Test 03

ultimate kcet crash course 2026

Maths

Q1 If $\tan\theta = -\frac{4}{3}$, then sin q is
(A) -4/5 but not 4/5
(B) -4/5 or 4/5
(C) 4/5 but not -4/5
(D) None of these

Q2 $\sin 20^o \cdot \sin 40^o \ \sin 60^o \sin 80^o =$
(A) -3/16
(B) 5/16
(C) 3/16
(D) -1/16

Q3 The value of $\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)$ is
(A) 1/2
(B) cos p/8
(C) 1/8
(D) $\frac{1+\sqrt{2}}{2\sqrt{2}}$

Q4 $x+iy=\sqrt{\frac{a+ib}{c+id}}$, then x^2+y^2 is equal to
(A) $\frac{a^2-b^2}{c^2+d^2}$
(B) $\left(\frac{a^2+b^2}{c^2+d^2}\right)^{1/2}$
(C) $\frac{a^2+b^2}{c^2+d^2}$
(D) None of these

Q5 If $z=\frac{4}{1-i}$, then $\overline{z}$ is equal to (where $\overline{z}$ is complex conjugate of z)
(A) 2(1 + i)
(B) (1 + i)
(C) 2(1-i)
(D) 4(1+i)

Q6 The real part of $\frac{(1+i)^2}{(3-i)}$ is
(A) 1/3
(B) 1/5
(C) -1/3
(D) None of these

Q7 If $\frac{|x-2|}{x-2} \geq 0$, then
(A) $x \in [2,\ \infty)$
(B) $x \in (2,\ \infty)$
(C) $x \in (-\infty,\ 2)$
(D) $x \in (-\infty,\ 2]$

Q8 If $|x+3| \geq 10$, then
(A) $x \in (-13,\ 7]$
(B) $x \in (-13,\ 7)$
(C) $x \in (-\infty,\ -13] \cup [7,\ \infty)$
(D) $x \in (-\infty,\ -13) \cup [7,\ \infty)$

Q9 In an examination there are three multiple choice questions and each questions have 4 choices. Number of ways in which a student can fail to get all answers correct, is
(A) 11
(B) 12
(C) 27
(D) 63

Q10 The number of ways in which the letters of the word TRIANGLE can be arranged such that two vowels do not occur together is
(A) 1200
(B) 2400
(C) 14400
(D) None of these

Q11 If $\left(1+x-2x^2\right)^6 = 1+a_1x+a_2x^2+\ldots.+a_{12}x^{12}$, then the expression $a_2+a_4+a_6+\ldots+a_{12}$ has the value
(A) 32
(B) 63
(C) 64
(D) None of these

Q12 Which term of the G.P. 2, 1 1/2, 1/4,.... is 1/128?
(A) 9th
(B) 8th
(C) 7th
(D) 5th

Q13 The third term of a G.P. is 42. Find the product of its first five terms.

(A) 42 (B) $(42)^5$

(C) 98 (D) $(25)^5$

Q14 The slopes of the line which makes an angle 45^o with the line 3x - y = -5 are

(A) 1, -1 (B) 1/2, -1

(C) 1, 1/2 (D) -2, 1/2

Q15 Find the angle between the x-axis and the line joining the points (3, 1) and (4, -2)

(A) 130^o (B) 135^o

(C) 150^o (D) None of these

Q16 If p is the length of perpendicular from origin to the line whose intercepts on the axes are a and b, $\frac{1}{a^2}+\frac{1}{b^2}$ is equal to

(A) p^2 (B) $1/p^2$

(C) $2p^2$ (D) $1/2p^2$

Q17 The eccentricity of the hyperbola $36x^2-25y^2=900$ is

(A) $\sqrt{61}/5$ (B) 5

(C) 6 (D) $\frac{\sqrt{31}}{5}$

Q18 $\lim\limits_{x\to 0}\frac{\tan x-\sin x}{x^3}=$

(A) 0 (B) 1

(C) 1/2 (D) -1/2

Q19 $\lim\limits_{x\to 0}\frac{\sin(2+x)-\sin(2-x)}{x}=$

(A) cos 1 (B) cos 2

(C) 2 cos 2 (D) 0

Q20 The standard deviation of 9, 16, 23, 30, 37, 44, 51 is

(A) 7 (B) 9

(C) 12 (D) 14

Q21 If x_1, x_2,.....x_n are n observations such that $\sum_{i=1}^{n} x_i^2=400$ and $\sum_{i=1}^{n} x_i=80$ then least value of n is

(A) 18 (B) 12

(C) 15 (D) 16

Q22 The relation R = {(1, 1), (2, 2), (3, 3)} on the set {1, 2, 3} is

(A) Symmetric only

(B) Reflexive only

(C) An equivalence relation

(D) Transitive only

Q23 Let A = {1, 2, 3, 4} and B = {1, 2}. Then the number of onto functions from A onto B is

(A) 14 (B) 16

(C) 12 (D) 8

Q24 The value of $\tan\left[2\tan^{-1}\frac{1}{5}-\frac{\pi}{4}\right]$ is

(A) 0 (B) 1

(C) -7/17 (D) None of these

Q25 For formula $\cos^{-1}\frac{1-x^2}{1+x^2}=2\tan^{-1}x$ holds only for

(A) $x\in R$

(B) $|x|\le 1$

(C) $x\in(-1,1]$

(D) $x\in[0,\ \infty)$

Q26 If $A=\begin{bmatrix}1&0&0\\0&1&0\\a&b&-1\end{bmatrix}$ then A^2 =

(A) Unit matrix (B) Null matrix

(C) A (D) -A

Q27 If $\begin{bmatrix}1&x&1\end{bmatrix}\begin{bmatrix}1&2&3\\0&5&1\\0&3&2\end{bmatrix}\begin{bmatrix}x\\1\\-2\end{bmatrix}=0$, then x =

(A) 3/4 (B) 1

(C) 5/4 (D) 1/4

Q28 The maximum value of $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1+\cos\theta & 1 & 1 \end{vmatrix}$ is (q is real number)

(A) 1/2
(B) $\sqrt{3}/2$
(C) $\sqrt{2}$
(D) $\frac{2\sqrt{3}}{4}$

Q29 If $f(x)=\begin{vmatrix} 0 & x-a & x-b \\ x+a & 0 & x-c \\ x+b & x-c & 0 \end{vmatrix}$, then

