ULTIMATE KCET

CRASH COURSE 2026

Mathematics | Lecture – 04

Integrals

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Recap *of previous lecture*

1. Definite Integrals
2.
3.
4.

Topics to be covered

1. Definite Integrals – continue
2.
3.
4.

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

Indefinite Integrals

$$\rightarrow \int \frac{1}{1+\tan x}\,dx = \int \frac{\cos x}{\cos x+\sin x}\,dx = \frac{1}{2}\int \frac{\cos x+\cos x+\sin x-\sin x}{\cos x+\sin x}\,dx$$

$$\rightarrow \int_0^{\pi/2} \frac{1}{1+\tan x}\,dx = \int \frac{\cos x}{\cos x+\sin x}\,dx$$

$b = \pi/2$

$a = 0$

$x \rightarrow a+b-x$

$\Downarrow$

$\frac{\pi}{2}-x$

Definite Integrals

observation

$\sin x = \sin\left(\frac{\pi}{2}-x\right) = \cos x$

$\cos x = \cos\left(\frac{\pi}{2}-x\right) = \sin x$

$$I = \frac{b-a}{2}$$

$$I = \frac{\frac{\pi}{2} - 0}{2}$$

$$I = \pi/4$$

$$I = \int_{a=0}^{b=\pi/2} \frac{\cos x}{\cos x + \sin x}\,dx \to (1)$$

Replace x by $\frac{\pi}{2} - x \to a + b - x$

$$I = \int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x}\,dx \to (2)$$

$$(1) + (2)$$

$$I + I = \int_0^{\pi/2} \frac{\cos x + \sin x}{\cos x + \sin x}\,dx$$

$$2I = \int_0^{\pi/2} 1\,dx$$

$$2I = x\Big]_0^{\pi/2}$$

$$2I = \frac{\pi}{2}$$

$$I = \frac{\pi}{4}$$

$$\int_a^b f(x)\,dx \longrightarrow \int_a^b f(a+b-x)\,dx$$

$$I = \frac{b-a}{2}$$ → shortcut

① $\int_{\pi/6}^{\pi/3} \frac{1}{1+\tan x}\,dx$

$\int_{\pi/6}^{\pi/3} \frac{\cos x}{\cos x+\sin x}\,dx$

$x \to \frac{\pi}{3}+\frac{\pi}{6}-x$

$x \to \frac{\pi}{2}-x$

$\Downarrow$

$\sin x \to \cos x$

$\cos x \to \sin x$

$I = \frac{b-a}{2} = \frac{\frac{\pi}{3}-\frac{\pi}{6}}{2}$

$= \frac{\pi}{12}$

② $\int_{\pi/12}^{5\pi/12} \frac{1}{1+\cot x}\,dx$

$I = \int_{\pi/12}^{5\pi/12} \frac{\sin x}{\sin x+\cos x}\,dx$

$x \to \frac{5\pi}{12}+\frac{\pi}{12}-x$

$x \to \frac{\pi}{2}-x$

$\Downarrow$

$\sin x \to \cos x$

$\cos x \to \sin x$

$I = \frac{b-a}{2} = \frac{75-15}{2} = 30^\circ$

$I = \frac{\pi}{6}$

③ $\int_{2}^{6} \frac{\sqrt{x}}{\sqrt{8-x}+\sqrt{x}}$

$a+b-x$

$= 6+2-x = 8-x$

$\sqrt{x} \to \sqrt{8-x}$

$\sqrt{8-x} \to \sqrt{x}$

$I = \frac{b-a}{2} = \frac{6-2}{2}$

$I = 2$

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

I use this

$$\Downarrow$$

if $a=0$ & $b=a$

$$\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$$ → Instead of this

Property ②

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

lower limit $= 0$

Property 3

if $a = 0$ & $b = 2a$

$$\int_0^{2a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx & \text{if } f(2a-x) = f(x) \\ 0 & \text{if } f(2a-x) = -f(x) \end{cases}$$

lower limit $= -a$

$a = -a$ & $b = a$

Property ④

$$\int_{-a}^{a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx & \text{if } f(-x) = f(x) \quad f(x) \to \text{even} \\ 0 & \text{if } f(-x) = -f(x) \quad f(x) = \text{odd} \end{cases}$$

