ULTIMATE KCET

CRASH COURSE 2026

Mathematics | Lecture - 01

Combinations

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Topics *to be covered*

1. MCQ Discussion
2.
3.
4.

① $^nC_r = \dfrac{n!}{(n-r)!\, r!}$ $\quad 0 \le r \le n$

or

$$^nC_r = \frac{n}{r}\,\frac{n-1}{r-1}\,\frac{n-2}{r-2}\cdots$$

$$^{10}C_4 = \frac{10}{\cancel{4}}\,\frac{\cancel{9}^{\,3}}{\cancel{3}_{\,1}}\,\frac{\cancel{8}}{\cancel{2}}\,\frac{7}{1} = 30 \times 7 = 210$$

(1) $\dfrac{^nC_r}{^nC_{r-1}} = \dfrac{n-r+1}{r}$

(2) $^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

(3) $^nC_r = {}^nC_{n-r}$

Exi- $^{100}C_{97} = {}^{100}C_3$

(4) if $^nC_r = {}^nC_p$

Then (1) $r+p=n$ (2) $r=p$

(5) $^nC_0 = 1$

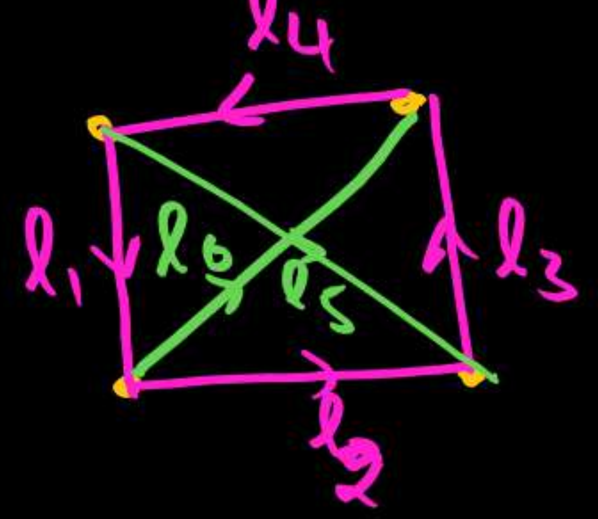
(6) $^nC_n = 1$

(*) If there are 'n' non-collinear Vertices:-

Then

(1) no of lines can be constructed $= {}^nC_2$

(2) no of Triangles that can be constructed $= {}^nC_3$

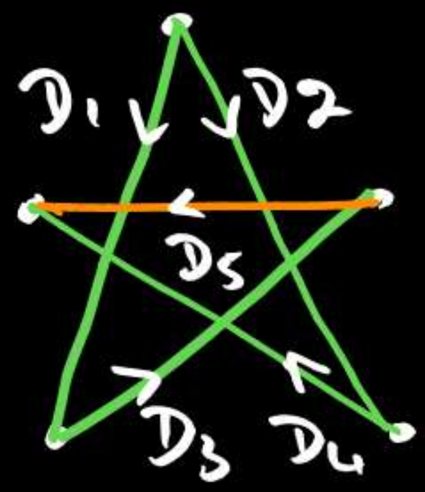


(3) no of Quadrilaterals that can be constructed $= {}^nC_4$

(4) no of Diagonals $= {}^nC_2 - n$

no of lines $= {}^4C_2 = 6$

no of Diagonals $= {}^4C_2 - 4 = 6 - 4$
$= 2$



no of Diagonals

$$= {}^{5}C_{2} - 5$$

$$= \left(\frac{5}{2} \times 4\right) - 5$$

$$= 10 - 5$$

$$= 5$$

(*) If there are n persons in a room, no. of handshakes

$= {}^{n}C_{2}$

(*) If there are n teams in a tournament, no. of matches played b/w them $= {}^{n}C_{2}$

$^{5}C_{2} = 10$

$$(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + {}^nC_3 x^3 + \dots + {}^nC_n x^n \rightarrow (1)$$

Put $x=1$

$$2^n = {}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + \dots + {}^nC_n$$

