(C) 2, 6 (D) 2, 4

Q6 Particular solution of the differential equation $e^{-x}(y+1)dy+(\cos^2 x-\sin 2x)y\,dx=0$ subjected to the condition that $y=1$ when $x=0$ is

(A) $y+\log y+e^x\cos^2 x=2$
(B) $\log(y+1)+e^x\cos^2 x=1$
(C) $y+\log y=e^x\cos^2 x$
(D) $\log(y+1)+e^x\cos^2 x=2$

Q7 Solve the differential equation

$\tan y\frac{dy}{dx}=\sin(x+y)+\sin(x-y)$

(A) sin x + sec y = c
(B) 2 cos x + sec y = c
(C) cos x + sin x = c
(D) 2 sin x + sec y = c

Q8 Solution of differential equation

$\frac{dy}{dx}=\sin(x+y)+\cos(x+y)$ is equal to

(A) $\log\left(2+\sec\frac{x+y}{2}\right)=x+c$
(B) log {1 + tan (x + y)} = y + c
(C) $\log\left(1+\tan\frac{x+y}{2}\right)=y+c$
(D) $\log\left(1+\tan\frac{x+y}{2}\right)=x+c$

Q9 The general solution of the differential equation

$(x+y+3)\frac{dy}{dx}=1$ is

(A) $x+y+3=Ce^y$
(B) $x+y+4=Ce^y$
(C) $x+y+3=Ce^{-y}$
(D) $x+y+4=Ce^{-y}$

Q10 Solve : $(x+y)^2\frac{dy}{dx}=a^2$

(A) $y=a\tan^{-1}\left(\frac{x-y}{a}\right)+c$
(B) $y=a\tan^{-1}\left(\frac{y}{a}\right)+c$
(C) $y=a\tan^{-1}\left(\frac{x+y}{a}\right)+c$
(D) $y=a\tan^{-1}\left(\frac{x}{a}\right)+c$

Q11 The solution of $\frac{dy}{dx}=2^{y-x}$ is

(A) $2^x+2^y=c$
(B) $2^x-2^y=c$
(C) $\frac{1}{2^x}-\frac{1}{2^y}=c$
(D) $x+y=c$

Q12 The solution of $\frac{dy}{dx}+y\tan x=0$ is:

(A) y = a cos x
(B) y = a sin x
(C) y = log cos x + c
(D) y = a tan x + c

Q13 The solution of the differential equation $x\left(e^{2y}-1\right)dy+\left(x^2-1\right)e^{y}dx=0$ is
(A) $e^{y}+e^{-y}=\log x-\frac{x^2}{2}+c$
(B) $e^{y}-e^{-y}=\log x-\frac{x^2}{2}+c$
(C) $e^{y}+e^{-y}=\log x+\frac{x^2}{2}+c$
(D) None of these

Q14 The solution of $\frac{dy}{dx}=\frac{1-y}{1-x}$ is:
(A) (1 - x) (1 + y) = c
(B) $\frac{1-x}{1-y}=c$
(C) $\frac{1+x}{1-y}=c$
(D) (1 - x) (1 - y) = c

Q15 The solution of the differential equation $\left(x^2-yx^2\right)\frac{dy}{dx}+y^2+xy^2=0$ is
(A) $\log\left(\frac{x}{y}\right)=\frac{1}{x}+\frac{1}{y}+c$
(B) $\log\left(\frac{y}{x}\right)=\frac{1}{x}+\frac{1}{y}+c$
(C) $\log\left(xy\right)=\frac{1}{x}+\frac{1}{y}+c$
(D) $\log\left(xy\right)+\frac{1}{x}+\frac{1}{y}=c$

Q16 The solution of the equation $x\frac{dy}{dx}=y+x\tan\frac{y}{x}$ is
(A) sin x/y = cx
(B) sin y/x = cx
(C) sin x/y = cy
(D) sin y/x = cy