① $I = \int_0^{\pi/2} \frac{e^{\cos x}}{e^{\cos x} + e^{\sin x}} dx$

$x \to a+b-x$

$\cos x \to \sin x$

$\sin x \to \cos x$

$I = \frac{\frac{\pi}{2}}{2} = \frac{\pi}{4}$

② $I = \int_{a=0}^{b=\pi} \frac{1}{1+e^{+\cos x}} dx \to$ ①

Apply Prop ②

$x \to a+b-x$

$\cos x \to \cos(\pi - x)$

$= -\cos x$

$= \int_0^{\pi} \frac{1}{1+e^{-\cos x}} dx$

$= \int_0^{\pi} \frac{1}{e^{-\cos x}[e^{\cos x}+1]} dx$

$I = \int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x}+1} dx \to$ ②

① + ②

$2I = \int_0^{\pi} \frac{1+e^{\cos x}}{1+e^{\cos x}} dx$

$2I = \int_0^{\pi} 1\, dx$

$2I = x \Big]_0^{\pi} = \pi$

$I = \frac{\pi}{2}$

method ①

$$I = \int_0^{\pi} \frac{\sin x}{1+\cos^2 x}\,dx$$

Put $\cos x = t$

$\sin x\,dx = -dt$

As $x \to 0$; $t \to 1$

$x \to \pi$; $t \to -1$

$$I = -\int_1^{-1} \frac{dt}{1+t^2}$$

$$= -\left[\tan^{-1} t\right]_1^{-1}$$

$$= -\left[-\frac{\pi}{4} - \frac{\pi}{4}\right] = \frac{\pi}{2}$$

method ②

$$I = \int_0^{\pi} \frac{\sin x}{1+\cos^2 x}\,dx$$

Property ② Apply $(x \to \pi - x$

$\sin(\pi - x) = \sin x$

$[\cos(\pi - x)]^2 = (-\cos x)^2 = \cos^2 x$

$$I = \int_0^{\pi} \frac{\sin x}{1+\cos^2 x}\,dx$$

we got same

$\Downarrow$

Think about property ③

if $f(2a-x) = f(x)$

$$\int_0^{2a} = 2\int_0^{a}$$

$2a = \pi$

$\Downarrow$

$a = \frac{\pi}{2}$

$$I = 2\int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x}\,dx$$

Put $\cos x = t$

$\sin x\,dx = -dt$

$0 \to 1$	
$\frac{\pi}{2} \to 0$	

$$I = -2\int_1^{0} \frac{dt}{1+t^2} = -2\left[\tan^{-1} t\right]_1^{0}$$

$$= -2\left[0 - \frac{\pi}{4}\right] = \frac{\pi}{2}$$

QUESTION

Substitution method ✗

#Q. The value of $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x}\, dx$ is

(A) $\pi/4$

(B) $\pi/2$

(C) $\pi^2/4$

(D) $\pi^2/2$

Property (2) $\Rightarrow x \to \pi - x$

$$I = \int_0^{\pi} \frac{(\pi - x)\sin x}{1+\cos^2 x}$$

$$I = \int_0^{\pi} \frac{\pi \sin x}{1+\cos^2 x}\, dx - \int_0^{\pi} \frac{x \sin x}{1+\cos^2 x}\, dx$$

$$I = \pi \int_0^{\pi} \frac{\sin x}{1+\cos^2 x}\, dx - I$$

$$2I = \pi \int_0^{\pi} \frac{\sin x}{1+\cos^2 x}\, dx$$

Proper (3) $\Rightarrow f(2a-x) = f(x)$

$$2I = 2\pi \int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x}\, dx$$

Put $\cos x = t$ | $0 \to 1$

$\sin x\, dx = -dt$ | $\frac{\pi}{2} \to 0$

$$2I = 2\pi \int_1^{0} \frac{-1}{1+t^2}\, dt$$

$$I = \pi \left[\tan^{-1} t\right]_1^0 = -\pi\left[-\frac{\pi}{4}\right]$$

$$I = \pi^2/4$$

(*)

