ULTIMATE KCET

CRASH COURSE 2026

Mathematics | Lecture – 05

Integrals

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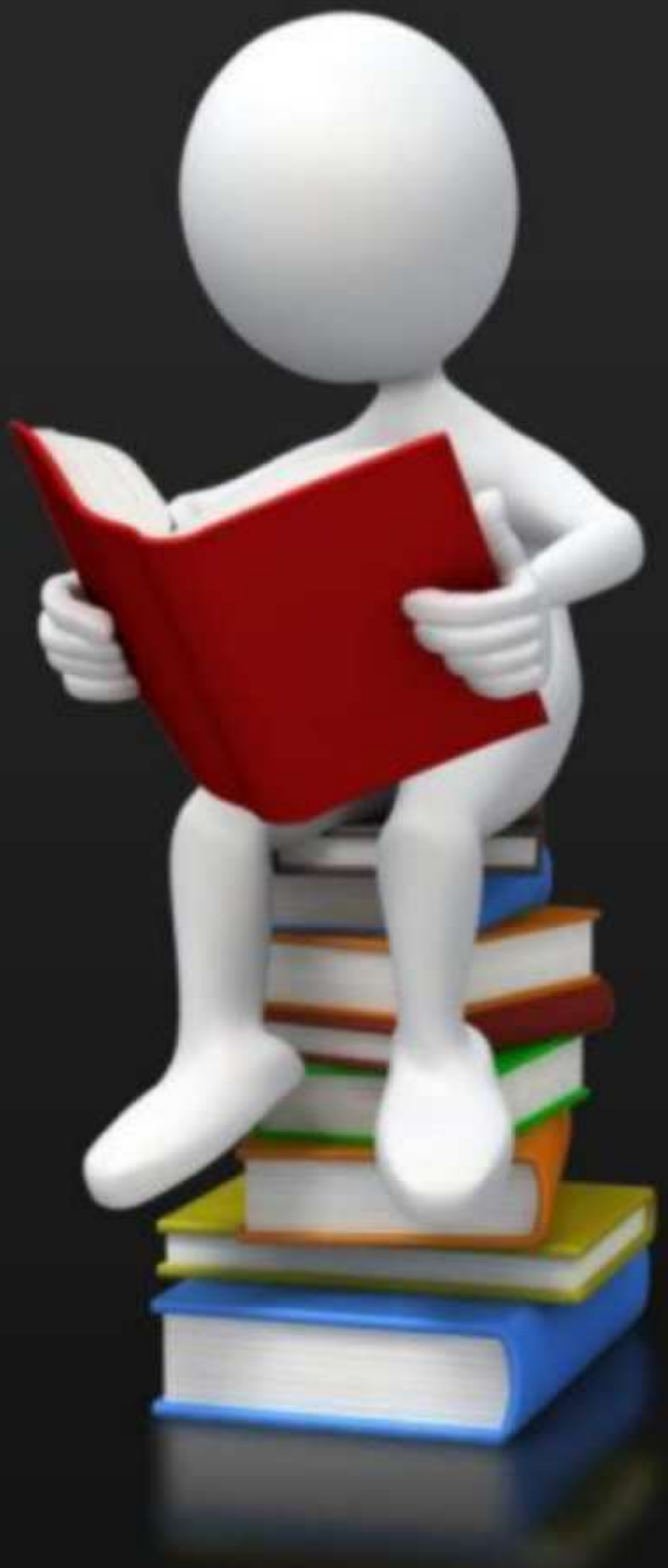
Recap of previous lecture

1. Definite Integrals
2.
3.
4.

Topics *to be covered*

1. Definite Integrals – continue
2.
3.
4.



QUESTION

$\sqrt{a^2-x^2} = \sqrt{a^2(1-\sin^2\theta)} = a\cos\theta$

#Q. The value of $\int_0^a \frac{dx}{x+\sqrt{a^2-x^2}}$ is

(A) 0.25

(B) 0.5

(C) 0.25π

(D) 0.5π

Put $x = a\sin\theta$ | $0 \to 0$

$dx = a\cos\theta\, d\theta$ | $a \to \frac{\pi}{2}$

$I = \int_0^{\pi/2} \frac{\cancel{a}\cos\theta\, d\theta}{\cancel{a}\sin\theta + \cancel{a}\cos\theta}$

