ULTIMATE KCET

CRASH COURSE 2026

Mathematics | Lecture – 02

Methods of Differentiation

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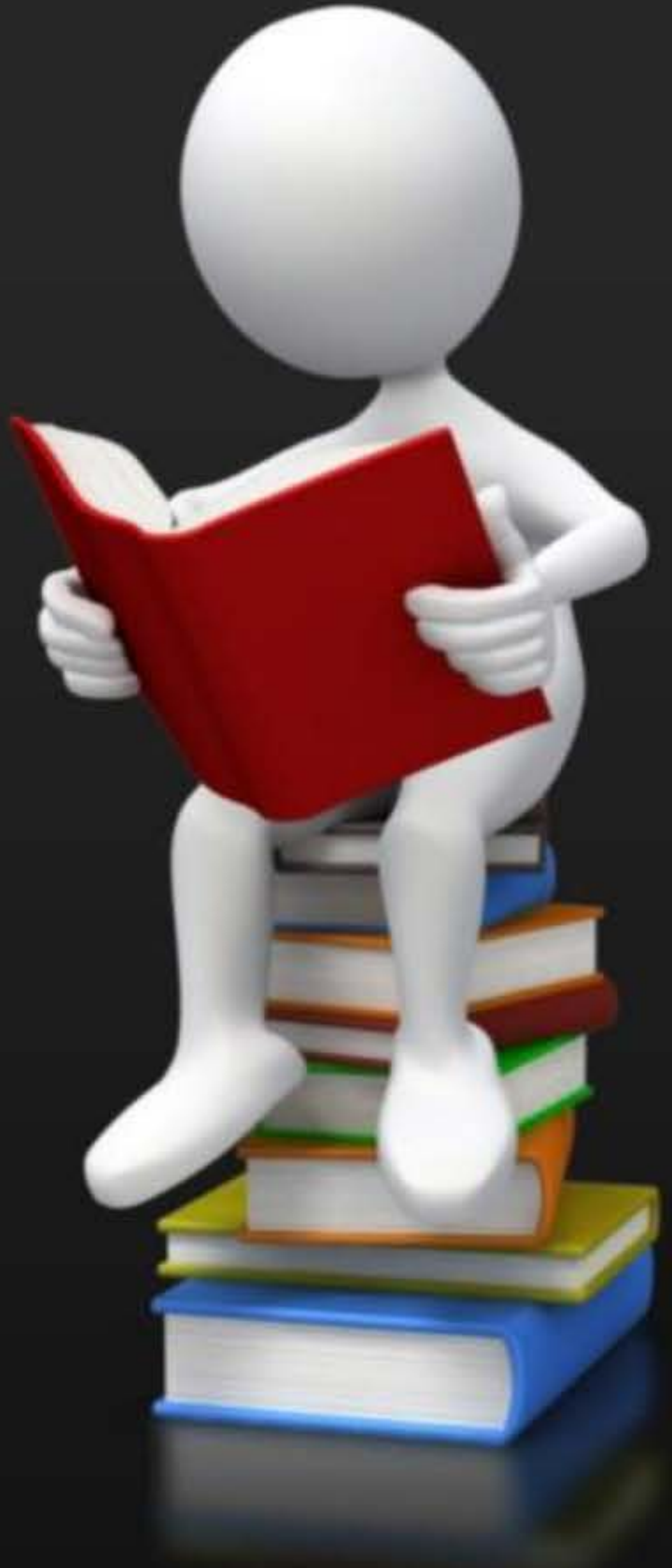
Recap of previous lecture

1. Differention ITF
2.
3.
4.

Topics to be covered

1. Differentiation of Parametric func
2. Differentiation of Implicit func
3.
4.



QUESTION

#Q. If $y = \tan^{-1}\left(\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right)$ then y' is

$\div$ by $b\cos x$

A 1

B -1

C $\frac{1}{1+x^2}$

D $-\frac{1}{1+x^2}$

$$y = \tan^{-1}\left[\frac{\frac{a}{b} - \tan x}{1 + \frac{a}{b}\tan x}\right]$$

$$y = \tan^{-1}\frac{a}{b} - \tan^{-1}(\tan x)$$

$$y = \tan^{-1}\frac{a}{b} - x$$

$$\frac{dy}{dx} = 0 - 1 = -1$$

QUESTION

#Q. If $y = \tan^{-1}\frac{x}{\sqrt{1+x^2}} + \tan^{-1}\frac{\sqrt{1+x^2}}{x}$ then $\frac{dy}{dx}$ is

A 1

B -1

C [✓ 0]

D $\frac{1}{1+x^2}$

$$y = \tan^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right) + \cot^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$$

$$y = \frac{\pi}{2}$$

$$\frac{dy}{dx} = 0$$

QUESTION

$\log\frac{A}{B} = \log A - \log B$ | $\log AB = \log A + \log B$

#Q. If $y = \tan^{-1}\left[\frac{\log_e\left(\frac{e}{x^2}\right)}{\log_e(ex^2)}\right] + \tan^{-1}\left(\frac{5+2\log_e x}{1-10\log_e x}\right)$ then y'' is

