
ULTIMATE KCET
CRASH COURSE 2026
Mathematics
Lecture - 01
Three dimensional geometry
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Recap of previous lecture

1. Vector Algebra
2.
3.
4.

Topics *to be covered*

1. 3 D - Class 12th
2.
3.
4.

Angle b/w 2 lines

① If D.R's of 2 lines are given

ie, D.R of l_1 : a_1, b_1, c_1 &

D.R of l_2 : a_2, b_2, c_2

① If eqⁿ of line is given in cartesian form

Line l_1: $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$

Line l_2: $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$

$$\cos\theta = \left| \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}} \right|$$

(3) if vector eq^n are given

line l_1: $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$

line l_2: $\vec{r} = a_2 + \mu \vec{b}_2$

$$\cos\theta = \left| \frac{\vec{b}_1 \cdot \vec{b}_2}{|\vec{b}_1| \, |\vec{b}_2|} \right|$$

(4) If D.C's are given

line l_1: l_1, m_1, n_1

line l_2: l_2, m_2, n_2

$$\cos\theta = |l_1 l_2 + m_1 m_2 + n_1 n_2|$$

(*) If 2 lines are $\perp^r$ to each other

Then

(1) $a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$ [If D.R's are given]

(2) $\vec{b_1} \cdot \vec{b_2} = 0$ [If vector eq^n is given]

Question

$\alpha = \beta = \gamma$ | $l = m = n$ $\Rightarrow l^2 + m^2 + n^2 = 1$ | $3l^2 = 1$ $l^2 = \frac{1}{3}$ | $l = \frac{1}{\sqrt{3}} = m = n$

$\therefore \vec{a} = \frac{1}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$

A vector $\vec{a}$ makes equal acute angles on the coordinate axis. Then the projection of vector $\vec{b} = 5\hat{i} + 7\hat{j} - \hat{k}$ on $\vec{a}$ is

$|\vec{a}| = \frac{1}{\sqrt{3}}\sqrt{1+1+1}$

$= \frac{\sqrt{3}}{\sqrt{3}} = 1$

$\therefore$ Projection of $\vec{b}$ on $\vec{a} = \frac{\vec{b} \cdot \vec{a}}{|\vec{a}|}$

$= \frac{1}{\sqrt{3}}\left[\frac{5(1) + 7(1) + (-1)(1)}{(1)}\right]$

$= \frac{11}{\sqrt{3}}$

(A) $\frac{11}{15}$

(B) $\frac{11}{\sqrt{3}}$ ✓

(C) $\frac{4}{5}$

(D) $\frac{3}{5\sqrt{3}}$

Question

The component of $\hat{i}$ $\boxed{\text{in the direction}}$ of vector $\hat{i}+\hat{j}+2\hat{k}$ is

$$\vec{a} = \hat{i}+\hat{j}+2\hat{k}$$

$$|\vec{a}| = \sqrt{1+1+4} = \sqrt{6}$$

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{1}{\sqrt{6}}\hat{i} + \frac{1}{\sqrt{6}}\hat{j} + \frac{2}{\sqrt{6}}\hat{k}$$

$\therefore$ component of $i = \frac{1}{\sqrt{6}}$

$$= \frac{1}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \frac{\sqrt{6}}{6}$$

