
ULTIMATE KCET
CRASH COURSE 2026
Mathematics
Lecture - 02
Inverse Trigonometric Functions
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Recap of previous lecture

1. ITF
2.
3.
4.

Topics *to be covered*

1. ITF - (Part - 2)
2.
3.
4.

$\sin(\sin^{-1} x) = x \; \forall x \in [-1,1]$

$\cos(\cos^{-1} x) = x \; \forall x \in [-1,1]$

$\tan(\tan^{-1} x) = x \; \forall x \in R$

$\cot(\cot^{-1} x) = x \; \forall x \in R$

$\sec(\sec^{-1} x) = x \; \forall x \in R(-1, 1)$

$\operatorname{cosec}(\operatorname{cosec}^{-1} x) = x \; \forall x \in R - (-1, 1)$

$$\sin^{-1}(\sin x) = x \quad \forall x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$\cos^{-1}(\cos x) = x \quad \forall x \in [0, \pi]$$

$$\tan^{-1}(\tan x) = x \quad \forall x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$$\cot^{-1}(\cot x) = x \quad \forall x \in (0, \pi)$$

$$\sec^{-1}(\sec x) = x \quad \forall x \in [0, \pi] - \left\{\frac{\pi}{2}\right\}$$

$$\operatorname{cosec}^{-1}(\operatorname{cosec} x) = x \quad \forall x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$$

$$2\tan^{-1}x = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

$$2\tan^{-1} = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$

$$2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

$$2\sin^{-1}x = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$$

$$2\cos^{-1}x = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$$

$$3\sin^{-1}x = \sin^{-1}(3x - 4x^3)$$

$$3\cos^{-1}x = \cos^{-1}(4x^3 - 3x)$$

$$3\tan^{-1}x = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)$$

$$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x + y}{1 - xy}\right)$$

$$\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x - y}{1 + xy}\right)$$

$$\sin^{-1}(-x) = -\sin^{-1}x$$
$$\tan^{-1}(-x) = -\tan^{-1}x$$
$$\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}x$$

$$\cos^{-1}(-x) = \pi - \cos^{-1}x$$
$$\cot^{-1}(-x) = -\cot^{-1}x$$
$$\sec^{-1}(-x) = \pi - \sec^{-1}x$$

$$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$$

or

$$\sin^{-1}x = \frac{\pi}{2} - \cos^{-1}x$$

or

$$\cos^{-1}x = \frac{\pi}{2} - \sin^{-1}x$$

$$\sec^{-1}x + \operatorname{cosec}^{-1}x = \frac{\pi}{2}$$

or

$$\sec^{-1}x = \frac{\pi}{2} - \operatorname{cosec}^{-1}x$$

or

$$\operatorname{cosec}^{-1}x = \frac{\pi}{2} - \sec^{-1}x$$

$$\sin^{-1}\left(\frac{a}{b}\right) = \operatorname{cosec}^{-1}\left(\frac{b}{a}\right)$$

or

$$\cos^{-1}\left(\frac{a}{b}\right) = \sec^{-1}\left(\frac{b}{a}\right)$$

or

$$\tan^{-1}\left(\frac{a}{b}\right) = \cot^{-1}\left(\frac{b}{a}\right)$$

$$\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$$

or

$$\tan^{-1}x = \frac{\pi}{2} - \cot^{-1}x$$

or

$$\cot^{-1}x = \frac{\pi}{2} - \tan^{-1}x$$

$$\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$$

or

$$\tan^{-1}x = \frac{\pi}{2} - \cot^{-1}x$$

or

$$\cot^{-1}x = \frac{\pi}{2} - \tan^{-1}x$$

Trigonometric functions with following Domain and Range will be bijective

	Domain	**Range**
$\sin x$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[-1, 1]$
$\cos x$	$[0, \pi]$	$[-1, 1]$
$\tan x$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$	R
$\cot x$	$(0, \pi)$	R
$\sec x$	$[0, \pi] - \{\pi/2\}$	$R - (-1, 1)$
$\operatorname{cosec} x$	$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$	$R - (-1, 1)$

Inverse Trigonometric Functions with following Domain and Range will be bijective

Note : Here range is also called as Principal value branch.

	Domain	Range	Quadrant in which solution lies
$\sin^{-1} x$	$[-1, 1]$	$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$	I and IV
$\cos^{-1} x$	$[-1, 1]$	$[-1, 1]$	I and II
$\tan^{-1} x$	R	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$	I and IV
$\cot^{-1} x$	R	$(0, \pi)$	I and II
$\sec^{-1} x$	$R - (-1, 1)$	$[0, \pi] - \{\pi/2\}$	I and II
$\text{cosec}^{-1} x$	$R - (-1, 1)$	$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$	I and IV

QUESTION

The domain of the function $\operatorname{cosec}^{-1}\left(\frac{1+x}{x}\right)$ is

A $\left[-\frac{1}{2}, \infty\right) - \{0\}$

B $\left(-1, -\frac{1}{2}\right] \cup (0, \infty)$

C $\left[-\frac{1}{2}, 0\right) \cup [1, \infty)$

D $\left(-\frac{1}{2}, \infty\right) - \{0\}$

QUESTION

The domain of the function $f(x) = \sin^{-1}\left(\frac{x^2-3x+2}{x^2+2x+7}\right)$ is

A $[1, \infty]$

B $[-1, 2]$

C $[-1, \infty]$

D $[-\infty, 2]$

QUESTION

The domain of the function $\cos^{-1}\left(\frac{2\sin^{-1}\left(\frac{1}{4x^2-1}\right)}{x}\right)$ is

A $R-\left\{-\frac{1}{2}, \frac{1}{2}\right\}$

B $(-\infty, -1) \cup (1, \infty) \cup \{0\}$

C $\left(-\infty, \frac{-1}{2}\right) \cup \left(\frac{1}{2}, \infty\right) \cup \{0\}$

D $\left(-\infty, \frac{-1}{\sqrt{2}}\right) \cup \left(\frac{1}{\sqrt{2}}, \infty\right) \cup \{0\}$

QUESTION

The range of the function $y = \left(\frac{\cos^{-1}(3x-1)}{\pi} + 1\right)^2$ is