A $\frac{1}{1+x^2}$

B $\frac{2}{1+x^2}$

C [✓ 0]

D $\tan^{-1}(4)$

$$y = \tan^{-1}\left[\frac{\log_e e - \log x^2}{\log_e e + \log x^2}\right] + \tan^{-1}\left[\frac{5+2\log x}{1-5(2)\log_e x}\right]$$

$$y = \tan^{-1}\left[\frac{1-2\log x}{1+2\log x}\right] + \tan^{-1}5 + \tan^{-1}(2\log_e x)$$

$$y = \tan^{-1}(1) - \tan^{-1}(2\cancel{\log_e} x) + \tan^{-1}5 + \cancel{\tan^{-1}}(2\log x)$$

$$\frac{dy}{dx} = 0 + 0 = \underline{0}$$

QUESTION

#Q. If $y = \tan^{-1}\left[\frac{\sqrt{1+x^3}+\sqrt{1-x^3}}{\sqrt{1+x^3}-\sqrt{1-x^3}}\right]$ then y' at $x = 0$ is

- A [✓ 0]
- B $-\frac{3}{2}$
- C $\frac{1}{2}$
- D $\frac{3}{2}$

$\theta = \frac{1}{2}\cos x^3$

Put $x^3 = \cos 2\theta$

$\sqrt{1+x^3} = \sqrt{1+\cos 2\theta} = \sqrt{2\cos^2\theta} = \sqrt{2}\cos\theta$

$\sqrt{1-x^3} = \sqrt{1-\cos 2\theta} = \sqrt{2\sin^2\theta} = \sqrt{2}\sin\theta$

$$y = \tan^{-1}\left[\frac{\cancel{\sqrt{2}}(\cos\theta+\sin\theta)}{\cancel{\sqrt{2}}(\cos\theta-\sin\theta)}\right]$$

÷ by $\cos\theta$

$$y = \tan^{-1}\left(\frac{1+\tan\theta}{1-\tan\theta}\right)$$

$$y = \tan^{-1}\left(\tan\left(\frac{\pi}{4}+\theta\right)\right]$$

$$y = \frac{\pi}{4}+\theta$$

$$y = \frac{\pi}{4}+\frac{1}{2}\cos x^3$$

$$\frac{dy}{dx} = \frac{-1}{2\sqrt{1-x^3}}(3x^2)$$

At $x = 0$

$$\frac{dy}{dx} = 0$$

QUESTION

#Q. If $u = cos(\log_e x), v = \log_e cos\, x$ then

A $\frac{du}{dx} = \frac{dv}{dx}$

B $\frac{du}{dx} = \log_e x \cdot \frac{dv}{dx}$

C $cos\, x \cdot \frac{du}{dx} = \frac{dv}{dx}$

D $\frac{du}{dv} = \frac{1}{x} \cdot cot\, x \cdot sin(log\, x)$

$$\frac{du}{dx} = \frac{-\sin(\log x)}{x} \quad \Big| \quad \frac{dv}{dx} = -\frac{1}{\cos x}\sin x = -\tan x$$

$$\frac{du}{dv} = \frac{du/dx}{dv/dx} = \frac{\frac{-\sin(\log x)}{x}}{-\tan x} = +\underline{\frac{1}{x}\cot x\cdot\sin(\log x)}$$

if $u = 3\cos^{-1}\left(\frac{2x}{1+x^2}\right)$ & $v = 5\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ Find $\frac{du}{dv}$

Soln :-

$u = 3\left[\frac{\pi}{2} - \sin^{-1}\left(\frac{2x}{1+x^2}\right)\right]$

$u = \frac{3\pi}{2} - 3(2\tan^{-1}x)$

$u = \frac{3\pi}{2} - 6\tan^{-1}x$

$v = 5(2\tan^{-1}x)$

$\frac{v}{10} = \tan^{-1}x$

$u = \frac{3\pi}{2} - 6\left(\frac{v}{10}\right)$

Diff wrt v

$\frac{du}{dv} = 0 - \frac{6}{10}(1) = -\frac{3}{5}$

QUESTION

$\Rightarrow \sec^2\theta + \cos^2\theta = x^2+2$

$\Rightarrow \sec^{2n}\theta + \cos^{2n}\theta = y^2+2$

#Q. If $x = \sec\theta - \cos\theta$, $y = \sec^n\theta - \cos^n\theta$ then $\left(\frac{dy}{dx}\right)^2 (x^2+4)$ is