(A) $6\sqrt{6}$

(B) $\sqrt{6}$

(C) $\frac{\sqrt{6}}{6}$

(D) 6

Arbitrary point of a given line

Arbitrary point is a point which can be anywhere on the line

$(3r+2, 4r-3, 2r+1)$

$$\frac{x-2}{3} = \frac{y+3}{4} = \frac{z-1}{2} = r$$

$$x = 3r+2$$

$$y = 4r-3$$

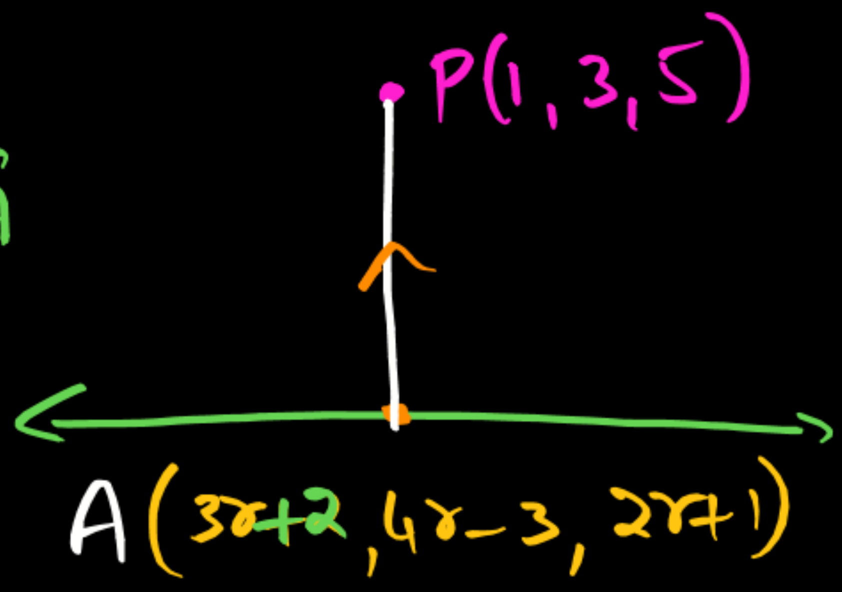
$$z = 2r+1$$

D.R of $\vec{PA}$

$3r+2-1$

$4r-3-3$

$2r+1-5$



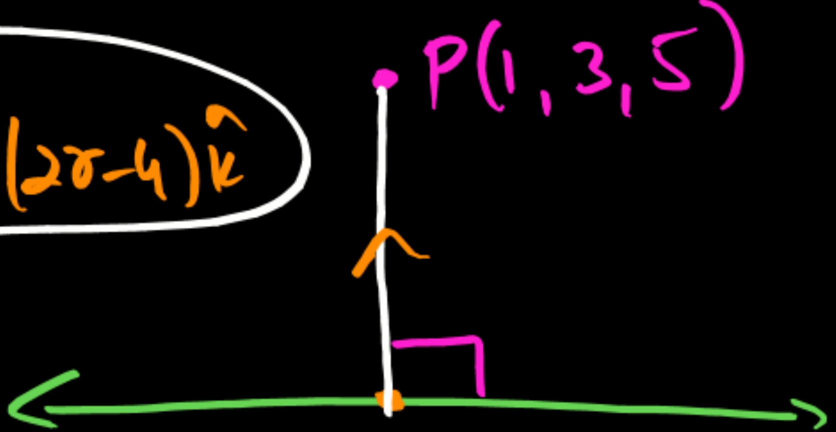
$$\frac{x-2}{3}=\frac{y+3}{4}=\frac{z-1}{2}=r$$

$$\vec{PA}=(3r+1)\hat{i}+(4r-6)\hat{j}+(2r-4)\hat{k}$$

$x=3r+2$

$y=4r-3$

$z=2r+1$



$A(3r+2,\ 4r-3,\ 2r+1)$

$\vec{b} = 3\hat{i}+4\hat{j}+2\hat{k}$

$$\frac{x-2}{3} = \frac{y+3}{4} = \frac{z-1}{2} = r$$

$\vec{b} = 3\hat{i}+4\hat{j}+2\hat{k}$

$x = 3r+2$

$y = 4r-3$

$z = 2r+1$

$$\vec{PA} = (3r+1)\hat{i} + (4r-6)\hat{j} + (2r-4)\hat{k}$$

$\Downarrow$

$\vec{PA} \perp$ Line L

$\Downarrow$

$\vec{PA} \perp \vec{b}$

$$\Rightarrow (3r+1)(3) + (4r-6)(4) + (2r-4)(2) = 0$$

$$9r + 3 + 16r - 24 + 4r - 8 = 0$$

$$29r - 29 = 0$$

$$r = 1$$

$\because$ Arbitrary Point

$(3r+2, 4r-3, 2r+1)$

$\Downarrow$

Put $r=1$

$(5, 1, 3)$

$\hookrightarrow$ Foot of $\perp^{r}$ of the Point

P(1, 3, 5) to the given line

Foot of perpendicular of a given point on a line:-

(1) Find arbitrary point

(2) If P is the given & A is the Arbitrary Point

Find $\vec{PA}$

(3) Here $\vec{PA} \perp$ line L

$\Rightarrow \vec{PA} \perp \vec{b}$

(4) using $\vec{PA} \perp \vec{b}$

Find 'r'

& Hence coordinates of Point A, which is the foot of $\perp^{r}$ of the point P.

(2) Find the Foot of $\perp^r$ of the point $P(1,6,3)$ to the line

$$\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$$

Soln:

$x = r$

$y = 2r+1$

$z = 3r+2$

$\vec{PA} = (r-1)\hat{i} + (2r-5)\hat{j} + (3r-1)\hat{k}$

$\vec{PA} \perp \vec{b}$

$(r-1)(1) + (2r-5)(2) + (3r-1)(3) = 0$

$r - 1 + 4r - 10 + 9r - 3 = 0$

$14r = 14$

$r = 1$

Arbitrary Point

$\Downarrow$

$(1, 3, 5)$

Question

midpoint formula $(x, y, z) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$

The $\boxed{\text{image}}$ of the point (1,6,3) in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ is [2018]

A (1, 0, 7) ✓

B (7, 0, 1)

C (2, 7, 0)

D (−1, −6, −3)

$P(1,6,3)$

$A(1,3,5)$ ↪ Previous slide

$\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$

P' (a, b, c)

<u>To find P'</u> :-

Here A is the midpoint of P & P'

$(1,3,5) = \left(\frac{1+a}{2}, \frac{6+b}{2}, \frac{3+c}{2}\right)$

$\frac{1+a}{2} = 1$ | $3 = \frac{6+b}{2}$ | $5 = \frac{3+c}{2}$

$a = 1$ | $b = 0$ | $c = 7$

$(1, 0, 7)$

Question

$\frac{x}{1/2} = \frac{y}{1/3} = \frac{z}{-1}$

LCM = −6 LCM = 12

The angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$ is [2018]

(A) 0°

(B) 45°

(C) 90°

(D) 30°

$2x = 3y = -z$

$\Downarrow$

$\div$ by (-6)

$\frac{x}{-3} = \frac{y}{-2} = \frac{z}{+6}$

$6x = -y = -4z$

$\div$ by 12

$\frac{x}{2} = \frac{y}{-12} = \frac{z}{-3}$

$\cos\theta = \left|\frac{-3(2)+(-2)(-12)+(+6)(-3)}{\sqrt{9+4+36}\sqrt{4+144+9}}\right|$

$\cos\theta = \left|\frac{-6+24-18}{(7)\sqrt{157}}\right| = 0$

$\cos\theta = 0$

$\theta = \frac{\pi}{2}$

Find the angle b/w the lines

$$\frac{2x+6}{-2} = \frac{9y-3}{-1} = \frac{4-2z}{3} \quad \& \quad 3x = -2y = -z$$