$$I = \int_0^{\pi} \frac{1}{1+\cos^2 x}\,dx$$

$\div$ both N^r & D^r by $\cos^2 x$

$$I = \int_0^{\pi} \frac{\sec^2 x}{\sec^2 x + 1}$$

$$= \int_0^{\pi} \frac{\sec^2 x}{2+\tan^2 x}\,dx \rightarrow \text{next Page}$$

This doesn't work ✗

Put $\tan x = t$

lower limit = upper limit = 0

Substitution

$\downarrow$

Manipulation

($\times$ & $\div$, Add & Subtract, Rationalise)

$\Downarrow$

Property

$$I = \int_0^{\pi} \frac{\sec^2 x}{2+\tan^2 x}\,dx$$

$[\sec(\pi - x)]^2 = (-\sec x)^2 = \sec^2 x$

$[\tan(\pi - x)]^2 = \tan^2 x$

same

Prop ②

$$I = \int_0^{\pi} \frac{\sec^2 x}{2+\tan^2 x}\,dx$$

Proper ③

$f(2a-x) = f(x)$

$2a = \pi$

$\Downarrow$

$a = \frac{\pi}{2}$

$$\int_0^{\pi} = 2\int_0^{\pi/2}$$

$$I = 2\int_0^{\pi/2} \frac{\sec^2 x}{2+\tan^2 x}\,dx$$

$2 \to (\sqrt{2})^2$

Put $\tan x = t$

$\sec^2 x\,dx = dt$

$\tan x = t$

$\Downarrow$

$x = 0 \Rightarrow t = \tan 0 = 0$

$x = \frac{\pi}{2} \Rightarrow t = \tan\frac{\pi}{2} = N.D\,(\infty)$

$$I = 2\int_0^{\infty} \frac{dt}{(\sqrt{2})^2 + t^2}$$

$$I = 2\left[\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{t}{\sqrt{2}}\right)\right]_0^{\infty}$$

$$= \sqrt{2}\left[\tan^{-1}\infty - \tan^{-1} 0\right]$$

$$I = \sqrt{2}\left[\frac{\pi}{2} - 0\right] = \frac{\pi}{\sqrt{2}}$$

$\tan\frac{\pi}{2} = ND\,(\infty)$

$\Downarrow$

$\tan^{-1}\infty = \frac{\pi}{2}$

$\frac{\infty}{\sqrt{2}} = \infty$

$\frac{0}{\sqrt{2}} = 0$

(*) $I = \int_0^{\pi} \frac{x \sin^2 x}{\sin^2 x + \cos^2 x} dx$

Prop

Prop (2) $\Rightarrow x \rightarrow \pi - x$

$I = \pi \int_0^{\pi} \frac{\sin^2 x}{\sin^2 x + \cos^2 x} dx - I$

$\int \frac{x}{- + -}$

$2I = \pi \int_0^{\pi} \frac{\sin^2 x}{\sin^2 x + \cos^2 x} dx$

Prop (2)

$\Downarrow$

we get same

$\Downarrow$

$\therefore$ Prop (3)

$\cancel{2}I = \cancel{2}\pi \int_0^{\pi/2} \frac{\sin^2 x}{\sin^2 x + \cos^2 x} dx$

$I = \pi \left[\frac{b-a}{2} \right]$

$I = \pi \left[\frac{\frac{\pi}{2} - 0}{2} \right]$

$I = \frac{\pi^2}{4}$

(*) $I = \int_0^{\pi} \frac{x \sin^4 x}{\sin^4 x + \cos^4 x} dx$

Prop (2)

$I = \pi \int_0^{\pi} \frac{\sin^4 x}{\sin^4 x + \cos^4 x} dx - I$

$2I = \pi \int_0^{\pi} \frac{\sin^4 x}{\sin^4 x + \cos^4 x} dx$

Prop (2)

$\Downarrow$

Same results

$\therefore$ Property (3)