A $y^2 + 4$

B n^2y^2

C $n^2(4 - y^2)$

D $n^2(y^2 + 4)$

$$\frac{dx}{d\theta} = \sec\theta\tan\theta + \sin\theta$$

$$= \sec\theta\frac{\sin\theta}{\cos\theta} + \sin\theta$$

$$= \frac{\sin\theta}{\cos\theta}(\sec\theta + \cos\theta)$$

$$\frac{dy}{d\theta} = n\sec^n\theta\tan\theta + n\cos^{n-1}\theta\sin\theta$$

$$= n\sin\theta\left[\frac{\sec^n\theta}{\cos\theta} + \cos^{n-1}\theta\right]$$

$$= n\frac{\sin\theta}{\cos\theta}(\sec^n\theta + \cos^n\theta]$$

$$\frac{dy}{dx} = \frac{n(\sec^n\theta + \cos^n\theta)}{\sec\theta + \cos\theta}$$

$$\left(\frac{dy}{dx}\right)^2 = \frac{n^2[\sec^{2n}\theta + \cos^{2n}\theta + 2(1)]}{\sec^2\theta + \cos^2\theta + 2} = \frac{n^2(y^2+2+2)}{x^2+2+2}$$

$$\left(\frac{dy}{dx}\right)^2 = \frac{n^2(y^2+4)}{x^2+4}$$

$$(x^2+4)\left(\frac{dy}{dx}\right)^2 = n^2(y^2+4)$$

$$x = \sec\theta - \cos\theta$$

$$x^2 = (\sec\theta - \cos\theta)^2$$

$$x^2 = \sec^2\theta + \cos^2\theta - 2\sec\theta\cos\theta$$

$$x^2 = \sec^2\theta + \cos^2\theta - 2$$

$$x^2 + 2 = \sec^2\theta + \cos^2\theta$$

$$y = \sec^n\theta - \cos^n\theta$$

$$\Downarrow$$

$$y^2 + 2 = \sec^{2n}\theta + \cos^{2n}\theta$$

Diff wrt θ

$$\frac{dx}{d\theta} = \sec\theta\tan\theta + \sin\theta$$

$$= \tan\theta\left[\sec\theta + \frac{\sin\theta}{\tan\theta}\right)$$

$$= \tan\theta\left[\sec\theta + \cos\theta\right]$$

Diff wrt θ

$$\frac{dy}{d\theta} = n\sec^n\theta\tan\theta + n\cos^{n-1}\theta\sin\theta$$

$$= n\tan\theta\left[\sec^n\theta + \cos^n\theta\right]$$

$$\frac{dy}{dx} = \frac{n \cancel{\tan\theta}\left(\sec^n\theta + \cos^n\theta\right)}{\cancel{\tan\theta}\left(\sec\theta + \cos\theta\right)}$$

$$\left(\frac{dy}{dx}\right)^2 = n^2 \frac{\overbrace{\sec^{2n}\theta + \cos^{2n}\theta} + 2}{\underbrace{\sec^2\theta + \cos^2\theta} + 2}$$

$$= \frac{n^2\left(y^2 + 2 + 2\right)}{x^2 + 2 + 2}$$

$$= \frac{n^2\left(y^2 + 4\right)}{x^2 + 4}$$

$$\underline{(x^2+4)\left(\frac{dy}{dx}\right)^2 = n^2\left(y^2+4\right)}$$

$y = \sec^n\theta$

$$\frac{dy}{d\theta} = n\sec^{n-1}\theta \frac{d}{d\theta}(\sec\theta)$$

$$= n\sec^{n-1}\theta\,(\sec\theta\tan\theta)$$

$$= n\sec^n\theta\,\tan\theta$$

QUESTION

$\to f'\left(\frac{2x+3}{3-2x}\right) = \sin\log\left[\frac{2x+3}{3-2x}\right]$

#Q. If $f'(x) = \sin(\log_e x)$ and $y = f\left(\frac{2x+3}{3-2x}\right)$ then $\frac{dy}{dx}$ is

$6-4x+4x+6$

$=12$

A $\sin(\log_e x)$

B $\sin\left(\log_e \frac{2x+3}{3-2x}\right) \cdot \frac{12}{(3-2x)^2}$

C $\frac{12\cos(\log_e x)}{(3-2x)^2}$

D $\frac{12}{(3-2x)^2}$

Diff wrt x

$$\frac{dy}{dx} = f'\left(\frac{2x+3}{3-2x}\right)\left(\frac{(3-2x)\overset{6}{(2)}-(2x+3)(-2)}{(3-2x)^2}\right)$$

$$= \sin\log\left(\frac{2x+3}{3-2x}\right)\left[\frac{12}{(3-2x)^2}\right]$$

QUESTION

#Q. If $x^{2/3} + y^{2/3} = a^{2/3}$ then $\frac{dy}{dx}$ is

- **A** ✓ $\left[-\frac{y}{x}\right]^{1/3}$
- **B** $\frac{y^{1/3}}{x^{1/3}}$
- **C** $-\frac{x^{1/3}}{y^{1/3}}$
- **D** $\frac{x^{1/3}}{y^{1/3}}$