(A) $[1, \pi]$

(B) $[0, \pi]$

(C) $[1, \pi]$

(D) $[0, \pi^2]$

QUESTION

The domain of the function $f(x) = \frac{\sin^{-1}(x-3)}{\sqrt{9-x^2}}$ is

A $[1, 2]$

B $[2, 3]$

C $[2, 3)$

D $[1, 2)$

QUESTION

The domain of the function $\cos^{-1}(\log_2(x^2 + 5x + 8))$ is

(A) $[2, 3]$

(B) $[-2, 2]$

(C) $[3, 1]$

(D) $[-3, -2)$

QUESTION

Range of the function $y = \sin^{-1}\left(\frac{x^2}{1+x^2}\right)$ is

A $\left(0, \frac{\pi}{2}\right)$

B $\left[0, \frac{\pi}{2}\right)$

C $\left(0, \frac{\pi}{2}\right]$

D $\left[0, \frac{\pi}{2}\right]$

QUESTION

The domain of the function $f(x) = \sqrt{\cos^{-1}\left(\frac{1-|x|}{2}\right)}$

A $(-3, 3)$

B $[-3, 3]$

C $(-\infty, -3) \cup (3, \infty)$

D $(-\infty, -3] \cup [3, \infty)$

QUESTION

The domain of the real valued function $f(x) = \sqrt{1-2x} + 2\sin^{-1}\left(\frac{3x-1}{2}\right)$ is

A $\left[\frac{-1}{3}, \frac{1}{2}\right]$

B $\left[\frac{1}{2}, 1\right]$

C $\left[\frac{-1}{2}, \frac{1}{3}\right]$

D $\left[-1, \frac{1}{3}\right]$

QUESTION

Find the domain and range of $f(x) = \tan^{-1}\left(\log_{4/5}(5x^2 - 8x + 4)\right)$ respectively.

A $f_D = (-\infty, \infty), f_R = \left(\frac{-\pi}{2}, \frac{\pi}{4}\right]$

B $f_D = [-\pi, \pi], f_R = \left(0, \frac{\pi}{2}\right]$

C $f_D = R - \left\{\frac{\pi}{2}\right\}, f_R = \left[\frac{-\pi}{4}, \frac{\pi}{4}\right]$

D $f_D = (-\infty, 0), f_R = (0, \pi]$

QUESTION

Range of $\tan^{-1}\left(\frac{2x}{1+x^2}\right)$ is

(A) $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

(B) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(C) $\left(-\frac{\pi}{2}, \frac{\pi}{4}\right]$

(D) $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

QUESTION

The $\boxed{\text{minimum value of } n}$ for which $\tan^{-1}\frac{n}{\pi} > \frac{\pi}{4}$, $n \in N$ is

A 3

B 4

C 5

D 6

$\tan^{-1}\frac{n}{\pi} > \frac{\pi}{4}$

$\frac{n}{\pi} > \tan\frac{\pi}{4}$

$\frac{n}{\pi} > 1$

$n > \pi$

$n > 3.14$

$n > 3.14$

smallest natural to satisfy the above statement

$n = 4$ since $4 > 3$

QUESTION

The largest & smallest value of n for which $\cos^{-1}\frac{n}{\pi} > 0, n \in N$ is

$$\cos^{-1}\frac{n}{\pi} > 0$$

Apply cos on B.S

$$\frac{n}{\pi} < \cos 0$$ | cos func is a decreasing func

$$\frac{n}{\pi} < 1$$

$$n < \pi$$

$$n < 3.14$$

$n < 3.14$

smallest = 1 & largest = 3

$\downarrow$ min $\downarrow$ max

$N = \{1, 2, 3, \ldots\}$

$\cos^{-1}\frac{n}{\pi} > \frac{\pi}{4}, \quad n \in N$

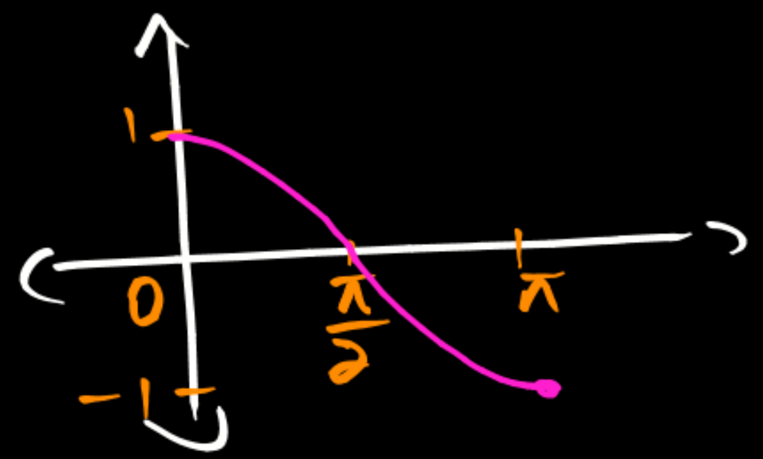
$\frac{n}{\pi} < \cos\left(\frac{\pi}{4}\right)$

$\frac{n}{\pi} < \frac{1}{\sqrt{2}}$

$n < \frac{\pi}{\sqrt{2}}$

$n < \frac{3.14}{1.41}$

$n < 2.22$



$f(x) = \cos x$

$n < 2.22$

min $= 1$

max $= 2$

$f(x) \geq A$

$\hookrightarrow A \rightarrow$ min value

$f(x) \leq B$

$\hookrightarrow B \rightarrow$ max value

QUESTION

The domain of the function $f(x)=\sin^{-1}\left[\log_2\left(\frac{x^2}{2}\right)\right]$ is

$\rightarrow g(x)>0 \Rightarrow \frac{x^2}{2}>0$

$x^2>0$

$x\in(-\infty,0)\cup(0,\infty) \rightarrow (1)$

(A) $1\le x\le 2$

(B) $1\le x\le 3$

(C) $2\le x\le 4$

(D) $1\le x\le 5$

$-1\le \log_2\frac{x^2}{2}\le 1$

$2^{-1}\le \frac{x^2}{2}\le 2^1$

$\frac{1}{2}\le \frac{x^2}{2}\le 2$

$1\le x^2\le 4$

$1\le |x|\le 2$

$x\in[-2,-1]\cup[1,2] \rightarrow (2)$

$(1)\cap(2)$

$x\in[-2,-1]\cup\underline{[1,2]}$

QUESTION

If $x \neq n\pi, x \neq (2n+1)\frac{\pi}{2}, n \in Z$ then $\frac{\sin^{-1}(\cos x)+\cos^{-1}(\sin x)}{\tan^{-1}(\cot x)+\cot^{-1}(\tan x)} =$

(A) 1

(B) 2

(C) 3

(D) 0

$$\frac{\frac{\pi}{2}-\cos^{-1}(\cos x)+\frac{\pi}{2}-\sin^{-1}(\sin x)}{\frac{\pi}{2}-\cot^{-1}(\cot x)+\frac{\pi}{2}-\tan^{-1}(\tan x)}$$

$$\frac{\frac{\pi}{2}-x+\frac{\pi}{2}-x}{\frac{\pi}{2}-x+\frac{\pi}{2}-x} = 1$$

QUESTION

If $\tan^{-1} x = \frac{\pi}{4} - \tan^{-1}\left(\frac{1}{3}\right)$, then x is

A $\frac{1}{3}$

B $\frac{1}{2}$

C $\frac{1}{4}$

D $\frac{1}{6}$

$x = \tan\left[\frac{\pi}{4} - \tan^{-1}\frac{1}{3}\right]$

$$\frac{\tan\frac{\pi}{4} - \tan\left(\tan^{-1}\frac{1}{3}\right)}{1 - \tan\frac{\pi}{4}\tan\left(\tan^{-1}1/3\right)}$$