<u>Ex:-</u> Find ${}^5C_0 + {}^5C_1 + {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5 = 2^n = 2^5 = 32$

$\Downarrow$

$n = 5$

(2) Find ${}^6C_1 + {}^6C_2 + {}^6C_3 + {}^6C_4 + {}^6C_5$

Soln:

WKT

$${}^6C_0 + {}^6C_1 + {}^6C_2 + {}^6C_3 + {}^6C_4 + {}^6C_5 + {}^6C_6 = 2^6$$

$${}^6C_0 = 1, \quad {}^6C_6 = 1$$

$${}^6C_1 + {}^6C_2 + \dots + {}^6C_5 = 2^6 - 2$$

$$= 64 - 2$$

$$= \underline{62}$$

WKT $(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + {}^nC_3 x^3 + \dots + {}^nC_n x^n$

Put $x = -1$

$$0^n = {}^nC_0 - {}^nC_1 + {}^nC_2 - {}^nC_3 + \dots + (-1)^n\, {}^nC_n$$

#Q. The value of $^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + \dots + {}^{10}C_9$ is

(A) $2^{10} - 1$

(B) 2^{10}

(C) 2^{11}

(D) $2^{10} - 2$

$$\Downarrow$$

$$2^{10} - {}^{10}C_0 - {}^{10}C_{10}$$

$$2^{10} - 1 - 1$$

$$\underline{2^{10} - 2}$$

#Q

$${}^{10}C_6 + {}^{10}C_5 + {}^{11}C_5 + {}^{12}C_5 + {}^{13}C_5$$

Soln:-

$${}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r$$

$${}^{10}C_6 + {}^{10}C_5$$

$$\Downarrow$$

$${}^{11}C_6 + {}^{11}C_5$$

$$\Downarrow$$

$${}^{12}C_6 + {}^{12}C_5$$

$$\Downarrow$$

$${}^{13}C_6 + {}^{13}C_5 = {}^{14}C_6 \text{ or } {}^{14}C_8$$

$${}^{n}C_r = {}^{n}C_{n-r}$$

#Q The value of ${}^{16}C_9 + {}^{16}C_{10} - {}^{16}C_6 - {}^{16}C_7$ is

(A) 1

(B) ${}^{17}C_{10}$

(C) ${}^{17}C_3$

(D) 0

$$\Downarrow$$

$$({}^{16}C_{10} + {}^{16}C_9) - ({}^{16}C_7 + {}^{16}C_6)$$

$${}^{17}C_{10} - {}^{17}C_7$$

$${}^{17}C_7 - {}^{17}C_7$$

$$= 0$$

$${}^{n}C_r = {}^{n}C_{n-r}$$

$$\Downarrow$$

$${}^{17}C_{10} = {}^{17}C_7$$

#Q The value of $^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 + {}^{46}C_3 + {}^{45}C_3 + {}^{45}C_4$

(A) $^{50}C_4$

(B) $^{50}C_3$

(C) $^{50}C_2$

(D) $^{50}C_1$

Soln:

$^{45}C_4 + {}^{45}C_3$

$\Downarrow$

$^{46}C_4 + {}^{46}C_3$

$\Downarrow$

$^{47}C_4 + {}^{47}C_3$

$\Downarrow$

$^{48}C_4 + {}^{48}C_3$

$\Downarrow$

$^{49}C_4 + {}^{49}C_3 = {}^{50}C_4$ or $^{50}C_{46}$

$${}^{n}C_r = {}^{n}C_{n-r}$$

#Q The sum ${}^{20}C_0 + {}^{20}C_1 + {}^{20}C_2 + \dots + {}^{20}C_{10} =$

(A) $2^{20} - \frac{20!}{(10!)^2}$

(B) $2^{19} + \frac{1}{2}\frac{20!}{(10!)^2}$

(C) $2^{19} + {}^{20}C_{10}$

(D) $2^{19} - {}^{20}C_{10}$

WKT

$$2^{20} = {}^{20}C_0 + {}^{20}C_1 + {}^{20}C_2 + \dots + {}^{20}C_{10} +$$

$$+ {}^{20}C_{11} + {}^{20}C_{12} + {}^{20}C_{13} + \dots + {}^{20}C_{20}$$

$$= {}^{20}C_0 + {}^{20}C_1 + {}^{20}C_2 + \dots + {}^{20}C_9 + {}^{20}C_{10} +$$