Diff wrt x

$$\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{x^{-1/3}}{y^{-1/3}} = -\frac{y^{+1/3}}{x^{+1/3}}$$

$$= \left(-\frac{y}{x}\right)^{1/3}$$

$$\cancel{\frac{2}{3}}y^{-\frac{1}{3}}\frac{dy}{dx} = -\cancel{\frac{2}{3}}x^{-1/3}$$

$$\frac{dy}{dx} = -\frac{x^{-1/3}}{y^{-1/3}}$$

$$= -\left(\frac{x}{y}\right)^{-1/3}$$

$$= -\left(\frac{y}{x}\right)^{1/3}$$

$$\left(\frac{a}{b}\right)^{-1} = \frac{b}{a}$$

QUESTION

#Q. If $x = \log_e \cos\theta, y = \log_e \sin\theta$ then $\frac{dy}{dx}$ is

A $\tan^2\theta$

B $-\tan^2\theta$

C $\cot^2\theta$

D [✓ $-\cot^2\theta$]

$\frac{dx}{d\theta} = -\tan\theta \quad \Big| \quad \frac{dy}{d\theta} = \cot\theta$

$$\frac{dy}{dx} = \frac{\cot\theta}{-\tan\theta}$$

$$= \underline{-}\cot^2\theta$$

QUESTION

#Q. If $x = e^t; y = log(te^t)$ then $\frac{dy}{dx}$ is

A $\frac{t}{e^t}$

B $\frac{t+1}{e^t}$

C [✓ $\frac{t+1}{te^t}$]

D $\frac{t}{(t+1)e^t}$

$$\frac{dx}{dt} = e^t$$

$$y = \log t + \log_e e^t$$

$$y = \log t + t \log_e e$$

$$y = \log t + t$$

$$\frac{dy}{dt} = \frac{1}{t} + 1 = \frac{1+t}{t}$$

$$\frac{dy}{dx} = \frac{\frac{1+t}{t}}{e^t} = \frac{1+t}{t(e^t)}$$

QUESTION

$\Rightarrow \sin at = \log x$

$\rightarrow \cos at = \log y$

#Q. If $x = e^{\sin at}, y = e^{\cos at}$ then $\frac{dy}{dx}$ is

A $\frac{x \log_e x}{y \log_e y}$

B $\frac{x \log_e y}{y \log_e x}$

C $-\frac{x \log_e y}{y \log_e x}$

D [✓ $-\frac{y\log_e x}{x\log_e y}$]

$x = e^{\sin at}$

Take $\log_e$ on BS

$\log x = \log_e e^{\sin at}$

$\boxed{\log x = \sin at}$

$\frac{dx}{dt} = e^{\sin at}(a\cos at)$

$= x(a \log y)$

$= ax \log y$

$\frac{dy}{dt} = e^{\cos at}(-a\sin at)$

$= -a y \log x$

$\frac{dy}{dx} = \frac{-a y \log x}{a x \log y} = \frac{-y \log x}{x \log y}$

$\log e^{\sin at}$

$\sin at \log e$

$\Downarrow$

$\sin at\,(1)$

$\sin at$

QUESTION

#Q. If $x = t\log_e t, y = t$ then $\frac{dy}{dx}$ is

$\log t = \frac{x}{t}$

- **A** $\frac{t+x}{t}$
- **B** [✓ $\frac{t}{t+x}$]
- **C** $\frac{t}{1+log_e t}$
- **D** $\frac{x}{1+log_e t}$

$$\frac{dx}{dt} = 1+\log t$$

$$\frac{dy}{dt} = 1$$

$$\frac{dy}{dx} = \frac{1}{1+\log t}$$

$$= \frac{1}{1+\frac{x}{t}} = \frac{t}{t+x}$$

$$x = t\log t$$

$$\frac{dx}{dt} = t\left(\frac{1}{t}\right)+\log t(1)$$

$$= 1+\log t$$

QUESTION

#Q. If $x = e^{\sin^{-1} t}$, $y = \sin^{-1} e^t$ then $\frac{dy}{dx}$ is

(A) $\frac{\tan y}{x}\sqrt{1+t^2}$

(B) $\frac{x}{\tan y\sqrt{1-t^2}}$

(C) $\frac{x\sqrt{1-t^2}}{\tan y}$

(D) $\frac{\tan y\cdot\sqrt{1-t^2}}{x}$

$e^t = \sin y \quad \Big| \quad \sqrt{1-(e^t)^2} = \sqrt{1-\sin^2 y} = \cos y$

$$\frac{dx}{dt} = e^{\sin^{-1} t}\frac{1}{\sqrt{1-t^2}}$$

$$= \frac{x}{\sqrt{1-t^2}}$$

$$\frac{dy}{dt} = \frac{e^t}{\sqrt{1-(e^t)^2}} = \frac{\sin y}{\cos y} = \tan y$$

$$\frac{dy}{dx} = \frac{\tan y}{\frac{x}{\sqrt{1-t^2}}} = \frac{\tan y\sqrt{1-t^2}}{x}$$

QUESTION

#Q. If $x = \sin^{-1}(3t - 4t^3), y = \cos^{-1}(1 - 2t^2)$ then $\frac{dy}{dx}$ is equal to