(A) f(a) = 0
(B) f(b) = 0
(C) f(0) = 0
(D) f(1) = 0

Q30 If $y = ae^{mx} + be^{-mx}$, then $\frac{d^2y}{dx^2} - m^2y =$

(A) $m^2\left(ae^{mx} - be^{-mx}\right)$
(B) 1
(C) 0
(D) None of these

Q31 If $x = a\sin\theta$ and $y = b\cos\theta$, then $\frac{d^2y}{dx^2}$ is

(A) $\frac{a}{b^2}\sec^2\theta$
(B) $\frac{-b}{a^2}\sec^2\theta$
(C) $\frac{-b}{a^2}\sec^3\theta$
(D) $\frac{-b^2}{a^2}\sec^2\theta$

Q32 If $f(x)=\begin{cases} \frac{\log x}{x-1} & \text{if } x \neq 1 \\ k & \text{if } x = 1 \end{cases}$, is continuous at x = 1, then the value of k is

(A) 0
(B) -1
(C) 1
(D) e

Q33 The function f(x) = $\cot^{-1}$x + x increases in the interval

(A) (1, ¥)
(B) (-1, ¥)
(C) (-¥, ¥)
(D) (0, ¥)

Q34 The function f(x) = ax + b is strictly decreasing for all $x \in R$, if and only if,

(A) a = 0
(B) a < 0
(C) a > 0
(D) None of these

Q35 $\int \frac{\sin x - \cos x}{\sqrt{1-\sin 2x}}dx,\ x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right) =$

(A) x + c
(B) -x + c
(C) ±x + c
(D) None

Q36 $\int \frac{dx}{1+\sin 2x} =$

(A) $\frac{1}{1+\tan x} + c$
(B) $\frac{-1}{1+\tan x} + c$
(C) $\frac{1}{1+\cot x} + c$
(D) $\frac{-1}{1+\cot x} + c$

Q37 $\int_{-1/2}^{1/2} \left[[x] + \log\left(\frac{1+x}{1-x}\right)\right]dx =$

(A) -1/2
(B) 0
(C) 1
(D) 2 log 1/2

Q38 $\int_0^6 |x-3|\ dx =$

(A) 6
(B) 9
(C) 0
(D) 12

Q39 Area bounded by one arch of y = sin 4x and x-axis is

(A) 1/2
(B) 3/2
(C) 2
(D) 1

Q40 Area under the curve y = sin 2x + cos 2x between the ordinates x = 0 and x = p/4 is

(A) 2
(B) 1
(C) 1/2
(D) $\sqrt{2}$

Q41 If the integrating factor of the differential equation $\frac{dy}{dx} + P(x)y = Q(x)$ is x, then P(x) is

(A) x
(B) $x^2/2$
(C) 1/x
(D) $1/x^2$

Q42 The slope at any point of a curve y = f(x) is given $\frac{dy}{dx} = 3x^2$ and it passes through (-1, 1). The equation of the curve is

(A) y = x^3 + 2 (B) y = $-x^3$ - 2

(C) y = $3x^3$ + 4 (D) y = $-x^3$ + 2

Q43 if q be the angle between the unit vectors $\vec{a}$ and $\vec{b}$, then $\vec{a} - \sqrt{2}b$ will be a unit vector if q =

(A) p/6 (B) p/4

(C) p/3 (D) 2p/3

Q44 if |a| = 3, |b| = 4 and the angle between a and b be 120^o, then |4a + 3b| =

(A) 25 (B) 12

(C) 13 (D) 7

Q45 What are the DR's of vector parallel to (2, -1, 1) and (3, 4, -1)?

(A) (1, 5, -2) (B) (-2, -5, 2)

(C) (-1, 5, 2) (D) (-1, -5, 2)

Q46 The point of intersection of lines $\frac{x-5}{3} = \frac{y-7}{-1} = \frac{z+2}{1}$ and $\frac{x+3}{-36} = \frac{y-3}{2} = \frac{z-6}{4}$ is

(A) (5, 7, -2)

(B) (-3, 3, 6)

(C) (2, 10, 4)

(D) (21, 5/3, 10/3)

Q47 If P(A ∪ B)=0.83, P(A) = 0.3 and P(B) = 0.6 then the events will be

(A) Dependent

(B) Independent

(C) Cannot say anything

(D) None of these

Q48 The probability of India winning a test match against south Africa is 1/2 assuming independence from match to match played. The probability that in a match series India's second win occurs on the third match played

(A) 1/8 (B) 1/2

(C) 1/4 (D) 2/3

Q49 If four persons independently solve a certain problem correctly with probabilities 1/2, 3/4, 1/4 and 1/8. Then, the probability that the problem is solved correctly by at least one of them, is

(A) $\frac{235}{256}$ (B) $\frac{21}{256}$

(C) $\frac{3}{256}$ (D) $\frac{253}{256}$

Q50 A and B throw a dice alternatively till one of them gets a six and wins the game. If A starts the game first, then the probability of A winning the game is

(A) 3/11 (B) 5/11

(C) 6/11 (D) 7/11

Q51 The general solution of the differential equation $\frac{dy}{dx} = e^{\frac{x^2}{2}} + xy$ is

(A) $y = ce^{-\frac{x^2}{2}}$

(B) $y = ce^{\frac{x^2}{2}}$

(C) $y = (x + c)e^{\frac{x^2}{2}}$

(D) $y = (c - x)e^{\frac{x^2}{2}}$

Q52 General solution of $\frac{dy}{dx} + y \tan x = \sec x$ is

(A) y sec x = tan x + c

(B) y tan x = sec x + c

(C) tan x = y tan x + c

(D) x sec x = tan y + c

Q53 $\int \frac{dx}{x(x+1)}$ equals

(A) $\ln\left|\frac{x+1}{x}\right| + c$

(B) $\ln\left|\frac{x}{x+1}\right| + c$

(C) $\ln\left|\frac{x-1}{x}\right| + c$

(D) $\ln\left|\frac{x-1}{x+1}\right| + c$

Q54 $\int \frac{dx}{\sqrt{2x-x^2}} =$

(A) $\cos^{-1}(x - 1) + c$

(B) $\sin^{-1}(x - 1) + c$

(C) $\cos^{-1}(1 + x) + c$

(D) $\sin^{-1}(1 - x) + c$

Q55 If $\sin^2 x + 2\cos y + xy = 0,$ then $\frac{dy}{dx} =$

(A) $\frac{y+2\sin x}{2\sin y+x}$ (B) $\frac{y+\sin 2x}{2\sin y+x}$

(C) $\frac{y+2\sin x}{\sin y+x}$ (D) None of these

Q56 If x = 2 cos t + cos 2t, y = 2 sin t - sin 2t, then $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$ is

(A) $-\left(1+\sqrt{2}\right)$ (B) $1-\sqrt{2}$

(C) $1/\sqrt{2}$ (D) $\sqrt{2}$

Q57 Let $f(x)= \frac{1-\cos px}{x\cdot\sin x}$, $x \neq 0$ $f(0)= \frac{1}{2}$.. If f is continuous at x = 0 then p =

(A) 1 or -1 (B) -2

(C) 2 (D) 1/2

Q58 Let $f(x)=\begin{cases} x^2 & x \leq 0 \\ ax+b & x > 0 \end{cases}$ The values of a and b for which the function f(x) is continuous on the whole real line is

(A) a = 1, b = 0

(B) a = 0, b = 1

(C) a = 0, b = 2

(D) b = 0, a any real number

Q59 Solve the system of equations x + 2y + z = 4, -x + y + z = 0 and x - 3y + z = 4.