$I = \int_0^{\pi/2} \frac{\cos\theta}{\sin\theta+\cos\theta}$

$I = \frac{\frac{\pi}{2}-0}{2} = \frac{\pi}{4} = 0.25\pi$

Expression	Std substitution
a^2-x^2	→ Put $x = a\sin\theta$
x^2-a^2	→ Put $x = a\sec\theta$
a^2+x^2	→ Put $x = a\tan\theta$

$x = a\sin\theta$

$\sin\theta = \frac{x}{a}$

$\theta = \sin^{-1}\left(\frac{x}{a}\right)$

$x = 0 \mid \theta = \sin^{-1}0 = 0$

$x = a \mid \theta = \sin^{-1}(1) = \frac{\pi}{2}$

QUESTION

#Q. The value of $\int_0^a \sqrt{a^2-x^2}\,dx$ is

(A) πa

(B) $\pi a^2/4$ ✓

(C) $4\pi a^2$

(D) πa^2

Put $x = a\sin\theta$

$dx = a\cos\theta\, d\theta$

x	θ
$0 \to$	0
$a \to$	$\frac{\pi}{2}$

$$I = \int_0^{\pi/2} \underbrace{a\cos\theta}_{\sqrt{a^2-x^2}}\,\underbrace{(a\cos\theta)\,d\theta}_{dx}$$

$$I = a\int_0^{\pi/2} \cos^2\theta\, d\theta \to (1)$$

Prop (2)

$$I = a\int_0^{\pi/2} \sin^2\theta\, d\theta \to (2)$$

(1) + (2)

$$2I = a^2\int_0^{\pi/2} \sin^2\theta + \cos^2\theta\, d\theta$$

$$2I = a^2\int_0^{\pi/2} 1\, d\theta = a^2(\theta)_0^{\pi/2}$$

$$2I = a^2\left(\frac{\pi}{2}\right)$$

$$I = \frac{\pi a^2}{4}$$

method 2

$$\int_0^a \sqrt{a^2-x^2}\,dx$$

$$= \left.\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]_0^a$$

$$= \left(0+\frac{a^2}{2}\frac{\pi}{2}\right)-0$$

$$= \frac{\pi a^2}{4}$$

Prop 2 :- $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

$a = -a$

$b = a$

Play b/w these 2 property

$f(x) \to$ even

Prop (4) $\int_{-a}^{a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx & \text{if } f(-x) = f(x) \\ 0 & \text{if } f(-x) = -f(x) \end{cases}$