A $\frac{3}{2}$

[✓ B] $\frac{2}{3}$

C $\frac{-4t}{3-4t^2}$

D $\frac{-4t}{3-12t^2}$

$x = 3\sin^{-1}t$

$y = 2\sin^{-1}t$

$\frac{y}{2} = \sin^{-1}t$

$x = 3\left(\frac{y}{2}\right)$

$y = \frac{2}{3}x$

Diff wrt x

$\frac{dy}{dx} = \frac{2}{3}$

QUESTION

$\Rightarrow \frac{du}{dx} = f'(x^3)(3x^2) = \cos(x^3)(3x^2)$

#Q. If $u = f(x^3), v = g(x^2), f'(x) = \cos x$ and $g'(x) = \sin x$ then $\frac{du}{dv}$ is

A $\cos x^3 \cdot \operatorname{cosec} x^2$

B $\left(\frac{2}{3}\right) \sin x^3 \cdot \sec x^2$

C $\tan x$

[✓ **D**] $\left(\frac{3}{2}\right) x . \cos(x^3) \cdot \operatorname{cosec}(x^2)$

$\frac{dv}{dx} = g'(x^2)(2x)$

$= \sin(x^2)(2x)$

$= \frac{3x^2 \cos(x^3)}{2x \sin(x^2)}$

$= \frac{3}{2} x \cos(x^3) \operatorname{cosec}(x^2)$

QUESTION

$$\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \sqrt{\frac{2\sin^2\theta/2}{2\cos^2\theta/2}} = \tan\theta/2$$

#Q. If $x = \tan^{-1}\sqrt{\frac{1-t}{1+t}}, y = \cos^{-1}(4t^3 - 3t)$ then $\frac{dx}{dy}$ is equal to

Put $t = \cos\theta$

(A) $\frac{2}{3}$

(B) $\frac{3}{2}$

(C) 6

[✓ (D) $\frac{1}{6}$]

$x = \tan^{-1}\left(\tan\frac{\theta}{2}\right)$

$x = \frac{\theta}{2}$

$\theta = 2x$

$y = \cos^{-1}(4\cos^3\theta - 3\cos\theta)$

$y = \cos^{-1}(\cos 3\theta)$

$y = 3\theta$

$y = 3(2x)$

$y = 6x$

$x = \frac{y}{6}$

$\frac{dx}{dy} = \frac{1}{6}$

if $x=f(t)$ & $y=g(t)$

$$\frac{dy}{dx}=\frac{dy/dt}{dx/dt}$$

$$\frac{dy}{dx}=\frac{g'(t)}{f'(t)}$$

Diff wrt x

$$\frac{d}{dx}\left(\frac{dy}{dx}\right)=\frac{d}{dx}\left[\frac{g'(t)}{f'(t)}\right]$$

if

$x=\sin t$ & $y=\cos t$

Find $\frac{d^2y}{dx^2}$

Soln:

$x=\sin t$ & $y=\cos t$

$$\frac{dx}{dt}=\cos t \qquad \frac{dy}{dt}=-\sin t$$

$$\frac{dy}{dx}=\frac{-\sin t}{\cos t}=-\tan t$$

Diff wrt x

$$\frac{d^2y}{dx^2}=\frac{d}{dx}(-\tan t)=-\sec^2 t\left(\frac{dt}{dx}\right)$$

$$\frac{d^2y}{dx^2} = -\sec^2\left(\frac{dt}{dx}\right)$$

$$= -\sec^2 t\,(\sec t)$$

$$= \underline{-\sec^3 t}$$

$$\frac{dx}{dt} = \cos t$$

$$\frac{dt}{dx} = \frac{1}{\cos t}$$

$$= \sec t$$

$$y^2 = x^3$$

Diff wrt x

$$2y\boxed{\frac{dy}{dx}} = 3x^2$$

$$y_1 = \frac{3x^2}{2y}$$

$$y = \tan t$$

$$\frac{dy}{dx} = +\sec^2 t\left(\frac{dt}{dx}\right)$$

QUESTION

#Q. If $x = at^2$ and $y = 2at$ then $\frac{d^2y}{dx^2}$ is

A [✓ $-\frac{1}{2at^3}$]