$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

$x = \frac{1 - 1/3}{1 + 1/3} = \frac{2/3}{4/3} = \frac{2}{4} = \frac{1}{2}$

$\Downarrow$

$\tan\left(\frac{\pi}{4} - x\right) = \frac{1 - \tan x}{1 + \tan x}$

$\tan\left(\frac{\pi}{4} + x\right) = \frac{1 + \tan x}{1 - \tan x}$

QUESTION

$$\cos\left[2\cos^{-1}\frac{1}{5}+\sin^{-1}\frac{1}{5}\right]=$$

(A) $\frac{1}{5}$

(B) $\frac{-2\sqrt{6}}{5}$

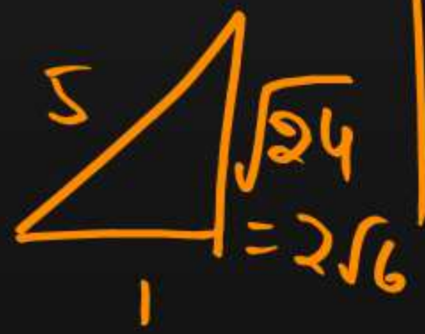
(C) $-\frac{1}{5}$

(D) $\frac{\sqrt{6}}{5}$

$$\cos\left[\cos^{-1}\frac{1}{5}+\underbrace{\cos^{-1}\frac{1}{5}+\sin^{-1}\frac{1}{5}}_{\frac{\pi}{2}}\right]$$

$$\cos\left[\frac{\pi}{2}+\cos^{-1}\frac{1}{5}\right]$$

$$=-\sin\left[\cos^{-1}\frac{1}{5}\right]$$



$$=-\sin\left[\sin^{-1}\frac{2\sqrt{6}}{5}\right]$$

$$=-\frac{2\sqrt{6}}{5}$$

QUESTION

If $\alpha \leq 2\sin^{-1}x + \cos^{-1}x \leq \beta$, then

A $\alpha = \frac{-\pi}{2}, \beta = \frac{3\pi}{2}$

B $\alpha = 0, \beta = 2\pi$

C $\alpha = \frac{-\pi}{2}, \beta = \frac{\pi}{2}$

D $\alpha = 0, \beta = \pi$

$\alpha \leq \sin^{-1}x + \underbrace{\sin^{-1}x + \cos^{-1}x}_{\frac{\pi}{2}} \leq \beta$

$\alpha \leq \sin^{-1}x + \frac{\pi}{2} \leq \beta$

$\alpha - \frac{\pi}{2} \leq \sin^{-1}x \leq \beta - \frac{\pi}{2} \rightarrow (1)$

WKT $-\frac{\pi}{2} \leq \sin^{-1}x \leq \frac{\pi}{2} \rightarrow (2)$

(1) & (2)

$\alpha - \frac{\pi}{2} = -\frac{\pi}{2}$ & $\beta - \frac{\pi}{2} = \frac{\pi}{2}$

$\alpha = 0$ & $\beta = \pi$

if $\alpha < 3\tan^{-1}x + \cot^{-1}x < \beta$

Then α & β = ?

Soln:-

$$\alpha < 2\tan^{-1}x + \frac{\pi}{2} < \beta$$

$$\frac{\alpha - \frac{\pi}{2}}{2} < \tan^{-1} < \frac{\beta - \frac{\pi}{2}}{2} \rightarrow (1)$$

Range of $\tan^{-1}x$

WKT

$$-\frac{\pi}{2} < \tan^{-1}x < \frac{\pi}{2}$$

$$\frac{\alpha - \frac{\pi}{2}}{2} = -\frac{\pi}{2}$$

$$\alpha - \frac{\pi}{2} = -\pi$$

$$\alpha = -\frac{\pi}{2}$$

$$\frac{\beta - \frac{\pi}{2}}{2} = \frac{\pi}{2}$$

$$\beta - \frac{\pi}{2} = \pi$$

$$\beta = 3\pi/2$$

QUESTION

If $3\tan^{-1}x + \cot^{-1}x = \pi$ then x equal to

$\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$

$2\tan^{-1}x + \underbrace{\tan^{-1}x + \cot^{-1}x}_{\downarrow} = \pi$

$2\tan^{-1}x + \frac{\pi}{2} = \pi$

$2\tan^{-1}x = \frac{\pi}{2}$

$\tan^{-1}x = \frac{\pi}{4}$

$x = 1$

A) 0

B) 1

C) −1

D) $\frac{1}{2}$

QUESTION

If $\tan^{-1} x + \tan^{-1} y = \frac{4\pi}{5}$, then $\cot^{-1} x + \cot^{-1} y$ is equal to

(A) $\frac{\pi}{5}$

(B) $\frac{3\pi}{5}$

(C) $\frac{2\pi}{5}$

(D) π

$\frac{\pi}{2} - \cot^{-1} x + \frac{\pi}{2} - \cot^{-1} y = \frac{4\pi}{5}$

$\pi - (\cot^{-1} x + \cot^{-1} y) = \frac{4\pi}{5}$

$\cot^{-1} x + \cot^{-1} y = \pi - \frac{4\pi}{5}$

$= \frac{\pi}{5}$

QUESTION

If $\sin^{-1} x + \cos^{-1} y = \frac{2\pi}{5}$, then $\cos^{-1} x + \sin^{-1} y$ is