$$+ {}^{20}C_9 + {}^{20}C_8 + {}^{20}C_7 + \dots + {}^{20}C_1 + {}^{20}C_0$$

$$2^{20} = 2\left[{}^{20}C_0 + {}^{20}C_1 + {}^{20}C_2 + \dots + {}^{20}C_9\right] + {}^{20}C_{10}$$

$$2^{20} = 2\left[{}^{20}C_0 + {}^{20}C_1 + {}^{20}C_2 + \cdots + {}^{20}C_9\right] + \underbrace{{}^{20}C_{10} + {}^{20}C_{10}} - {}^{20}C_{10}$$

$$2^{20} = 2\left[{}^{20}C_0 + {}^{20}C_1 + {}^{20}C_2 + \cdots + {}^{20}C_9 + {}^{20}C_{10}\right] - {}^{20}C_{10}$$

$$\therefore 2\left[{}^{20}C_0 + {}^{20}C_1 + {}^{20}C_2 + \cdots + {}^{20}C_9 + {}^{20}C_{10}\right] = 2^{20} + {}^{20}C_{10}$$

$\div$ by 2

$${}^{20}C_0 + {}^{20}C_1 + {}^{20}C_2 + \cdots + {}^{20}C_9 + {}^{20}C_{10} = 2^{19} + \frac{{}^{20}C_{10}}{2}$$

$$\Downarrow$$

$$2^{19} + \frac{1}{2}\frac{20!}{(10!)^2}$$

QUESTION

#Q. If ${}^nC_r + {}^nC_{r+1} = {}^{(n+1)}C_x$ then $x =$

(A) r

(B) $r - 1$

(C) n

(D) $r + 1$

${}^nC_{r+1} + {}^nC_r = {}^{(n+1)}C_x$

${}^{(n+1)}C_{r+1} = {}^{(n+1)}C_x$

$r+1 = x$

$x = r+1$

${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

$P = r+1$

${}^nC_{r+1} + {}^nC_r = {}^{n+1}C_{r+1}$

${}^nC_r = {}^nC_r$

(1) $r + r = n$ (2) $r = r$

QUESTION

$2x = x + x$

#Q. ${}^{n}C_{r+1} + {}^{n}C_{r-1} + \underline{2\,{}^{n}C_{r}} =$

$\left({}^{n}C_{r+1} + {}^{n}C_{r}\right) + \left({}^{n}C_{r} + {}^{n}C_{r-1}\right)$