Q17 Solve the differential equation $y^2+x^2\frac{dy}{dx}=xy\frac{dy}{dx}$.
(A) $y = e^{x} + c$
(B) y = x (c + log |y|)
(C) y = log |y| + c
(D) $y = x + ye^{x} + c$

Q18 The solution of $\frac{dy}{dx}=\frac{y}{x}+\sin\left(\frac{y}{x}\right)$ is:
(A) $\tan\left(\frac{y}{2x}\right)-cx=0$
(B) $\tan\left(\frac{y}{x}\right)-cx=0$
(C) $\tan\left(\frac{y}{x}\right)+cx-x=0$
(D) $\sin\left(\frac{y}{2x}\right)-cx=0$

Q19 Solve the differential equation $x\frac{dy}{dx}-y=2\sqrt{y^2-x^2}$
(A) $y-\sqrt{x+y}=x+c$
(B) $y+\sqrt{y^2-x^2}=cx^3$
(C) $y+\sqrt{y-x}=cx^2$
(D) $y-\sqrt{y^2+x^2}=cx$

Q20 By substituting $x = vy$ the transformed equation of $\left(1+e^{x/y}\right)dx+e^{x/y}\left(1-\frac{x}{y}\right)dy=0$ is:
(A) $y\frac{dv}{dy}-\left(\frac{1+e^{v}}{1-e^{v}}\right)=0$
(B) $y\frac{dv}{dy}+\left(\frac{v+e^{v}}{1+e^{v}}\right)=0$
(C) $y\frac{dv}{dy}+x\left(v+e^{v}\right)=0$
(D) $y\frac{dy}{dv}+x\left(v+e^{v}\right)=0$

Q21 I.F. of $\frac{dy}{dx}-y\cot x=\operatorname{cosec}x$ is:
(A) sin x
(B) tan x
(C) sec x
(D) cosec x

Q22 I.F of $\cos^2 x\frac{dy}{dx}+y=\tan x$ is:
(A) $e^{\sec x}$
(B) $e^{\tan x}$
(C) $e^{\sin x}$
(D) $e^{\cos x}$

Q23 The solution of the differential equation $x\frac{dy}{dx}+y=x^2+3x+2$ is
(A) $xy=\frac{x^3}{3}+\frac{3}{2}x^2+2x+c$
(B) $xy=\frac{x^4}{4}+x^3+x^2+c$
(C) $xy=\frac{x^4}{4}+\frac{x^3}{3}+x^2+c$
(D) $xy=\frac{x^4}{4}+x^3+x^2+cx$

Q24 The solution of dy = cos x(2 – y cosec x)dx where y = 1 when $x=\frac{\pi}{2}$ is
(A) y = sin x + cosec x
(B) $y=\tan\frac{x}{2}+\cot\frac{x}{2}$
(C) $y=\frac{1}{\sqrt{2}}\sec\frac{x}{2}+\sqrt{2}\cos\frac{x}{2}$
(D) None of these

Q25 The solution of the differential equation $\left(1+y^2\right)+\left(x-e^{\tan^{-1}y}\right)\frac{dy}{dx}=0$ is
(A) $x^2e^{2\tan^{-1}y}=e^{\tan^{-1}y}+k$
(B) $(x-2)=ke^{-\tan^{-1}y}$
(C) $2xe^{\tan^{-1}y}=e^{2\tan^{-1}y}+k$

(D) $xe^{\tan^{-1} y} = \tan^{-1} y + k$

Q26 A square plate decreases in area at the rate of 2 sq. cm/sec. The rate at which the perimeter decreases when the side is 16 is

(A) 4 (B) 1/4

(C) 1/2 (D) 2

Q27 The length of a rectangle is increasing at the rate of 3.5 cm/sec and its breadth is decreasing at the rate of 3 cm/sec. Find the rate of change of the area of the rectangle when length is 12 cm and breadth is 8 cm .