A $\frac{2\pi}{5}$

B $\frac{3\pi}{5}$

C $\frac{4\pi}{5}$

D $\frac{3\pi}{10}$

$\pi - \frac{2\pi}{5}$

$= \frac{3\pi}{5}$

$\frac{\pi}{2} - \cos^{-1} x + \frac{\pi}{2} - \sin^{-1} y = \frac{2\pi}{5}$

$\pi - \frac{2\pi}{5} = \frac{3\pi}{5} = \cos^{-1} x + \sin^{-1} y$

QUESTION

If $\sin^{-1} x + \sin^{-1} y = \frac{\pi}{2}$, then x^2 is equal to

A $1 - y^2$

B y^2

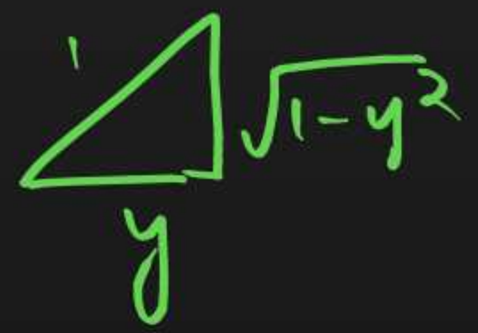
C 0

D $\sqrt{1-y}$

$$\sin^{-1} x = \frac{\pi}{2} - \sin^{-1} y$$

$$\sin^{-1} x = \cos^{-1} y$$

$$x = \sin[\cos^{-1} y]$$



$$x = \sin[\sin^{-1}\sqrt{1-y^2}]$$

$$\therefore\ x = \sqrt{1-y^2}$$

$$x^2 = 1 - y^2$$

if $\sin^{-1}\frac{1}{5} + \cos^{-1}x = \frac{\pi}{2}$

$\sin^{-1}A + \cos^{-1}A = \frac{\pi}{2}$

Then $x =$

$x = \frac{1}{5}$

(A) $\frac{2\sqrt{6}}{5}$

(B) $-\frac{2\sqrt{6}}{5}$

(C) $\frac{1}{5}$

(D) $-\frac{1}{5}$

if $\sin^{-1}\frac{1}{5} + \sin^{-1}x = \frac{\pi}{2}$

Then $x =$

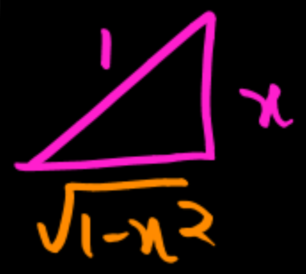
(A) $\cancel{\frac{2\sqrt{6}}{5}}$

(B) $-\frac{2\sqrt{6}}{5}$

(C) $\frac{1}{5}$

(D) $-\frac{1}{5}$

$\sin^{-1}\left(\frac{x}{1}\right)$



$\Downarrow$

$\cos^{-1}\left(\sqrt{1-x^2}\right)$

$\Downarrow$

$\sin^{-1}x = \cos^{-1}\sqrt{1-x^2}$

$\sin^{-1}\frac{1}{5} + \underline{\cos^{-1}\sqrt{1-x^2}} = \frac{\pi}{2}$

$\sqrt{1-x^2} = \frac{1}{5}$

$1 - x^2 = \frac{1}{25}$

$1 - \frac{1}{25} = x^2$

$x^2 = \frac{24}{25}$

$x = \frac{2\sqrt{6}}{5}$

QUESTION

$$\cos\left[2\sin^{-1}\frac{3}{4}+\cos^{-1}\frac{3}{4}\right]=$$

(A) $\frac{-3}{4}$

$\cos\left[\sin^{-1}\frac{3}{4}+\underbrace{\sin^{-1}\frac{3}{4}+\cos^{-1}\frac{3}{4}}_{\frac{\pi}{2}}\right]$

(B) $\frac{3}{5}$

$\cos\left[\frac{\pi}{2}+\sin^{-1}\frac{3}{4}\right]$

$=-\sin\left[\sin^{-1}\frac{3}{4}\right]$

(C) $\frac{3}{4}$

$=-\frac{3}{4}$

(D) Does not exist

QUESTION

$2x+3y = 2x+2y+y = 2(x+y)+y$

If $a+\frac{\pi}{2}<2\tan^{-1}x+3\cot^{-1}x<b$ then 'a' and 'b' are respectively

Range of $\cot^{-1}x$

(A) 0 and π

(B) 0 and 2π

(C) $\frac{\pi}{2}$ and 2π

(D) $\frac{-\pi}{2}$ and $\frac{\pi}{2}$

$a+\frac{\pi}{2}<2(\tan^{-1}x+\cot^{-1}x)+\cot^{-1}x<b$

$a+\frac{\pi}{2}<2\left(\frac{\pi}{2}\right)+\cot^{-1}x<b$

$a+\frac{\pi}{2}<\pi+\cot^{-1}x<b$

$a-\frac{\pi}{2}<\cot^{-1}x<b-\pi \rightarrow (1)$

WKT

$0<\cot^{-1}x<\pi \rightarrow (2)$

From (1) & (2)

$a-\frac{\pi}{2}=0$	$b-\pi=\pi$
$a=\frac{\pi}{2}$	$b=2\pi$

QUESTION

$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2} \quad \forall x \in [-1, 1]$

The value of $\cos\left(\sin^{-1}\frac{\pi}{3} + \cos^{-1}\frac{\pi}{3}\right)$

$\frac{\pi}{3} = \frac{3.14}{3} > 1$

does not exist

$\sin^{-1}x$
$\downarrow$
$x \in [-1, 1]$

$\cos\left[\sin^{-1}\frac{\pi}{4} + \cos^{-1}\frac{\pi}{4}\right]$

$= \cos\left(\frac{\pi}{2}\right)$

$= 0$

$\frac{\pi}{4} = \frac{3.14}{4} < 1$

(A) 1

(B) −1

(C) Does not exist

(D) 0

QUESTION

The domain of the function defined by $f(x) = \cos^{-1}\sqrt{x-1}$ is

A [0, 2]

B [−1, 1]

C [0, 1]

D [1, 2]

$-1 \le \sqrt{x-1} \le 1$

But sqrt func ≥ 0

$0 \le \sqrt{x-1} \le 1$

$0 \le x-1 \le 1$

$1 \le x \le 2$

QUESTION

$$\cos\left[\cot^{-1}(-\sqrt{3}) + \frac{\pi}{6}\right] =$$

(A) 0

(B) 1

(C) $\frac{1}{\sqrt{2}}$

(D) -1

$\cos\left[\pi - \cot^{-1}\sqrt{3} + \frac{\pi}{6}\right]$

$\cos\left[\pi - \frac{\pi}{6} + \frac{\pi}{6}\right]$

$\cos\pi = -1$

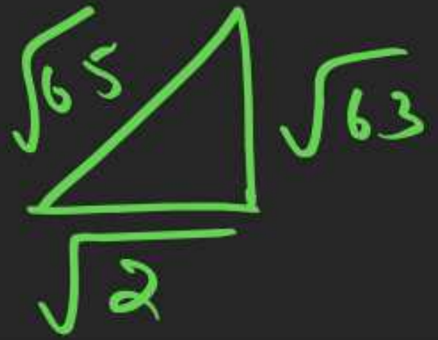
$\sin^{-1} x$ $\qquad\qquad$ $\sin x$

$\downarrow$ $\qquad\qquad$ $\downarrow$

$x \in [-1, 1]$ $\qquad\qquad$ $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