$f(x) \to$ odd

Lower limit $= -a$

Even func

$\Downarrow$

whenever

$f(-x) = f(x)$

Then

$f(x) \rightarrow$ even

odd func

$\Downarrow$

whenever

$\Downarrow$

$f(-x) = -f(x)$

$\Downarrow$

Then $f(x) \rightarrow$ odd

Ex: $f(x) = x^2$

$x \to -x$

$f(-x) = (-x)^2$

$= x^2$

$\therefore f(-x) = f(x)$

$f(x) = x^2$ is even

② $f(x) = x^3$

$x \to -x$

$f(-x) = (-x)^3$

$= -x^3$

$\therefore f(-x) = -f(x)$

$f(x) = x^3$ is odd

③ $f(x) = \cos x$

$x \to -x$

$f(-x) = \cos(-x)$

$= \cos x$

$\therefore f(-x) = f(x)$

$f(x) = \cos x$ is even

④ $f(x) = \sin x$

$x \to -x$

$f(-x) = \sin(-x)$

$= -\sin x$

$\therefore f(-x) = -f(x)$

$f(x) = \sin x$ is odd

(5) $f(x) = \sin^2 x$

$\Downarrow$

$x \to -x$

$f(-x) = [\sin(-x)]^2$

$= (-\sin x)^2$

$= \sin^2 x$

$\therefore f(-x) = f(x)$

$f(x) = \sin^2 x$ is even

(6) $f(x) = \log\left(\frac{3-x}{3+x}\right)$

$x \to -x$

$f(-x) = \log\left(\frac{3+x}{3-x}\right)$

$= -\log\left(\frac{3-x}{3+x}\right)$ since $\log\frac{A}{B} = -\log\frac{B}{A}$

$\therefore f(-x) = -f(x)$

$\Rightarrow f(x) = \log\left(\frac{3-x}{3+x}\right)$ is odd

(7) $f(x) = 1 + 3^x$

$x \to -x$

$f(-x) = 1 + 3^{-x}$

$\neq f(x)$ or $\neq -f(x)$

$\therefore$ $f(x)$ is neither even nor odd

(1) $I = \int_{-2}^{2} \frac{1}{1+e^{x}}\,dx$

Observe :-

L.L = −2

U.L = 2 $\Rightarrow \int_{-a}^{a}$ form $\longrightarrow$ Here we need first check when the func is even or odd

$f(x) = \frac{1}{1+e^{x}}$

$f(-x) = \frac{1}{1+e^{-x}} \neq f(x)$

or

$\neq -f(x)$

$\therefore f(x) \to$ neither even nor odd

$\Downarrow$

Apply Prop (2)

$I = \int_{-2}^{2} \frac{1}{1+e^{-x}}\,dx$

$x \to a+b-x$

$\downarrow$

$x \to -2+2-x$

$\downarrow$

$x \to -x$

(1) if $f(x) \to$ even

$\int_{-a}^{a} = 2\int_{0}^{a}$

(2) if $f(x) \to$ odd

$\int_{-a}^{a} = 0$

(3) if $f(x) \to$ neither even nor odd

$\Downarrow$

Apply prop (2)

$$I = \int_{-2}^{2} \frac{1}{1+e^x}\,dx \longrightarrow (1)$$

$\Downarrow$

neither even nor odd

$\Downarrow$

Prop (2)

$x \to a+b-x$

$\Downarrow$

$x \to -x$

$$I = \int_{-2}^{2} \frac{1}{1+e^{-x}}\,dx$$

$$I = \int_{-2}^{2} \frac{1}{e^{-x}(e^x+1)}\,dx$$

$$I = \int_{-2}^{2} \frac{e^x}{1+e^x}\,dx \longrightarrow (2)$$

(1) + (2)

$$2I = \int_{-2}^{2} \frac{1+e^x}{1+e^x}\,dx = \int_{-2}^{2} 1\,dx = x\Big)_{-2}^{2}$$

$$2I = 2-(-2) = 4$$

$$I = 2$$

(2) $I = \int_{-2\pi}^{2\pi} \frac{1}{1+3^{\sin x}}\,dx \rightarrow (1)$

$\downarrow$

neither even nor odd

$\Downarrow$

Apply Prop (2)

$= \int_{-2\pi}^{2\pi} \frac{1}{1+3^{-\sin x}}\,dx$

$I = \int_{-2\pi}^{2\pi} \frac{3^{\sin x}}{3^{\sin x}+1}\,dx \rightarrow (2)$

(1) + (2)

$2I = \int_{-2\pi}^{2\pi} \frac{1+3^{\sin x}}{1+3^{\sin x}}\,dx = \int_{-2\pi}^{2\pi} 1\,dx = 4\pi$

$I = 2\pi$

(3) $I = \int_{-3}^{3} \sec^{-1} x \, dx$

$f(x) = \sec^{-1} x$

$x \to -x$

$f(-x) = \sec^{-1}(-x)$

$= \pi - \sec^{-1} x$

$\neq f(x)$

or

$\neq -f(x)$

neither even nor odd

Apply Prop (2)