$(27)^2$

$= 4\ \underline{28}\ \underline{49} = 729$

Soln:

$$\frac{2[x-(-3)]}{-2} = \frac{9[y-1/3]}{-1} = \frac{-2[z-2]}{3}$$

LCM $(2, 9, -2) = -18$

$\div$ by -18

$$\frac{x-(-3)}{18} = \frac{y-1/3}{2} = \frac{z-2}{+27}$$

$\& \quad 3x = -2y = -z$

LCM$(3, -2, -1) = 6$

$\div$ by 6

$$\frac{x}{2} = \frac{y}{-3} = \frac{z}{-6}$$

$$\cos\theta = \left|\frac{36-6-162}{\sqrt{324+4+729}\sqrt{4+9+36}}\right| = \left|\frac{-132}{7\sqrt{1057}}\right|$$

$$\cos\theta = \left|\frac{-132}{7\sqrt{1057}}\right| = \frac{132}{7\sqrt{1057}}$$

$$\theta = \cos^{-1}\left(\frac{132}{7\sqrt{1057}}\right)$$

$$\frac{(1)x - x_1}{a} = \frac{(1)y - y_1}{b} = \frac{(1)z - z_1}{c}$$

observations:- ① coefficient of x, y & z should be ± 1

② b/w x & x_1

y & y_1

z & z_1 There should be -ve sign

Question

The point (1,-3,4) lies in the octant [2020]

A Third

B Fourth

C Eighth

D Second

Question

The angle between the lines whose $\boxed{\text{direction cosines}}$ are $\left(\frac{\sqrt{3}}{4}, \frac{1}{4}, \frac{\sqrt{3}}{2}\right)$ and $\left(\frac{\sqrt{3}}{4}, \frac{1}{4}, \frac{-\sqrt{3}}{2}\right)$ is [2021]

(A) π

(B) $\pi/2$

(C) $\pi/3$

(D) $\pi/4$

$\cos\theta = |l_1 l_2 + m_1 m_2 + n_1 n_2|$

$= \left|\frac{3}{16} + \frac{1}{16} - \frac{3}{4}\right|$

$= \left|\frac{4}{16} - \frac{3}{4}\right|$

$= \left|\frac{1}{4} - \frac{3}{4}\right|$

$= \left|-\frac{1}{2}\right| = \frac{1}{2}$

$\cos\theta = 1/2$

$\theta = 60^\circ = \pi/3$

Question

The mid points of the sides of triangle are $(1, 5, -1), (0, 4, -2)$ and $(2, 3, 4)$, then centroid of the triangle is [2021]

A $(1, 4, 3)$

B $(1, 4, 1/3)$

C $(-1, 4, 3)$

D $(1/3, 2, 4)$

Question

The octant in which the point $(2, -4, -7)$ lies is [2022]

(A) Eighth

(B) Fourth

(C) Third

(D) Fifth

Question

The angle between the pair of lines $\frac{x+3}{3}=\frac{y-1}{5}=\frac{z+3}{4}$ and $\frac{x+1}{1}=\frac{y-4}{4}=\frac{z-5}{2}$ is [2022]

(A) $\theta=\cos^{-1}\left[\frac{27}{5}\right]$

(B) $\theta=\cos^{-1}\left[\frac{19}{21}\right]$

(C) $\theta=\cos^{-1}\left[\frac{8\sqrt{3}}{15}\right]$

(D) $\theta=\cos^{-1}\left[\frac{5\sqrt{3}}{16}\right]$

Question

If a line makes an angle of $\pi/3$ with each X and Y axis then the acute angle made by Z-axis is [2023]

$n = +ve$

$\Downarrow$

$\alpha = \beta = \frac{\pi}{3}$

$l = m = \cos\frac{\pi}{3} = \frac{1}{2}$

WKT

$l^2 + m^2 + n^2 = 1$

$\frac{1}{4} + \frac{1}{4} + n^2 = 1$

$\frac{1}{2} + n^2 = 1$

$n^2 = 1/2$

$n = 1/\sqrt{2}$

$\cos\gamma = \frac{1}{\sqrt{2}}$

$\gamma = \pi/4$

(A) $\pi/2$

(B) $\pi/6$

(C) $\pi/4$

(D) $\pi/3$

Question

If lines $\frac{x-1}{-3}=\frac{y-2}{2k}=\frac{z-3}{2}$ and $\frac{x-1}{3k}=\frac{y-5}{1}=\frac{z-6}{-5}$ are mutually perpendicular, then k is equal to

$$-3(3k)+2k(1)+(2)(-5)=0$$

$$-9k+2k-10=0$$

$$-7k=10$$

$$k=-10/7$$

A) $-10/7$

B) $-7/10$

C) -10

D) -7

Question

If a lines makes angle α, β, γ with the positive direction of coordinate axes, then write the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$

$\Downarrow \quad \Downarrow \quad \Downarrow$

$1-\cos^2\alpha + 1-\cos^2\beta + 1-\cos^2\gamma$

$= 3-(\cos^2\alpha+\cos^2\beta+\cos^2\gamma)$

$= 3-1$

$= 2$

$l = \cos\alpha$

$m = \cos\beta$

$n = \cos\gamma$

$l^2+m^2+n^2=1$

$\Downarrow$

$\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$

(A) 2

(B) 3

(C) 0

(D) 1

Question

If a line makes angles 90°, 135°, 45° with the x, y and z axes respectively, then find its direction cosines.