$f(2a - x) = f(x)$ | $2a = \pi \Rightarrow a = \frac{\pi}{2}$

$\cancel{2} I = \cancel{2} \pi \int_0^{\pi/2} \frac{\sin^4 x}{\sin^4 x + \cos^4 x} dx$

$I = \pi \left[\frac{\frac{\pi}{2} - 0}{2} \right]$

$I = \pi \left(\frac{\pi}{4} \right)$

$I = \pi^2/4$

(*) $$I = \int_0^{2\pi} \frac{x \sin^6 x}{\sin^6 x + \cos^6 x}\,dx$$

(A) 0

(B) π^2

(C) $\frac{\pi^2}{4}$

(D) $\frac{\pi^2}{2}$

Soln :- $x \to 2\pi - x$

$$I = 2\pi \int_0^{2\pi} \frac{\sin^6 x}{\sin^6 x + \cos^6 x}\,dx - I$$

$$\cancel{2}I = \cancel{2}\pi \int_0^{2\pi} \frac{\sin^6 x}{\sin^6 x + \cos^6 x}\,dx$$

Prop (2)

$\Downarrow$

Same

$\Downarrow$

Prop (3)

$f(2a-x) = f(x)$ | $2a = 2\pi$, $a = \pi$

$$I = 2\pi \int_0^{\pi} \frac{\sin^6 x}{\sin^6 x + \cos^6 x}\,dx$$

$\Downarrow$

Prop (2)

$\Downarrow$

Same

$\Downarrow$

Prop (3) | $2a = \pi$ $\Downarrow$ $a = \frac{\pi}{2}$

$$\int_0^{2a} = 2\int_0^{a}$$

$$I = 2\pi(2) \int_0^{\pi/2} \frac{\sin^6 x}{\sin^6 x + \cos^6 x}\,dx$$

$$I = 4\pi\left[\frac{\pi/2 - 0}{2}\right]$$

$$I = 4\pi\left[\frac{\pi}{4}\right] = \pi^2$$

QUESTION

#Q. The value of $I = \int_0^{\frac{\pi}{2}} \frac{a \sin x+b \cos x}{\sin x+\cos x} dx$ is

Prop ②

A $\frac{\pi(a+b)}{4}$

B $\frac{\pi(a-b)}{4}$

C $\frac{\pi(a+b)}{2}$

D $\frac{\pi(a-b)}{2}$

$$I = \int_0^{\pi/2} \frac{a\cos x + b\sin x}{\cos x + \sin x} dx \rightarrow ②$$

① + ②

$$2I = \int_0^{\pi/2} \frac{\cos x(a+b) + \sin x(a+b)}{\cos x + \sin x} dx$$

$$2I = (a+b)\int_0^{\pi/2} 1\, dx$$

$$2I = (a+b)\frac{\pi}{2}$$

$$I = \frac{\pi}{4}(a+b)$$

QUESTION

× Substitution

⇓

✓ Manipulation

#Q. The value of $\int_0^{\pi} \frac{x \cdot dx}{a^2\cos^2 x + b^2\sin^2 x}$ is

(A) $\pi^2 ab$

(B) $\frac{\pi^2 ab}{2}$

(C) $\pi^2/2ab$ ✓

(D) $\pi^2/4ab$

Prop ② $\Rightarrow x \to \pi - x$

$$I = \pi\int_0^{\pi} \frac{1}{a^2\cos^2 x + b^2\sin^2 x}\,dx - I$$

$$2I = \pi\int_0^{\pi} \frac{1}{a^2\cos^2 x + b^2\sin^2 x}\,dx$$

÷ by $\cos^2 x$ in N^r & D^r

$$2I = \pi\int_0^{\pi} \frac{\sec^2 x}{a^2 + b^2\tan^2 x}\,dx$$

$$2I = \pi\int_0^{\pi} \frac{\sec^2 x}{a^2 + b^2\tan^2 x}\,dx$$

× $\tan x = t$ | $\frac{L.L}{U.L} = 0$

∴ Prop ②

⇓

same

⇓

∴ Prop ③

$$\therefore \cancel{2}I = \cancel{2}\pi\int_0^{\pi/2} \frac{\sec^2 x}{a^2 + b^2\tan^2 x}\,dx$$

$$I = \pi \int_0^{\pi/2} \frac{\sec^2 x}{a^2 + b^2 \tan^2 x}\, dx$$

Put $\tan x = t$

$$I = \pi \int_0^{\infty} \frac{dt}{a^2 + (bt)^2}$$

$$= \pi \left[\frac{1}{a} \frac{\tan^{-1}\left(\frac{bt}{a}\right)}{b} \right]_0^{\infty}$$

$$= \frac{\pi}{ab} \left[\tan^{-1}\infty - \tan^{-1} 0 \right]$$

$$I = \frac{\pi}{ab} \left[\frac{\pi}{2} \right] = \frac{\pi^2}{2ab}$$

QUESTION

$I \Rightarrow$

#Q. The value of $\int_0^{\frac{\pi}{2}} \ln \tan x \, dx$ is $\rightarrow ①$

Prop ② $x \rightarrow \frac{\pi}{2} - x$

$\Rightarrow \tan x \rightarrow \cot x$

A $\pi/4$

$$I = \int_0^{\pi/2} \log \cot x \, dx \rightarrow ②$$

B 0

① + ②

C $\pi/2$

$$2I = \int_0^{\pi/2} \log \tan x + \log \cot x \, dx$$

D 1

$$2I = \int_0^{\pi/2} \log(\tan x \cdot \cot x) \, dx$$

$$2I = \int_0^{\pi} \log 1 \, dx$$

$$2I = 0 \qquad \log 1 = 0$$

$$I = 0$$

$$I = \int_0^{\pi} \log \tan^2 x \, dx$$

Prop (2)

$\Downarrow$

same

$\Downarrow$

Prop (3)

$f(2a-x) = f(x)$

$\Downarrow$

$2a = \pi$

$\Downarrow$

$a = \frac{\pi}{2}$

$$I = 2\int_0^{\pi/2} \log \tan^2 x \, dx \rightarrow (1)$$

Again Prop (2) $x \rightarrow \frac{\pi}{2} - x$ | $\tan x \rightarrow \cot x$

$$I = 2\int_0^{\pi/2} \log \cot^2 x \, dx \rightarrow (2)$$

(1) + (2)

$$2I = 2\int_0^{\pi/2} \log(\tan^2 x \cdot \cot^2 x)\, dx$$

$$2I = 2\int_0^{\pi/2} \log 1 \, dx = 0$$

$$2I = 0$$

$$I = 0$$

QUESTION

#Q. The value of $\int_0^{\frac{\pi}{4}} \ln(1+\tan\theta)d$ is

Prop ②

$$I = \int_0^{\pi/4} \log\left[1+\tan\left(\frac{\pi}{4}-\theta\right)\right]d\theta$$

$$= \int_0^{\pi/4} \log\left[1+\frac{1-\tan\theta}{1+\tan\theta}\right]d\theta$$

$$I = \int_0^{\pi/4} \log\left[\frac{2}{1+\tan\theta}\right]d\theta$$

(A) $\frac{\pi}{2}\ln 2$

(B) $\frac{\pi}{4}\ln 2$

(C) $\frac{\pi}{8}\ln 2$

(D) $\frac{\pi}{8}$

$$I = \int_0^{\pi/4} \log 2 - \underbrace{\log(1+\tan\theta)}_{I}\,d\theta$$

$$2I = \log 2\int_0^{\pi/4} 1\,dx = \log 2\left(\frac{\pi}{4}\right)$$

$$I = \frac{\pi}{8}\log 2$$

QUESTION

#Q. The value of $\int_0^\infty \frac{x\,dx}{(1+x)(1+x^2)}$ is

↳ Hint $(1+x^2)$

(A) $\pi/2$

(B) 0

(C) 1

(D) $\pi/4$

Put $x = \tan\theta$ $\quad$ $0 \to 0$

$dx = \sec^2\theta\, d\theta$ $\quad$ $\frac{\pi}{2} \to \infty$

$$I = \int_0^{\pi/2} \frac{\tan\theta \cancel{\sec^2\theta}\, d\theta}{(1+\tan\theta)\cancel{(1+\tan^2\theta)}}$$

$$I = \int_0^{\pi/2} \frac{\tan\theta}{1+\tan\theta}\, d\theta \qquad \frac{\frac{\sin\theta}{\cos\theta}}{1+\frac{\sin\theta}{\cos\theta}}$$

$$I = \int_0^{\pi/2} \frac{\sin\theta}{\cos\theta+\sin\theta}\, d\theta$$

$$I = \frac{\frac{\pi}{2}-0}{2}$$

$$I = \frac{\pi}{4}$$

QUESTION

#Q. The value of $\int_0^{\frac{\pi}{2}} \frac{\phi(x)}{\phi(x)+\phi\left(\frac{\pi}{2}-x\right)} dx$ is

(A) $\pi/4$

(B) $\pi/2$

(C) π

(D) 0

QUESTION

#Q. The value of $\int_0^{\pi/2} \frac{(\sin x)^{3/2}}{(\sin x)^{\frac{3}{2}}+(\cos x)^{3/2}} dx$ is

(A) $\pi/2$

(B) $\pi/3$

(C) $\pi/4$

(D) 0

QUESTION

#Q. The value of $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$ is

(A) 4π

(B) $\pi/4$

(C) $4/\pi$

(D) 2π

QUESTION

#Q. The value of $\int_0^{\pi/2} \frac{dx}{1+\tan x}$ is

(A) 4π

(B) $\pi/4$

(C) $4/\pi$

(D) 2π

QUESTION

#Q. The value of $I = \int_0^{\pi} \frac{dx}{1+3^{\cos x}}$ is $\rightarrow (1)$

Prop (2)

$$I = \int_0^{\pi} \frac{1}{1+3^{-\cos x}}\, dx$$

$$= \int_0^{\pi} \frac{1}{3^{-\cos x}[3^{\cos x}+1]}\, dx$$

$$I = \int_0^{\pi} \frac{3^{\cos x}}{3^{\cos x}+1}\, dx \rightarrow (2)$$

A. π

B. 0

C. $\pi/2$

D. $\pi/4$

(1) + (2)

$$2I = \int_0^{\pi} \frac{1+3^{\cos x}}{1+3^{\cos x}}\, dx$$

$$2I = \int_0^{\pi} 1\, dx = x\Big)_0^{\pi} = \pi$$

$$I = \frac{\pi}{2}$$

QUESTION

#Q. The value of $\int_0^{\pi/2} \frac{dx}{1+\tan^3 x}$ is $= \int_0^{\pi/2} \frac{\cos^3 x}{\cos^3 x+\sin^3 x} dx$

A $\pi/2$

B $\pi/4$

C π

D 2π

$I = \frac{\frac{\pi}{2}-0}{2}$

$I = \frac{\pi}{4}$

QUESTION

#Q. The value of $\int_0^{\frac{\pi}{2}} \frac{\sin^3 x}{\sin^3 x+\cos^3 x} dx$ is

(A) $\pi/6$

(B) $\pi/3$

(C) $\pi/2$

(D) $\pi/4$

QUESTION

#Q. The value of $\int_0^{\frac{\pi}{2}} \frac{\cos x}{1+\sin^2 x} dx$ is

Put $\sin x = t$

(A) $\pi/2$

(B) $\pi/4$

$I = \int_0^1 \frac{1}{1+t^2} dt$

(C) $\pi/3$

$= \frac{\pi}{4}$

(D) $\pi/6$

QUESTION

#Q. The value of $I = \int_0^{\frac{\pi}{2}} \frac{\sin^3 x - \cos^3 x}{\sin x + \cos x} dx$ is → (1)

A 0

B $\pi/4$

C $\pi/2$

D π

$x \to \left(\frac{\pi}{2} + 0\right) - x$

$x \to \frac{\pi}{2} - x$

$I = \int_0^{\pi/2} \frac{\cos^3 x - \sin^3 x}{\cos x + \sin x} dx \to (2)$

(1) + (2)

$I + I = \int_0^{\pi/2} \frac{\sin^3 x - \cos^3 x + \cos^3 x - \sin^3 x}{\cos x + \sin x} dx$

$2I = 0$

$I = 0$

QUESTION

#Q. The value of $\int_0^{\frac{\pi}{2}} \frac{e^{\sin x}}{e^{\sin x}+e^{\cos x}} dx$ is

(A) $\pi/4$

(B) $\pi/2$

(C) 0

(D) π

$I = \frac{\pi}{4}$

(4) $I = \int_{\pi/6}^{\pi/3} \frac{4^{\sin x}}{4^{\sin x}+4^{\cos x}} dx$

$I = \frac{\frac{\pi}{3}-\frac{\pi}{6}}{2} = \frac{\pi/6}{2} = \frac{\pi}{12}$

QUESTION

#Q. The value of $\int_1^2 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{3-x}}dx$ is

(A) $\pi/4$

$I = \frac{b-a}{2} = \frac{2-1}{2} = \frac{1}{2}$

(B) $\pi/2$

(C) $\pi/6$

(D) $1/2$

QUESTION

#Q. The value of $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1+\sqrt{\cot x}}\,dx$ is

$$I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$$

(A) $\pi/4$

(B) $\pi/6$

(C) $\pi/12$

(D) $\pi/3$

QUESTION

#Q. The value of $\int_0^1 x(1-x)^4 dx$ is

Prop ②

$x \to 1-x$

(A) 1/5

(B) 1/6

$I = \int_0^1 (1-x)x^4 dx$

(C) 1/30

$= \int_0^1 x^4 - x^5 dx = \left.\frac{x^5}{5} - \frac{x^6}{6}\right]_0^1$

(D) 1/25

$I = \frac{1}{5} - \frac{1}{6} = \frac{1}{30}$

$$I = \int_0^2 x(2-x)^5$$

Prop ②

$x \to 2-x$

$$I = \int_0^2 (2-x)x^5$$

$$= \int_0^2 2x^5 - x^6\,dx$$

$$= 2\left[\frac{x^6}{6}\right]_0^2 - \left(\frac{x^7}{7}\right)_0^2$$

$$= \frac{1}{3}\left[2^6\right] - \frac{1}{7}\left[2^7\right]$$

$$= 2^6\left[\frac{1}{3} - \frac{2}{7}\right]$$

$$= 2^6\left[\frac{7-6}{21}\right]$$

$$I = \frac{64}{21}$$

QUESTION

$x \to a+b-x$

$= 0+3-x = 3-x$

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

$I = $

#Q. The value of $\int_0^3 \frac{\sqrt{x+2}}{\sqrt{x+2}+\sqrt{5-x}}\,dx$ is $\to (1)$

$b = 3$, $a = 0$

$x \to 3-x$

$$I = \int_0^3 \frac{\sqrt{5-x}}{\sqrt{5-x}+\sqrt{x+2}}\,dx \to (2)$$

(A) 2

(B) 3

(C) 3/2 ✓

(D) 2/3

$$I = \frac{3-0}{2}$$

$\therefore I = 3/2$

QUESTION

#Q. The value of $\int_0^{2\pi} \frac{x \sin^{2n} x}{\sin^{2n} x + \cos^{2n} x} dx$ is

A) $\pi/4$

B) $\pi^2/4$

C) π^2

D) π

$I = 2\pi \int_0^{2\pi} \frac{\sin^{2n} x}{\sin^{2n} x + \cos^{2n} x} dx$

Prop (2)
$\Downarrow$
Same
$\Downarrow$
Prop (2)

$I = 2\pi \int_0^{\pi} \frac{\_\_}{\_\_ + \_\_}$

Prop (2)
$\Downarrow$
Same

Prop (3)

$I = 2\pi (2) \int_0^{\pi/2} \frac{\_\_}{\_\_ + \_\_}$

$I = 4\pi \left[\frac{\pi}{4}\right]$

$I = \pi^2$

QUESTION

#Q. The value of $\int_a^b \frac{f(x)dx}{f(x)+f(a+b-x)}$ is

(A) $\frac{a+b}{2}$

(B) $\frac{a-b}{2}$

(C) $\frac{b-a}{2}$

(D) 0

Thank

You