$I = \int_{-3}^{3} \sec^{-1}(-x) dx$

$= \int_{-3}^{3} \pi - \sec^{-1} x \, dx$

$I = \pi \int_{-3}^{3} 1 \, dx - \int_{-3}^{3} \sec^{-1} x \, dx$

$I = \pi (x)_{-3}^{3} - I$

$2I = \pi(3+3) = 6\pi$

$I = 3\pi$

(4) $I = \int_{-1/2}^{1/2} \tan^{-1}x \, dx$

$f(x) = \tan^{-1}x$

$x \to -x$

$f(-x) = \tan^{-1}(-x)$

$= -\tan^{-1}x$

$\therefore f(-x) = -f(x)$

$f(x) \to$ odd

$I = 0$

QUESTION

#Q. The value of $\int_{-\pi}^{\pi} \sin^3 x dx$ is

odd

A 0

$I = 0$

B 2

C 4

D 1

QUESTION

#Q. The value of $\int_{-\pi}^{\pi} \frac{x\cos x}{1+\sin^2 x} dx$ is

(A) $\pi/4$

(B) $\pi/4(\pi-3)$

(C) $\pi/2$

(D) 0 ✓

$f(x) = \frac{x\cos x}{1+\sin^2 x}$

$f(-x) = \frac{(-x)\cos(-x)}{1+[\sin(-x)]^2}$

$= -\frac{x\cos x}{1+\sin^2 x}$

$f(-x) = -f(x)$

$f(x) \rightarrow \text{odd}$

$I = 0$

(*) $I = \int_{-\pi}^{\pi} \frac{x \sin x}{1+\cos^2 x}\, dx$

$\Downarrow$

even

$f(x) = \frac{x \sin x}{1+\cos^2 x}$

$\Downarrow$

$f(-x) = \frac{(-x)\sin(-x)}{1+[\cos(-x)]^2}$

$f(-x) = \frac{x \sin x}{1+\cos^2 x}$

$f(-x) = f(x)$

$I = 2\int_{0}^{\pi} \frac{x \sin x}{1+\cos^2 x}\, dx$

$\Downarrow$

prop (2)

$I = 2\left[\pi \int_0^{\pi} \frac{\sin x}{1+\cos^2 x}\, dx - I\right]$

$3I = 2\pi \int_0^{\pi} \frac{\sin x}{1+\cos^2 x}\, dx$

Put $\cos x = t$

$0 \to 1$

$\pi \to -1$

$3I = -2\pi \int_{1}^{-1} \frac{dt}{1+t^2}$

$3I = -2\pi \left[\tan^{-1} t\right]_1^{-1}$

$I = -\frac{2}{3}\pi\left[-\frac{\pi}{4} - \frac{\pi}{4}\right]$

$I = -\frac{2}{3}\pi\left[-\frac{\pi}{2}\right]$

$I = \frac{\pi^2}{3}$

$$I = 2\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}$$

Prop 2

$$I = 2\int_0^{\pi} \frac{(\pi - x)\sin x}{1+\cos^2 x}\,dx$$

$$I = 2\left\{\pi\int_0^{\pi} \frac{\sin x}{1+\cos^2 x}\,dx - I\right\}$$

(*) $I = \int_0^{2\pi} \frac{\sin x}{1+\cos^2 x}\,dx$

$\Downarrow$

Prop (3)

$x \to 2a - x$

$\Downarrow$

$x \to 2\pi - x$

$\Downarrow$

$f(2\pi - x) = \frac{\sin(2\pi - x)}{1+[\cos(2\pi - x)]^2}$

$= \frac{-\sin x}{1+\cos^2 x}$

$f(2\pi - x) = -f(x)$

$I = 0$

$$\int_0^{2a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx & \text{if } f(2a-x) = f(x) \\ 0 & \text{if } f(2a-x) = -f(x) \end{cases}$$

QUESTION

#Q. The value of $\int_{-1}^{1} \ln\left(\frac{2-x}{2+x}\right) dx$ is

$\Downarrow$

odd

A. 1

B. 0 ✓

C. 2

D. 3

$f(x) = \log\left(\frac{2-x}{2+x}\right)$

$x \to -x$

$f(-x) = \log\left(\frac{2+x}{2-x}\right)$

$= -\log\left(\frac{2-x}{2+x}\right)$

$\log\frac{A}{B} = -\log\frac{B}{A}$

$f(-x) = -f(x)$

QUESTION

#Q. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \ln\left(\frac{2-\sin\theta}{2+\sin\theta}\right) d\theta$ is

$\Downarrow$

odd

(A) $\pi/4$

(B) $\pi/2$

(C) π

(D) 0

QUESTION

$\Rightarrow \int_1^3 2\,dx - 6 = \int_1^3 f(x)\,dx$ | $\int_1^3 f(x)\,dx = 2(x)_1^3 - 6$
$= 2(2) - 6 = -2$

#Q. If $\int_{-2}^{3} f(x)dx = 5$ and $\int_{1}^{3}\{2 - f(x)\}dx = 6$, the value of $\int_{-2}^{1} f(x)dx$ is

$$\int_{-2}^{1} f(x)\,dx + \int_{1}^{3} f(x)\,dx = 5$$

(A) 7

(B) 3

(C) −7

(D) −3

$$\int_{-2}^{1} f(x)\,dx - 2 = 5$$

$$\int_{-2}^{1} f(x)\,dx = 7$$

QUESTION

$x \in [-2, 2]$

$|x| \le a$

$x \in [-a, a]$

#Q. If $f(x) = \begin{cases} e^{\cos x} \sin x, & |x| \le 2 \\ 2, & \text{otherwise} \end{cases}$ then $\int_{-2}^{3} f(x)dx$ is

(A) 0

(B) 1

(C) 2

(D) 3

$$\int_{-2}^{3} f(x)dx = \int_{-2}^{2} e^{\cos x} \sin x \, dx + \int_{2}^{3} 2\,dx$$

even × odd

= odd

$$\int_{-2}^{3} f(x)dx = 0 + 2(x)_2^3$$

$$= \underline{2}$$

QUESTION

#Q. The value of $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$ is

(A) $\pi/4$

(B) $\pi^2/4$

(C) $\pi/2$

(D) 0

QUESTION

#Q. The value of $\int_0^{\frac{\pi}{2}} 2\ln\sin x\,dx$ is

(A) $-\frac{\pi}{2}\ln 2$

(B) $-\pi\ln 2$ ✓

(C) $\frac{\pi}{4}\ln 2$

(D) $\frac{\pi}{4}$

$I = 2\int_0^{\pi/2} \log\sin x\,dx \rightarrow (1)$

Prop (2) $x \rightarrow a+b-x \Downarrow x \rightarrow \frac{\pi}{2}-x$

$I = 2\int_0^{\pi/2} \log\cos x\,dx \rightarrow (2)$

(1) + (2)

$\cancel{2}I = \cancel{2}\int_0^{\pi/2} \log\sin x\cdot\cos x\,dx$

$I = \int_0^{\pi/2} \log\frac{\sin 2x}{2}\,dx$

$I = \int_0^{\pi/2} \log\sin 2x\,dx - \int_0^{\pi/2} \log 2\,dx$

$\log 2(x)_0^{\pi/2}$

$\frac{\pi}{2}\log 2$

$\Downarrow$

Put $2x = t$

$dx = \frac{dt}{2}$

$0 \rightarrow 0$

$\frac{\pi}{2} \rightarrow \pi$

$I = \frac{1}{2}\int_0^{\pi} \log\sin t\,dt - \frac{\pi}{2}\log 2$

$\Downarrow$

Prop (3)

Prop 3

$$\int_0^{2a} = 2\int_0^{a}$$

$\Downarrow$

$$\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$$

$$I = \frac{1}{2}\left(2\int_0^{\pi/2} \log\sin t\,dt\right) - \frac{\pi}{2}\log 2$$

$$I = \frac{1}{2}I - \frac{\pi}{2}\log 2$$

$$I - \frac{1}{2}I = -\frac{\pi}{2}\log 2$$

$$\frac{1}{2}I = -\frac{\pi}{2}\log 2$$

$$I = -\pi\log 2$$

QUESTION

#Q. The value of $I = \int_0^{\frac{\pi}{2}} \ln \cos x \, dx$ is $\rightarrow (1)$

Prop (2)

A $\pi \ln 2$

B $\pi/2 \ln 2$

C $\pi/2 \ln 1/2$

D $\pi \ln 1/2$

$I = \int_0^{\pi/2} \log \sin x \, dx \rightarrow (2)$

$(1) + (2)$

$2I = \int_0^{\pi/2} \log \sin x \cos x \, dx$

$2I = \int_0^{\pi/2} \log \frac{\sin 2x}{2} dx$

$2I = \int_0^{\pi/2} \log \sin(2x) \, dx - \int_0^{\pi/2} \log_2 2 \, dx$

$\Downarrow$

$2x = t$

$2I = \frac{1}{2}\int_0^{\pi} \log \sin t \, dt - \log 2 \left(\frac{\pi}{2}\right)$

$\Downarrow$

Prop (3)

$2I = \cancel{\frac{1}{2}}(\cancel{2})\int_0^{\pi/2} \log \sin t \, dt - \frac{\pi}{2}\log 2$

$\Downarrow$

From (2)

$2I = I - \frac{\pi}{2}\log_2 2$

$I = -\frac{\pi}{2}\log 2$

$$\boxed{\begin{aligned} I &= \frac{\pi}{2}(-\log 2) \\ I &= \frac{\pi}{2}\log \frac{1}{2} \end{aligned}}$$

QUESTION

#Q. The value of $\int_0^{\pi} \sqrt{\frac{1+\cos 2x}{2}}\, dx$ is

(A) 0

(B) 1

(C) 2 ✓

(D) 4

$I = \int_0^{\pi} \sqrt{\frac{2\cos^2 x}{2}}\, dx$

$= \int_0^{\pi} \sqrt{\cos^2 x}\, dx$

In definite integrals we should use

$\sqrt{x^2} = |x|$

$I = \int_0^{\pi} |\cos x|\, dx$

$= \int_0^{\pi/2} \cos x\, dx + \int_{\pi/2}^{\pi} -\cos x\, dx$

$\cos x > 0$ 1st Quad $\qquad \cos x < 0$ 2nd Quad

$I = \left(\sin x\right)_0^{\pi/2} - \left(\sin x\right)_{\pi/2}^{\pi}$

$I = (1-0) - (0-1)$

$= 1+1$

$\underline{I = 2}$

QUESTION

#Q. The value of $\int_0^{\log 5} \frac{e^x\sqrt{e^x-1}}{e^x+3}\,dx$ is

(A) $4-\tan^{-1}(2)$

(B) $4-\pi/2$

(C) $4-\pi$

(D) $4-2\pi/3$

$e^x = t+1 \Rightarrow e^x+3 = t+4$

$t = e^x - 1$

Put $e^x - 1 = t$

$e^x\,dx = dt$

$x \to 0$

$t \to e^0 - 1$

$= 1-1$

$= 0$

$x \to \log 5$

$t \to e^{\log 5} - 1$

$5-1$

$= 4$

$I = \int_0^4 \frac{\sqrt{t}}{t+4}\,dt$

splitting is not possible

(next Page)

QUESTION

#Q. The value of $\int_0^{\log 5} \frac{e^x\sqrt{e^x-1}}{e^x+3}dx$ is

(A) $4-\tan^{-1}(2)$

(B) $4-\pi/2$

(C) $4-\pi$

(D) $4-2\pi/3$

Put $\sqrt{e^x-1}=t$

$e^x-1=t^2$

$e^x=t^2+1$

$e^x dx = 2t\,dt$

$e^x=t^2+1$

$e^x+3=t^2+4$

x	t
$0 \to$	0
$\log 5 \to$	2

$I=2\int_0^2 \frac{t(t)}{t^2+4}dt$

$I=2\int_0^2 \frac{t^2+4-4}{t^2+4}dt$

$I=2\left[\int_0^2 1-\frac{4}{4+t^2}dt\right]$

$=2\left[(x)_0^2-4\left(\frac{1}{2}\right)\left(\tan^{-1}\left(\frac{t}{2}\right)\right)_0^2\right]$

$=2\left[2-2\left[\frac{\pi}{4}\right]\right]$

$=4-4\left(\frac{\pi}{4}\right)$

$=4-\pi$

QUESTION

#Q. The value of $\int_{-20}^{20}(x + x^3 + x^5 + \cdots + x^{19})dx$ is

$\Downarrow$

odd

(A) 0

$I = 0$

(B) 2/19

(C) 1059/1253

(D) 4/53

$$I = \int_{3\pi/4}^{5\pi/4} |\sin x|\,dx$$

$$= \int_{3\pi/4}^{\pi} \sin x\,dx + \int_{\pi}^{5\pi/4} (-\sin x)$$

$(\sin x > 0)$ 2nd Quad

$(\sin x < 0)$ 3rd Quad

$$I = (-\cos x)_{\frac{3\pi}{4}}^{\pi} + (\cos x)_{\pi}^{5\pi/4}$$

$$I = -\left[-1 + \frac{1}{\sqrt{2}}\right] + \left[-\frac{1}{\sqrt{2}} - (-1)\right]$$

$$= 1 - \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} + 1$$

$$= 2 - \frac{2}{\sqrt{2}}$$

$$= \underline{2 - \sqrt{2}}$$

QUESTION

#Q. The value of the integral $\int_0^{3/2}[x^2]dx$ [ ] denotes the greatest integer function, is

(A) $2+\sqrt{2}$

(B) $2-\sqrt{2}$

(C) $4+2\sqrt{2}$

(D) $4-2\sqrt{2}$

QUESTION

#Q. The value of $\int_0^{\pi} [\cos x + |\cos x|] dx$ is

(A) 1

(B) 1

(C) -1

(D) 2

QUESTION

#Q. The value of $\int_0^1 (|x| + |x-1|)dx$ is

A 0

B 1

C 2

D –2

QUESTION

#Q. $\int_0^{\pi/2} \sqrt{1-\sin 2x}\,dx$ is equal to

(A) $2\sqrt{2}$

(B) $2(\sqrt{2}+1)$

(C) 2

(D) $2(\sqrt{2}-1)$

QUESTION

#Q. Let $[x]$ denote the greatest integer less than or equal to x, then the value of the integral $\int_{-1}^{1}(|x| - 2[x])dx$ is equal to

(A) 3

(B) 2

(C) -2

(D) -3

QUESTION

#Q. Find the value of $\int_0^{\sqrt{2}} [x^2] dx$.

A $2-\sqrt{2}$

B $2+\sqrt{2}$

C $\sqrt{2}-1$

D $\sqrt{2}-2$

QUESTION

#Q. $\int_0^1 \frac{\sin x + \cos x}{\sqrt{1 + \sin 2x}} dx =$

(A) 1

(B) 0

(C) 2

(D) 5

QUESTION

#Q. $\int_{-1}^{1} |1 + x^2| dx$

(A) $8/3$

(B) 1

(C) $-1/3$

(D) $-4/3$

QUESTION

#Q. $\int_0^1 \sqrt{x(1-x)}dx =$

A $\pi/2$

B $\pi/4$

C $\pi/6$

D $\pi/8$

QUESTION

#Q. $\int_0^{50} (x - [x])\, dx =$

(A) 25

(B) 20

(C) 15

(D) 15

QUESTION

#Q. $\int_0^{88\pi} \sqrt{1-\cos 2x}\,dx =$

(A) $176\sqrt{2}$

(B) $88\sqrt{2}$

(C) $44\sqrt{2}$

(D) $22\sqrt{2}$

$\int_0^{88\pi} \sqrt{2\sin^2 x}\,dx$

$= \sqrt{2}\int_0^{88\pi} |\sin x|\,dx$

$= \sqrt{2}(88)\int_0^{\pi} |\sin x|\,dx$

$= 88\sqrt{2}(2)$

$= 176\sqrt{2}$

Hint

$\int_0^{\pi} |\sin x|\,dx = 2$

QUESTION

#Q. $\int_0^2 (|x| + |x-1|)dx =$

A) 1

B) –1

C) 2

D) 3

QUESTION

#Q. $\int_1^4 \log[x]dx =$ $\longrightarrow$ GIF

$\int_1^2 \log 1\, dx + \int_2^3 \log 2\, dx + \int_3^4 \log 3\, dx$

(A) $\log 4$

(B) $\log 5$

(C) $\log 6$

(D) Zero

$= 0 + \log 2(x)_2^3 + \log 3(x)_3^4$

$= \log 2(1) + \log 3(1)$

$= \log 2 + \log 3$

$= \log(2)(3)$

$= \log 6$

QUESTION

#Q. If $0 < a < b$, Then $\int_a^b \frac{|x|}{x} dx =$

A a–b

B a+b

C 0

D B–a

QUESTION

#Q. $\int_{2}^{5}(|x-2|+|x-5|)dx$

A 0

B 3

C 9/2

D 9

QUESTION

#Q. $\int_{2.1}^{3.5} [x]dx =$

A) 1

B) 2

C) 3.4

D) 3.3

QUESTION

#Q. $\int_0^2 |x-2|dx =$

A ½

B 3

C 2

D 3/4

QUESTION

#Q. $\int_0^4 |2-x|dx =$

A. 0

B. 4

C. ½

D. 3/2

QUESTION

#Q. $\int_{-1}^{1} |1 - x^2| dx =$

(A) 4/3

(B) 1

(C) −1/3

(D) −4/3

b/w −1 to 1

$|1-x^2| = 1-x^2$

$I = \int_{-1}^{1} 1-x^2 \, dx$

$= x - \frac{x^3}{3}\Big]_{-1}^{1}$

$= 2 - \frac{1}{3}(2) = \frac{4}{3}$

$$|1-x^2| = \begin{cases} 1-x^2 & 1-x^2 \geq 0 \\ & x^2 \leq 1 \Rightarrow x \in [-1,1] \\ -(1-x^2) & 1-x^2 < 0 \\ = x^2-1 & x^2 > 1 \\ & x \in (-\infty,-1) \cup (1,\infty) \end{cases}$$

QUESTION

#Q. $\int_0^2 |x^2 + 2x - 3| dx =$

(A) -1

(B) 4

(C) 2

(D) 0

$|(x+3)(x-1)| = \begin{cases} x^2+2x-3 & \text{if } (x+3)(x-1) \geq 0 \\ & x \in (-\infty,-3) \cup (1,\infty) \\ 3-2x-x^2 & \text{if } (x+3)(x-1) < 0 \\ & x \in (-3,1) \end{cases}$

$\int_0^1 3-2x-x^2\,dx + \int_1^2 x^2+2x-3\,dx$

$x \in (-3,1)$ $\quad$ $x \in (1,\infty)$

$3x - x^2 - \frac{x^3}{3}\Big]_0^1 + \left(\frac{x^3}{3} + x^2 - 3x\right)_1^2$

$= \left(3 - 1 - \frac{1}{3}\right) + \frac{1}{3}(8-1) + 3 - 3$

$= 2 - \frac{1}{3} + \frac{7}{3} = \frac{6-1+7}{3} = \frac{12}{3} = 4$

QUESTION

#Q. $\int_2^3 |x^2 - 5x + 6|\,dx =$

(A) $1/6$

(B) $2/3$

(C) $4/7$

(D) $3/7$

QUESTION

#Q. $\int_{-1}^{1}[x]dx$, where $[x]$ is the greatest integer function not greater than x is

(A) 0

(B) 1

(C) −1

(D) 2

QUESTION

#Q. $\int_{-1}^{1} \frac{|x|}{x} dx =$

$\downarrow$ odd

A ✓ 0

B ½

C 1/3

D –1

$\int_{-1}^{0} -1 \, dx + \int_{0}^{1} 1 \, dx$

$-(x)_{-1}^{0} + (x)_{0}^{1}$

$= -[1] + \underline{1 = 0}$

$$I = \int_{-3}^{2} \frac{|x|}{x}\,dx$$

$$= \int_{-3}^{-2} \frac{|x|}{x}\,dx + \underbrace{\int_{-2}^{2} \frac{|x|}{x}\,dx}_{\text{odd}}$$

$$= \int_{-3}^{-2} \frac{-x}{x}\,dx + 0$$

$$= \int_{-3}^{-2} -1\,dx$$

$$= -(x)_{-3}^{-2} = -[-2+3] = -1$$

method (2)

$$I = \int_{-3}^{0} -1\,dx + \int_{0}^{2} 1\,dx$$

$$= -(x)_{-3}^{0} + (x)_{0}^{2}$$

$$= -[0+3] + (2)$$

$$= -3+2$$

$$= -1$$

Thank

You