$\downarrow \alpha \quad \downarrow \beta \quad \downarrow \gamma$

(A) $\frac{1}{\sqrt{2}}, 0, \frac{-1}{\sqrt{2}}$

(B) $-1, 0, 1$

(C) $0, \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}$

(D) None of these

$l = \cos 90 = 0$

$m = \cos 135 = -\cos 45 = \frac{-1}{\sqrt{2}}$

$n = \cos 45 = \frac{1}{\sqrt{2}}$

Question

The cartesian equation of a line passing through the point (3,2,1) and is parallel to the vector $2\hat{i}+2\hat{j}-3\hat{k}$ is

x_1, y_1, z_1

$a=2 \mid b=2 \mid c=-3$

(A) $\frac{x+3}{2}=\frac{y+2}{2}=\frac{z+1}{-3}$

(B) $\frac{x-3}{2}=\frac{y-2}{2}=\frac{z-1}{-3}$

(C) $\frac{x+2}{-3}=\frac{y+2}{2}=\frac{z-3}{1}$

(D) $\frac{x-2}{3}=\frac{y-2}{2}=\frac{z+3}{1}$

Question

The equation of the line passing through the point (a, b, c) and parallel to y-axis, is

$\Downarrow$

$x_1 = a$

$y_1 = b$

$z_1 = c$

$\Downarrow$

D.R's $\Rightarrow 0, 1, 0$

(A) $\frac{x-a}{1} = \frac{y-b}{1} = \frac{z-c}{0}$

(B) $\frac{x-a}{0} = \frac{y-b}{0} = \frac{z-c}{1}$

(C) $\frac{x-a}{0} = \frac{y-b}{1} = \frac{z-c}{0}$

(D) $\frac{x-a}{1} = \frac{y-b}{0} = \frac{z-c}{1}$

eq^n is

$$\frac{x-a}{0} = \frac{y-b}{1} = \frac{z-c}{0}$$

Question

$DR \Rightarrow -4, 4, -7$

The angle between the lines passing through the points $(8,2,0),(4,6,-7)$ and $(-3,1,2),(-9,-2,4)$ is

$D.R \Rightarrow -6, -3, 2$

(A) $\cos^{-1}\left(\frac{2}{63}\right)$

(B) $\cos^{-1}\left(\frac{20}{63}\right)$

(C) $\frac{\pi}{2}$

(D) $\frac{\pi}{5}$

$$\cos\theta = \left|\frac{24-12-14}{\sqrt{16+16+49}\sqrt{36+9+4}}\right|$$

$$= \left|\frac{-2}{\sqrt{81}\sqrt{49}}\right|$$

$$= \frac{2}{9(7)} = \frac{2}{63}$$

$$\cos\theta = \frac{2}{63}$$

$$\theta = \cos^{-1}\left(\frac{2}{63}\right)$$

Question

The angle between the lines

$\vec{r} = (2\hat{i} - 5\hat{j} + \hat{k}) + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$ and $\vec{r} = (7\hat{i} - 6\hat{k}) + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$ is

(A) $\cos^{-1}\left(\frac{19}{21}\right)$

(B) $\cos^{-1}\left(\frac{1}{21}\right)$

(C) $\cos^{-1}\left(\frac{9}{19}\right)$

(D) $\cos^{-1}\left(\frac{9}{21}\right)$

Question

The $\boxed{\text{direction ratios of the line}}$ which is perpendicular to the lines

$\underbrace{\frac{x-7}{2} = \frac{y+17}{-3} = \frac{z-6}{1}}_{l_1}$ and $\underbrace{\frac{x+5}{1} = \frac{y+3}{2} = \frac{z-4}{-2}}_{l_2}$ are

A (4, 5, 7)

B (4, −5, 7)

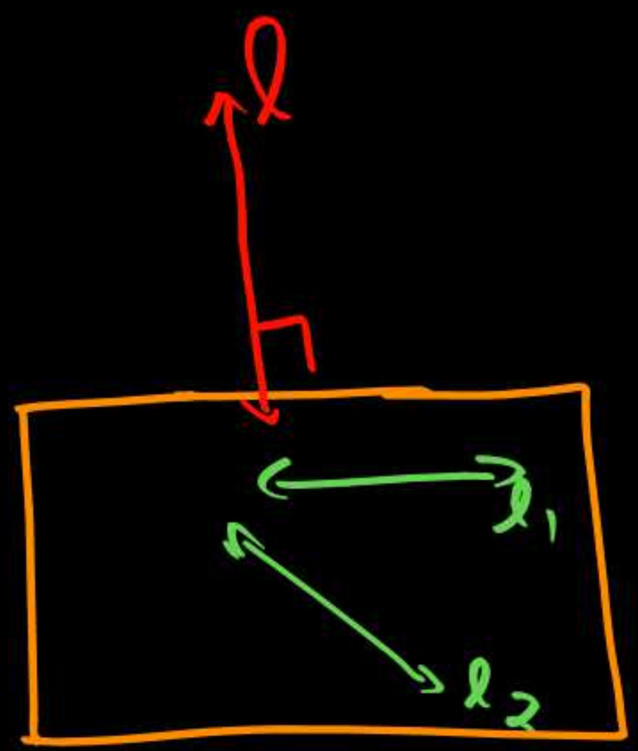
C (4, −5, −7)

D (−4, 5, 7)