QUESTION

$$\sin\left(2\sin^{-1}\sqrt{\frac{63}{65}}\right) =$$



(A) $\frac{8\sqrt{63}}{65}$

(B) $\frac{\sqrt{63}}{65}$

(C) $\frac{2\sqrt{126}}{65}$

(D) $\frac{4\sqrt{65}}{65}$

$$2\sin\left(\sin^{-1}\sqrt{\frac{63}{65}}\right)\cos\left(\cos^{-1}\sqrt{\frac{2}{65}}\right)$$

$$2\,\frac{\sqrt{63}}{\sqrt{65}}\,\frac{\sqrt{2}}{\sqrt{65}} = \frac{2\sqrt{126}}{65}$$

QUESTION

If $\tan^{-1}x = \frac{\pi}{10}$ for some $x \in R$, then the value of $\cot^{-1}x$ is

$\frac{\pi}{2} - \cot^{-1}x = \frac{\pi}{10}$

$\cot^{-1}x = \frac{\pi}{2} - \frac{\pi}{10}$

$= \frac{5\pi - \pi}{10}$

$= \frac{4\pi}{10} = \frac{2\pi}{5}$

(A) $\pi/5$

(B) $2\pi/5$

(C) $3\pi/5$

(D) $4\pi/5$

QUESTION

The domain of the function $y = \sin^{-1}(-x^2)$ is

(A) $[0, 1]$

(B) $(0, 1)$

(C) $[-1, 1]$

(D) ϕ

$-1 \le -x^2 \le 1$

x^{by} by -1

$1 \geq x^2 \geq -1$

$\Downarrow$

$-1 \le x^2 \le 1$

WKT $x^2 \geq 0$

$0 \le x^2 \le 1$

$\sqrt{x^2} \le 1$

$|x| \le 1$

$x \in [-1, 1]$

$f(x) = x^2$

$\Downarrow$

Range $= [0, \infty)$

$\Downarrow$

$x^2 \geq 0$

QUESTION

If $3\tan^{-1}x + \cot^{-1}x = \pi$, then x equals

(A) 0

(B) 1

(C) -1

(D) $1/2$

① Find x if

$$\sin^{-1}x - \cos^{-1}x = \frac{\pi}{6}$$

Soln:-

method 1:-

$$\sin^{-1}x - \left(\frac{\pi}{2} - \sin^{-1}x\right) = \frac{\pi}{6}$$

$$2\sin^{-1}x - \frac{\pi}{2} = \frac{\pi}{6}$$

$$2\sin^{-1}x = \frac{2\pi}{3}$$

$$\sin^{-1}x = \frac{\pi}{3}$$

$$x = \sqrt{3}/2$$

method 2:-

Given $\sin^{-1}x - \cos^{-1}x = \frac{\pi}{6} \rightarrow ①$

WKT

$$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2} \rightarrow ②$$

① + ②

$$2\sin^{-1}x = \frac{\pi}{6} + \frac{\pi}{2}$$

$$2\sin^{-1}x = \frac{2\pi}{3}$$

$$\sin^{-1}x = \frac{\pi}{3}$$

$$x = \sqrt{3}/2$$

QUESTION

The equation $\tan^{-1} x - \cot^{-1} x = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ has

$\tan^{-1} x - \cot^{-1} x = \frac{\pi}{6} \rightarrow (1)$

A no solution

WKT $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2} \rightarrow (2)$

B unique solution

$(1) + (2)$

C infinite number of solutions

$2\tan^{-1} x = \frac{\pi}{6} + \frac{\pi}{2} = \frac{2\pi}{3}$

D two solutions

$\tan^{-1} x = \frac{\pi}{3}$

$x = \sqrt{3} \longrightarrow$ one solution

QUESTION

If $\cos\left(\sin^{-1}\frac{2}{5}+\cos^{-1}x\right)=0$, then x is equal to

(A) $1/5$

(B) $2/5$

(C) 0

(D) 1

$\sin^{-1}\left(\frac{2}{5}\right)+\cos^{-1}(x)=\cos^{-1}(0)$

$\sin^{-1}\frac{2}{5}+\cos^{-1}x=\frac{\pi}{2}$

$x=\frac{2}{5}$

WKT

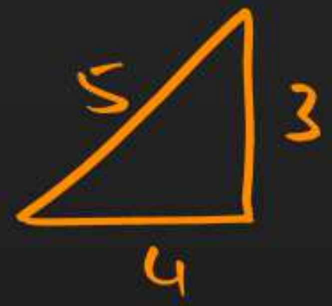
$\sin^{-1}A+\cos^{-1}A=\frac{\pi}{2}$

$$\frac{24}{25} \times \frac{4}{4} = \frac{96}{100} = 0.96$$

QUESTION

The value of $\sin(2\tan^{-1}(0.75))$ is equal to

$$\sin\left[2\tan^{-1}\frac{3}{4}\right]$$



$$\sin\left[\underbrace{2\sin^{-1}\frac{3}{5}}_{\theta}\right]$$

(A) 0.75

(B) 1.5

(C) 0.96

(D) 5

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$= 2\sin\left(\sin^{-1}\frac{3}{5}\right)\cos\left(\cos^{-1}\frac{4}{5}\right)$$

$$= 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right) = \frac{24}{25} = 0.96$$

QUESTION

If $|x| \le 1$, then $2\tan^{-1}x + \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ is equal to

$$2\tan^{-1}x = \begin{cases} (1)\ \sin^{-1}\left(\frac{2x}{1+x^2}\right) \\ (2)\ \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) \\ (3)\ \tan^{-1}\left(\frac{2x}{1-x^2}\right) \end{cases}$$

A) $4\tan^{-1}x$

$2\tan^{-1}x + 2\tan^{-1}x$

$= 4\tan^{-1}x$

B) 0

C) $\pi/2$

D) π

QUESTION

$2\tan^{-1}A = \tan^{-1}\left(\frac{2A}{1-A^2}\right)$

If $\sin^{-1}\left(\frac{2a}{1+a^2}\right) + \cos^{-1}\left(\frac{1-a^2}{1+a^2}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, where $a, x \in]0, 1$, then the value of x is

A 0

B $a/2$

C a

D $2a/1 - a^2$

$2\tan^{-1}a + 2\tan^{-1}a = 2\tan^{-1}x$

$\cancel{4}^{2}\tan^{-1}a = \cancel{2}\tan^{-1}x$

$2\tan^{-1}a = \tan^{-1}x$

$\tan^{-1}\left(\frac{2a}{1-a^2}\right) = \tan^{-1}x$

$\Downarrow$

$x = \frac{2a}{1-a^2}$

QUESTION

$\sqrt{x^2} = |x|$

$1+\cos 2x = 2\cos^2 x$

The number of real solutions of the equation $\sqrt{1+\cos 2x} = \sqrt{2}\cos^{-1}(\cos x)$ in $\left[\frac{\pi}{2}, \pi\right]$ is

(A) 1

(B) 2

(C) 0

(D) infinite

$\sqrt{2\cos^2 x} = \sqrt{2}\, x$

$\cancel{\sqrt{2}}\sqrt{\cos^2 x} = \cancel{\sqrt{2}}\, x$

$|\cos x| = x$

Here $x \in \left[\frac{\pi}{2}, \pi\right] \Rightarrow x \in 2^{nd}$ Quad

$\cos x = -ve$

$-\cos x = x$

$\cos x = -x$

(not Possible)

Find the $\underline{\underline{no}}$ of $\underline{\underline{soln}}$ of $\sqrt{1-\cos 2x} = \sqrt{2}\sin^{-1}(\sin x)$ $\quad x \in \left[0, \frac{\pi}{2}\right]$

(A) 1

(B) 2

(C) 0

(D) Infinite

Soln:

$$\sqrt{2\sin^2 x} = \sqrt{2}\,(x)$$

$$\sqrt{2}\,|\sin x| = \sqrt{2}\,(x)$$

$$\sin x = x$$

Since $x \in \left[0, \frac{\pi}{2}\right]$

$\sin x = +ve$

$\Downarrow$

is true for

$x = 0$

$\therefore$ $\underline{\underline{no}}$ of $\underline{\underline{soln}} = 1$

$$\sin^{-1}[\sin x] = x \qquad \forall x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$\sin 0 = 0$

If $\cos x = x$

$\Downarrow$

input & output are same value

QUESTION

If $\cos^{-1}x > \sin^{-1}x$ Then

(A) $\frac{1}{\sqrt{2}} < x \leq 1$

(B) $0 \leq x < \frac{1}{\sqrt{2}}$

(C) ✓ $-1 \leq x < \frac{1}{\sqrt{2}}$

(D) $x > 0$

$\cos^{-1}x > \frac{\pi}{2} - \cos^{-1}x$

$2\cos^{-1}x > \frac{\pi}{2}$

$\cos^{-1}x > \frac{\pi}{4}$

$\Downarrow$

$\frac{\pi}{4} < \cos^{-1}x \leq \pi$ since range of $\cos^{-1}x = [0, \pi]$

Apply cos

$\cos\frac{\pi}{4} > x \geq \cos\pi$

$\frac{1}{\sqrt{2}} > x \geq -1$

$\Downarrow$

$-1 \leq x < \frac{1}{\sqrt{2}}$

PYQ

QUESTION

The value of the expression $\tan\left(\frac{1}{2}\cos^{-1}\frac{2}{\sqrt{5}}\right)$ is

$\xrightarrow{} \theta$

(A) $2+\sqrt{5}$

(B) $\sqrt{5}-2$

(C) $\frac{\sqrt{5}+2}{2}$

(D) $5+\sqrt{2}$

Let $\cos^{-1}\frac{2}{\sqrt{5}}=\theta$

$\cos\theta=\frac{2}{\sqrt{5}}$

$$\tan\left(\frac{\theta}{2}\right)=\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}=\sqrt{\frac{1-2/\sqrt{5}}{1+2/\sqrt{5}}}$$

$$=\sqrt{\frac{\sqrt{5}-2}{\sqrt{5}+2}}$$

$$=\sqrt{\frac{\sqrt{5}-2}{\sqrt{5}+2}\times\frac{\sqrt{5}-2}{\sqrt{5}-2}}$$

$$=\sqrt{\frac{(\sqrt{5}-2)^2}{5-4}}$$

$$=\underline{\sqrt{5}-2}$$

QUESTION

$$\tan\left[\frac{1}{2}\underbrace{\sin^{-1}\frac{3}{4}}_{\theta}\right] =$$

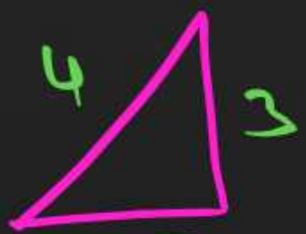
A) $\dfrac{4+\sqrt{7}}{3}$

B) $\dfrac{4-\sqrt{7}}{3}$ ✓

C) $\sqrt{3}+1$

D) 2

$\sin^{-1}\frac{3}{4} = \theta$



$\sqrt{16-9} = \sqrt{7}$

$\cos^{-1}\frac{\sqrt{7}}{4} = \theta$

$\cos\theta = \frac{\sqrt{7}}{4}$

$$\tan\left(\frac{\theta}{2}\right) = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \sqrt{\frac{1-\sqrt{7}/4}{1+\sqrt{7}/4}}$$

$$= \sqrt{\frac{4-\sqrt{7}}{4+\sqrt{7}}} = \sqrt{\frac{4-\sqrt{7}}{4+\sqrt{7}} \times \frac{4-\sqrt{7}}{4-\sqrt{7}}}$$

$$= \sqrt{\frac{(4-\sqrt{7})^2}{16-7}} = \frac{4-\sqrt{7}}{\sqrt{9}} = \frac{4-\sqrt{7}}{3}$$

QUESTION

If $\sin^{-1}x + \sin^{-1}(1-x) = \cos^{-1}x$, then x is equal to

option verification

(A) 0

(B) $1/2$

(C) $0, 1/2$

(D) $0, \sqrt{3}/2$

$x = 0$

LHS: $0 + \frac{\pi}{2}$

RHS: $\frac{\pi}{2}$

$x = \frac{1}{2}$

LHS: $\frac{\pi}{6} + \frac{\pi}{6} = \frac{\pi}{3}$

RHS: $\frac{\pi}{3}$

$\cos 2\theta = 1 - 2\sin^2\theta$

If $\sin^{-1}x + \sin^{-1}(1-x) = \cos^{-1}x$

Then no of solutions =

(A) 0

(B) 1

(C) 2

(D) Infinite

Soln:-

$\sin^{-1}x + \sin^{-1}(1-x) = \cos^{-1}x$

$\sin^{-1}(1-x) = \cos^{-1}x - \sin^{-1}x$

$= \frac{\pi}{2} - \sin^{-1}x - \sin^{-1}x$

$\sin^{-1}(1-x) = \frac{\pi}{2} - 2\sin^{-1}x$

$1 - x = \sin\left[\frac{\pi}{2} - 2\sin^{-1}x\right]$

$1 - x = \cos[2\underbrace{\sin^{-1}x}_{\theta}]$

$= 1 - 2[\sin(\underbrace{\sin^{-1}x}_{\theta})]^2$

$1 - x = 1 - 2x^2$

$x = 2x^2$

$2x^2 - x = 0$

$x(2x-1) = 0$

$x = 0 \mid x = 1/2$

$\therefore$ no of soln = 2

QUESTION

$6x \to \frac{1}{2}$

$x = \frac{1}{12} \Rightarrow \frac{1}{2}$

If $\sin^{-1} 6x + \sin^{-1} 6\sqrt{3}x = -\frac{\pi}{2}$ then x is equal to

(A) $-1/12$

(B) $1/12$

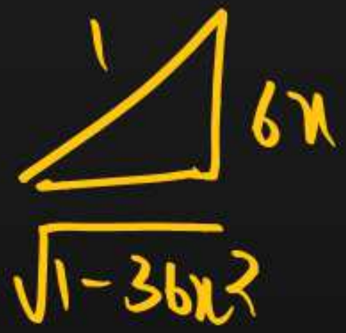
(C) -1/12, 1/12

(D) None of these

$\sin^{-1} 6\sqrt{3}x = -\frac{\pi}{2} - \sin^{-1} 6x$

$6\sqrt{3}x = -\sin\left[\frac{\pi}{2} + \sin^{-1} 6x\right]$

$6\sqrt{3}x = -\cos\left(\sin^{-1} 6x\right]$



$6\sqrt{3}x = -\cos\left[\cos^{-1}\sqrt{1-36x^2}\right]$

$6\sqrt{3}x = -\sqrt{1-36x^2}$

on squaring

$108x^2 = 1 - 36x^2$

$144x^2 = 1$

$x^2 = \frac{1}{144}$

$x = \pm\frac{1}{12}$

Shortcut

Cross verify :-

$\frac{6\sqrt{3}}{12} = \frac{\sqrt{3}}{2}$

$\times \quad x = \frac{1}{12}$

LHS

$\sin^{-1}\frac{1}{2} + \sin^{-1}\frac{\sqrt{3}}{2}$

$= \frac{\pi}{6} + \frac{\pi}{3}$

$= \frac{\pi}{2}$

RHS

$= -\frac{\pi}{2}$

$\checkmark \quad x = \frac{-1}{12}$

LHS

$\sin^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

$= -\sin^{-1}\frac{1}{2} - \sin^{-1}\frac{\sqrt{3}}{2}$

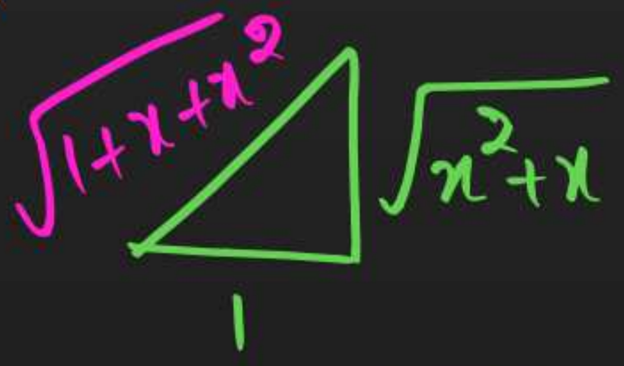
$= -\frac{\pi}{6} - \frac{\pi}{3}$

$= -\frac{\pi}{2} =$ RHS

QUESTION

PYQ

The real solutions of the equation $\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{2}$ is



$$\cos^{-1}\frac{1}{\sqrt{1+x+x^2}} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{2}$$

$$\frac{1}{\sqrt{1+x+x^2}} = \sqrt{x^2+x+1}$$

$$1 = 1+x+x^2$$

$$0 = x+x^2$$

$$x(x+1) = 0$$

$$x = 0 \mid x = -1$$

(A) 0

(B) -1

(C) $0, -1$

(D) 1

verify

(1) $x = 0$

LHS

$\tan^{-1}(0) + \sin^{-1}(1)$

$= \frac{\pi}{2} = \text{RHS}$

$x = -1$

LHS

$\tan^{-1}\sqrt{(-1)(-1+1)} + \sin^{-1}\sqrt{(-1)^2 + -1 + 1}$

$\tan^{-1}(0) + \sin^{-1}(1)$

$= \frac{\pi}{2} = \text{RHS}$

(*) The no of soln of $\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{2}$ is

(A) 1

(B) 0

(C) 2

(D) Infinite

QUESTION

In the following which is greater

(i) $\tan 1$ & $\tan^{-1}1$ (ii) $\sin 1$ & $\sin^{-1}1$ (iii) $\cos 1$ & $\cos^{-1}1$

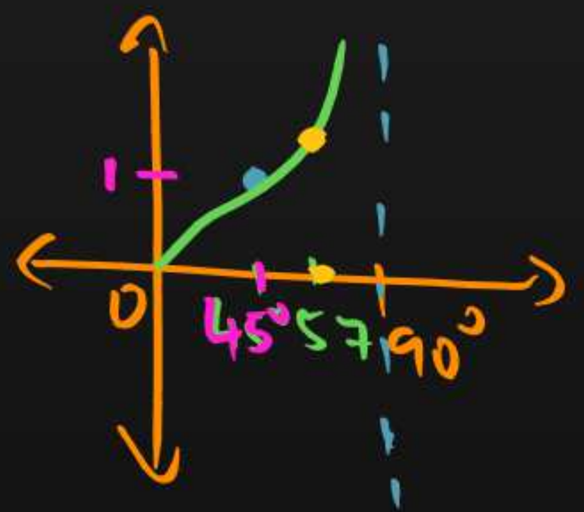
$1 = 1^c$

$\simeq 57^\circ$

(1) Which is greater out of $\underline{\tan 1}$ & $\tan^{-1}1$

$\downarrow \qquad \downarrow$

$\tan 57^\circ$ & $\frac{\pi}{4}$



$\tan 57^\circ > 1$ & $\frac{\pi}{4} = \frac{3.14}{4} < 1$

$\therefore \tan 57^\circ > \tan^{-1}1$

$\Downarrow$

$\underline{\tan 1 > \tan^{-1}1}$

tan 1 is greater

② which is greater

$\sin 1$ & $\sin^{-1} 1$

$\Downarrow$ $\Downarrow$

$\sin 57^\circ$ $\frac{\pi}{2} = \frac{3.14}{2} > 1$

$\Downarrow$

$\sin 57^\circ < 1$

$\boxed{1 = \sin 90}$

$\sin 1 < \sin^{-1} 1$

$\therefore \sin^{-1} 1$ is greater

③ which is greater

$\cos 1$ & $\cos^{-1} 1$

$\Downarrow$ $\Downarrow$

$\cos 57^\circ$ 0

$\Downarrow$

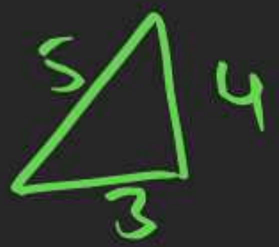
$\cos 57 > 0$

$\cos 57 > \cos^{-1} 1$

$\cos 1 > \cos^{-1} 1$

$\therefore \cos 1$ is greater

QUESTION



If $\cos[\tan^{-1}x] = \sin\left[\cot^{-1}\frac{3}{4}\right]$, then x =

$$\cos\left[\cos^{-1}\frac{1}{\sqrt{1+x^2}}\right] = \sin\left[\sin^{-1}\frac{4}{5}\right]$$

$$\frac{1}{\sqrt{1+x^2}} = \frac{4}{5}$$

$$\frac{1}{1+x^2} = \frac{16}{25}$$

$$1+x^2 = \frac{25}{16}$$

$$x^2 = \frac{9}{16}$$

$$x = \pm\frac{3}{4}$$

(A) 0

(B) -1

(C) 3/4

(D) 3/4, −3/4 ✓

QUESTION

$2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

$2\tan^{-1}(-3) =$

(A) $\tan^{-1}\frac{4}{3}$

(B) $\frac{\pi}{2} - \tan^{-1}\frac{4}{3}$

(C) $\frac{-\pi}{2} - \tan^{-1}\frac{4}{3}$

(D) $\tan^{-1}\frac{3}{4}$

$2\tan^{-1}(-3)$

$= -2\tan^{-1}(3)$

$= -\tan^{-1}\left[\frac{2(3)}{1-3^2}\right]$

$= -\tan^{-1}\left(\frac{6}{-8}\right)$

$= -\tan^{-1}\left(\frac{-3}{4}\right)$

$= +\tan^{-1}\frac{3}{4}$

Find $2\cot^{-1}(-3)$

$$\cot^{-1}(-x) = \pi - \cot^{-1}x$$

$$= 2\left[\pi - \cot^{-1}3\right]$$

$$= 2\pi - 2\cot^{-1}3$$

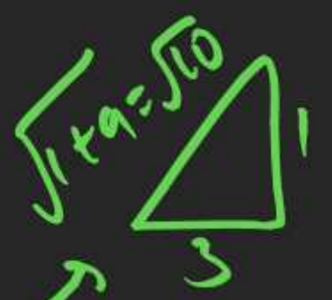
$$= 2\pi - 2\tan^{-1}\left(1/3\right)$$

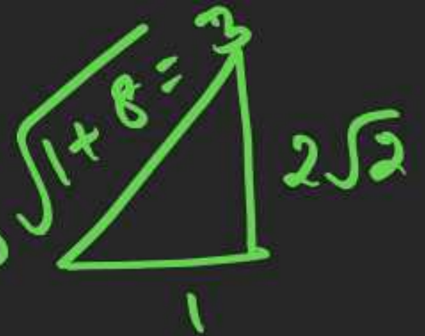
$$= 2\pi - \tan^{-1}\left(\frac{2/3}{1 - 1/9}\right)$$

$$= 2\pi - \tan^{-1}\left(\frac{2}{\cancel{3}} \times \frac{\cancel{9}^{\,3}}{\cancel{8}_{\,4}}\right)$$

$$= 2\pi - \tan^{-1}\left(\frac{3}{4}\right)$$

QUESTION





The value of $\sin\left[2\tan^{-1}\frac{1}{3}\right]+\cos\left[\tan^{-1}2\sqrt{2}\right]$ is

(A) 24/25

(B) 14/15

(C) 2/3

(D) 5

$\sin\left[\underbrace{2\sin^{-1}\frac{1}{\sqrt{10}}}_{\theta}\right]+\cos\left[\cos^{-1}\frac{1}{3}\right]$

$\sin 2x$

$2\sin\left(\sin^{-1}\frac{1}{\sqrt{10}}\right)\cos\left(\cos^{-1}\frac{3}{\sqrt{10}}\right)+\frac{1}{3}$

$=2\left(\frac{1}{\sqrt{10}}\right)\left(\frac{3}{\sqrt{10}}\right)+\frac{1}{3}$

$=\frac{6}{10}+\frac{1}{3}=\frac{18+10}{30}=\frac{28}{30}=\frac{14}{15}$

$\cot\left[\frac{\pi}{4} - 2\cot^{-1}3\right] =$

(A) 6

(B) −5

(C) 7

(D) 10

$$= \frac{\cot\left[2\cot^{-1}3\right] + 1}{\cot\left[2\cot^{-1}3\right] - 1} = \frac{\frac{4}{3}+1}{\frac{4}{3}-1} = \frac{7}{1} = 7$$

$\cot(A-B)$

$$= \frac{\cot A \cot B + 1}{\cot B - \cot A}$$

$\cot\frac{\pi}{4} = 1$

$$\cot\left[2\cot^{-1}3\right] = \cot\left[2\tan^{-1}\frac{1}{3}\right]$$

$$= \cot\left[\tan^{-1}\left(\frac{2/3}{1-1/9}\right)\right]$$

$$= \cot\left[\tan^{-1}\left(\frac{2}{3}\times\frac{9}{8}\right)\right]$$

$$= \cot\left[\tan^{-1}\left(\frac{3}{4}\right)\right] = \cot\left[\frac{\pi}{2} - \cot^{-1}\frac{3}{4}\right]$$

$$= \tan\left(\cot^{-1}\frac{3}{4}\right) = \tan\left(\tan^{-1}\frac{4}{3}\right) = \frac{4}{3}$$

QUESTION

$$\tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right]=$$

(A) $\frac{1}{2}\cos^{-1}x^2$

(B) $\cos^{-1}x$

(C) $\frac{\pi}{4}+\frac{1}{2}\cos^{-1}x^2$

(D) $\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x^2$

$\cos 2\theta = x^2$

$2\theta = \cos^{-1}(x^2)$

$\theta = \frac{1}{2}\cos^{-1}x^2$

$\Uparrow$

Put $x^2 = \cos 2\theta$

$\sqrt{1+x^2} = \sqrt{1+\cos 2\theta} = \sqrt{2\cos^2\theta} = \sqrt{2}\cos\theta$

$\sqrt{1-x^2} = \sqrt{1-\cos 2\theta} = \sqrt{2\sin^2\theta} = \sqrt{2}\sin\theta$

$\tan^{-1}\left[\frac{\sqrt{2}(\cos\theta+\sin\theta)}{\sqrt{2}(\cos\theta-\sin\theta)}\right]$

$= \tan^{-1}\left[\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}\right]$

$\div$ N^r & D^r by $\cos\theta$

$= \tan^{-1}\left[\frac{1+\tan\theta}{1-\tan\theta}\right]$

$= \tan^{-1}\left[\tan\left(\frac{\pi}{4}+\theta\right)\right]$

$= \frac{\pi}{4}+\theta$

$= \frac{\pi}{4}+\frac{1}{2}\cos^{-1}x^2$

Thank

You