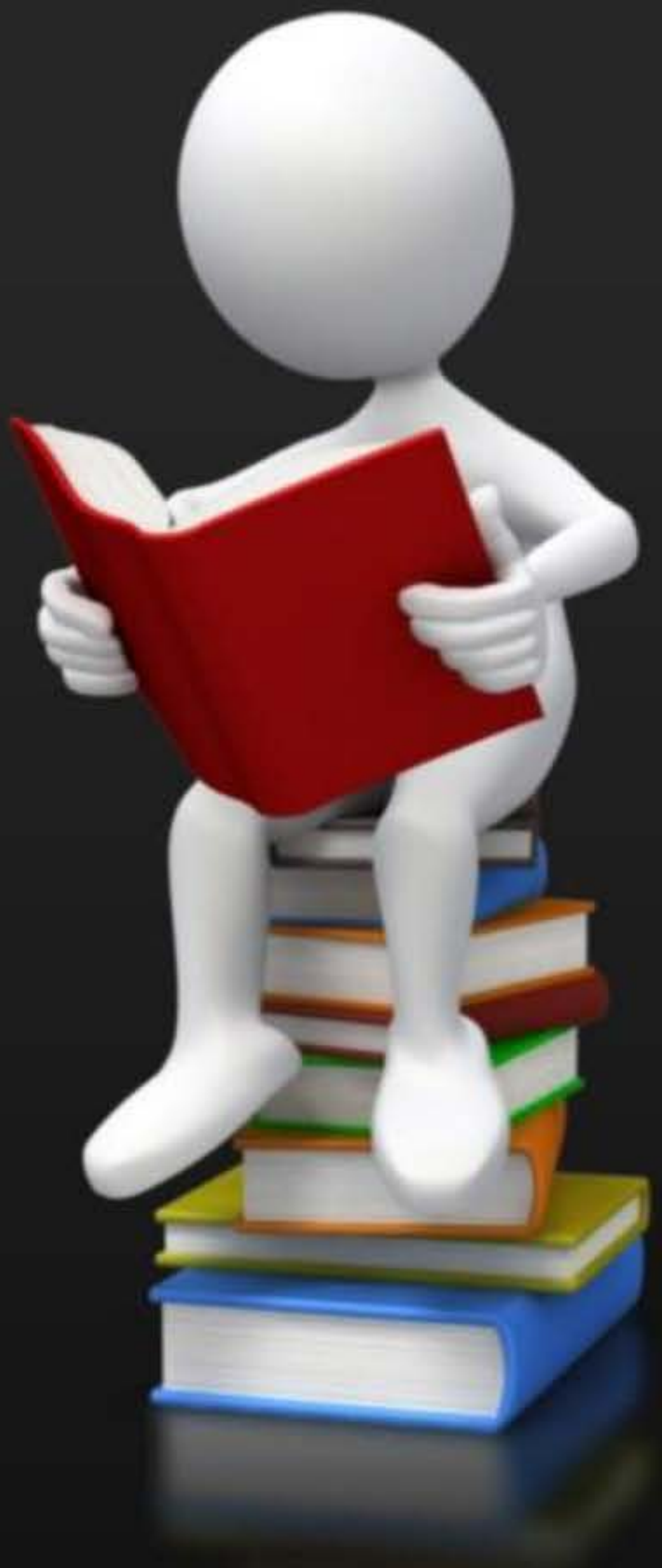

ULTIMATE KCET
CRASH COURSE 2026
Mathematics
Lecture – 03
Methods of Differentiation
By – Guru sir

Recap of previous lecture

1. Differentiation
2.
3.
4.

Topics *to be covered*

1. Differentiation → continue
2.
3.
4.



Limit, Continuity, Differentiability, Methods of Diff

1m

1m

1m

3 to 4 m

2 easy

1 moderate

1 Diff

QUESTION

#Q. If $x = a(1+t), y = a(a-t^2)$ then $\frac{dy}{dx}$ at $t = 0$ is

$\frac{dx}{dt} = a$ | $\frac{dy}{dt} = -2at$

A 1

B −1

$$\frac{dy}{dx} = \frac{-2at}{a} = -2t$$

C 0

$$t = 0$$

D 2

$$\frac{dy}{dx} = 0$$

QUESTION

#Q. If $x = \sin t \cdot \cos 2t, y = \cos t \cdot \sin 2t$ then $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$ is

(A) -2

(B) 2

(C) $-\frac{1}{2}$

(D) $\frac{1}{2}$

$$\frac{dy}{dx} = \frac{2\cos t\cos 2t - \sin t\sin 2t}{-2\sin t\sin 2t + \cos t\cos 2t}$$

Put $t = \frac{\pi}{4}$

$$= \frac{2\left(\frac{1}{\sqrt{2}}\right)(0) - \frac{1}{\sqrt{2}}(1)}{-2\left(\frac{1}{\sqrt{2}}\right)(1)} = \frac{\frac{1}{\sqrt{2}}}{+\sqrt{2}} = +\frac{1}{2}$$

QUESTION

#Q. If $x = a\left(\cos t + \log \tan\frac{t}{2}\right), y = a\sin t$, then $\frac{dy}{dx}$ is

(A) tan t

(B) cot t

(C) -tan t

(D) sec t

$$\frac{dy}{dx} = \frac{\cancel{a}\cos t}{\cancel{a}\left[-\sin t + \frac{1}{\tan\frac{t}{2}}\sec^2 t/2\left(\frac{1}{2}\right)\right]}$$

$$= \frac{\cos t}{-\sin t + \frac{1}{\sin t}}$$

$$= \frac{\cos t \sin t}{\cos^2 t} = \tan t$$

QUESTION

#Q. If $x = a(\sin\theta - \theta\cos\theta)$, $y = a(\cos\theta + \theta\sin\theta)$ then $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$ is

(A) 1

(B) -1

(C) $\sqrt{3}$

(D) $\frac{1}{\sqrt{3}}$

$$\frac{dy}{dx} = \frac{a[-\sin\theta + \theta\cos\theta + \sin\theta]}{a[\cos\theta + \theta\sin\theta - \cos\theta]}$$

$$= \frac{\theta\cos\theta}{\theta\sin\theta} = \cot\theta$$

Put $\theta = \frac{\pi}{4}$

$$= 1$$

After April 14

1. Revision
2. Mock test
3. Analysis

Improve
1. Speed
2. Accuracy
3. More MCQ's on weaker Areas

QUESTION

#Q. $x^m \cdot y^n = (x+y)^{m+n}$ then $\frac{dy}{dx}$ is

(A) $\frac{x^m}{y^n}$

(B) $\frac{x^n}{y^m}$

(C) $-\frac{x}{y}$

(D) $\frac{y}{x}$

Take log

$m\log x + n\log y = (m+n)\log(x+y)$

Diff

$\frac{m}{x} + \frac{n}{y}y_1 = \frac{(m+n)}{x+y}(1+y_1)$

$y_1\left[\frac{n}{y} - \frac{m+n}{x+y}\right] = \frac{m+n}{x+y} - \frac{m}{x}$

$y_1\left[\frac{nx+ny-my-ny}{y(x+y)}\right] = \frac{mx+nx-mx-my}{x(x+y)}$

$e^{x+y} = e^x + e^y$ Find $\frac{dy}{dx}$

$y_1\left[\frac{nx+\cancel{ny}-my-\cancel{ny}}{y\cancel{(x+y)}}\right]$

$= \frac{\cancel{mx}+nx-\cancel{mx}-my}{x\cancel{(x+y)}}$

$y_1 = \frac{\cancel{(nx-my)}y}{x\cancel{(nx-my)}}$

$y_1 = \frac{y}{x}$

$$\frac{m}{x}+\frac{n}{y}y_1=(m+n)\frac{1}{(x+y)}(1+y_1)$$

$$\frac{m}{x}+\frac{n}{y}y_1=\frac{m+n}{x+y}+\frac{m+n}{x+y}y_1$$

$$y_1\left[\frac{n}{y}-\frac{m+n}{x+y}\right]=\frac{m+n}{x+y}-\frac{m}{x}$$

QUESTION

#Q. If $y = a\sin(\log x)$ then

$$y_1 = \frac{a\cos(\log x)}{x}$$

$$x y_1 = a\cos(\log x)$$

$$x y_2 + y_1 = -\frac{a\sin(\log x)}{x}$$

$$x^2 y_2 + x y_1 + y = 0$$

(A) $y_2 + a^2 y = 0$

(B) $x^2 y_2 + x y_1 - y = 0$

(C) $x^2 y_2 + x y_1 + y = 0$

(D) $x^2 y_2 - x y_1 + y = 0$

QUESTION

#Q. If $x\sin y + y\sin x = 0$ then $\frac{dy}{dx}$ is

(A) $\frac{x\cos y+\sin x}{\sin y+y\cos x}$

(B) $-\frac{(\sin y+y\cos x)}{\sin x+x\cos y}$

(C) $\frac{\sin x}{\cos y}$

(D) $\frac{\cos x}{\sin y}$

$x\cos y\cdot y_1 + \sin y + y\cos x + \sin x\, y_1 = 0$

$y_1 = -\frac{(\sin y + y\cos x)}{(x\cos y + \sin x)}$

QUESTION

#Q. If $\tan^{-1}\left(\frac{y}{x}\right)-\frac{1}{2}\log_e(x^2+y^2)=0$ then $\frac{dy}{dx}$ is

(A) $\frac{x-y}{x+y}$

(B) $\frac{x+y}{x-y}$

(C) $(x+y)(x-y)$

(D) 1

$$\frac{1}{1+\frac{y^2}{x^2}}\left(\frac{xy_1-y}{x^2}\right)-\frac{1}{\cancel{2}}\left(\frac{1}{x^2+y^2}\right)(\cancel{2}x+\cancel{2}yy_1)=0$$

$$\frac{xy_1-y}{x^2+y^2}-\frac{x+yy_1}{x^2+y^2}=0$$

$$y_1[x-y]=x+y$$

$$y_1=\frac{x+y}{x-y}$$

QUESTION

$e^y = \frac{x}{y}$

#Q. If $ye^y = x$ then $\frac{dy}{dx} =$

$ye^y y_1 + e^y y_1 = 1$

$y_1 = \frac{1}{e^y(y+1)}$

$y_1 = \frac{y}{x(y+1)}$

(A) $-\frac{1}{y}$

(B) $\frac{y}{x(y+1)}$

(C) $\frac{y}{x}$

(D) $\frac{y}{y+1}$

QUESTION

#Q. If $y = \left(x^{x^{x}}\right)$ then $\frac{dy}{dx} =$

$\log y = \log x^{x^x}$

(A) $y(1 + 2\log_e x)$

(B) $xy(1 + 2\log_e x)$

(C) $xy(2 + \log_e x)$

(D) $x^x \cdot y\left[\frac{1}{x} + \log_e x \cdot (1 + \log_e x)\right]$

$\log AB = \log A + \log B$

Take log

$\log y = x^x \log x$

Take log

$\log(\log y) = \log(x^x)(\log x)$

$\log x^x + \log(\log x)$

$\log(\log y) = x\log x + \log\log x$

$\frac{1}{\log y}\frac{1}{y}y_1 = (1+\log x) + \frac{1}{x\log x}$

$y_1 = y\log y\left[\frac{x\log x(1+\log x)+1}{x\log x}\right]$

$y_1 = y(x^x \cancel{\log x})\left[\frac{x\log x(1+\log x)+1}{x\cancel{\log x}}\right]$

$y = yx^x\left[\log x(1+\log x) + \frac{1}{x}\right]$

QUESTION

#Q. If $y = x^y$ then $x(1 - y\log_e x) \cdot \frac{dy}{dx} =$

Take log

(A) x^2

(B) y^2

(C) xy^2

(D) x^2y

$\log y = y\log x$

$\frac{1}{y} y_1 = \frac{y}{x} + \log x \, y_1$

$y_1\left(\frac{1}{y} - \log x\right) = \frac{y}{x}$

$y_1\left[\frac{1 - y\log x}{y}\right] = \frac{y}{x}$

$y_1 = \frac{y^2}{x(1 - y\log x)}$

$x(1 - y\log x)\, y_1 = y^2$

QUESTION

#Q. If $y = x^{\log_e x}$, then dy/dx is

Take log

(A) $\frac{2\log_e x}{xy}$

(B) $\frac{2y\log_e x}{x}$

(C) $\frac{2x\log_e x}{y}$

(D) $\frac{y}{x\log_e x}$

$\log y = (\log x)^2$

$\frac{1}{y} y_1 = \frac{2\log x}{x}$

$y_1 = \frac{2y\log x}{x}$

QUESTION

$e^x = \frac{y}{x}$

#Q. If $xe^x = y$ then $\frac{dy}{dx}$ is equal to

(A) $\frac{x}{y}$

(B) $\frac{y}{x}$

(C) $1 + \frac{1}{x}$

(D) $y\left(1 + \frac{1}{x}\right)$ ✓

$xe^x + e^x = y_1$

$y_1 = e^x(1+x)$

$= \frac{y}{x}(1+x)$

$y_1 = y\left(\frac{1}{x}+1\right)$

QUESTION

#Q. If $y^x = x$ then $\frac{dy}{dx}$ vanishes when x is equal to

$\frac{dy}{dx} = 0$

(A) 0

(B) 1

(C) e

(D) 1/e

$x \log y = \log x$

$\frac{x}{y} y_1 + \log y = \frac{1}{x}$

$y_1 = \frac{y}{x}\left[\frac{1}{x} - \log y\right]$

if $y_1 = 0$

$\frac{y}{x} = 0$ or $\frac{1}{x} - \log x = 0$

Not possible

$\frac{x}{y} y_1 + \log y = \frac{1}{x}$

$\frac{x}{y} y_1 = \left[\frac{1}{x} - \log y\right]$

$y_1 = \frac{y}{x}\left[\frac{1}{x} - \log y\right]$

if $a \cdot b = 0$

$\Rightarrow$ either $a = 0$ or $b = 0$

$\frac{1}{x} = \log y$

$x = \frac{1}{\log y}$

$x = \frac{1}{\frac{\log x}{x}}$

$x = \frac{x}{\log x}$

$\log x = 1$

$x = e$

$\log_e x = 1$

$x = e^1$

$x = e$

QUESTION

#Q. If $y^x = e^x$ then $\frac{dy}{dx}$ is

A. $\frac{\log_e x}{\log_e x - 1}$

B. $\frac{(\log_e x)^2}{\log_e x - 1}$

C. $\frac{\log_e y}{\log_e y - 1}$

D. $\frac{y(1 - \log_e y)}{x}$

$x \log y = x$

$\frac{x}{y} y_1 + \log y = 1$

$y_1 = \frac{y}{x}[1 - \log y]$

QUESTION

#Q. If $y = a^{x+y}$ then y' is

(A) $\frac{y\log_e a}{1-y\log_e a}$

(B) $\frac{x\log_e a}{1-x\log_e a}$

(C) $\frac{y\log_e a}{y-\log_e a}$

(D) $\frac{x\log_e a}{x-\log_e a}$

$y_1 = a^{x+y}\log a(1+y_1)$

$y_1 = y\log a(1+y_1)$

$y_1[1-y\log a] = y\log a$

$y_1 = \frac{y\log a}{1-y\log a}$

$x^2 = 3x$, $x = ?$

$\Downarrow$

$x^2 - 3x = 0$

$x(x-3) = 0$

$x = 0 \mid x = 3$

QUESTION

#Q. If $y = x^2 + \frac{1}{x^2 + \frac{1}{x^2 + \cdots}}$ then $\frac{dy}{dx}$ is

(A) $\frac{2x}{2-x^2}$

(B) $\frac{2xy}{2y-x^2}$

(C) $\frac{2xy}{x^2-2y}$

(D) $\frac{xy}{x^2-y^2}$

$$y = x^2 + \frac{1}{y}$$

$$y^2 = x^2 y + 1$$

$$2yy_1 = x^2 y_1 + y(2x)$$

$$y_1(2y - x^2) = 2xy$$

$$y_1 = \frac{2xy}{2y - x^2}$$

QUESTION

#Q. If $x\sqrt{1+y}+y\sqrt{1+x}=0$ then y' is

(A) $\frac{1}{1+x}$

(B) $-\frac{1}{(1+x)^2}$

(C) $\frac{y}{1+x}$

(D) $\frac{2y}{1+x}$

$x\sqrt{1+y} = -y\sqrt{1+x}$

on squaring

$x^2(1+y) = y^2(1+x)$

$x^2+x^2y = y^2+y^2x$

$x^2-y^2 = y^2x - x^2y$

$(x-y)(x+y) = xy(y-x)$

$\cancel{(x-y)}\, x+y = -xy\cancel{(x-y)}$

$x+y = -xy$

$y+xy = -x$

$y(1+x) = -x$

$y = \frac{-x}{1+x}$

$y_1 = -\left[\frac{1+x-x}{(1+x)^2}\right]$

$y_1 = \frac{-1}{(1+x)^2}$

QUESTION

#Q. If $x^y = e^{x-y}$ then $y' =$

$y\log_e x + y = x$

$y(\log_e x + 1) = x$

$y = \dfrac{x}{\log_e ex}$

(A) $\dfrac{\log_e x}{\log_e ex}$

(B) $\dfrac{(\log_e ex)^2}{\log_e x}$

(C) $\dfrac{\log_e x}{(\log_e ex)^2}$

(D) $\dfrac{\log_e x}{x+y}$

$y\log x = x - y$

$\dfrac{y}{x} + \log x \, y_1 = 1 - y_1$

$y_1[\log x + 1] = 1 - \dfrac{y}{x}$

$y_1[\log x + \log_e e] = \dfrac{x-y}{x}$

$y_1[\log_e ex] = \dfrac{y\log_e x}{x}$

$y_1 = \dfrac{y\log x}{x(\log_e ex)}$

$y_1 = \dfrac{\dfrac{\cancel{x}}{\log_e ex}\log x}{\cancel{x}\log_e ex}$

$y_1 = \dfrac{\log x}{(\log_e ex)^2}$

$$x^y = e^{x-y}$$

$$y\log_e x = x - y \Longrightarrow y\log_e x + y = x$$

$$y(\log x + 1) = x$$

$$y[\log_e x + \log_e e] = x$$

$$y[\log_e ex] = x$$

$$y = \frac{x}{\log_e ex}$$

$$y_1 = \frac{(\log_e ex)(1) - x\left[0 + \frac{1}{x}\right]}{(\log_e x)^2}$$

$$= \frac{(\log_e x) - 1}{(\log_e x)^2} = \frac{(\log e + \log_e x) - \log e}{(\log_e x)^2}$$

$$= \frac{\log x}{(\log_e x)^2}$$

$$\frac{d}{dx}(\log_e x)$$

$$\frac{d}{dx}[\log e + \log x]$$

$$= 0 + \frac{1}{x}$$

QUESTION

#Q. If $y = ae^{mx} + be^{-mx}$ then y_2 is

(A) $-m^2y$

(B) m^2y

(C) my^2

(D) m^2y^2

$y_1 = ame^{mx} - bme^{-mx}$

$y_2 = am^2e^{mx} + bm^2e^{-mx}$

$y_2 = m^2[y]$

QUESTION

#Q. If $y = 2 + \log_e x$ then

$y_1 = \frac{1}{x}$

(A) $xy_2 + y_1 = 0$

$xy_1 = 1$

(B) $x^2y_2 + y_1 = 0$

$xy_2 + y_1 = 0$

(C) $x^2y_2 - y_1 = 0$

(D) $xy_2 - y_1 = 0$

$\log x + 1$

$\log x + \log e$

$\log ex$

$1 = \log_e e$

$\log A + \log B$

$= \log AB$

QUESTION

#Q. If $f(x) = \log_5 x$ then $f''(1) =$

(A) 1

(B) $-\log_5 e$

(C) $\frac{1}{\log_e 5}$

(D) $\log_5 e$

$f(x) = \frac{\log x}{\log 5}$

$f'(x) = \frac{1}{\log 5}\left(\frac{1}{x}\right)$

$f''(x) = \frac{1}{\log 5}\left(\frac{-1}{x^2}\right)$

$f''(1) = \frac{-1}{\log_e 5} = -\log_5 e$

QUESTION

#Q. If $y = \frac{A}{x} + Bx^2$, then $x^2 \frac{d^2y}{dx^2} =$

(A) $2y$

(B) y^2

(C) y^3

(D) y^4

$y_1 = \frac{-A}{x^2} + 2Bx$

$y_2 = \frac{2A}{x^3} + 2B$

$x^2 y_2 = \frac{2A}{x} + 2Bx^2$

$= 2\left[\frac{A}{x} + Bx^2\right]$

$x^2 y_2 = 2y$

QUESTION

#Q. Let $f(x) = \tan^{-1} x$. Then $f'(x) + f''(x) = 0$, when x is equal to

(A) 0

(B) 1

(C) i

(D) $-i$

$$f'(x) = \frac{1}{1+x^2}$$

$$f''(x) = \frac{-1}{(1+x^2)^2}(2x)$$

$$\frac{1}{1+x^2} - \frac{2x}{(1+x^2)^2} = 0$$

LCM

$$\Rightarrow \frac{1+x^2-2x}{(1+x^2)^2} = 0$$

$$\Rightarrow (x-1)^2 = 0$$

$$x = 1$$

QUESTION

#Q. If $\log y = m\tan^{-1} x$, then

$y = e^{m\tan^{-1}x}$

$y_1 = \frac{me^{m\tan^{-1}x}}{1+x^2}$

(A) $(1 + x^2)y_2 + (2x + m)y_1 = 0$

(B) $(1 + x^2)y_2 + (2x - m)y_1 = 0$

(C) $(1 + x^2)y_2 - (2x + m)y_1 = 0$

(D) $(1 + x^2)y_2 - (2x - m)y_1 = 0$

$(1+x^2)y_1 = my$

$(1+x^2)y_2 + \underline{2xy_1} - my_1 = 0$

QUESTION

$t = e^x$

#Q. Let $x = \log t, t > 0$ and $y + 1 = t^2$. Then $\frac{d^2x}{dy^2} =$

$\frac{dt}{dy} = \frac{1}{2t}$

$\frac{dx}{dt} = \frac{1}{t}$ | $\frac{dy}{dt} = 2t$

(A) $4e^{2x}$

(B) $-\frac{1}{2}e^{-4x}$

(C) $-\frac{3}{4}e^{-5x}$

(D) $4e^{x}$

$\frac{dx}{dy} = \frac{1/t}{2t} = \frac{1}{2t^2}$

$\frac{d^2x}{dy^2} = \frac{1}{\cancel{2}}\left(-\frac{1}{t^4}\right)\left(\cancel{2}t\,\frac{dt}{dy}\right)$

$= -\frac{1}{t^3}\left(\frac{1}{2t}\right)$

$= -\frac{1}{2t^4}$

$\frac{d}{dt}\left(\frac{1}{t^2}\right) = \frac{-1}{(t^2)^2}(2t) = \frac{-2}{t^3}$

$= -\frac{1}{2}t^{-4}$

$= -\frac{1}{2}(e^x)^{-4}$

$= -\frac{1}{2}e^{-4x}$

$$y = \frac{1}{x}$$

$$\frac{dy}{dx} = \frac{-1}{x^2}$$

$$y = \frac{1}{x^{10}}$$

$$y_1 = \frac{-1}{(x^{10})^2}(10x^9)$$

$$= \frac{-10x^9}{x^{20}}$$

$$= \frac{-10}{x^{11}}$$

$$y = \frac{1}{(3x^2+4x+5)^6}$$

$$y_1 = \frac{-1}{(3x^2+4x+5)^{12}}(6x+4)$$

QUESTION

#Q. The second order derivative of $\overset{u}{a\sin^3 t}$ with respect to $\overset{v}{a\cos^3 t}$ at $t = \frac{\pi}{4}$ is

(A) 2

(B) $\frac{1}{12a}$

(C) $\frac{4\sqrt{2}}{3a}$

(D) $\frac{3a}{4\sqrt{2}}$

$u = a\sin^3 t \qquad v = a\cos^3 t$

$\frac{du}{dt} = \cancel{3a}\sin^{\cancel{2}} t \cancel{\cos t} \quad \Big| \quad \frac{dv}{dt} = -\cancel{3}a\cos^{\cancel{2}} t \sin\cancel{t}$

$\frac{du}{dv} = -\tan t$

Diff wrt v

$\frac{d^2u}{dv^2} = -\sec^2 t \, \frac{dt}{dv}$

$= \frac{\cancel{+}\sec^2 t}{\cancel{+}3a\cos^2 t \sin t}$

Put $t = \frac{\pi}{4}$

$= \frac{2}{3a\left(\frac{1}{\sqrt{2}}\right)^3} = \frac{2 \cdot 2^{3/2}}{3a}$

$= \frac{2(2)\sqrt{2}}{3a} = \frac{4\sqrt{2}}{3a}$

QUESTION

#Q. If $x = \frac{1-t^2}{1+t^2}$ and $y = \frac{2t}{1+t^2}$, then $\frac{dy}{dx}$ is equal to

Put $t = \tan\theta$

(A) $-\frac{y}{x}$

$x = \cos 2\theta$ & $y = \sin 2\theta$

(B) $\frac{y}{x}$

$\frac{dx}{d\theta} = -2\sin 2\theta = -2y$ | $\frac{dy}{d\theta} = 2\cos 2\theta = 2x$

(C) $-\frac{x}{y}$

(D) $\frac{x}{y}$

$$\frac{dy}{dx} = -\frac{x}{y}$$

Thank

You