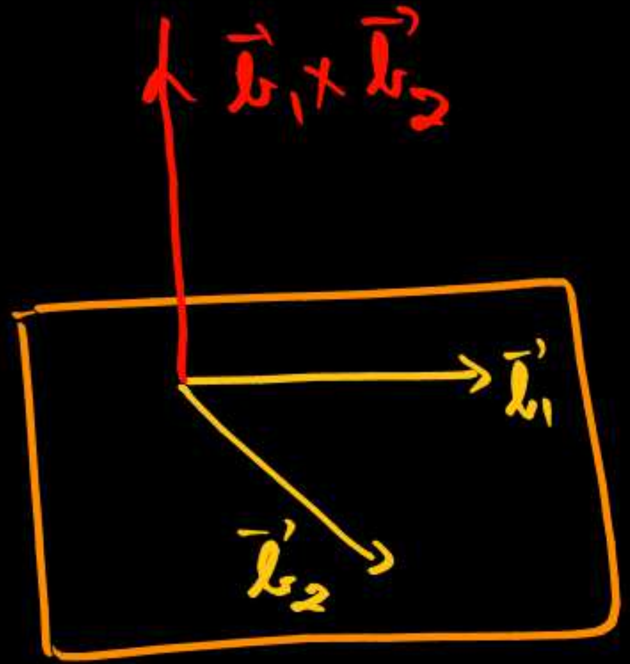
$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} i & j & k \\ 2 & -3 & 1 \\ 1 & 2 & -2 \end{vmatrix}$$

$$= i(4) - j(-5) + \hat{k}(7)$$

$$= 4\hat{i} + 5\hat{j} + 7\hat{k}$$



l
l1
l2



$\vec{b}_1 \times \vec{b}_2$
$\vec{b}_1$
$\vec{b}_2$

Question

$\vec{a}_1 = -i + 3j - 2\hat{k}$

Find the distance of point $(0,7,-7)$ from the line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$.

$\vec{a}_2 = 0i + 7j - 7\hat{k}$

$\vec{b} = -3i + 2j + \hat{k}$

(A) $\sqrt{42}$ units

(B) $\sqrt{40}$ units

(C) $\sqrt{24}$ units

(D) $\sqrt{41}$ units

$$(\vec{a}_2 - \vec{a}_1) \times \vec{b} = \begin{vmatrix} i & j & k \\ 1 & 4 & -5 \\ -3 & 2 & 1 \end{vmatrix}$$

$$= i(14) - j(-14) + k(14)$$

$$= 14(i + j + \hat{k})$$

$$|(\vec{a}_2 - \vec{a}_1) \times \vec{b}| = 14\sqrt{1+1+1} = 14\sqrt{3}$$

$$|\vec{b}| = \sqrt{9+4+1} = \sqrt{14}$$

$$d = \left| \frac{(\vec{a}_2 - \vec{a}_1) \times \vec{b}}{|\vec{b}|} \right|$$

$$= \frac{14\sqrt{3}}{\sqrt{14}} = \sqrt{14}\sqrt{3} = \sqrt{42}$$

Question

$\cos^2\theta = \dfrac{1+\cos 2\theta}{2}$ Replace θ by 3α

$\cos^2 3\alpha = \dfrac{1+\cos 6\alpha}{2}$

If a line makes angles $3\alpha, 3\beta$ and 3γ with the coordinate axes, find the value of $\cos 6\alpha + \cos 6\beta + \cos 6\gamma$.

(A) -1

(B) 0

(C) 1

(D) 8

$x = 3\alpha \Rightarrow l = \cos x = \cos 3\alpha$

$y = 3\beta \quad m = \cos y = \cos 3\beta$

$z = 3\gamma \quad n = \cos z = \cos 3\gamma$

WKT $l^2 + m^2 + n^2 = 1$

$\cos^2 3\alpha + \cos^2 3\beta + \cos^2 3\gamma = 1$

$\dfrac{1+\cos 6\alpha}{2} + \dfrac{1+\cos 6\beta}{2} + \dfrac{1+\cos 6\gamma}{2} = 1$

$\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}(\cos 6\alpha + \cos 6\beta + \cos 6\gamma) = 1$

$\frac{3}{2} + \frac{1}{2}(\cos 6\alpha + \cos 6\beta + \cos 6\gamma) = 1$

$\frac{1}{2}(\cos 6\alpha + \cos 6\beta + \cos 6\gamma) = 1 - \frac{3}{2} = -\frac{1}{2}$

$\cos 6\alpha + \cos 6\beta + \cos 6\gamma = \underline{-1}$

Question

$\cos^2 x = \frac{1+\cos 2x}{2}$ | $\cos^2 2x = \frac{1+\cos 4x}{2}$

If a line is making same angle 2θ with the positive direction of X, Y and Z-axes, then the value of $\theta =$

(A) $\frac{\pi}{4}$

(B) $\cos^{-1}\left(\frac{\sqrt{2}}{3}\right)$

(C) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

(D) none of these

$\boxed{\alpha = \beta = \gamma = 2\theta}$

$l = m = n = \cos 2\theta$

$\therefore l^2 + m^2 + n^2 = 1$

$\cos^2 2\theta + \cos^2 2\theta + \cos^2 2\theta = 1$

$3\cos^2 2\theta = 1$

$\cos^2 2\theta = \frac{1}{3}$

$\cos 2\theta = \frac{1}{\sqrt{3}}$

$2\theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$\theta = \frac{1}{2}\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$l = \cos\alpha = \cos 2\theta$

$m = \cos\beta = \cos 2\theta$

$n = \cos\gamma = \cos 2\theta$

$$\cos^2 2\theta = \frac{1}{3}$$

$$\frac{1+\cos 4\theta}{2} = \frac{1}{3}$$

$$1+\cos 4\theta = \frac{2}{3}$$

$$\cos 4\theta = \frac{2}{3} - 1$$

$$\cos 4\theta = -1/3$$

$$4\theta = \cos^{-1}|-1/3|$$

$$\theta = \frac{1}{4}\cos^{-1}\left(\frac{1}{3}\right)$$

$$\frac{x-3}{2}=\frac{y-4}{5}=\frac{z+2}{6}$$

$\Downarrow$

Arbitrary Point (The point which can lie anywhere on the line)