(A) 6 cm^2/sec (B) -6 cm^2/sec

(C) 8 cm^2/sec (D) -8 cm^2/sec

Q28 The volume V and depth x of water in a vessel are connected by the relation $V = 5x - \frac{x^2}{6}$ and the volume of water is increasing at the rate of 5 cm^3/sec, when x = 2 cm. The rate at which the depth of water is increasing is

(A) $\frac{5}{18} \mathrm{cm/sec}$ (B) $\frac{1}{4} \mathrm{cm/sec}$

(C) $\frac{5}{16} \mathrm{cm/sec}$ (D) None of these

Q29 If the rate of change of tan *t* is twice the rate of change of $\log_e(\cos t)$ then *t* is

(A) $\frac{\pi}{4}$ (B) $-\frac{\pi}{4}$

(C) $-\frac{\pi}{3}$ (D) $\frac{\pi}{6}$

Q30 The sides of an equilateral triangle are increasing at the rate of 4 cm/sec. The rate at which its area is increasing when the side is 14 cm is

(A) $10\sqrt{3}\ \mathrm{cm}^2/\mathrm{sec}$

(B) 42 cm^2/sec

(C) $28\sqrt{3}\ \mathrm{cm}^2/\mathrm{sec}$

(D) 14 cm^2/sec

Answer Key

Q1	(A)
Q2	(B)
Q3	(C)
Q4	(B)
Q5	(A)
Q6	(A)
Q7	(B)
Q8	(D)
Q9	(B)
Q10	(C)
Q11	(C)
Q12	(A)
Q13	(A)
Q14	(B)
Q15	(A)
Q16	(B)
Q17	(B)
Q18	(A)
Q19	(B)
Q20	(B)
Q21	(D)
Q22	(B)
Q23	(A)
Q24	(D)
Q25	(C)
Q26	(B)
Q27	(D)
Q28	(D)
Q29	(B)
Q30	(C)

Hints & Solutions

Note: scan the QR code to watch video solution

Q1 Text Solution:

We have, highest order derivative is $\frac{d^2y}{dx^2}$.

So, its order is 2.

degree is 2

Video Solution:

Q2 Text Solution:

$$y'' + 9(y')^{5/2} = e^x \Rightarrow \frac{d^2y}{dx^2} + 9\left(\frac{dy}{dx}\right)^{5/2} = e^x$$

$$\Rightarrow \frac{d^2y}{dx^2} - e^x = -9\left(\frac{dy}{dx}\right)^{3/2}$$

$$\text{(squaring both sides we get)}$$

$$\Rightarrow \left(\frac{d^2y}{dx^2} - e^x\right)^2 = 81\left(\frac{dy}{dx}\right)^3$$

Here degree = 2 (power of highest derivative)

Video Solution:

Q3 Text Solution:

The highest order derivative which occurs in the given differential equation is $\frac{d^4y}{dx^4}$, therefore its order is 4.

As the given differential equation is not a polynomial equation in dy/dx, therefore its degree is not defined.

Video Solution:

Q4 Text Solution:

Since the given differential has only '1' arbitrary constant i.e. 'a', hence the order of differential equation is '1'.

Video Solution:

Q5 Text Solution:

$$\text{Given } \frac{d^2y}{dx^2} + x^{1/4} = -\left(\frac{dy}{dx}\right)^{1/3}$$

Taking cube, $\left[\left(\frac{d^2y}{dx^2}\right) + x^{1/4}\right]^3 = -\frac{dy}{dx}$

Order of highest derivative = 2

Degree of highest derivative = 3

Video Solution:

Q6 Text Solution:

$$e^{-x}(y+1)dy = -(\cos^2 x - \sin 2x)y\ dy$$

$$\int \frac{y+1}{y} dy = -e^x(\cos^2 x - \sin 2x)dy$$

$$\int 1 + \frac{1}{y} dy = -\int e^x\left[\cos^2 x + (-\sin 2x)\right]dx$$

$$y + \log y = -e^x\left[\cos^2 x\right] + c$$

$$\Rightarrow\ y + \log y + e^x \cos^2 x = c$$

Given y = 1 when x = 0

1 + 1 = c

c = 2

$\therefore$ We get $y + \log y = e^x \cos^2 x = 2$

Video Solution:

Q7 Text Solution:

$\tan y \frac{dy}{dx} = \sin(x+y) + \sin(x-y)$

$\tan \frac{dy}{dx} = 2\sin[x]\cos[y]$

$\int \sec y \ \tan \ y \ dy = 2\int \sin x \ dx$

$\sec y \ = -2\cos \ x + c$

$2\cos x + \sec y = c$

Video Solution:

Q8 Text Solution:

Consider $\frac{dy}{dx} = \sin(x+y) + \cos(x+y)$

Put x + y = t

y = t - x

$\frac{dy}{dx} = \frac{dt}{dx} - 1$

$\frac{dt}{dx} - 1 = \sin t + \cos t$

$\frac{dt}{dx} = 2\sin\frac{t}{2}\cos\frac{t}{2} + 2\cos^2\frac{t}{2}$

$\frac{dt}{dx} = 2\cos^2\frac{t}{2}\left[\tan\frac{t}{2} + 1\right]$

$\therefore \frac{1}{2}\int \frac{\sec^2 t/2}{1+\tan t/2} dt = \int dx$

$= \log\left[1 + \tan\left(\frac{x+y}{2}\right)\right] = x + c$

Video Solution:

Q9 Text Solution:

$(x+y+3)\frac{dy}{dx} = 1$

Put $x + y + 3 = 1$

$\frac{dy}{dx} = \frac{dt}{dx} - 1$

$t\left[\frac{dt}{dx} - 1\right] = 1$

$\frac{dt}{dx} = \frac{1}{t} + 1$

$\int \frac{t}{t+1} dt = \int dx$

$\Rightarrow \int 1 - \frac{1}{t+1} dt = \int dx$

$\Rightarrow t - \log(t+1) + x + c$

$\Rightarrow y + 3 = \log(t+1)c$

$c_1 e^{(y+3)} = x + y + 4$

$x + y + 4 = c_2 e^y$

Video Solution:

Q10 Text Solution:

$(x+y)^2\frac{dy}{dx} = a^2$

Put $x + y = t$

$\frac{dy}{dx} = \frac{dt}{dx} = 1$

$t^2\left[\frac{dt}{dx} - 1\right] = a^2$

$\frac{dt}{dx} = \frac{a^2}{t^2} + 1$

$\int \frac{t^2}{a^2+t^2} dt = \int dx$

$\int 1 - \frac{a^2}{a^2+t^2} dt = \int dx$

$t - a\tan^{-1}\left(\frac{t}{a}\right) = x + c$

$y = a\tan^{-1}\left(\frac{x+y}{a}\right) + c$

Video Solution:

Q11 Text Solution:

Given $\frac{dy}{dx} = 2^{y-x} \Rightarrow \frac{dy}{2^y} = \frac{dx}{2^x}$

Integrating on both sides, $\int \frac{dy}{2^y} = \int \frac{dx}{2^x}$

$-2^{-\mathrm{y}} \log 2 = -2^{-\mathrm{x}} \log 2 + \mathrm{c}_1$

$\frac{\log 2}{2^{\mathrm{x}}} - \frac{\log 2}{2^{\mathrm{y}}} = c_1 \Rightarrow \frac{1}{2^{\mathrm{x}}} - \frac{1}{2^{\mathrm{y}}} = \frac{c_1}{\log 2} = c$

Video Solution:

Q12 Text Solution:

Solving the given DE by Variable-Seperable Method:

$\int \frac{dy}{y} = -\int \tan x \, dx$

$\log y = -\left(-\log\left|\cos x\right|\right) + \log C$

$\log y = \log\left|\cos x\right| + \log C$

$y = C \cos x$

$\Rightarrow y = a \cos x$

Video Solution:

Q13 Text Solution:

$x\left(e^{2y} - 1\right)dy + \left(x^2 - 1\right)e^y dx = 0$

$\Rightarrow \int \frac{e^{2y}-1}{e^y} dy = \int \frac{1-x^2}{x} dx$

$\Rightarrow e^y + e^{-y} = \log x - \frac{x^2}{2} + c$

Video Solution:

Q14 Text Solution:

Solving the given DE by Variable Seperable method:

$\int \frac{dy}{1-y} = \int \frac{dx}{1-x}$

On solving we get;

$\frac{1-x}{1-y} = c$

Video Solution:

Q15 Text Solution:

The given equation

$\left(x^2 - yx^2\right)\frac{dy}{dx} + y^2 + xy^2 = 0$

$\Rightarrow \frac{1-y}{y^2} dy + \frac{1+x}{x^2} dx = 0$

$\Rightarrow \left(\frac{1}{y^2} - \frac{1}{y}\right)dy + \left(\frac{1}{x^2} + \frac{1}{x}\right)dx = 0$

On integrating, we get the required solution

$\log\left(\frac{\mathrm{x}}{\mathrm{y}}\right) = \frac{1}{\mathrm{x}} + \frac{1}{\mathrm{y}} + \mathrm{c}$

Video Solution:

Q16 Text Solution:

Put $y=vx \Rightarrow v+x\frac{dv}{dx}=v+\tan v$

$\Rightarrow \cot v\ dv=\frac{dx}{x}$

Integrating both sides we get, log sin v = log x + log c $\Rightarrow$ $\sin\frac{y}{x}=cx$

Video Solution:

Q17 Text Solution:

$\frac{dy}{dx}=\frac{-y^2}{x^2-xy}$

$\frac{dx}{dy}=\frac{x^2-xy}{-y^2}$

$\frac{dx}{dy}=\frac{x}{y}-\left(\frac{x}{y}\right)^2$

on solving

$y=x\left(c+\log|y|\right)$

Video Solution:

Q18 Text Solution:

The solution of given differential equation is:

$\tan\left(\frac{y}{2x}\right)-cx=0$

Video Solution:

Q19 Text Solution:

We have, $\frac{dy}{dx}=\frac{2\sqrt{y^2-x^2}+y}{x}$

Which is a homogeneous differential equation

Putting $y=vx \Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$, we get

$v+x\frac{dv}{dx}=\frac{2\sqrt{v^2x^2-x^2}+vx}{x}=2\sqrt{v^2-1}+v$

$\Rightarrow x\frac{dv}{dx}=2\sqrt{v^2-1} \Rightarrow \int\frac{dv}{\sqrt{v^2-1}}=2\int\frac{dx}{x}$

$\Rightarrow \log\left|v+\sqrt{v^2-1}\right|=2\log|x|+\log c$

$\Rightarrow \log\left|v+\sqrt{v^2-1}\right|=\log\left|cx^2\right|$

$\Rightarrow \frac{y}{x}+\sqrt{\frac{y^2-x^2}{x^2}}=cx^2 \Rightarrow y+\sqrt{y^2-x^2}$
$=cx^3$

Video Solution:

Q20 Text Solution:

By substituting $x = vy$ the given differential equation can be transformed as

$y\frac{dv}{dy}+\left(\frac{v+e^v}{1+e^v}\right)=0$

Video Solution:

Q21 Text Solution:

The Integrating factor of Linear differential equation is given by:

$e^{\int P\,dx}=e^{\int(-cotx)dx}=e^{\log|\cos ecx|}=\cos ecx$

Video Solution:

Q22 Text Solution:

Required Integration factor is

$e^{\int sec^2x\,dx}=e^{\tan x}$

Video Solution:

Q23 Text Solution:

$x\frac{dy}{dx}+y=x^2+3x+2$

$\Rightarrow \frac{dy}{dx}+\frac{y}{x}=x+3+\frac{2}{x}$

Here $P=\frac{1}{x}, Q=x+3+\frac{2}{x}$, therefore I.F.