A) ${}^{n+2}C_{r}$

B) ${}^{n+2}C_{r+1}$ ✓

$\left({}^{n+1}C_{r+1}\right) + \left({}^{n+1}C_{r}\right)$

$\Downarrow$

$\underline{{}^{n+2}C_{r+1}}$

C) ${}^{n+1}C_{r}$

D) ${}^{n+1}C_{r+1}$

QUESTION

#Q. If ${}^{n}C_3 + {}^{n}C_4 > {}^{n+1}C_3$, then

A) $n > 6$

B) $n > 7$

C) $n < 6$

D) $n < 4$

$${}^{n}C_4 + {}^{n}C_3 > {}^{n+1}C_3$$

$${}^{n+1}C_4 > {}^{n+1}C_3$$

$$\frac{{}^{n+1}C_4}{{}^{n+1}C_3} > 1$$

$$\frac{(n+1)-4+1}{4} > 1$$

$$\frac{n-3+1}{4} > 1$$

$$n-2 > 4$$

$$n > 6$$

$$\frac{{}^{m}C_{r}}{{}^{m}C_{r-1}} = \frac{m-r+1}{r}$$

QUESTION

#Q. If a polygon has 44 diagonals, then the number of its sides are

Let n be no of sides of a Polygon

(A) 11

(B) 7

(C) 8

(D) 9

$\Rightarrow {}^nC_2 - n = 44$

$\frac{n(n-1)}{2} - n = 44$

$\frac{n^2 - n - 2n}{2} = 44$

$n^2 - 3n = 88$

$n^2 - 3n - 88 = 0$

-88

$-11 \quad +8$

$(n-11)(n+8) = 0$

$n = 11 \mid n = -8$

Not Possible

QUESTION

HW

#Q. In a polygon the number of diagonals is 54. The number of sides of the polygon is

(A) 10

(B) 12

(C) 9

(D) None of these

QUESTION

#Q. The number of triangles that can be formed by 5 points in a line and 3 points on a parallel line is

(A) $^8C_3$

(B) $^8C_3 - {}^5C_3$

(C) $^8C_3 - {}^5C_3 - 1$

(D) 55

Total no of vertices $= 8$

no of triangles $= {}^8C_3 - {}^5C_3 - {}^3C_3$

$= {}^8C_3 - {}^5C_3 - 1$

QUESTION

#Q. If ${}^nC_r = 84$, ${}^nC_{r-1} = 36$ and ${}^nC_{r+1} = 126$ then find the value of n

(A) 9

(B) 11

(C) 10

(D) 12

$$\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{\cancel{84}^{7}}{\cancel{36}_{3}}$$

$$\frac{n-r+1}{r} = \frac{7}{3}$$

$$3n-3r+3 = 7r$$

$$10r-3n = 3 \rightarrow (1)$$

$$\frac{{}^nC_{r+1}}{{}^nC_r} = \frac{\cancel{126}^{21}}{\cancel{84}_{14}}$$

$$\frac{n-(r+1)+1}{r+1} = \frac{3}{2}$$

$$2n-2r-2+2 = 3r+3$$

$$2n-2r = 3r+3$$

$$5r-2n = -3 \rightarrow (2)$$

$$\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}$$

$$① \times 1 \Rightarrow 10x - 3n = 3$$

$$② \times 2 \Rightarrow 10x - 4n = -6$$

$$\quad (-) \quad (+) \quad (+)$$

$$n = 9$$

$$② \Rightarrow 5x - 2(9) = -3$$

$$5x = -3 + 18 = 15$$

$$x = 3$$

$${}^{n}C_{x} = {}^{9}C_{3} = \frac{\cancel{9}^{3}}{\cancel{3}} \times \frac{\cancel{8}^{4}}{\cancel{2}_{1}} \times 7$$

$$= 12 \times 7$$

$$= \underline{84}$$

4G & 7B

QUESTION

#Q. A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if the team has atleast one boy and one girl

(1G & 4B) or (2G & 3B) or (3G & 2B) or (4G & 1B)

(A) 371

$({}^4C_1 \times {}^7C_4) + ({}^4C_2 \times {}^7C_3) + ({}^4C_3 \times {}^7C_2) + ({}^4C_4 \times {}^7C_1)$

(B) 512

$(4 \times {}^7C_3) + (6 \times \frac{7 \times 6 \times 5}{3 \times 2}) + (4 \times \frac{7}{2} \times 6) + (1 \times 7)$

(C) 441 ✓

$(4 \times 35) + (6 \times 35) + (2 \times 42) + 7$

(D) None of these

$140 + 210 + 84 + 7$

$= 441$

QUESTION

$$\begin{array}{l} 2W \\ 3B \\ 4R \end{array}$$

#Q. A box contains 2 white balls, 3 black balls and 4 red balls. In how many ways can 3 balls be drawn from the box, if at least one black ball is to be included in the draw?