(A) x = 2, y = 0, z = 2

(B) x = 2, y = 0, z = -2

(C) x = -2, y = 2, z = 0

(D) x = -2, y = 0, z = 2

Q60 If $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ and AB = O, then B =

(A) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ (D) $\begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}$

Vernacular

Answer Key

Q1	B
Q2	C
Q3	C
Q4	B
Q5	C
Q6	D
Q7	B
Q8	C
Q9	D
Q10	C
Q11	D
Q12	A
Q13	B
Q14	D
Q15	D
Q16	B
Q17	A
Q18	C
Q19	C
Q20	D
Q21	D
Q22	C
Q23	A
Q24	C
Q25	D
Q26	A
Q27	C
Q28	A
Q29	C
Q30	C
Q31	C
Q32	C
Q33	C
Q34	B
Q35	A
Q36	B
Q37	A
Q38	B
Q39	A
Q40	B
Q41	C
Q42	A
Q43	B
Q44	B
Q45	A
Q46	D
Q47	A
Q48	C
Q49	A
Q50	C
Q51	C
Q52	A
Q53	B
Q54	B
Q55	D
Q56	B
Q57	A
Q58	D
Q59	A
Q60	D

Hints & Solutions

Note: scan the QR code to watch video solution

Q1 Text Solution:

$\tan\theta = -\frac{4}{3}$

$\sec^2\theta = 1 + \tan^2\theta$

$\sec^2\theta = 1 + \left(-\frac{4}{3}\right)^2 = 1 + \frac{16}{9} = \frac{25}{9}$

$\cos^2\theta = \frac{9}{25}$

$\cos\theta = \pm\frac{3}{5}$

$\sin\theta = \tan\theta \cdot \cos\theta$

$\sin\theta = \left(-\frac{4}{3}\right)\left(\pm\frac{3}{5}\right)$

$\sin\theta = \mp\frac{4}{5}$

$\sin\theta = -\frac{4}{5}\text{or}\frac{4}{5}$

Video Solution:

Q2 Text Solution:

$\sin 20^\circ \sin 40^\circ \sin 60^\circ \sin 80^\circ$

$= \frac{\sqrt{3}}{2}[\sin 20^\circ \sin(60^\circ - 20^\circ)\sin(60^\circ + 20^\circ)]$

$= \frac{\sqrt{3}}{2}\left[\frac{1}{4}\sin(3 \times 20^\circ)\right]$

$= \frac{\sqrt{3}}{2} \times \frac{1}{4} \times \frac{\sqrt{3}}{2} = \frac{3}{16}$

Video Solution:

Q3 Text Solution:

$\cos\frac{7\pi}{8} = \cos\left(\pi - \frac{\pi}{8}\right) = -\cos\frac{\pi}{8}$

$\cos\frac{5\pi}{8} = \cos\left(\pi - \frac{3\pi}{8}\right) = -\cos\frac{3\pi}{8}$

$\left(1 + \cos\frac{\pi}{8}\right)\left(1 + \cos\frac{3\pi}{8}\right)\left(1 - \cos\frac{3\pi}{8}\right)\left(1 - \cos\frac{\pi}{8}\right)$

$\left(1 - \cos^2\frac{\pi}{8}\right)\left(1 - \cos^2\frac{3\pi}{8}\right)$

$\sin^2\frac{\pi}{8} \cdot \sin^2\frac{3\pi}{8}$

$\frac{1}{4}\left(2\sin^2\frac{\pi}{8}\right)\left(2\sin^2\frac{3\pi}{8}\right)$

$\frac{1}{4}\left(1 - \cos\frac{\pi}{4}\right)\left(1 - \cos\frac{3\pi}{4}\right)$

$\frac{1}{4}\left(1 - \frac{1}{\sqrt{2}}\right)\left(1 + \frac{1}{\sqrt{2}}\right)$

$\frac{1}{4}\left(1 - \frac{1}{2}\right) = \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8}$

Video Solution:

Q4 Text Solution:

$x + iy = \sqrt{\frac{a+ib}{c+id}}$

$\left|x + iy\right|^2 = \left|\sqrt{\frac{a+ib}{c+id}}\right|^2$

$x^2 + y^2 = \frac{|a+ib|}{|c+id|}$

$x^2 + y^2 = \frac{\sqrt{a^2+b^2}}{\sqrt{c^2+d^2}} = \left(\frac{a^2+b^2}{c^2+d^2}\right)^{1/2}$

Video Solution:

Q5 Text Solution:

$$z = \frac{4}{1-i} = \frac{4(1+i)}{(1-i)(1+i)} = \frac{4(1+i)}{2} = 2 + 2i$$

$$\overline{z} = 2 - 2i = 2(1 - i)$$

Video Solution:

Q6 Text Solution:

$$\frac{(1+i)^2}{3-i}$$

$$(1 + i)^2 = 1 + 2i + i^2 = 1 + 2i - 1 = 2i$$

$$\frac{2i}{3-i} \cdot \frac{3+i}{3+i} = \frac{6i+2i^2}{9-i^2}$$

$$\frac{6i-2}{9+1} = \frac{-2+6i}{10}$$

$$-\frac{2}{10} + \frac{6}{10}i = -\frac{1}{5} + \frac{3}{5}i$$

$$\mathrm{Re}\left(-\frac{1}{5} + \frac{3}{5}i\right) = -\frac{1}{5}$$

Video Solution:

Q7 Text Solution:

$$\frac{|x-2|}{x-2} \geq 0$$

$$|x - 2| \geq 0 \text{ for all } x \in \mathbb{R}$$

$$x - 2 \neq 0 \quad \Rightarrow x \neq 2$$

$$\frac{|x-2|}{x-2} > 0 \ \Rightarrow x - 2 > 0$$

$$x > 2$$

$$x \in (2, \infty)$$

Video Solution:

Q8 Text Solution:

$$|x + 3| \geq 10$$

$$x + 3 \geq 10 \text{ or } x + 3 \leq -10$$

$$x \geq 7 \text{ or } x \leq -13$$

$$x \in (-\infty, -13] \cup [7, \infty)$$

Video Solution:

Q9 Text Solution:

N = 3 questions

Choices per question = 4

Total ways = 4 x 4 x 4 = 64

Correct ways = 1 x 1 x 1 = 1

Ways to fail = 64 - 1 v= 63

Video Solution:

Q10 Text Solution:

Word: TRIANGLE

Total letters: 8

Vowels (A, I, E): 3

Consonants (T, R, N, G, L): 5

Consonants arrangement: 5! = 120

Gaps for vowels: _C_C_C_C_C_(6 gaps)

Vowel placement: $^6P_3$ = 6 x 5 x 4 = 120

Total ways: 120 x 120 = 14400

Video Solution:

Q11 Text Solution:

$$\left(1+x-2x^2\right)^6=1+a_1x+a_2x^2+a_3x^3+\ldots+a_{12}x^{12}$$

$x=1:$

$$(1+1-2)^6=1+a_1+a_2+a_3+\ldots+a_{12}$$

$$0=1+a_1+a_2+a_3+\ldots+a_{12} \quad \ldots.(1)$$

$x=-1:$

$$\left(1-1-2(-1)^2\right)^6=1-a_1+a_2-a_3+\ldots+a_{12}$$

$$(-2)^6=1-a_1+a_2-a_3+\ldots+a_{12}$$

$$64=1-a_1+a_2-a_3+\ldots+a_{12} \quad \ldots.(2)$$

$(1)+(2)$

$$64=2(1+a_2+a_4+\ldots+a_{12})$$

$$32=1+a_2+a_4+\ldots+a_{12}$$

$$a_2+a_4+a_6+\ldots+a_{12}=32-1=31$$

Video Solution:

Q12 Text Solution:

$2,\ 1,\ \frac{1}{2},\ \frac{1}{4},\ldots$

$a=2$

$r=\frac{1}{2}$

$a_n=\frac{1}{128}$

$a_n=ar^{n-1}$

$\frac{1}{128}=2\left(\frac{1}{2}\right)^{n-1}$

$\frac{1}{256}=\left(\frac{1}{2}\right)^{n-1}$

$\left(\frac{1}{2}\right)^8=\left(\frac{1}{2}\right)^{n-1}$

$8=n-1$

$n=9$

Video Solution:

Q13 Text Solution:

$a^3 = ar^2 = 42$

Product = $a \cdot (ar) \cdot (ar^2) \cdot (ar^3) \cdot (ar^4)$

Product = a^5r^{10}

Product = $(ar^2)^5$

Product = $(42)^5$

Video Solution:

Q14 Text Solution:

$3x - y = -5 \Rightarrow y = 3x + 5 \Rightarrow m_1 = 3$

$\tan 45^\circ = \left|\frac{m-3}{1+3m}\right|$

$1 = \left|\frac{m-3}{1+3m}\right|$

$1 = \frac{m-3}{1+3m} \Rightarrow 1 + 3m = m - 3 \Rightarrow 2m = -4 \Rightarrow m = -2$

$-1 = \frac{m-3}{1+3m} \Rightarrow -1 - 3m = m - 3 \Rightarrow -4m = -2 \Rightarrow m = \frac{1}{2}$

Video Solution:

Q15 Text Solution:

$m = \frac{-2-1}{4-3} = \frac{-3}{1} = -3$

$\tan \theta = -3$

$\theta = \tan^{-1}(-3)$

$135^\circ \Rightarrow \tan 135^\circ = -1$

$150^\circ \Rightarrow \tan 150^\circ = -1/\sqrt{3}$

Video Solution:

Q16 Text Solution:

$\frac{x}{a} + \frac{y}{b} = 1 \Rightarrow \frac{x}{a} + \frac{y}{b} - 1 = 0$

$p = \frac{\left|\frac{0}{a} + \frac{0}{b} - 1\right|}{\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2}}$

$p = \frac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}}$

$p^2 = \frac{1}{\frac{1}{a^2} + \frac{1}{b^2}}$

$\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{p^2}$

Video Solution:

Q17 Text Solution:

$36x^2 - 25y^2 = 900$

$\frac{36x^2}{900} - \frac{25y^2}{900} = 1$

$\frac{x^2}{25} - \frac{y^2}{36} = 1$

$a^2 = 25, b^2 = 36$

$e = \sqrt{1 + \frac{b^2}{a^2}}$

$e = \sqrt{1 + \frac{36}{25}}$

$e = \sqrt{\frac{25+36}{25}}$

$e = \sqrt{\frac{61}{25}}$

$e = \frac{\sqrt{61}}{5}$

Video Solution:

Q18 **Text Solution:**

$$\lim_{x\to 0} \frac{\tan x - \sin x}{x^3}$$

$$= \lim_{x\to 0} \frac{\frac{\sin x}{\cos x} - \sin x}{x^3}$$

$$= \lim_{x\to 0} \frac{\sin x(1-\cos x)}{x^3 \cos x}$$

$$= \lim_{x\to 0} \left(\frac{\sin x}{x}\right)\left(\frac{1-\cos x}{x^2}\right)\left(\frac{1}{\cos x}\right)$$

$$= 1 \cdot \frac{1}{2} \cdot 1 = \frac{1}{2}$$

Video Solution:

Q19 **Text Solution:**

$$\lim_{x\to 0} \frac{\sin(2+x) - \sin(2-x)}{x}$$

$$= \lim_{x\to 0} \frac{2\cos\left(\frac{2+x+2-x}{2}\right)\sin\left(\frac{2+x-2+x}{2}\right)}{x}$$

$$= \lim_{x\to 0} \frac{2\cos 2 \sin x}{x}$$

$$= 2\cos 2 \lim_{x\to 0} \frac{\sin x}{x}$$

$$= 2\cos 2 \cdot 1 = 2\cos 2$$

Video Solution:

Q20 **Text Solution:**

Data 9, 16, 2, 30, 37, 44, 51

n = 7

d = 7

$$\sigma^2 = \frac{d^2(n^2-1)}{12}$$

$$\sigma^2 = \frac{7^2(7^2-1)}{12}$$

$$\sigma^2 = \frac{49\times 48}{12} = 49 \times 4 = 196$$

$$\sigma = \sqrt{196} = 14$$

Video Solution:

Q21 **Text Solution:**

$${}_{i=1}^{n} x_i^2 = 400$$

$${}_{i=1}^{n} x_i = 80$$

$$\sigma^2 = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2 \geq 0$$

$$\frac{400}{n} - \left(\frac{80}{n}\right)^2 \geq 0$$

$$\frac{400}{n} \geq \frac{6400}{n^2}$$

$$400 \geq \frac{6400}{n}$$

$$n \geq \frac{6400}{400}$$

$$n \geq 16$$

Video Solution:

Q22 Text Solution:

S = {1, 2, 3\

R = {(1, 1), (2, 2), (3, 3)}

∀x   S, (x, x)   R   Reflexive

(x, y)   R   (y, x)   R   Symmetric

(x, y)   R, (y, z)   R   (x, z)   R   Transitive

Reflexive + Symmetric + Transitive = Equivalence Relation

Video Solution:

Q23 Text Solution:

n(A) = 4, n(B) = 2

Number of onto functions = 2^n - 2

= 2^4 - 2

= 16 - 2 = 14

Video Solution:

Q24 Text Solution:

$\tan\left[2\tan^{-1}\frac{1}{5}-\frac{\pi}{4}\right]$

$2\tan^{-1}\frac{1}{5}=\tan^{-1}\left(\frac{2\cdot\frac{1}{5}}{1-\left(\frac{1}{5}\right)^2}\right)$

$=\tan^{-1}\left(\frac{2/5}{24/25}\right)=\tan^{-1}\frac{5}{12}$

$\tan\left[\tan^{-1}\frac{5}{12}-\tan^{-1}1\right]=\frac{\frac{5}{12}-1}{1+\frac{5}{12}\cdot1}=\frac{-7/12}{17/12}=-\frac{7}{17}$

Video Solution:

Q25 Text Solution:

$\cos^{-1}\frac{1-x^2}{1+x^2}=2\tan^{-1}x$

$\text{Let } x=\tan\theta \ \Rightarrow \theta=\tan^{-1}x$

$\frac{1-\tan^2\theta}{1+\tan^2\theta}=\cos 2\theta$

$\cos^{-1}\left(\cos 2\theta\right)=2\theta$

$\text{Domain of } \cos^{-1}\left(\cos\alpha\right)=\alpha \text{ is } 0\le\alpha\le\pi$

$0\le 2\theta\le\pi \ \Rightarrow 0\le\theta\le\pi/2$

$0\le\tan^{-1}x\le\pi/2 \ \Rightarrow x\ge 0$

Considering standard range restrictions and given options.

(for $x\ge 0$, which includes $x\in[0,\ \infty)$)

Video Solution:

Q26 Text Solution:

$$A=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ a & b & -1\end{bmatrix}$$

$$A^2=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ a & b & -1\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ a & b & -1\end{bmatrix}$$

$$A^2=\begin{bmatrix}(1+0+0) & (0+0+0) & (0+0+0)\\ (0+1+0) & (0+1+0) & (0+0+0)\\ (a+0-a) & (0+b-b) & (0+0+1)\end{bmatrix}$$

$$A^2=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=I$$

Video Solution:

Q27 Text Solution:

$$\begin{bmatrix}1 & x & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\ 0 & 5 & 1\\ 0 & 3 & 2\end{bmatrix}\begin{bmatrix}x\\ 1\\ -2\end{bmatrix}=0$$

$$\begin{bmatrix}1 & (2+5x+3) & (3+x+2)\end{bmatrix}\begin{bmatrix}x\\ 1\\ -2\end{bmatrix}=0$$

$$\begin{bmatrix}1 & (5x+5) & (x+5)\end{bmatrix}\begin{bmatrix}x\\ 1\\ -2\end{bmatrix}=0$$

$1(x)+1(5x+5)-2(x+5)=0$

$x+5x+5-2x-10=0$

$4x-5=0$

$4x=5$

$x=\frac{5}{4}$

Video Solution:

Q28 Text Solution:

$R_2\to R_2-R_1$

$R_3\to R_3-R_2$

$$\Delta=\begin{vmatrix}1 & 1 & 1\\ 0 & \sin\theta & 0\\ \cos\theta & 0 & 0\end{vmatrix}$$

$\Delta=1(0-0)-1(0-0)+1\left(0-\sin\theta\ \cos\theta\right)$

$\Delta=-\sin\theta\ \cos\theta=-\frac{1}{2}\sin 2\theta$

$\text{Max Value}=\left|-\frac{1}{2}(-1)\right|=\frac{1}{2}$

Video Solution:

Q29 Text Solution:

$$f\left(x\right)=\begin{vmatrix}0 & x-a & x-b\\ x+a & 0 & x-c\\ x+b & x-c & 0\end{vmatrix}$$

$$f\left(0\right)=\begin{vmatrix}0 & -a & -b\\ a & 0 & -c\\ b & c & 0\end{vmatrix}$$

$$\text{Let } A=\begin{bmatrix}0 & -a & -b\\ a & 0 & -c\\ b & c & 0\end{bmatrix}$$

$$A^T=\begin{bmatrix}0 & a & b\\ -a & 0 & c\\ -b & -c & 0\end{bmatrix}=-A$$

A is a skew-symmetric matrix of odd order (3 x 3).

$|A|=0\ \Rightarrow f(0)=0$

Video Solution:

Q30 Text Solution:

$y = ae^{mx} + be^{-mx}$

$\frac{dy}{dx} = mae^{mx} - mbe^{-mx}$

$\frac{d^2y}{dx^2} = m^2ae^{mx} + m^2be^{-mx}$

$\frac{d^2y}{dx^2} = m^2\left(ae^{mx} + be^{-mx}\right)$

$\frac{d^2y}{dx^2} = m^2y$

$\frac{d^2y}{dx^2} - m^2y = m^2y - m^2y = 0$

Video Solution:

Q31 Text Solution:

$x = a\sin\theta, \quad y = b\cos\theta$

$\frac{dx}{d\theta} = a\cos\theta$

$\frac{dy}{d\theta} = -b\sin\theta$

$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{-b\sin\theta}{a\cos\theta} = -\frac{b}{a}\tan\theta$

$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(-\frac{b}{a}\tan\theta\right) = \frac{d}{d\theta}\left(-\frac{b}{a}\tan\theta\right)\cdot\frac{d\theta}{dx}$

$\frac{d^2y}{dx^2} = \left(-\frac{b}{a}\sec^2\theta\right)\cdot\left(\frac{1}{a\cos\theta}\right)$

$\frac{d^2y}{dx^2} = -\frac{b}{a^2}\sec^2\theta\cdot\sec\theta$

$\frac{d^2y}{dx^2} = -\frac{b}{a^2}\sec^3\theta$

Video Solution:

Q32 Text Solution:

$$f(x) = \begin{cases} \frac{\log x}{x-1} & \text{if } x \neq 1 \\ k & \text{if } x = 1 \end{cases}$$

$\lim_{x\to 1} f(x) = f(1)$

$\lim_{x\to 1} \frac{\log x}{x-1} = k$

Let x - 1 = h $\Rightarrow x = 1 + h$

As $x \to 1, \ h \to 0$

$\lim_{x\to 0} \frac{\log(1+h)}{h} = k$

1 = k

Value of k = 1

Video Solution:

Q33 Text Solution:

$f(x) = \cot^{-1}x + x$

$f'(x) = \frac{-1}{1+x^2} + 1$

$f'(x) = \frac{-1+(1+x^2)}{1+x^2}$

$f'(x) = \frac{x^2}{1+x^2}$

$\frac{x^2}{1+x^2} \geq 0 \quad \forall x \in \mathbb{R}$

$x \in (-\infty, \infty)$

Video Solution:

Q34 Text Solution:

f(x) = ax + b

f'(x) = a

f'(x) < 0 ⇒ a < 0

Video Solution:

Q35 Text Solution:

$\int \frac{\sin x - \cos x}{\sqrt{1-\sin 2x}} dx,\ x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

$1 - \sin 2x = (\sin x - \cos x)^2$

$\sqrt{1 - \sin 2x} = \left|\sin x - \cos x\right|$

For $x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right), \sin x > \cos x \Rightarrow \left|\sin x - \cos x\right| = \sin x - \cos x$

$\int \frac{\sin x - \cos x}{\sin x - \cos x} dx$

$\int 1 dx = x + c$

Video Solution:

Q36 Text Solution:

$\int \frac{1}{(\sin x + \cos x)^2} dx$

$\int \frac{1}{\cos^2 x (1 + \tan x)^2} dx$

$\int \frac{\sec^2 x}{(1 + \tan x)^2} dx$

Let $u = 1 + \tan x \Rightarrow du = \sec^2 x dx$

$\int \frac{1}{u^2} du = -\frac{1}{u} + c$

$-\frac{1}{1 + \tan x} + c$

Video Solution:

Q37 Text Solution:

$\int_{-1/2}^{1/2} \left[[x] + \log\left(\frac{1+x}{1-x}\right)\right] dx$

$\int_{-1/2}^{1/2} \left[x\right] dx + \int_{-1/2}^{1/2} \log\left(\frac{1+x}{1-x}\right) dx$

Let $f(x) = \log\left(\frac{1+x}{1-x}\right)$

$f(-x) = \log\left(\frac{1-x}{1+x}\right) = \log\left(\frac{1+x}{1-x}\right)^{-1} = -f(x)$

$\int_{-1/2}^{1/2} \log\left(\frac{1+x}{1-x}\right) dx = 0$ (Odd function)

$\int_{-1/2}^{1/2} \left[x\right] dx = \int_{-1/2}^{0} (-1) dx + \int_{0}^{1/2} (0) dx$

$= [1 - x]_{-1/2}^{0} = 0 - (-(-1/2)) = -1/2$

Video Solution:

Q38 Text Solution:

$\int_0^6 |x-3|\, dx =$

$= \int_0^3 -(x-3)dx + \int_3^6 (x-3)\, dx$

$= \left[3x - \frac{x^2}{2}\right]_0^3 + \left[\frac{x^2}{2} - 3x\right]_3^6$

$= (9 - \frac{9}{2}) + [(18-18) - (\frac{9}{2} - 9)]$

$= \frac{9}{2} + \frac{9}{2} = 9$

Video Solution:

Q39 Text Solution:

Area bounded by one arch of y = sin 4x and x-axis

$\sin 4x = 0 \Rightarrow 4x = 0, \pi \Rightarrow x = 0, \frac{\pi}{4}$

$A = \int_0^{\pi/4} \sin 4x dx$

$A = \left[-\frac{\cos 4x}{4}\right]_0^{\pi/4}$

$A = -\frac{1}{4}\left[\cos \pi - \cos 0\right]$

$A = -\frac{1}{4}\left[-1-1\right]$

$A = -\frac{1}{4}\left[-2\right] = \frac{1}{2}$

Video Solution:

Q40 Text Solution:

$A = \int_0^{\pi/4} \left(\sin 2x + \cos 2x\right) dx$

$A = \left[-\frac{\cos 2x}{2} + \frac{\sin 2x}{2}\right]_0^{\pi/4}$

$A = \left[\left(-\frac{\cos(\pi/2)}{2} + \frac{\sin(\pi/2)}{2}\right) - \left(-\frac{\cos 0}{2} + \frac{\sin 0}{2}\right)\right]$

$A = \left[\left(0 + \frac{1}{2}\right) - \left(-\frac{1}{2} + 0\right)\right]$

$A = \frac{1}{2} + \frac{1}{2} = 1$

Video Solution:

Q41 Text Solution:

$\frac{dy}{dx} + P\left(x\right)y = Q\left(x\right)$

$\text{I.F.} = e^{\int P(x)dx} = x$

$\int P\left(x\right)dx = \ln x$

$P\left(x\right) = \frac{d}{dx}\left(\ln x\right) = \frac{1}{x}$

Video Solution:

Q42 Text Solution:

$\frac{dy}{dx} = 3x^2$

$y = \int 3x^2 dx = x^3 + C$

$x = -1, y = 1 \Rightarrow 1 = (-1)^3 + C$

$1 = -1 + C \Rightarrow C = 2$

$y = x^3 + 2$

Video Solution:

Q43 Text Solution:

$\left|\vec{a}\right| = 1, \left|\vec{b}\right| = 1, \left|\vec{a} - \sqrt{2}\vec{b}\right| = 1$

$\left|\vec{a} - \sqrt{2}\vec{b}\right|^2 = 1^2$

$\left|\vec{a}\right|^2 + \left|\sqrt{2}\vec{b}\right|^2 - 2\left(\vec{a} \cdot \sqrt{2}\vec{b}\right) = 1$

$1^2 + 2(1)^2 - 2\sqrt{2}\left(\left|\vec{a}\right|\left|\vec{b}\right|\cos\theta\right) = 1$

$1 + 2 - 2\sqrt{2}\cos\theta = 1$

$2 = 2\sqrt{2}\cos\theta$

$\cos\theta = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\theta = \frac{\pi}{4}$

Video Solution:

Q44 Text Solution:

$\left|\vec{a}\right| = 3, \left|\vec{b}\right| = 4, \theta = 120^\circ$

$\left|4\vec{a} + 3\vec{b}\right|^2 = \left|4\vec{a}\right|^2 + \left|3\vec{b}\right|^2$