B $-\frac{1}{t^2}$

C $\frac{1}{t^2}$

D $\frac{1}{2at^2}$

$\frac{dx}{dt} = 2at$ | $\frac{dy}{dt} = 2a$

$\Downarrow$

$\frac{dt}{dx} = \frac{1}{2at}$

$\frac{dy}{dx} = \frac{2a}{2at} = \frac{1}{t}$

Diff wrt

$\frac{d^2y}{dx^2} = \frac{-1}{t^2}\frac{dt}{dx}$

$\frac{d^2y}{dx^2} = \frac{-1}{t^2}\left(\frac{1}{2at}\right)$

$= -\frac{1}{2at^3}$

QUESTION

$\sin^2 t\cos^2 t = (\sin t\cos t)^2$

$= \left(\frac{\sin 2t}{2}\right)^2 = \frac{\sin^2 2t}{4}$

$\cos^2 t - \sin^2 t = \cos 2t$

$\therefore \sin^2 t - \cos^2 t = -\cos 2t$

#Q. If $x = 2\log(\cot t), y = \tan t + \cot t$ then $\sin 2t.\frac{dy}{dx}$ is

Ⓐ ✓ $\cos 2t$

Ⓑ 1

Ⓒ $\cot 2t$

Ⓓ $\sin 2t$

$\Downarrow$

$\frac{dx}{dt} = \frac{-2}{\cot t}\operatorname{cosec}^2 t$

$= \frac{-2}{\cos t}(\sin t)(\operatorname{cosec}^2 t)$

$= \frac{-2\operatorname{cosec} t}{\cos t}$

$= \frac{-2}{\sin t\cos t} = \frac{-2}{\left(\frac{\sin 2t}{2}\right)}$

$\therefore \frac{dx}{dt} = \frac{-4}{\sin 2t}$

$\frac{dy}{dt} = \sec^2 t - \operatorname{cosec}^2 t$

$= \frac{\sin^2 t - \cos^2 t}{\cos^2 t\sin^2 t}$

$= \frac{-\cos 2t}{\left(\frac{\sin^2 2t}{4}\right)} = -\frac{4\cos 2t}{\sin^2 2t}$

$\frac{dy}{dx} = \frac{\cancel{-4}\cos 2t}{\sin^{\cancel{2}} 2t} \times \frac{\cancel{\sin 2t}}{\cancel{-4}} = \frac{\cos 2t}{\sin 2t}$

$\sin 2t\,\frac{dy}{dx} = \underline{\cos 2t}$

QUESTION

#Q. If $x = \frac{3at}{1+t^3}, y = \frac{3at^2}{1+t^3}$ then $\frac{dy}{dx}$ is

(A) $\frac{1-2t^3}{2t-t^4}$

(B) $\frac{1-2t^3}{1-3t^2}$

[✓ (C)] $\frac{2t-t^4}{1-2t^3}$

(D) $\frac{1-3t^2}{1-2t^3}$

$$\frac{dy}{dt} = (1+t^3)(6at) - 3at^2(3t^2)$$
$$= 6at + 6at^4 - 9at^4 = 6at - 3at^4$$
$$= 3at(2-t^3)$$

$$\frac{dx}{dt} = (1+t^3)(3a) - (3at)(3t^2)$$
$$= 3a + 3at^3 - 9at^3$$
$$= 3a - 6at^3$$
$$= 3a(1-2t^3)$$

$$\frac{dy}{dx} = \frac{\cancel{3a}\,t(2-t^3)}{\cancel{3a}(1-2t^3)} = \frac{2t-t^4}{1-2t^3}$$

QUESTION

#Q. If $x = e^t \cdot \sin t, y = e^{-t} \cos t$ then y' is $\to \frac{dy}{dx}$

A e^{-t}

B e^{-2t}

C [✓ $-e^{-2t}$]

D $-e^{2t}$

$\frac{dx}{dt} = e^t \cos t + e^t \sin t$ | $\frac{dy}{dt} = -e^{-t} \sin t - e^{-t} \cos t$

$$\frac{dy}{dx} = \frac{-e^{-t}(\sin t + \cos t)}{e^t(\sin t + \cos t)}$$

$$= -e^{-2t}$$

QUESTION

#Q. If $x = a\cos^3\theta, y = a\sin^3\theta$ then $\sqrt{1+\left(\frac{dy}{dx}\right)^2}$ is

A tan θ

B $\tan^2\theta$

[✓ C |sec θ|]

D $\sec^2\theta$

$$\frac{dx}{d\theta} = -\cancel{3a}\cos^2\theta\sin\theta \quad \Big| \quad \frac{dy}{d\theta} = \cancel{3a}\sin^2\theta\cos\theta$$

$$\frac{dy}{dx} = \frac{\sin^2\theta\cos\theta}{-\cos^2\theta\sin\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

$$1+\left(\frac{dy}{dx}\right)^2 = 1+\tan^2\theta = \sec^2\theta$$

$$\sqrt{1+\left(\frac{dy}{dx}\right)^2} = \sqrt{\sec^2\theta}$$

$$= |\sec\theta|$$

QUESTION

#Q. The derivative of $\overset{u}{e^{a\sin^{-1}x}}$ with respect to $\overset{v}{\cos^{-1}x}$ is

we need to find $\frac{du}{dv}$

$$\frac{du}{dx} = \frac{e^{a\sin^{-1}x}}{\sqrt{1-x}}(a) \qquad \frac{dv}{dx} = \frac{-1}{\sqrt{1-x^2}}$$

A $-e^{a\sin^{-1}x}$

B $e^{a\sin^{-1}x}$

C [✓ $-ae^{a\sin^{-1}x}$]

D $ae^{a\sin^{-1}x}$

$$\frac{du}{dv} = -e^{a\sin^{-1}x}(a)$$

QUESTION

#Q. The derivative of $\overset{u}{a^{\sin x}}$ w.r.t. $\overset{v}{\sin^2 x}$ is

- [✓ A] $\frac{1}{2}a^{\sin x}(\log_e \mathrm{a})\operatorname{cosec} x$
- B $\frac{1}{2}a^{\sin x}\log_e a$
- C $\frac{(e^{\sin x}\log_e a)}{\sin x}$
- D $\frac{a^{\sin x}\cdot\log_e a}{\sin 2x}$

$$\frac{dy}{dx} = a^{\sin x}\log a \cos x \quad \Big| \quad \frac{dv}{dx} = +2\cos x\sin x$$

$$\frac{dy}{dv} = \frac{a^{\sin x}\log a}{+2\sin x} = \frac{a^{\sin x}(\log a)(\operatorname{cosec} x)}{2}$$

QUESTION

$$\log_B A = \frac{1}{\log_A B}$$

#Q. The differential coefficient of $\overset{u}{\log_{10} x}$ w.r.t. $\overset{v}{x^3}$ is

A ✓ $\frac{\log_{10} e}{3x^3}$

B $\frac{3\log_{10} e}{x^3}$

C $3x^3 \log_e 10$

D $3x^3 \cdot \log_{10} e$

↓

$$u = \log_{10} x$$

$$u = \frac{\log_e x}{\log_e 10}$$

$$\frac{du}{dx} = \frac{1}{\log_e 10} \frac{1}{x}$$

$$= \frac{\log_{10} e}{x}$$

$$v = x^3$$

$$\frac{dv}{dx} = 3x^2$$

$$\frac{du}{dv} = \frac{\left(\frac{\log_{10} e}{x}\right)}{3x^2}$$

$$= \frac{\log_{10} e}{3x^3}$$

if $y = \log_{10} \sin x$ find $\frac{dy}{dx}$

Soln :

$$y = \frac{\log_e \sin x}{\log_e 10}$$

$$\frac{dy}{dx} = \frac{1}{\log_e 10} \frac{1}{\sin x} \cos x$$

$$= \log_{10} e \cot x$$

if $y = \log_5[\log \sin x]$ find $\frac{dy}{dx}$

Soln. :

$$y = \frac{\log_e[\log \sin x]}{\log_e 5}$$

$$\frac{dy}{dx} = \frac{1}{\log_e 5}\left[\frac{1}{\log \sin x}\right]\frac{1}{\sin x}\cos x$$

$$= \left(\log_5 e\right)\left(\log_{\sin x} e\right)\cot x$$

we know the differentiation

of log f(x) to the base ⓔ

ie, $\frac{d}{dx}\left[\log_e f(x)\right]$

$\Downarrow$

If given any other base

we need to convert it

into base e

$$\log_B A$$

$$= \frac{\log_e A}{\log_e B}$$

$$\log_{10} x = \frac{\log_e x}{\log_e 10}$$

QUESTION

$\theta = \sin^{-1}x$

Put $x = \sin\theta$

$\Uparrow$

$\rightarrow 1 - 2\sin^2\theta$

#Q. The derivative of $\cot^{-1}\left(\frac{1-2x^2}{2x\sqrt{1-x^2}}\right)$ with respect to $\sin^{-1}x$ is

$\downarrow$

$2\sin\theta\cos\theta$

A 1/2

B 1

[✓ C 2]

D −2

$$u = \cot^{-1}\left(\frac{1-2\sin^2\theta}{2\sin\theta\cos\theta}\right)$$

$$u = \cot^{-1}\left(\frac{\cos 2\theta}{\sin 2\theta}\right)$$

$$u = \cot^{-1}(\cot 2\theta)$$

$$u = 2\sin^{-1}x$$

$v = \sin^{-1}x$

$u = 2v$

$$\frac{du}{dv} = 2$$

⊛ if $u = \log_x 10$ & $v = \log_{10} x$ find $\frac{du}{dv}$

Sol:-

(A) $(\log_{10} x)^2$

(B) $(\log_x 10)^2$

[✓ (C)] $\frac{-1}{(\log_{10} x)^2}$

(D) $\frac{-1}{(\log_x 10)^2}$

$u = \log_x 10$

$u = \frac{\log 10}{\log x}$

$v = \log_{10} x$

$v = \frac{\log x}{\log 10}$

$\therefore uv = \log_x 10 \cdot \log_{10} x$

$uv = 1$

Diff wrt v

$u(1) + v\frac{du}{dv} = 0$

$\frac{du}{dv} = \frac{-u}{v}$

$= -\frac{\log_x 10}{\log_{10} x}$

$= -(\log_x 10)(\log_x 10)$

$= -(\log_x 10)^2$

$\frac{-1}{(\log_{10} x)(\log_{10} x)}$

$= -\frac{1}{(\log_{10} x)^2}$

method 2

$u = \log_x 10$ & $v = \log_{10} x$

$$u = \frac{1}{\log_{10} x}$$

$$u = \frac{1}{v}$$

Diff wrt v

$$\frac{du}{dv} = \frac{-1}{v^2} = \frac{-1}{(\log_{10} x)^2}$$

QUESTION

#Q. If $\sin(x+y) = \log_e(x+y)$ then $\frac{dy}{dx}$ is equal to

A 1

B [✓ −1]

C $\frac{y}{x}$

D $-\frac{y}{x}$

$$\cos(x+y)(1+y_1) = \frac{1}{x+y}(1+y_1)$$

$$y_1\left[\cos(x+y) - \frac{1}{x+y}\right] = \frac{1}{x+y} - \cos(x+y)$$

$$y_1 = \frac{-\left(\cos(x+y) - \frac{1}{x+y}\right)}{\cos(x+y) - \frac{1}{x+y}}$$

$$y_1 = -1$$

QUESTION

#Q. If $e^{x+y} = e^x + e^y$ then $\frac{dy}{dx}$ is

A e^{x+y}

B $-e^{x+y}$

C $-e^{x-y}$

D $-e^{y-x}$

$$e^x \cdot e^y = e^x + e^y$$

$\div$ by $e^x \cdot e^y$

$$1 = \frac{1}{e^y} + \frac{1}{e^x}$$

$$1 = e^{-y} + e^{-x}$$

Diff wrt x

$$0 = -e^{-y} y_1 - e^{-x}$$

$$e^{-x} = -e^{-y} y_1$$

$$y_1 = \frac{-e^{-x}}{e^{-y}}$$

$$\underline{y_1 = -e^{y-x}}$$

method 2

$$\underline{e^{x+y}} = \underline{e^x + e^y}$$

Diff wrt x

$$\underline{e^{x+y}}(1+y_1) = e^x + e^y y_1$$

$$\big\downarrow$$

$$(e^x+e^y)(1+y_1) = e^x + e^y y_1$$

$$y_1(e^x+e^y-e^y) = e^x - e^x - e^y$$

$$e^x y_1 = -e^y$$

$$y_1 = \frac{-e^y}{e^x}$$

$$\underline{y_1} = -e^{y-x}$$

QUESTION

#Q. If $y = \sqrt{x + \sqrt{x + \cdots \infty}}$ then $\frac{dy}{dx}$ is

A $\frac{1}{1-2y}$

B [✓ $\frac{1}{2y-1}$]

C $\frac{x}{1-2y}$

D $\frac{x}{2y-1}$

$$y = \sqrt{x + \underbrace{\sqrt{x + \sqrt{x + \cdots \infty}}}}$$

$$y = \sqrt{x + y}$$

$$y^2 = x + y$$

$$y^2 = x + y$$

$$2yy_1 = 1 + y_1$$

$$y_1(2y - 1) = 1$$

$$y_1 = \frac{1}{2y-1}$$

QUESTION

#Q. If $y = e^{x+e^{x+e^{x\ldots\infty}}}$ then $\frac{dy}{dx}$ is

A $\frac{e^x}{1-y}$

B $\frac{y}{1+y}$

[✓ C] $\frac{y}{1-y}$

D $\frac{e^x}{2y-1}$

$$y = e^{x+y}$$

$$y_1 = e^{x+y}(1+y_1)$$

$$\downarrow$$

$$y_1 = y(1+y_1)$$

$$y_1(1-y) = y$$

$$y_1 = \frac{y}{1-y}$$

QUESTION

#Q. If $y = x^{x^{x \dots \infty}}$ then $\frac{dy}{dx}$ is

A $\frac{y^2}{x(1-y\log_e x)}$ ✓

B $\frac{x}{y^2(1-x\log_e y)}$

C $\frac{x(1-y\log_e x)}{y^2}$

D $\frac{y^2}{x(1+y\log_e x)}$

$y = x^y$

Take log

$\log y = y \log x$

$\frac{1}{y} y_1 = \frac{y}{x} + \log x \cdot y_1$

$y_1 = \frac{y^2}{x} + y \log x \cdot y_1$

$y_1 (1 - y\log x) = \frac{y^2}{x}$

$y_1 = \frac{y^2}{x(1-y\log x)}$

QUESTION

$\tan y = \frac{y}{x} \Rightarrow \tan^2 y = \frac{y^2}{x^2}$

#Q. If $y = x \tan y$ then $\frac{dy}{dx}$ is

A $\frac{\tan y}{x-x^2-y^2}$

B ✓ $\frac{y}{x-x^2-y^2}$

C $\frac{\tan y}{y-x}$

D $\frac{y}{x+x^2+y^2}$

$y_1 = x \sec^2 y \, y_1 + \tan y$

$y_1 = x(1+\tan^2 y)\, y_1 + \tan y$

$y_1 = x\left(1+\frac{y^2}{x^2}\right) y_1 + \tan y$

$y_1 = \left(\frac{x^2+y^2}{x}\right) y_1 + \tan y$

$y_1\left[1-\frac{x^2+y^2}{x}\right] = \tan y$

$y_1\left[\frac{x-x^2-y^2}{x}\right] = \tan y$

$y_1 = \frac{x \tan y}{x-x^2-y^2}$

$y_1 = \frac{y}{x-x^2-y^2}$

Lakshya KCET 2027

Batch

⇓

April 14th to Dec 31st

Batch Access → upto May 31st 2027

Thank

You