Let $\frac{x-3}{2}=\frac{y-4}{5}=\frac{z+2}{6}=r$

$\frac{x-3}{2}=r$

$\boxed{x=2r+3}$ | $\boxed{y=5r+4}$ | $\boxed{z=6r-2}$

$\therefore$ Arbitrary Point

$(2r+3, 5r+4, 6r-2)$

Question

Two lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}=r$ and $\frac{x-3}{1}=\frac{y-k}{2}=z$ $\boxed{\text{intersect}}$ at a point, if k is equal to

Find arbitrary values

Substitute in (2)

$x=2r+1$

$y=3r-1$

$z=4r+1$

$\frac{2r+1-3}{1}=\frac{3r-1-k}{2}=4r+1$

Equate the Parts to find 'r'

(These Parts should not have 'k' in it)

$2r-2=4r+1$

$-3=2r$

$r=-3/2$

A) 2/9

B) 1/2

C) 9/2

D) 1/6

use $r = -\frac{3}{2}$ in ②

$$\frac{2(-3/2)-2}{1} = \frac{3(-3/2)-1-k}{2} = 4\left(-\frac{3}{2}\right)+1$$

$$\frac{-\frac{9}{2}-1-k}{2} = -6+1$$

$$-\frac{11}{2}-k = 2(-5) = -10$$

$$k = -\frac{11}{2}+10$$

$$k = \frac{-11+20}{2} = \frac{9}{2}$$

$$k = 9/2$$

Question

If the lines intersect, then find the value of k. give

$$\frac{x-4}{2}=\frac{y-k}{2}=\frac{z}{1} \rightarrow (2)$$

&

$$\frac{x-3}{2}=\frac{y+1}{3}=\frac{z-2}{4}=r$$
(1)

$x=2r+3$

$y=3r-1$

$z=4r+2$

$$\frac{2r-1}{2}=\frac{3r-1-k}{2}=4r+2$$

$2r-1=8r+4$

$-5=6r$

$r=-5/6$

(A) 9/2

(B) 2/9

(C) −3/2

(D) −5/6

use $r = \frac{-5}{6}$ in ②

$$\frac{2\left(-\frac{5}{6}\right)-1}{2} = \frac{3\left(-\frac{5}{6}\right)-1-k}{2} = 4\left(-\frac{5}{6}\right)+2$$

$$-\frac{5}{2}-1-k = 2\left[\frac{-20}{6}+2\right]$$

$$-\frac{7}{2}-k = \frac{-8}{3}$$

$$k = -\frac{7}{2}+\frac{8}{3}$$

$$k = \frac{-21+16}{6}$$

$$k = -5/6$$

Question

The lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}=r$ and $\frac{x-4}{5}=\frac{y-1}{2}=z$ intersect to each other. Find their point of intersection.

$x=2r+1$

$y=3r+2$

$z=4r+3$

$\Downarrow$

$x=-1$
$y=-1$
$z=-1$

$\frac{2r-3}{5}= \quad = 4r+3$

$2r-3=20r+15$

$-18=18r$

$r=-1$

(A) $(-1, 1, -1)$

(B) $(-1, -1, -1)$

(C) $(1, 1, -1)$

(D) $(1, -1, -1)$

Question

Find the shortest distance between the following lines whose vector equations are $\vec{r} = (1+\lambda)\hat{i} + (2-\lambda)\hat{j} + (-1+\lambda)\hat{k}$ and $\vec{r} = (1+\mu)\hat{i} - (1+\mu)\hat{j} + (-1+\mu)\hat{k}$

$\vec{a}_1 = \hat{i} + 2\hat{j} - \hat{k} \mid \vec{b}_1 = \hat{i} - \hat{j} + \hat{k}$ $\qquad$ $\vec{a}_2 = \hat{i} - \hat{j} - \hat{k} \mid \vec{b}_2 = \hat{i} - \hat{j} + \hat{k}$

(A) $2\sqrt{3}$ units

(B) $3\sqrt{3}$ units

(C) $\sqrt{3}$ units

(D) $\sqrt{6}$ units

lines are parallel

$(\vec{a}_2 - \vec{a}_1) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -3 & 0 \\ 1 & -1 & 1 \end{vmatrix}$

$= -3(\hat{i} - \hat{k}) = 3(-\hat{i} + \hat{k})$

$|(\vec{a}_2 - \vec{a}_1) \times \vec{b}| = 3\sqrt{1+1} = 3\sqrt{2}$

$|\vec{b}| = |\vec{b}_1| = |\vec{b}_2|$

$= \sqrt{1+1+1} = \sqrt{3}$

$d = \dfrac{3\sqrt{2}}{\sqrt{3}} = \sqrt{3}\sqrt{2} = \sqrt{6}$

Question

The value of β, so that line $\frac{x-1}{-4} = \frac{-3y-12}{2\beta} = \frac{z+9}{3}$ and $\frac{5-5x}{2\beta} = \frac{y-7}{2} = \frac{3-z}{4}$ intersect at right angle is

$\frac{x-1}{-4} = \frac{-3[y+4]}{2\beta} = \frac{z-(-3)}{3}$ | $\frac{-5(x-1)}{2\beta} = \frac{y-7}{2} = \frac{-1(z-3)}{4}$

÷ by 3 | ÷ by 5

$\frac{x-1}{-12} = \frac{y-(-4)}{-2\beta} = \frac{z-(-3)}{9}$ | $\frac{x-1}{-2\beta} = \frac{y-7}{10} = \frac{z-3}{-20}$

$24\beta - 20\beta - 180 = 0$

$4\beta = 180$

$\beta = 45$

(A) 40

(B) 0

(C) 45

(D) 12

if eq^n of lines

$$l_1 = \frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$$

$$l_2 = \frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$$

If l_1 is parallel to l_2

Then

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Question

The value of β for which the lines $\frac{x-7}{2}=\frac{2y-4}{3}=\frac{1-8z}{\beta}$ and $\frac{x+5}{2}=\frac{2y-8}{3}=\frac{z-2}{4}$ are parallel to each other is

$$\frac{x-7}{2}=\frac{2(y-2)}{3}=\frac{-8(z-1/8)}{\beta}$$

$$\frac{x-(-5)}{2}=\frac{2(y-4)}{3}=\frac{z-2}{4}$$

$3/2$, $-\beta/8$

(A) 32

(B) −32

(C) 4

(D) −4

$$\frac{2}{2}=\frac{3/2}{3/2}=\frac{-\beta/8}{4}$$

$$1=\frac{-\beta/8}{4}$$

$$-4=\beta/8$$

$$\beta=-32$$

$(47)^2$

4^2	$4(7)(2)$	7^2
16	56	49

$16+6=22$

$56+4=60$

2209

Question

The equation of a line is given by $\frac{4-x}{3} = \frac{y+3}{3} = \frac{z+2}{6}$. Write the direction cosines of a line parallel to the above line.

$\Downarrow$

$$\frac{x-4}{-3} = \frac{y-(-3)}{3} = \frac{z-(-2)}{6}$$

(A) $\frac{-1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}$

(B) $\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}$

(C) $\frac{-1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}$ ✓

(D) $\frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}}$

D.R ; $-3, 3, 6$

$$r = \sqrt{9+9+36} = \sqrt{54} = \sqrt{9 \times 6} = 3\sqrt{6}$$

$$l = \frac{-3}{3\sqrt{6}} = \frac{-1}{\sqrt{6}}$$

$$m = \frac{1}{\sqrt{6}}$$

$$n = \frac{2}{\sqrt{6}}$$

Question

D.R ⇒ 1, 2, −2

If the line joining $(2,3,-1)$ and $(3,5,-3)$ is perpendicular to the line joining $(1,2,3)$ and $(3,5,\lambda)$, then λ is

D.R ⇒ 2, 3, λ−3

$1(2)+2(3)+(-2)(\lambda-3)=0$

$2+6=2\lambda-6$

$14=2\lambda$

$\lambda=7$

(A) -3

(B) 2

(C) 5

(D) 7

Question

$\vec{r} = (\quad) + t(-3\hat{i} + 2\hat{j} + 6\hat{k})$

The angle between the straight lines $\vec{r} = (2-3t)\hat{i} + (1+2t)\hat{j} + (2+6t)\hat{k}$ and $\vec{r} = (1+4s)\hat{i} + (2-1s)\hat{j} + (8s-1)\hat{k}$ is

$\vec{r} = (\quad) + s(4\hat{i} - \hat{j} + 8\hat{k})$

(A) $\cos^{-1}\left(\frac{\sqrt{41}}{34}\right)$

(B) $\cos^{-1}\left(\frac{21}{34}\right)$

(C) $\cos^{-1}\left(\frac{43}{63}\right)$

(D) $\cos^{-1}\left(\frac{34}{63}\right)$

$$\cos\theta = \left|\frac{-12-2+48}{\sqrt{9+4+36}\,\sqrt{16+1+64}}\right|$$

$$= \left|\frac{34}{7(9)}\right|$$

$$\theta = \cos^{-1}\left(\frac{34}{63}\right)$$

Question

$\frac{x-1}{2}=\frac{y-(-1)}{3}=\frac{z-0}{-1}$

$\Rightarrow \frac{x-(-1)}{5}=\frac{y-2}{1}=\frac{z-2}{0}$

The lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z}{-1}$ and $\frac{x+1}{5}=\frac{y-2}{1}, z=2$

$\vec{a}_2-\vec{a}_1=-2\hat{i}+3\hat{j}+2\hat{k}$

$\vec{a}_1=\hat{i}-\hat{j}+0\hat{k} \mid \vec{b}_1=2\hat{i}+3\hat{j}-\hat{k}$

$\vec{a}_2=-\hat{i}+2\hat{j}+2\hat{k} \mid \vec{b}_2=5\hat{i}+\hat{j}+0\hat{k}$

$$\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & -1\\ 5 & 1 & 0\end{vmatrix}$$

$$=\hat{i}(1)-\hat{j}(+5)+\hat{k}(-7)$$

$$(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2)=-2+3(-5)+2(-7)$$
$$=-2-15-14$$
$$\neq 0$$

$d\neq 0$

(A) ~~Parallel~~ $\Rightarrow \frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

(B) Intersecting $\Rightarrow d=0$

$d=$ distance b/w skew lines

(C) Non intersecting $\Rightarrow d\neq 0$
(Skew lines)

(D) None of these

Question

The lines $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and $\frac{x-1}{-2}=\frac{y-2}{-4}=\frac{z-3}{-6}$ are

$a_1 \quad b_1 \quad c_1 \quad a_2 \quad b_2 \quad c_2$

(A) perpendicular

$\frac{1}{-2}=\frac{a_1}{a_2}$

(B) parallel

$\frac{b_1}{b_2}=\frac{2}{-4}=-\frac{1}{2}$

(C) intersecting

$\frac{c_1}{c_2}=\frac{3}{-6}=\frac{-1}{2}$

(D) skew (non intersection)

Question

Find the value of λ so that the lines $\frac{-(x-1)}{3} = \frac{7(y-2)}{2\lambda} = \frac{z-3}{2}$ and $\frac{-7(x-1)}{3\lambda} = \frac{y-5}{1} = \frac{-(z-6)}{5}$ are perpendicular to each other.