$+ 2\left(4\vec{a} \cdot 3\vec{b}\right)$

$= 16\left|\vec{a}\right|^2 + 9\left|\vec{b}\right|^2 + 24\left(\left|\vec{a}\right|\left|\vec{b}\right|\cos 120^\circ\right)$

$= 16(3)^2 + 9(4)^2 + 24(3)(4)\left(-\frac{1}{2}\right)$

$= 16(9) + 9(16) + 24(12)\left(-\frac{1}{2}\right)$

$= 144 + 144 - 144 = 144$

$\left|4\vec{a} + 3\vec{b}\right| = \sqrt{144} = 12$

Video Solution:

Q45 Text Solution:

$\vec{a} = (2, -1, 1)$

$\vec{b} = (3, 4, -1)$

$\vec{b} - \vec{a} = (3 - 2,\ 4 + 1,\ -1 - 1)$

$= (1,\ 5,\ -2)$

Video Solution:

Q46 Text Solution:

$\frac{x-5}{3} = \frac{y-7}{-1} = \frac{z+2}{1} = \lambda$

$\frac{x+3}{-36} = \frac{y-3}{2} = \frac{z-6}{4} = \mu$

$x = 3\lambda + 5 = -36\mu - 3$

$y = -\lambda + 7 = 2\mu + 3$

$z = \lambda - 2 = 4\mu + 6$

From $z : \lambda = 4\mu + 8$

Substitute l into y:

$-(4\mu + 8) + 7 = 2\mu + 3$

$-4\mu - 1 = 2\mu + 3$

$-6\mu = 4 \Rightarrow \mu = -2/3$

$\lambda = 4(-2/3) + 8 = 16/3$

$x = 3(16/3) + 5 = 21$

$y = -(16/3) + 7 = 5/3$

$z = (16/3) - 2 = 10/3$

Video Solution:

Q47 Text Solution:

We are given $P(A \cup B)= 0.83,\ P(A)= 0.3,\ \text{and}\ P(B) = 0.6$

Use the Addition Rule:

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.83 = 0.3 + 0.6 - P(A \cap B)$

$P(A \cap B) = 0.9 - 0.83 = 0.07$

Check for Independence :

Events are independent if

$P\left(A \cap B\right)= P(A)\times P(B)$

$P(A)\times P(B)= 0.3 \times 0.6 = 0.18$

Since $0.07 \neq 0.18$, the events are dependent.

Video Solution:

Q48 Text Solution:

Probability of winning a match (p) = 1/2 so losing (q) = 1/2

We need India's second win to occur exactly at the third match.

1. Condition : In the first 2 matches, India must win exactly once. Then, they must win the 3rd match.

2. Calculate:

P(1 win in first 2) = (2 1) x $(1/2)^1$ x $(1/2)^1$ = 2 x 1/4 = 1/2

P(Win on 3rd match) = 1/2

Total Probability = 1/2 x 1 / 2 = 1/4

Video Solution:

Q49 **Text Solution:**

Probabilities of four persons solving are: P(1) = 1/2 P(2) = 3/4 r(3) = 1/4 P(4) = 1/8

1. Calculate the probability that NO ONE solves it:

- P(not 1) = 1 - 1/2 = 1/2
- P(not 2) = 1 - 3/4 = 1/4
- P(not 3) = 1 - 1/4 = 3/4
- P(not 4) = 1 - 1/8 = 7/8
- P(None solve) = 1/2 × 1/4 x 3/4 x 7/8 = 21/256

2. Find the probability that at least one solves it:

P(At least one) = 1 P(None solve)

P = 1 - 21/256 = **235/256**

Video Solution:

Q50 **Text Solution:**

A and B throw a die; the first to get a '6' wins. A starts.

- P(getting 6)= p = 1/6
- P(not getting 6) = q = 5/6

A wins if he gets a 6 on the 1st, 3rd, 5th, ... throw.

P(A wins) = p + q^2p + q^4P + ...

This is an infinite geometric series with first term a = p and common ratio r = q^4

$$\text{Sum} = \frac{a}{1-r} = \frac{1/6}{1-(5/6)^2} = \frac{1/6}{1-25/36} = \frac{1/6}{11/36}$$

$$P(A \text{ wins}) = 1/6 \times 36/11 = 6/11$$

Video Solution:

Q51 **Text Solution:**

Rewrite in standard from $\frac{dy}{dx} + P(x)y = Q(x)$:

$$\frac{dy}{dx} - xy = e^{x^2/2}$$

Here, P(x) = -x and Q(x) = $e^{x^2/2}$

Calculate the Integrating Factor (I.F.) :

$$\text{I.F.} = e^{\int P(x)dx} = e^{\int -xdx} = e^{-x^2/2}$$

general solution is

$$y \cdot (I.F.) = \int Q(x) \cdot (I.F.)dx$$

$$-x^2/2 = \int x^2/2 - x^2/2dx$$

$$y \cdot (I.F.) = \int Q(x) \cdot (I.F.)dx$$

$$y \cdot e^{-x^2/2} = \int e^{x^2/2} \cdot e^{-x^2/2} dx$$

$$y \cdot e^{-x^2/2} = \int e^0 dx = \int 1\ dx$$

$$y \cdot e^{-x^2/2} = x + c$$

$$y = (x + c)e^{x^2/2}$$

Video Solution:

Q52 **Text Solution:**

Equation : $\frac{dy}{dx} + y \tan x = \sec x$

1. Identify P(x) and Q(x) :

P(x) = tan x and Q(x) = sec x.

2. Calculate the Integrating Factor (I.F.) :

$$\text{I.F.} = e^{\int \tan x\ dx}$$

3. Find the general solution:

$$y \cdot (I.F.) = \int Q(x) \cdot (I.F.)dx$$

$$y \cdot \sec x = \int \sec\ x \cdot \sec x\ dx$$

$$y \cdot (I.F.) = \int Q(x) \cdot (I.F.)dx$$

$$y \cdot \sec x = \int \sec x \cdot \sec\ x\ dx$$

$$y\ \sec\ x = \int \sec^2\ x\ dx$$

4. Integrate the RHS :

y sec x = tan x + c

Video Solution:

Q53 Text Solution:

1. Partial Fractions :

We can rewrite the integrand using partial fractions:

$\frac{1}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1}$

$1 = A(x+1) + Bx$

If $x = 0$, then $1 = A(1) \Rightarrow A = 1$

If $x = -1$, then $1 = B(-1) \Rightarrow B = -1$

So, $\frac{1}{x(x+1)} = \frac{1}{x} - \frac{1}{x+1}$

2. Integrate:

$\int \left(\frac{1}{x} - \frac{1}{x+1}\right) dx = \ln |x| - \ln|x+1| + c$

3. Simplify using Log Laws:

$\ln|x| - \ln|x+1| = \ln\left|\frac{x}{x+1}\right| + c$

Video Solution:

Q54 Text Solution:

1. Complete the Square:

Inside the square root :

$2x - x^2 = -\left(x^2 - 2x\right)$

$2x - x^2 = -\left(x^2 - 2x + 1 - 1\right) =$

$-\left((x-1)^2 - 1\right) = 1 - (x-1)^2$

2. Rewrite the Integral:

$\int \frac{dx}{\sqrt{1-(x-1)^2}}$

3. Apply Standard Formula :

Recall that $\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + c$

Here a = 1 and u = (x - 1)

$\int \frac{dx}{\sqrt{1-(x-1)^2}} = \sin^{-1}(x-1) + c$

Video Solution:

Q55 Text Solution:

Given : $\sin^2 x + 2\cos y + xy = 0$

1. Differentiate both sides with respect to x:

$\frac{d}{dx}\left(\sin^2 x\right) + \frac{d}{dx}(2\cos y) + \frac{d}{dx}(xy) = 0$

2. Apply chain rule and Product rule :

$\frac{d}{dx}\left(\sin^2 x\right) = 2\sin x \cos x = \sin 2x$

$\frac{d}{dx}(2\cos y) = -2\sin y \cdot \frac{dy}{dx}$

$\frac{d}{dx}(xy) = x \cdot \frac{dy}{dx} + y$

3. Combine and Solve for dy/dx :

$\sin 2x - 2\sin y \frac{dy}{dx} + x\frac{dy}{dx} + y = 0$

$(x - 2\sin y)\frac{dy}{dx} = -(y + \sin 2x)$

$\frac{dy}{dx} = \frac{-(y+\sin 2x)}{x-2\sin y} = \frac{y+\sin 2x}{2\sin y - x}$

Video Solution:

Q56 Text Solution:

Given : x = 2 cos t + cos 2t and y = 2 sin t - sin 2t

1. Find $\frac{dx}{dt}$ and $\frac{dy}{dt}$:

$\frac{dx}{dt} = -2\sin t - 2\sin 2t$

$\frac{dy}{dt} = 2\cos t - 2\cos 2t$

2. Find $\frac{dy}{dx}$:

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2\cos t - 2\cos 2t}{-2\sin t - 2\sin 2t} = -\frac{\cos t - \cos 2t}{\sin t + \sin 2t}$

3. Evaluate at $t = \frac{\pi}{4}$

$\cos(\pi/4) = 1/\sqrt{2}$, $\cos(\pi/2) = 0$

$\sin(\pi/4) = 1/\sqrt{2}$, $\sin(\pi/2) = 1$

$$\frac{dy}{dx} = -\frac{1/\sqrt{2}-0}{1/\sqrt{2}+1} = -\frac{1/\sqrt{2}}{\left(1+\sqrt{2}\right)/\sqrt{2}} = -\frac{1}{1+\sqrt{2}}$$

4. Rationalize the denominator:

$$-\frac{1}{1+\sqrt{2}} \times \frac{1-\sqrt{2}}{1-\sqrt{2}} = -\frac{1-\sqrt{2}}{1-2} = \frac{-\left(1-\sqrt{2}\right)}{-1} = 1$$

$-\sqrt{2}$

Video Solution:

Q57 Text Solution:

Given $f(x) = \frac{1-\cos px}{x \sin x}$ for $x \neq 0$ and $f(0) = \frac{1}{2}$

If f is continuous at x = 0, then $\lim_{x\to 0} f(x) = f(0)$

1. Set up the limit:

$$\lim_{x\to 0} \frac{1-\cos px}{x \sin x} = \frac{1}{2}$$

2. Use Trigonometric Identities :

Recall that $1 - \cos\theta = 2\sin^2(\theta/2)$.

$$\lim_{x\to 0} \frac{2\sin^2(px/2)}{x \sin x}$$

3. Apply Standard Limits :

Recall $\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$. Multiply and divide to create these forms :

$$\lim_{x\to 0}\left[\frac{2\sin^2(px/2)}{(px/2)^2} \cdot \frac{(px/2)^2}{x\cdot\frac{\sin x}{x}\cdot x}\right] = \frac{1}{2}$$

$$2\cdot(1)^2 \cdot \lim_{x\to 0}\left[\frac{p^2x^2/4}{x^2\cdot(1)}\right] = \frac{1}{2}$$

4. Solve for p:

$$2\cdot\frac{p^2}{4} = \frac{1}{2}$$

$$\frac{p^2}{2} = \frac{1}{2} \Rightarrow p^2 = 1 \Rightarrow p = \pm 1$$

Video Solution:

Q58 Text Solution:

Given : $f(x) = \begin{cases} x^2 & x \leq 0 \\ ax+b & x > 0 \end{cases}$

For f(x) to be continuous on the whole real line, it must be continuous at the transition point x = 0.

This means the Left-Hand Limit (LHL), Right-Hand Limit (RHL), and f(0) must all be equal.

1. Find the LHL and f(0) :

$$LHL = \lim_{x\to 0^-} x^2 = 0^2 = 0$$

$$f(0) = 0^2 = 0$$

2. Find the RHL :

$$\text{RHL} = \lim_{x\to 0^+}(ax+b) = a(0)+b = b$$

3. Equate for Continuity :

$$LHL = RHL \Rightarrow b = 0$$

4. Determine a:

Since the RHL calculation a(0) + b results in b regardless of what a is a can be any real number.

Video Solution:

Q59 Text Solution:

$$|A| = \begin{vmatrix} 1 & 2 & 1 \\ -1 & 1 & 1 \\ 1 & -3 & 1 \end{vmatrix} = 10$$

$$cofactor\ of\ A = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

$$adj\ of\ A = \begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix}$$

$$X = A^{-1}B$$

$$= \frac{1}{10}\begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$$

Video Solution:

Q60 **Text Solution:**

1. Perform the Matrix Multiplication AB

$$AB = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$= \begin{bmatrix} (0 \cdot a + 1 \cdot c) & (0 \cdot b + 1 \cdot d) \\ (0 \cdot a + 0 \cdot c) & (0 \cdot b + 0 \cdot d) \end{bmatrix}$$

$$AB = \begin{bmatrix} c & d \\ 0 & 0 \end{bmatrix}$$

2. Set AB equal to the Zero Matrix O

$$\begin{bmatrix} c & d \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

By comparing elements, we get:

c = 0

d = 0

on verification B= $\begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}$

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