$=e^{\int \frac{1}{x} dx}=x$

$\therefore yx=\int\left(x+3+\frac{2}{x}\right)\cdot x\ dx+c$

$\Rightarrow \quad xy=\frac{x^3}{3}+\frac{3x^2}{3}+2x+c$

Video Solution:

Q24 Text Solution:

$\frac{dy}{dx}=2\cos x-y\cot x$

$\Rightarrow \frac{dy}{dx}+y\cot x=2\cos x$

$I.F.=e^{\int \cot x\ dx}=\sin x$

$y\cdot\sin x=\int 2\cos x\cdot\sin x+c$

$y\sin x=-\frac{1}{2}\cos 2x+c$

at $x=\frac{\pi}{2}$ and $y=1$, we get $c=\frac{1}{2}$

$\therefore \quad y\sin x+\frac{\cos 2x}{2}=\frac{1}{2}$

Video Solution:

Q25 Text Solution:

Given : $\left(1-y^2\right)+\left(x-e^{\tan^{-1}y}\right)\frac{dy}{dx}=0$

The above D.E. can be rewritten in the form

$\frac{dx}{dy}+\frac{1}{1+y^2}x=\frac{e^{\tan^{-1}y}}{1+y^2}$

Which is of the form : $\frac{dx}{dy}+Px=Q$

Where $P=\frac{1}{1+y^2}$ and $Q=\frac{e^{\tan^{-1}y}}{1+y^2}$

Now, find the integrating factor and proceed to find required solution.

Video Solution:

Q26 Text Solution:

$Given,\ \frac{dA}{dt}=-2sqcm/s$

$Now;\ A=x^2\Rightarrow \frac{dA}{dt}=2x.\frac{dx}{dt}=\frac{-1}{16}$

$Now,\ P=4x\Rightarrow \frac{dP}{dt}=4.\frac{dx}{dt}=\frac{-1}{4}cm/sec$

Video Solution:

Q27 Text Solution:

Let at any instant of time t, the length of rectangle be x and the breadth be y and letA be the corresponding area, then A = xy

Differentiating (i) w.r.t. t, we get

$$\frac{dA}{dt}=\frac{d}{dt}(xy)=$$

$\because \frac{dx}{dt}=$

When x = 12 cm, y = 8 cm

$\frac{dA}{dt}=(-3\times 12+3.5\times 8)\mathrm{cm}^2/\mathrm{sec}=$

$-8\ \mathrm{cm}^2/\mathrm{sec}$

Hence, the area of the rectangle is decreasing at the rate of 8 cm^2/sec

Video Solution:

Q28 Text Solution:

$V=5x-\frac{x^2}{6} \Rightarrow \frac{dV}{dt}=5\frac{dx}{dt}-\frac{x}{3}\cdot\frac{dx}{dt}$

$\Rightarrow \frac{dx}{dt}=\frac{\frac{dV}{dt}}{\left(5-\frac{x}{3}\right)} \Rightarrow \left(\frac{dx}{dt}\right)_{x=2}=\frac{5}{5-\frac{2}{3}}$

$=\frac{15}{13}\ \mathrm{cm/sec.}$

Video Solution:

Q29 Text Solution:

According to the question

$\frac{d}{dt}\left(\tan t\right)=2\frac{d}{dt}\left(\log\left(\cos t\right)\right)$

$\Rightarrow sec^2 t=-2\tan t$

$\Rightarrow 1+\tan^2 t+2\tan t=0$

$\Rightarrow (1+\tan t)^2=0$

$\Rightarrow \tan t=-1$

$\Rightarrow t=\frac{-\pi}{4}$

Video Solution:

Q30 Text Solution:

Let ‘x’ be the side and ‘A’ be the area of an equilateral triangle.

$A=\frac{\sqrt{3}}{4}x^2,$

Given : $\frac{dx}{dt}=4\ \mathrm{cm/sec.}$

$\frac{dA}{dt}=\frac{\sqrt{3}}{2}\cdot x\cdot\frac{dx}{dt}$

When $x=14$ cm and $\frac{dx}{dt}=4\ \mathrm{cm/sec}$

$\frac{dA}{dt}=28\sqrt{3}\ \mathrm{cm}^2/\mathrm{sec}$

Video Solution: