ULTIMATE KCET

CRASH COURSE 2026

Mathematics | **Lecture-01**

Differentiability

By – Guru sir

Topics to be covered

1. Continuity
2. Differentiability
3.
4.

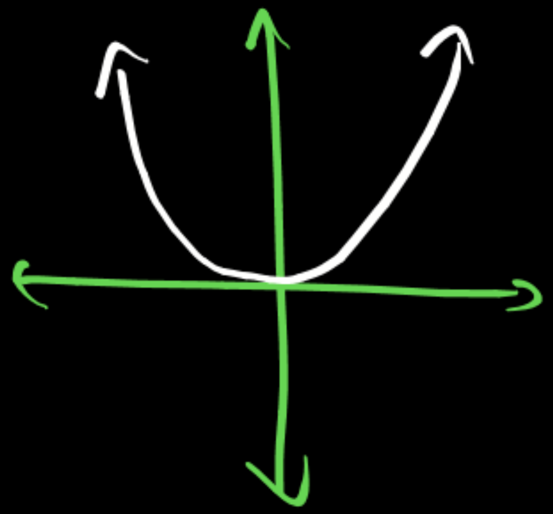
Discontinuity

$\Downarrow$

There should be

breakage or gap in the curve

$f(x)=x^2$



The func $f(x)=x^2$ is continuous & Differentiable everywhere

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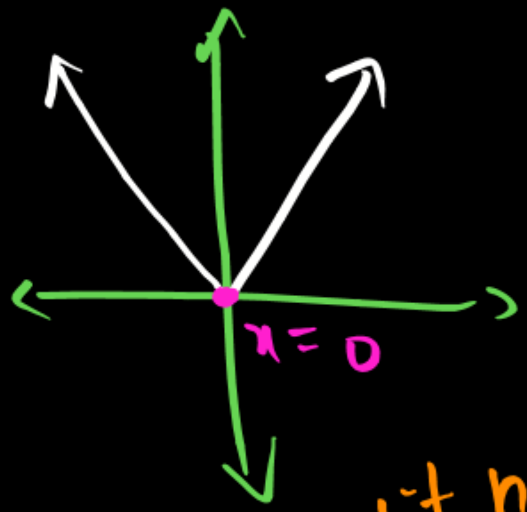
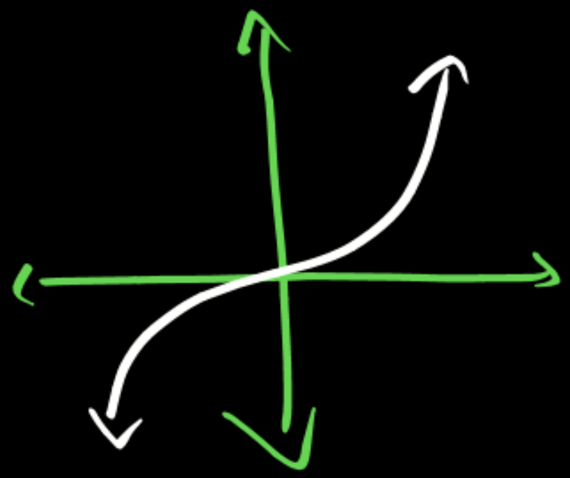
$f(x)=|x|$



At $x=0$ $\rightarrow$ split point , There is a sharp edge

$f(x)=x^3$



The func $f(x)=x^3$ is continuous & Differentiable everywhere.

$f(x) = |x|$

$\Downarrow$

only point of Non Differentiability is

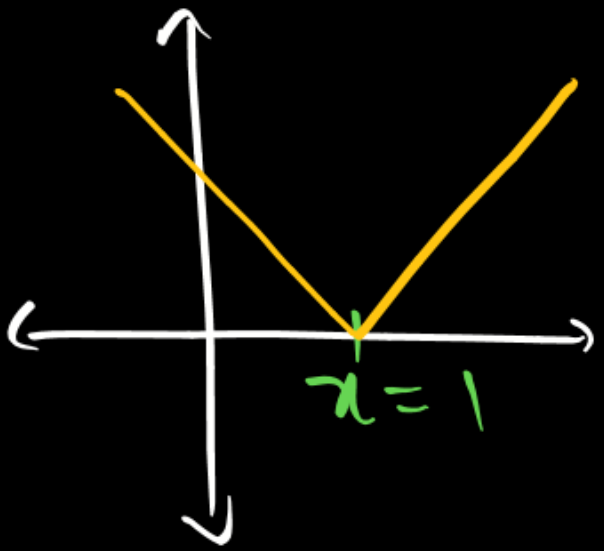
$x = 0$

$f(x) = |x-1|$

$\Downarrow$

only point of Non Differentiability is

$x = 1$

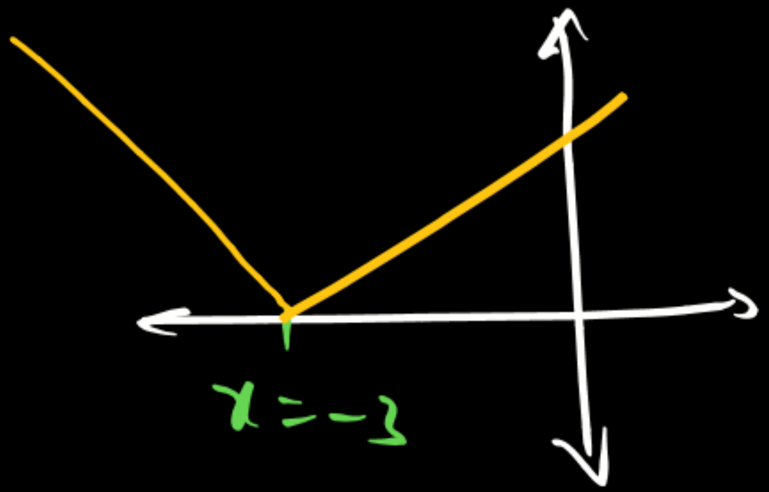


$f(x) = |x+3|$

$\Downarrow$

only point of Non Differentiability is

$x = -3$



All mod func are continuous

& are not differentiable

at split point

$$f(x) = |x+4| + |x-1| + |x^2-2x|$$

$$= |x+4| + |x-1| + |x(x-2)|$$

$$= |x+4| + |x-1| + |x|\,|x-2|$$

Point of non Differentiability

are $x=-4$ | $x=0$

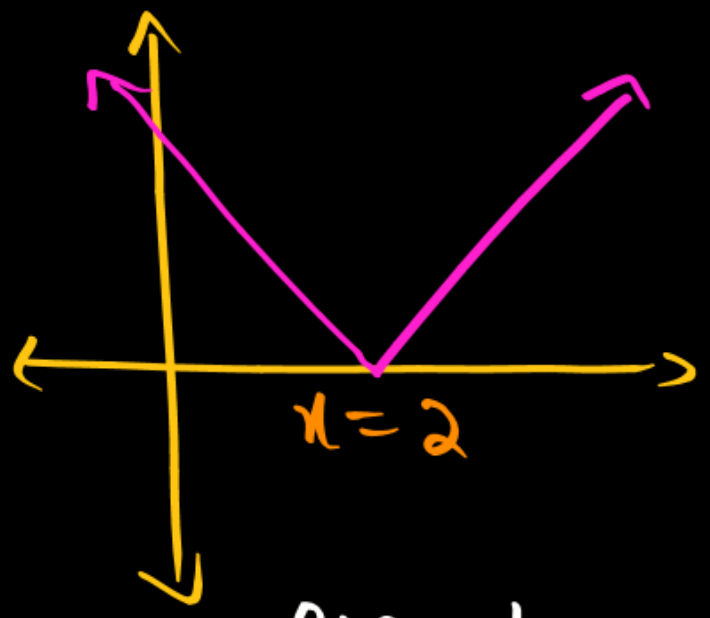
$x=1$ | $x=2$

⊛ ① All differentiable func are continuous

② If a func is continuous,
Then it doesnot imply that
it should be differentiable

Ex.: $f(x) = |x-2|$ is continuous everywhere

but it is not differentiable at $x=2$



$f(x) = |x-2|$

Differentiable at a point

⇓

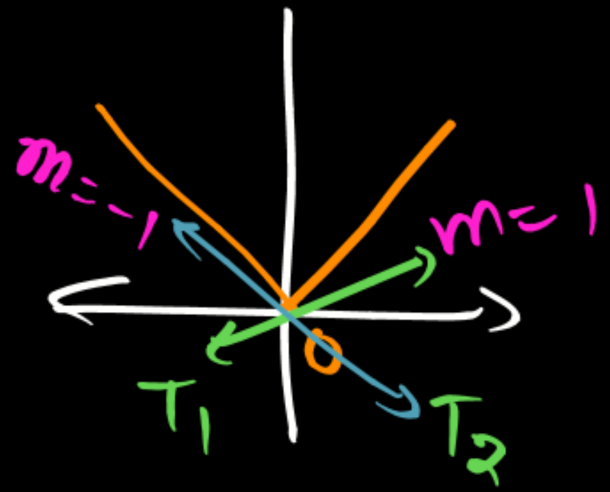
unique tangent to the curve at a point

⇓

we get unique slope

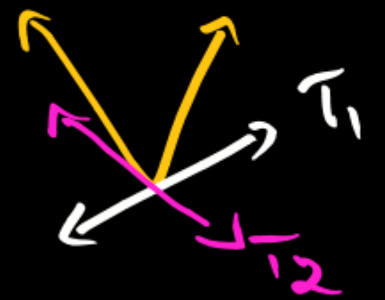
$$f(x) = |x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

$$f'(x) = \begin{cases} 1 & \text{RHD} & x \geq 0 \\ -1 & \text{LHD} & x < 0 \end{cases}$$



How can we know from a graph, whether it is differentiable or not?

① If the graph has sharp edge.

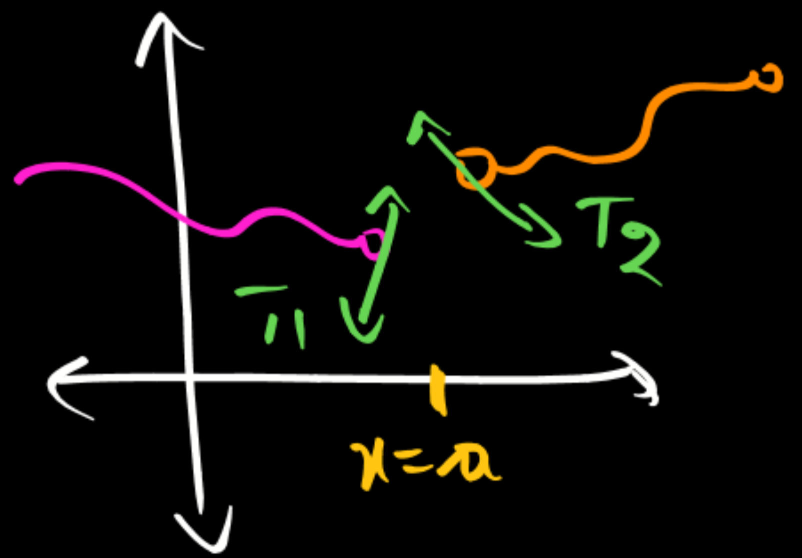


② If there is discontinuity in the graph

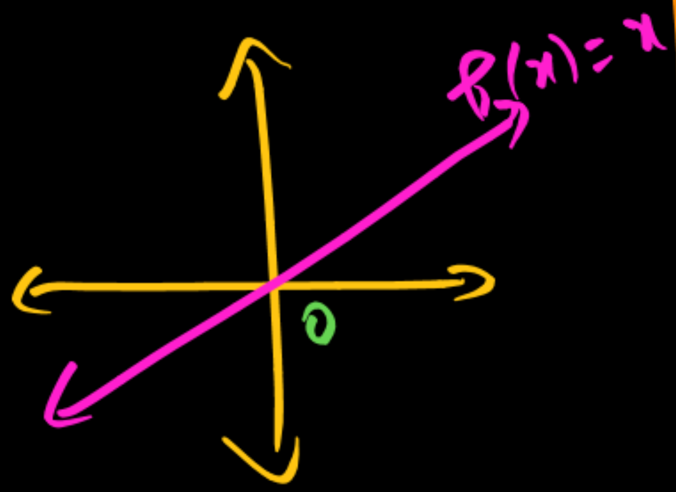
⇓

If a func is discontinuous at a point $x = a$

Then it is not differentiable at $x = a$.



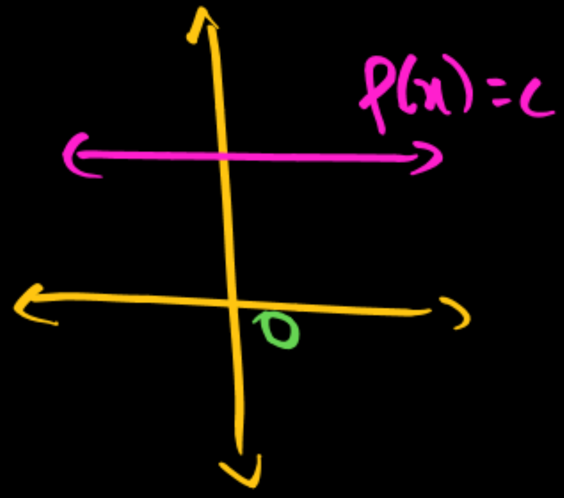
$f(x) = x$



continuity = ✓

Diff = ✓

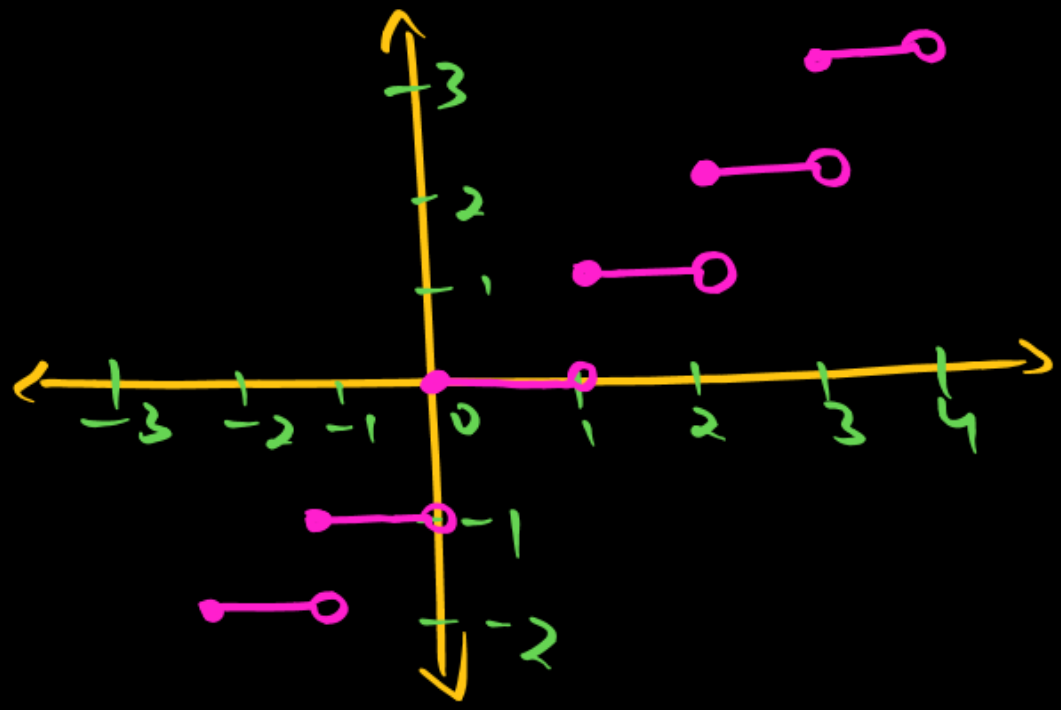
$f(x) = c$



conti = ✓

Diff = ✓

$f(x) = [x]$

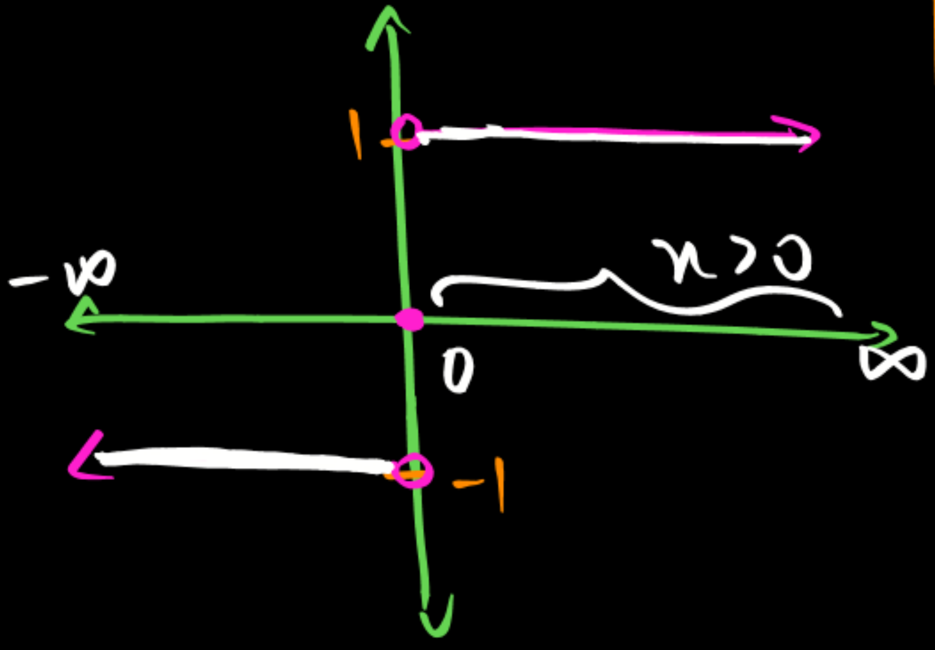


Contin :- X

Differen :- X

Discontinuous &
Not Differentiable
at Z (all
integer
values)

$f(x) =$ signum func

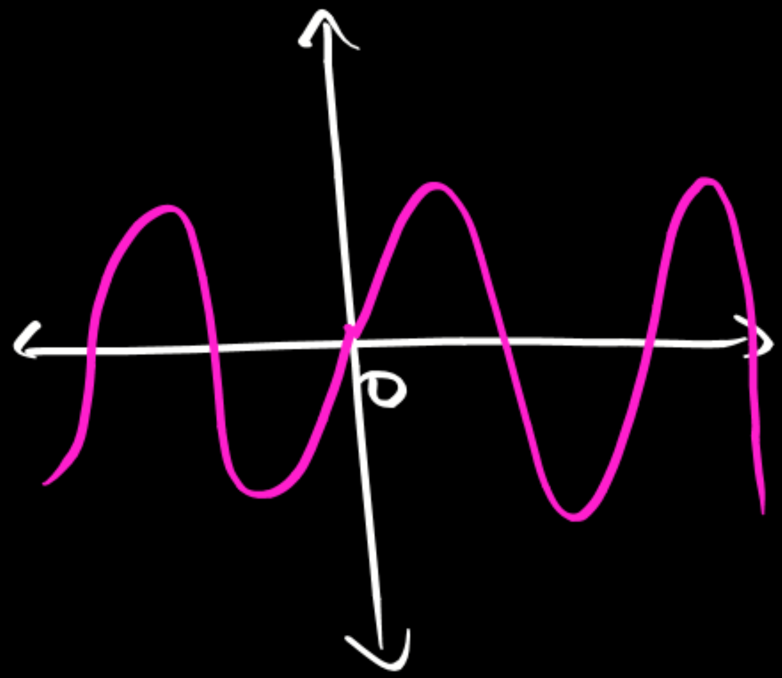


Cont : X

Diff : X

Not continuous &
hence not Differatiable
at $x = 0$

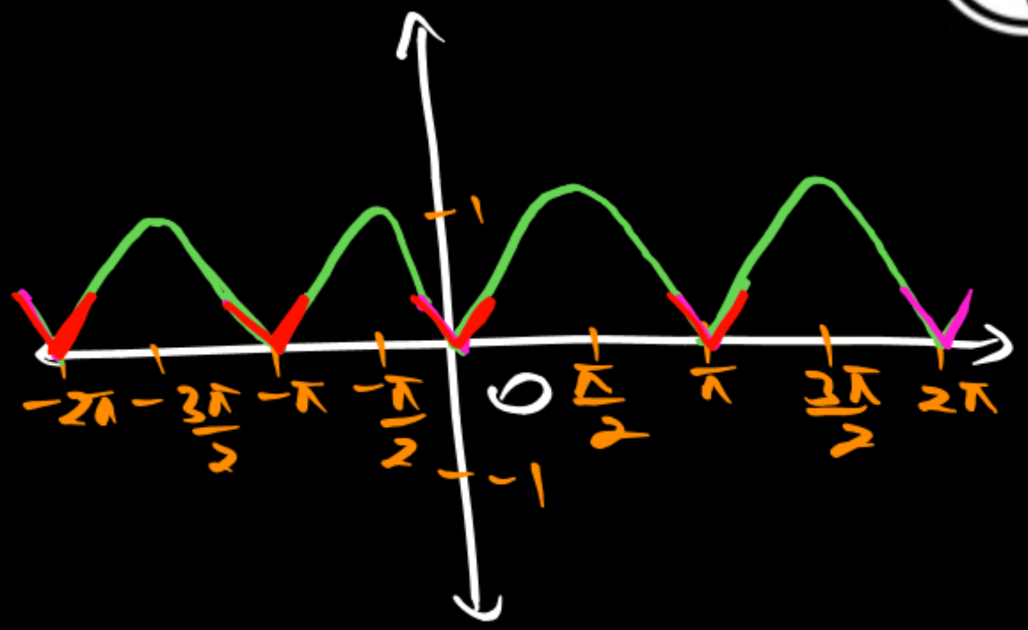
$f(x) = \sin x$



Conti : ✓ $\forall x \in R$

Diff : ✓ $\forall x \in R$

$f(x) = |\sin x|$



Conti : ✓ $\forall x \in R$

Diff : X

Not Diff at

$x = n\pi, n \in Z$.

$f(x) = |\sin x|$

split point

$0, \pi, 2\pi, 3\pi$

$-\pi, -2\pi, -3\pi$

$$|\sin x| = \begin{cases} \sin x & \text{if } \sin x \geq 0 \\ & x \in 1^{st} \text{ \& } 2^{nd} \text{ Quad} \\ -\sin x & \text{if } \sin x < 0 \\ & x \in 3^{rd} \text{ \& } 4^{th} \text{ Quad} \end{cases}$$

$$f(x) = |\sin x| = \begin{cases} \sin x & \text{if } x < \pi \\ & x \in 2^{nd} \\ -\sin x & \text{if } x > \pi \\ & x \in 3^{rd} \end{cases}$$

$$f'(x) = \begin{cases} \cos x = \cos \pi = -1 = LHD \\ -\cos x = -\cos \pi = -(-1) = RHD \\ \quad = 1 \end{cases}$$

$LHD \neq RHD$ at $\boxed{x = \pi}$

1^{st} & 2^{nd} Quad	3^{rd} & 4^{th} Quad
$[0, \pi]$	$[\pi, 2\pi)$
$[2\pi, 3\pi]$	$[3\pi, 4\pi]$
$[4\pi, 5\pi]$	$[5\pi, 6\pi]$
$[-\pi, -2\pi]$	$[-\pi, 0]$
$[-3\pi, -4\pi]$	$[-3\pi, -2\pi]$

$$f(x) = \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases}$$

$f(1) = 1$

$f(2) = 1$

$f(1.5) = 1$

$f(1.9) = 1$

$f(100) = 1$

$\Downarrow$

$\boxed{x > 0}$

Functional value

$f(0) = 0$

Limiting value

LHL $= -1$

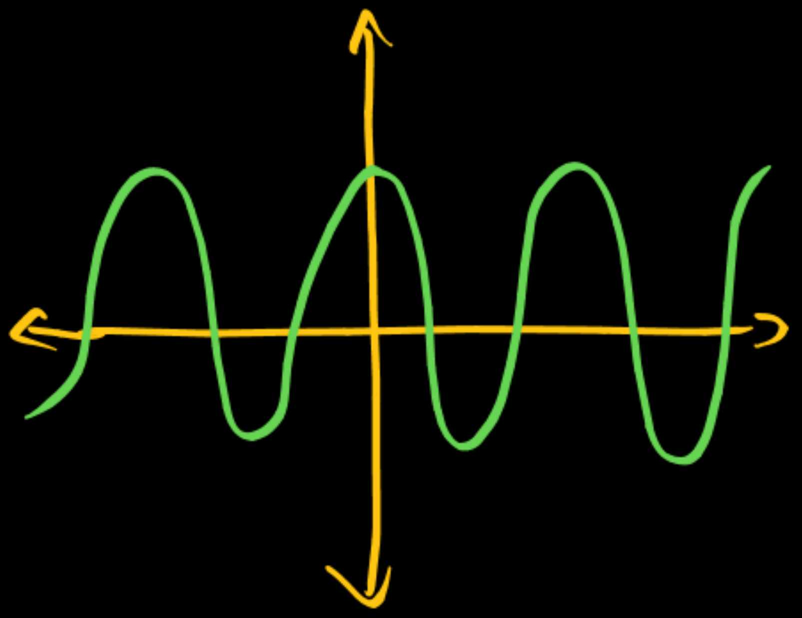
RHL $= 1$

LHL $\neq$ RHL

$\Downarrow$ limit does not exist

$\Rightarrow$ Discontinuous

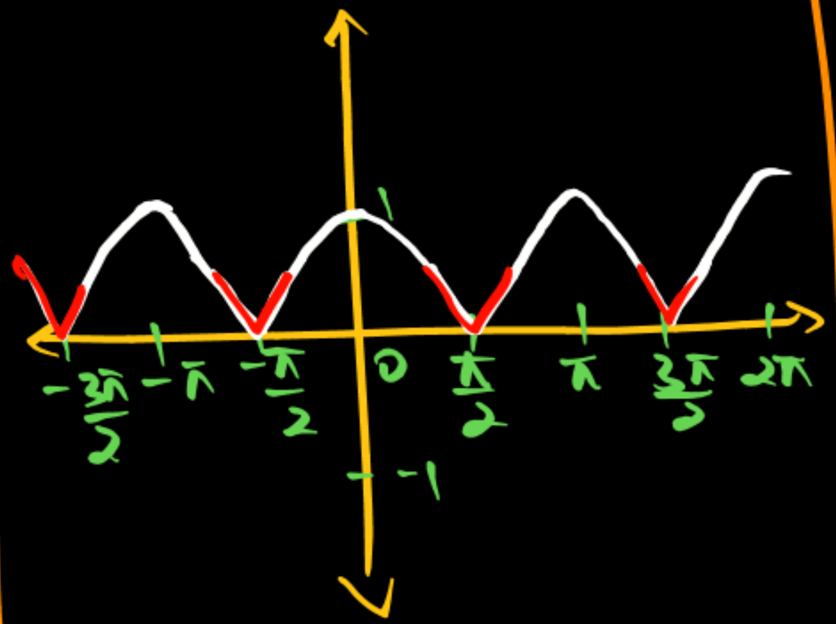
$f(x) = \cos x$



Cont : ✓

Diff : ✓

$f(x) = |\cos x|$



Contin : ✓

Diff : X

Not Differentiable at $x = (2n+1)\frac{\pi}{2}$

$n \in Z$

$f(x) = \tan x = \frac{\sin x}{\cos x}$

$\cos x = 0$

$\Rightarrow x = (2n+1)\frac{\pi}{2}, n \in Z$

$f(x) = \sec x = \frac{1}{\cos x}$

Not continuous & hence not Diff at

$x = (2n+1)\frac{\pi}{2}, n \in Z$

$f(x) = \cot x = \frac{\cos x}{\sin x}$

$\sin x = 0$

$x = n\pi, n \in Z$

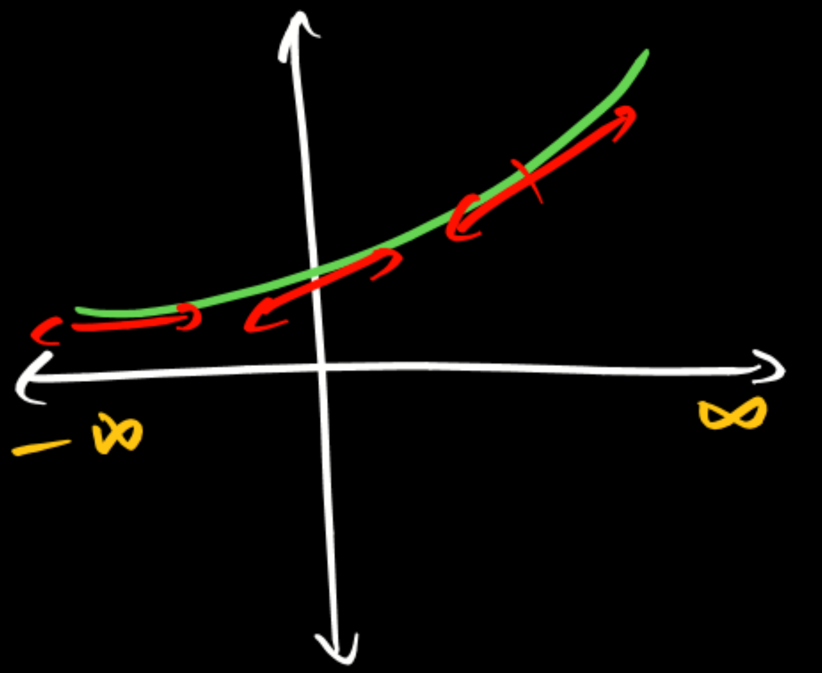
$f(x) = \operatorname{cosec} x = \frac{1}{\sin x}$

$\Downarrow$

Not continuous & hence not Diff

at $x = n\pi, n \in Z$

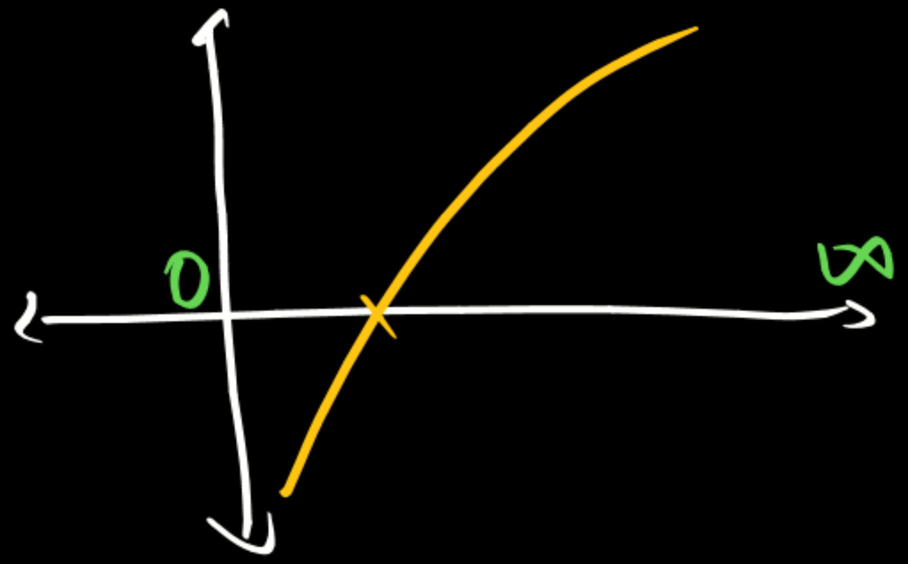
$f(x) = e^x$



cont : ✓ $\forall x \in R$

Diff : ✓ $\forall x \in R$

$f(x) = \log_e x$



cont :- ✓ $\forall x > 0$

Diff : ✓ $\forall x > 0$

→ $f(x)$ is Discontinuous if

① LHL ≠ RHL

limit does not exist

Ex:- Signum function

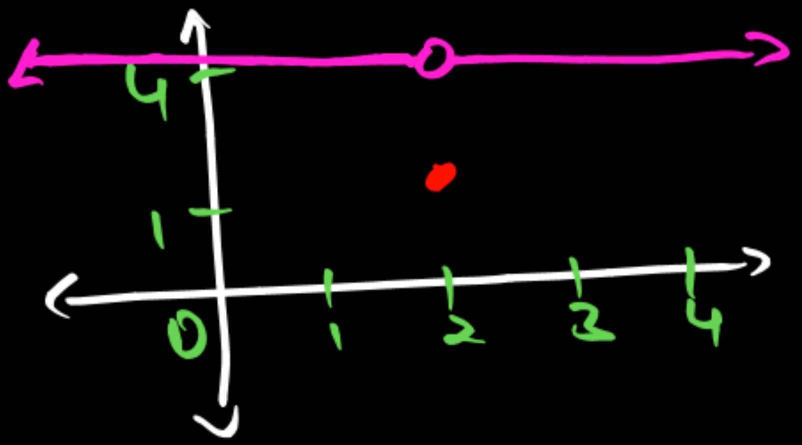
② $\lim_{x \to a} f(x) \neq f(a)$

limiting value ≠ functional value

③ if $f(x)$ is not defined at $x = a$.

Ex: $f(x) = \frac{1}{x-1}$

Here $f(x)$ is discontinuous at $x = 1$



→ Ex: $f(x) = \begin{cases} 4 & x \neq 2 \\ 1 & x = 2 \end{cases}$

$\lim_{x \to 2} f(x) = \lim_{x \to 2} 4 = 1$

$f(2) = 1$

Discontinuous at $x = 2$ & hence not Differentiable.

QUESTION

#Q. $f(x) = \sin x + \log_e x$ is continuous

⇓

at intersection of

Domain of sin x & $\log_e x$

↓ ↓

$(-\infty, \infty) \leftarrow R$ $\quad x > 0 \Rightarrow (0, \infty)$

intersection

$x > 0$

$(0, \infty)$

A $\forall x$

B [✓ $\forall x > 0$]

C $\forall x < 0$

D $\forall x \in (0, 1]$

$f(x) = \sin x$

⇓

Domain & Range

⇓ ⇓

R $\quad [-1, 1]$

$f(x) = \log_e x$

Domain & Range

↓ ↓

$(0, \infty)$ $\quad$ R

To check the points of Discontinuity

⇓

its enough if we check at what points the given function is not defined

$$x^2-3x+2=0 \qquad +2$$

$$(x-2)(x-1)=0 \qquad -2 \quad -1$$

$$x=2,\ x=1$$

$$f(x)=\frac{1}{x^2-3x+2}$$

① is discontinuous at $x=2$ & $x=1$

② is continuous at $R-\{1,2\}$

QUESTION

#Q. The number of points at which the function $\frac{1}{\log_e |x|}$ is discontinuous is

A 1

B 2

[✓ **C**] 3

D infinite

$f(x) = \log_{|x|} e$

$f(x)$ is not defined if

$|x| = 1$ & $|x| = 0$

$x = \pm 1$ | $x = 0$

$\therefore f(x)$ is not defined for $x = 0, 1, -1$

$\log_{\boxed{h(x)}} g(x)$

① $h(x) > 0$

② $h(x) \neq 1$

$f(x) = \log_{x+2} (x-1)$ is $\boxed{\text{discontinuous}}$ for.

↳ we need find the point where $f(x)$ is not defined

Ⓐ $x > 1$

~~Ⓑ~~ $x \leq 1$

Ⓒ $x < -2$

Ⓓ $(-2,-1) \cup (-1, \infty)$

Soln :-

is discontinuous if

① $x-1 \leq 0$

$x \leq 1$

② $x+2 \leq 0$

$x \leq -2$

③ $x+2 = 1$

$x = -1$

union

$x < 1$

$f(x) = \log_{h(x)} g(x)$

① $g(x) > 0$

② $h(x) > 0$

③ $h(x) \neq 1$

if $f(x) = \log_{x+2} x-1$, Domain

$x-1>0$

$x>1$

$x+2>0$

$x>-2$

$x+2\neq 1$

$x\neq -1$

intersection

$\boxed{x>1}$

QUESTION

$$\lim_{x\to 0} f(x) = f(0)$$

#Q. $f(x) = \begin{cases} (1+x)^{1/x} & \text{for } x \neq 0 \\ \underset{\searrow f(0)}{e} & \text{for } x = 0 \end{cases}$ then $f(x)$ is

[✓ A] continuous at $x = 0$

B discontinuous at $x = 0$

C continuous at $x = e$

D continuous, $\forall x \in R - \{0\}$

$$\lim_{x\to 0} (1+x)^{1/x} = K$$

Take log on B.S

$$\lim_{x\to 0} \log(1+x)^{1/x} = \log K$$

$$\lim_{x\to 0} \frac{1}{x}\log(1+x) = \log K$$

$$\lim_{x\to 0} \frac{\overset{\nearrow 0}{\log(1+x)}}{\underset{\searrow 0}{x}} = \log K$$

L Hospitals rule

$$\lim_{x\to 0} \frac{\frac{1}{1+x}}{1} = \log K$$

$$\frac{1}{1+0} = 1 = \log K$$

$$K = e^1 = e$$

$$= \underline{f(0)}$$

QUESTION

#Q. The function $f(x) = |x - 3|$ is

→ ① continous everywhere

② Not Differentiable at $x = 3$

A [✓ continuous at $x = 3$]

B not continuous at $x = 3$

C differentiable at $x = 3$

D neither continuous nor differentiable at $x = 3$

QUESTION

#Q. The function $f(x) = [x]$. (The greatest integer function) is

A continuous at all real points

B [✓ not continuous at integral values of x] & hence not Diff

C differentiable at integral values of x

D continuous but not differentiable at integral values of x

QUESTION

#Q. Let $f(x) = |x| + |x - 1|$ then

A $f(x)$ is continuous at $x = 0$ as well as at $x = 1$

B $f(x)$ is continuous at $x = 0$ but not at $x = 1$

C $f(x)$ is continuous at $x = 1$, but not at $x = 0$

D $f(x)$ is not continuous at $x = 0$ and $x = 1$

QUESTION

#Q. $f(x) = \begin{cases} -1 & : x < -1 \\ -x & : -1 \leq x \leq 1 \\ 1 & : x > 1 \end{cases}$ is continuous

A at $x = 1$ but not at $x = -1$

B at $x = -1$ but not at $x = 1$

C at both $x = 1$ and $x = -1$

[✓ **D**] at none of $x = 1$ and -1

$$f(x) = \begin{cases} -1 & \text{if } x < -1 \\ -x & \text{if } x \geq -1 \\ -x & \text{if } x \leq 1 \\ 1 & \text{if } x > 1 \end{cases}$$

At $x = -1$:-

LHL $= -1$

RHL:-

$f(x) = -x$

$\lim_{x \to -1} (-x) = -(-1) = 1$

LHL $\neq$ RHL

at $x = -1$

At $x = 1$:-

LHL:- $f(x) = -x$

$\lim_{x \to 1} -x = -1$

RHL $= 1$

LHL $\neq$ RHL

at $x = 1$

QUESTION

$$f(x) = \begin{cases} x + a\sqrt{2}\sin x \rightarrow \text{if } x < \frac{\pi}{4} \\ 2x\cot x + b \rightarrow x \geq \pi/4 \\ 2x\cot x + b \longrightarrow x \leq \frac{\pi}{2} \\ a\cos 2x - b\sin x \rightarrow x > \frac{\pi}{2} \end{cases}$$

#Q. The function $f(x) = \begin{cases} x + a\sqrt{2}\sin x & : 0 \leq x < \pi/4 \\ 2x\cot x + b & : \pi/4 \leq x \leq \pi/2 \\ a\cos 2x - b\sin x & : \pi/2 < x \leq \pi \end{cases}$ is continuous for $0 \leq x \leq \pi$. Then a, b are

- **A** $\frac{\pi}{6}, \frac{\pi}{12}$
- **B** $\frac{\pi}{3}, \frac{\pi}{6}$
- **C** $\frac{\pi}{6}, -\frac{\pi}{12}$
- **D** $\frac{\pi}{6}, \frac{\pi}{2}$

At $x = \frac{\pi}{4}$:-

LHL :-

$\frac{\pi}{4} + a\sqrt{2}\,\frac{1}{\sqrt{2}}$

$a + \frac{\pi}{4}$

RHL :-

$2\left(\frac{\pi}{4}\right)(1) + b$

$\frac{\pi}{2} + b$

LHL = RHL

$a + \frac{\pi}{4} = b + \frac{\pi}{2}$

$a - b = \frac{\pi}{4} \rightarrow$ ①

At $x = \frac{\pi}{2}$:-

LHL :-

$2\left(\frac{\pi}{2}\right)(0) + b = b$

RHL :-

$a(-1) - b(1)$

$-a - b$

LHL = RHL

$b = -a - b$

$a = -2b \rightarrow$ ②

$a - b = \frac{\pi}{4} \rightarrow ①$

$a = -2b \rightarrow ②$

$-2b - b = \frac{\pi}{4}$

$-3b = \frac{\pi}{4}$

$\boxed{b = -\frac{\pi}{12}}$

$a = -2b$

$a = -2\left(-\frac{\pi}{12}\right)$

$\boxed{a = \frac{\pi}{6}}$

QUESTION

#Q. The function defined by $f(x) = \begin{cases} |x-3| & \text{for } x \geq 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4} & \text{for } x < 1 \end{cases}$ is

(A) ✓ continuous at $x = 1$ and at $x = 3$

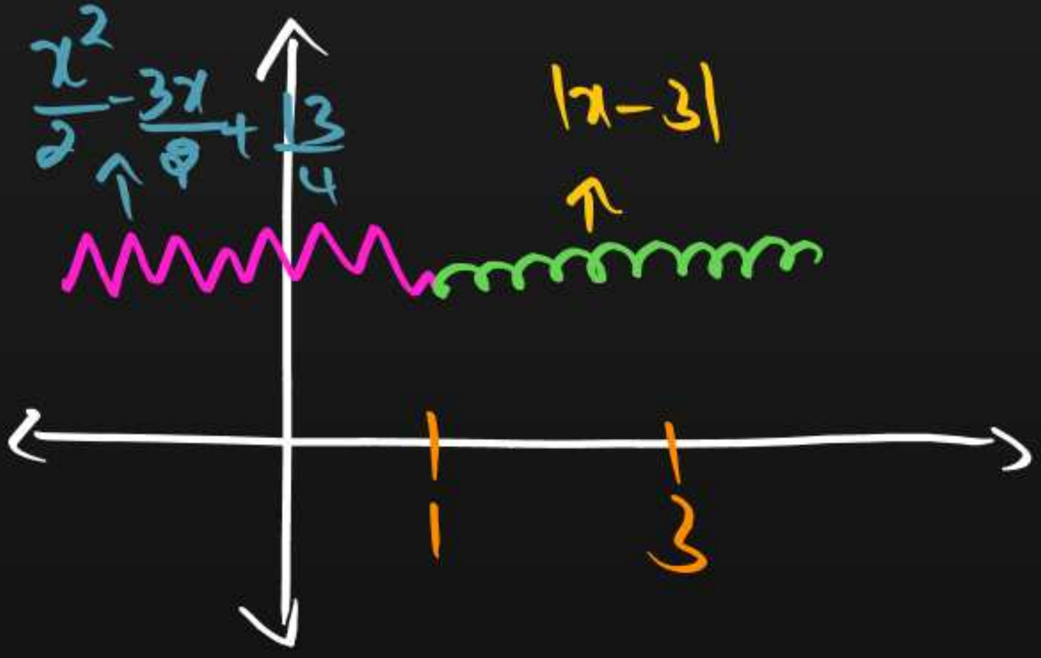
(B) ✗ continuous at $x = 1$ and discontinuous at $x = 3$

(C) continuous at $x = 3$ and discontinuous at $x = 1$

(D) ✗ discontinuous at $x = 1$, $x = 3$

At $x=3$

$f(x) = |x-3|$ which is always continuous



$$f(x)=\begin{cases}|x-3| & \text{for } x\geq 1\\ \frac{x^2}{2}-\frac{3x}{2}+\frac{13}{4} & \text{for } x<1\end{cases}$$

$$|x-3| = -(x-3)$$
$$= 3-x \quad \text{if } \boxed{x<3}$$

$$f(x)=\begin{cases}3-x & x\geq 1\\ \frac{x^2}{4}-\frac{3x}{2}+\frac{13}{4} & x<1\end{cases}$$

$$|x-3| = \begin{cases}x-3 & \text{if } x\geq 3\\ -(x-3) & \text{if } x<3\end{cases}$$

At $x=1$:

LHL:

$3-1=2$

RHL:

$$\frac{1}{4}-\frac{3}{2}+\frac{13}{4}=\frac{14}{4}-\frac{6}{4}=\frac{8}{4}=2$$

$$\boxed{\text{LHL}=\text{RHL}}$$

$x=1$ means $\boxed{x<3}$

$x > 1$

$\Downarrow$

means

$$\left.\begin{aligned} x &= 1.001 \\ &= 1.0001 \\ &= 1.01 \end{aligned}\right\} < 3 \Rightarrow \begin{aligned} |x-3| &= -(x-3) \\ &= 3-x \end{aligned}$$

QUESTION

#Q. Which one of the following is a $\boxed{\text{false statement?}}$

A ✗ If $f(x)$ is continuous at $x = a$, then $\lim_{x \to a} f(x)$ exists. True

Differentiability ⇒ continuity

B ✓ If $f'(a)$ exists, then $f(x)$ is continuous at $x = a$ True

C ✓ If $f(x)$ is continuous at $x = a$, then $\lim_{x \to a} f(x) = f(a)$ True

D ✓ If $\lim_{x \to a} f(x)$ exists, then $f(x)$ is continuous at $x = a$ False

limit exists

⇓

continuous

This information is half

Actually

limit exists & should be equal to $f(a)$

QUESTION

$$\begin{cases} \frac{x-x}{x} = \frac{0}{x} = 0 & \text{if } x>0 \Rightarrow RHL \\ \frac{x-(-x)}{x} = \frac{2x}{x} = 2 & \text{if } x<0 \Rightarrow LHL \\ 1 & \text{if } x=0 \end{cases}$$

#Q. If $f(x) = \begin{cases} \frac{x-|x|}{x} & \text{for } x \neq 0 \\ 1 & \text{for } x = 0 \end{cases}$, then at $x = 0$, the function is

LHL ≠ RHL

A continuous

B not continuous since LHL ≠ RHL

C not continuous since LHL or RHL does not exist

D discontinuous since the function is not defined at $x = 1$

QUESTION

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\frac{\lim_{x\to a}f(x)}{\lim_{x\to a}g(x)}$$

#Q. If $f(x)=\begin{cases}\frac{x^2-(k+2)x+k}{x-2} & \text{for } x\neq 2\\ 2 & \text{for } x=2\end{cases}$ is continuous at $x = 2$ then k is

[✓ A] 0

B 1

C −1

D 3

$\Downarrow$

$$\lim_{x\to 2}f(x)=f(2)$$

$$\lim_{x\to 2}\frac{x^2-(k+2)x+k}{x-2}=2$$

$$\frac{\lim_{x\to 2}x^2-(k+2)x+k}{\lim_{x\to 2}(x-2)}=2$$

$$4-(k+2)2+k=2\lim_{x\to 2}(x-2)$$

$$4-2k-4+k=2(0)$$

$$-k=0$$

$$\boxed{k=0}$$

QUESTION

#Q. The function $f(x) = |\sin x|$ is

A differentiable at $x = 0$

B not differentiable at $x = \pi/2$

[✓ C not differentiable at $x = 0$]

D differentiable at $x = 0$ and $x = \pi/2$

QUESTION

#Q. Find the points discontinuity of the functions $f(x)=\frac{1}{2\sin x-1}$ $x \in [0, 2\pi]$

(A) $\frac{\pi}{3}, \frac{2\pi}{3}$

[✓ (B)] $\frac{\pi}{6}, \frac{5\pi}{6}$

(C) $\frac{\pi}{4}, \frac{3\pi}{4}$

(D) $\frac{\pi}{2}, \pi$

$\rightarrow 2\sin x - 1 = 0$

$2\sin x = 1$

$\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}$ & $x = \frac{5\pi}{6}$

QUESTION

#Q. Number the points of discontinuity of the functions $f(x) = \frac{1}{x^2-3\underline{|x|}+2}$

(A) $\underline{1}$

(B) 2

(C) 3

[✓ (D) 4]

WKT $|x|^2 = x^2$

$\Downarrow$

$x^2 - 3|x| + 2 = 0$

$|x|^2 - 3|x| + 2 = 0$

$(|x|-2)(|x|-1) = 0$

+2

−2 −1

$\lvert x \rvert = 2$	$\lvert x \rvert = 1$
$x = \pm 2$	$x = \pm -1$

QUESTION

#Q. Number the points of discontinuity of the function $f(x) = \frac{1}{x^4+x^2+1}$

[✓ (A) 0]

(B) 1

(C) 3

(D) 4

$x^4+x^2+1=0$

Put $x^2=t$

$D<0 \Leftarrow t^2+t+1=0$

$\Downarrow$

no real root

$\underbrace{t^2+t+\frac{1}{4}}-\frac{1}{4}+1=0$

$\left(t+\frac{1}{2}\right)^2+\frac{3}{4}=0$

$\left(x^2+\frac{1}{2}\right)^2+\frac{3}{4}=0$

Which is not Possible for any $x \in R$

∴ There are no Points of Discontinuity

QUESTION

#Q. Let $f(x) = \left\{\frac{\log(1+x)^{1+x}-x}{x^2}\right\}$. Then find the value of $f(0)$ so that the function f is continuous at $x = 0$.

(A) 0

(B) 2

[✓ (C) $\frac{1}{2}$]

(D) 1

$$\lim_{x\to0}\frac{\log(1+x)^{1+x}-x}{x^2}=f(0)$$

$$\lim_{x\to0}\frac{(1+x)\overset{\to0}{\log(1+x)}-x}{\underset{\to0}{x^2}}=f(0)$$

L Hospital's rule

$$\lim_{x\to0}\frac{(1+x)\left(\frac{1}{1+x}\right)+\log(1+x)(0+1)-1}{2x}=f(0)$$

$$\lim_{x\to0}\frac{\cancel{1}+\overset{\to0}{\log(1+x)}-\cancel{1}}{\underset{\to0}{2x}}=f(0)$$

L Hospitals rule

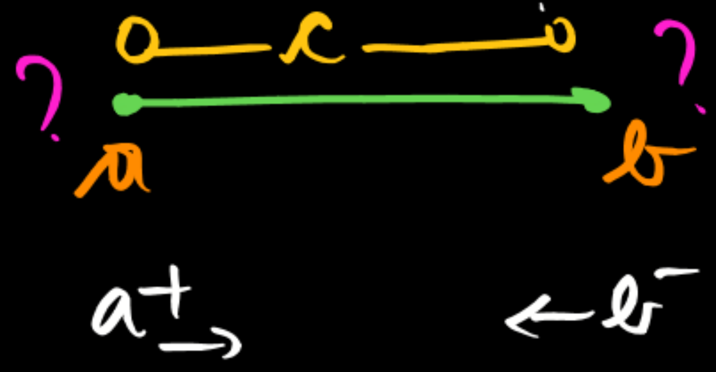
$$\lim_{x\to0}\frac{\frac{1}{1+x}}{2}=f(0)$$

$$\left(\frac{\frac{1}{1}}{2}\right)=f(0)$$

$$\boxed{f(0)=\frac{1}{2}}$$

if $x \in [a, b]$ Then $f(x)$ is continuous if

$$\left.\begin{array}{ll} \text{①} & \lim_{x \to a^+} f(x) = f(a) \\ & \\ \text{②} & \lim_{x \to b^-} f(x) = f(b) \end{array}\right\} \text{at end points}$$



③ $\lim_{x \to c} f(x) = f(c) \quad \forall c \in (a, b)$

QUESTION

#Q. Find the number of points where $f(x) = [x/3]$, $x \in [0, 30]$, is discontinuous (where [.] represents greatest integer function).

(A) 11

~~(B) 10~~

✗ (C) 30

✗ (D) 31

$x \in [0, 30]$

$\frac{x}{3} \in [0, 10]$

$$\text{integers are} = \{0, \underbrace{1, 2, 3, 4, 5, 6, 7, 8, 9}_{\text{Pakka Discontinuous}}, 10\} \rightarrow 11 \text{ integers}$$

① At $\frac{x}{3} = 0$:-

$\lim_{\frac{x}{3} \to 0^+} f(x) = \lim_{x \to 0^+} \left[\frac{x}{3}\right] = [0] = 0$

$f(0) = [0] = 0$

f(x) is continuous at $\frac{x}{3} = 0$

At $\frac{x}{3} = 10$:-

$\lim_{\frac{x}{3} \to 10^-} \left[\frac{x}{3}\right] = 9$

$f(10) = [10] = 10$

f(x) is discontinuous at $\frac{x}{3} = 10$

$\therefore$ Points of Discontinuity

$= \{1, 2, 3, \ldots \ldots 10\}$

$\Downarrow$

10 points of Discontinuity

To know the points at which $f(x)$ is differentiable

we find the points where $f'(x)$ exists

(*) if $f(x) = \sin^{-1}x$, Then the points at which $f(x)$ is differentiable

Ⓐ $[-1, 1]$

[✓ Ⓑ $(-1, 1)$]

Ⓒ R

Ⓓ $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Soln:

$$f(x) = \sin^{-1}x$$

$$f'(x) = \frac{1}{\sqrt{1-x^2}}$$

$f'(x)$ exists if

$$1 - x^2 > 0$$

$$1 > x^2$$

$$x^2 < 1 \implies \sqrt{x^2} < 1$$

$$|x| < 1$$

$$\underline{x \in (-1, 1)}$$

$$f(x) = \frac{1}{\sqrt{g(x)}}$$

$$\Rightarrow g(x) > 0$$

② if $f(x) = \cos^{-1}x$, Then the points at which which $f(x)$ is not differentiable

Ⓐ $(-1,1)$

Ⓑ $[-1,1]$

Ⓒ R

[✓ Ⓓ $(-\infty,-1] \cup [1,\infty)$]

$$f(x) = \cos^{-1}x$$

$$f'(x) = \frac{-1}{\sqrt{1-x^2}}$$

∴ $f'(x)$ doesnot exists if

$$1-x^2 \leq 0$$

$$1 \leq x^2$$

$$x^2 \geq 1$$

$$|x| \geq 1$$

$$x \in (-\infty,-1] \cup [1,\infty)$$

$$\Downarrow$$

$$R-(-1,1)$$

$$\frac{1}{\sqrt{1-x^2}}$$

exists

$$1-x^2 > 0$$

$$\frac{1}{\sqrt{1-x^2}}$$

doesnot exist

$$1-x^2 \leq 0$$

QUESTION

#Q. Let $f(x) = 1 + \sqrt[3]{(x-1)^2}$, the set of all points at which $f(x)$ is differentiable is

A R

B [✓ $R - \{1\}$]

C $R - \{0\}$

D R^+

$$f(x) = 1 + (x-1)^{2/3}$$

$$f'(x) = \frac{2}{3}(x-1)^{-1/3} = \frac{2}{3}\,\frac{1}{(x-1)^{1/3}}$$

$\therefore$ $f'(x)$ exists if

$$x - 1 \neq 0$$

$$x \neq 1$$

$\therefore$ $f(x)$ is differentiable

$\forall x \in \underline{R - \{1\}}$

QUESTION

#Q. If $y = |x + 1| + |x - 1|$ is differentiable at all points except

$x = 1$ & $x = -1$

A 0

B [✓ ± 1]

C R

D ± 2

QUESTION

#Q. The function $f(x)=\sqrt{x-1}+\sqrt{1-x}$ is differentialbe in the set

A $\{x : -1 < x < 1\}$

B $\{x : 1 < x < \infty\}$

C $\{x : -\infty < x < -1\}$

D [✓ $\{\}$]

$$f'(x) = \frac{1}{2\sqrt{x-1}} + \frac{-1}{2\sqrt{1-x}}$$

$f'(x)$ exists if

$x-1>0$ & $1-x>0$

$x>1$

$1>x$

$x<1$

intersection

$\{\}$

QUESTION

$x^2>0$

is true $\forall x\in R-\{0\}$

#Q. The set of all points where the function $f(x)=\sqrt{1-e^{-x^2}}$ is differentiable is

A $(0,\infty)$

B $(-\infty,\infty)$

[✓ **C**] $(-\infty,\infty)\sim\{0\}$

D $(-1,\infty)$

$$f'(x)=\frac{1}{2\sqrt{1-e^{-x^2}}}\left(0-e^{-x^2}(-2x)\right)$$

$$=\frac{2x\,e^{-x^2}}{2\sqrt{1-e^{-x^2}}}$$

$\therefore f'(x)$ exists if

$1-e^{-x^2}>0$

$1>e^{-x^2}$

$e^{-x^2}<1$

$\log e^{-x^2}<\log 1$

$-x^2\log e<0$

$-x^2<0$

$x^2>0$

is true $\forall x\in R-\{0\}$

Thank

You