(A) 64

(B) 24

(C) 3

(D) 12

$6 =$ 2white + 4Red

$(1B \; \& \; {}^6C_2)$ or $(2B \; \& \; {}^6C_1)$ or $(3B \; \& \; {}^6C_0)$

$({}^3C_1 \times {}^6C_2) + ({}^3C_2 \times {}^6C_1) + ({}^3C_3 \times {}^6C_0)$

$(3 \times 15) + (3 \times 6) + (1)$

$45 + 18 + 1$

$\underline{64}$

QUESTION

#Q. If ${}^nC_2 + {}^nC_3 = {}^6C_3$ and ${}^nC_x = {}^nC_3, x \neq 3$, then the value of x is equal to

(A) 5

(B) 4

(C) 2

(D) 6

${}^nC_2 + {}^nC_3 = {}^6C_3$

${}^nC_3 + {}^nC_2 = {}^6C_3$

${}^{n+1}C_3 = {}^6C_3$

$n+1 = 6$

$n = 5$

Here $n = 5$

$\therefore$ we get

${}^5C_x = {}^5C_3$

$x + 3 = 5$

$x = 2$

${}^nC_x = {}^nC_r$

(1) $x + r = n$ ✓

(2) $x = n$

QUESTION

#Q. The number of diagonals in a hexagon is

6 vertices

$\Rightarrow n = 6$

A 8

B 9

C 10

D 11

$^6C_2 - 6$

$\left(\frac{6}{2} \times 5\right) - 6$

$15 - 6$

$= \underline{9}$

QUESTION

#Q. $({}^7C_0 + {}^7C_1) + ({}^7C_2 + {}^7C_3) + \ldots + ({}^7C_6 + {}^7C_7) =$

$\Downarrow$

2^7

A $2^8 - 2$

B $2^7 - 1$

C 2^7

D $2^8 - 1$

QUESTION

#Q. If ${}^{x}C_{15} = {}^{x}C_{14}$, then the value of ${}^{x}C_{29}$ is equal to

A) 6

B) 1

C) 8

D) 9

$$^{x}C_{15} = {}^{x}C_{14}$$

$$\Rightarrow 15 + 14 = x$$

$$x = 29$$

$$\therefore {}^{x}C_{29} = {}^{29}C_{29}$$

$$= \underline{1}$$

QUESTION

#Q. The value of ${}^{10}C_1 - {}^{10}C_2 + {}^{10}C_3 - {}^{10}C_4 + {}^{10}C_5 - {}^{10}C_6 + {}^{10}C_7 - {}^{10}C_8 + {}^{10}C_9$ is

A 0

B ✓ 2

C 10

D 252

WKT

$$0^{10} = {}^{10}C_0 - \overbrace{{}^{10}C_1 + {}^{10}C_2 - {}^{10}C_3 + {}^{10}C_4 - {}^{10}C_5 + {}^{10}C_6 - {}^{10}C_7 + {}^{10}C_8 - {}^{10}C_9}^{} + {}^{10}C_{10}$$

$$0 = \underset{1}{{}^{10}C_0} + \underset{1}{{}^{10}C_{10}} - \left[{}^{10}C_1 - {}^{10}C_2 + {}^{10}C_3 - {}^{10}C_4 + {}^{10}C_5 - {}^{10}C_6 + {}^{10}C_7 - {}^{10}C_8 + {}^{10}C_9\right]$$

$$\therefore {}^{10}C_1 - {}^{10}C_2 + {}^{10}C_3 - \dots + {}^{10}C_9 = 1 + 1$$

$$= \underline{2}$$

QUESTION

#Q. The value of n if ${}^{2n}C_2 : {}^{n}C_2 = 9 : 2$

A 5

B 4

C 3

D 2

$$\frac{{}^{2n}C_2}{{}^{n}C_2} = \frac{9}{2}$$

$$\frac{\frac{2\cancel{n}}{\cancel{2}} \times \frac{2n-1}{1}}{\frac{\cancel{n}}{\cancel{2}} \times n-1} = \frac{9}{2}$$

$$\frac{2(2n-1)}{n-1} = \frac{9}{2}$$

$$(4n-2)2 = 9n-9$$

$$8n-4 = 9n-9$$

$$n = 5$$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