(A) 70/11

(B) 7/11

(C) 17/11

(D) 71/11

$-3\left(-\frac{3\lambda}{7}\right) + \left(\frac{2\lambda}{7}\right)(1) + 2(-5) = 0$

$\frac{9\lambda}{7} + \frac{2\lambda}{7} = 10$

$\frac{11\lambda}{7} = 10$

$\lambda = \frac{70}{11}$

Question

What is the value of m, such that $<\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{8}}, m>$ represent direction cosines of a line?

$$l^2+m^2+n^2=1$$

$$\frac{1}{3}+\frac{1}{8}+m^2=1$$

$$\frac{11}{24}+m^2=1$$

$$m^2=1-\frac{11}{24}$$

$$m^2=\frac{13}{24}$$

$$m=\pm\frac{\sqrt{13}}{2\sqrt{6}}$$

A. $\frac{\pm 11}{\sqrt{24}}$

B. $\frac{\pm\sqrt{13}}{2\sqrt{6}}$

C. $\frac{\pm 13}{2\sqrt{6}}$

D. $\frac{\pm\sqrt{11}}{2\sqrt{6}}$

These type of PYQ's are asked earlier in IIT

$\sin\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(1) If $2\sin^{-1}x = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$ Find the restrictions of x.

Soln:

Put $\sin^{-1}x = \theta$

$x = \sin\theta$

Range of sin

$\sin\theta = x$

$\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-x^2}$

WKT

$-\frac{\pi}{2} \le \sin^{-1}\left(2x\sqrt{1-x^2}\right) \le \frac{\pi}{2}$

Apply sin

$-1 \le 2x\sqrt{1-x^2} \le 1$

$-1 \le 2\sin\theta\cos\theta \le 1$

$-1 \le \sin 2\theta \le 1$

Apply $\sin^{-1}$

$-\frac{\pi}{2} \le 2\theta \le \frac{\pi}{2}$

$-\frac{\pi}{4} \le \theta \le \frac{\pi}{4}$

$$-\frac{\pi}{4} \le \theta \le \frac{\pi}{4}$$

$$\Downarrow$$

$$-\frac{\pi}{4} \le \sin^{-1}x \le \frac{\pi}{4}$$

Apply sin

$$-\frac{1}{\sqrt{2}} \le x \le \frac{1}{\sqrt{2}}$$

② If $3\sin^{-1}x = \sin^{-1}(3x - 4x^3)$ Find the restrictions of 'x'

Soln:

Put $\sin^{-1}x = \theta$

$x = \sin\theta$

WKT

$-\frac{\pi}{2} \le \sin^{-1}(3x - 4x^3) \le \frac{\pi}{2}$

$-1 \le 3x - 4x^3 \le 1$

$-1 \le 3\sin\theta - 4\sin^3\theta \le 1$

$-1 \le \sin 3\theta \le 1$

$-\frac{\pi}{2} \le 3\theta \le \frac{\pi}{2}$

$-\frac{\pi}{6} \le \theta \le \frac{\pi}{6}$

$-\frac{\pi}{6} \le \sin^{-1}x \le \frac{\pi}{6}$

$-\frac{1}{2} \le x \le \frac{1}{2}$

③ If $2\cos^{-1}x = \sin^{-1}(2x\sqrt{1-x^2})$ find the restrictions of x

Soln:

Let $\cos^{-1}x = \theta$

$\cos\theta = x$

$\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-x^2}$

WKT

$-\frac{\pi}{2} \le \sin^{-1}(2x\sqrt{1-x^2}) \le \frac{\pi}{2}$

$-1 \le 2x\sqrt{1-x^2} \le 1$

$-1 \le 2\cos\theta\sin\theta \le 1$

$-1 \le \sin 2\theta \le 1$

$-\frac{\pi}{2} \le 2\theta \le \frac{\pi}{2}$

$-\frac{\pi}{4} \le \theta \le \frac{\pi}{4}$

$-\frac{\pi}{4} \le \cos^{-1}x \le \frac{\pi}{4} \rightarrow ①$

WKT range of $\cos^{-1}x = [0, \pi]$

$0 \le \cos^{-1}x \le \pi \rightarrow ②$

$① \cap ②$

$0 \le \cos^{-1}x \le \frac{\pi}{4}$

$0 \le \cos^{-1} x \le \frac{\pi}{4}$

Apply cos

$\cos 0 \geq x \geq \cos\frac{\pi}{4}$

$1 \geq x \geq \frac{1}{\sqrt{2}}$

$\Downarrow$

$\frac{1}{\sqrt{2}} \le x \le 1$

$x \in \left[\frac{1}{\sqrt{2}}, 1\right]$

cos func is a decreasing func in $[0, \pi]$

$\therefore$ inequality reverses.

(4) If $3\cos^{-1}x = \cos^{-1}(4x^3 - 3x)$ find the restrictions of x

Soln:

Put $\cos^{-1}x = \theta$

$x = \cos\theta$

WKT

$$0 \le \cos^{-1}(4x^3 - 3x) \le \pi$$

$$1 \geq 4x^3 - 3x \geq -1$$

$$\Downarrow$$

$$-1 \le 4\cos^3\theta - 3\cos\theta \le 1$$

$$-1 \le \cos 3\theta \le 1$$

$$\Downarrow$$

$$\cos^{-1}(1) = 0$$

$$\pi \geq 3\theta \geq 0$$

$$\Downarrow$$

$$0 \le 3\theta \le \pi$$

$$0 \le \theta \le \frac{\pi}{3}$$

$$\Downarrow$$

$$0 \le \cos^{-1}x \le \frac{\pi}{3}$$

$$1 \geq x \geq \frac{1}{2}$$

$$\Downarrow$$

$$\frac{1}{2} \le x \le 1$$

Thank

You