VARUN

ADVANCE 2026

Physics

Lecture - 03

Center of Mass and Collision

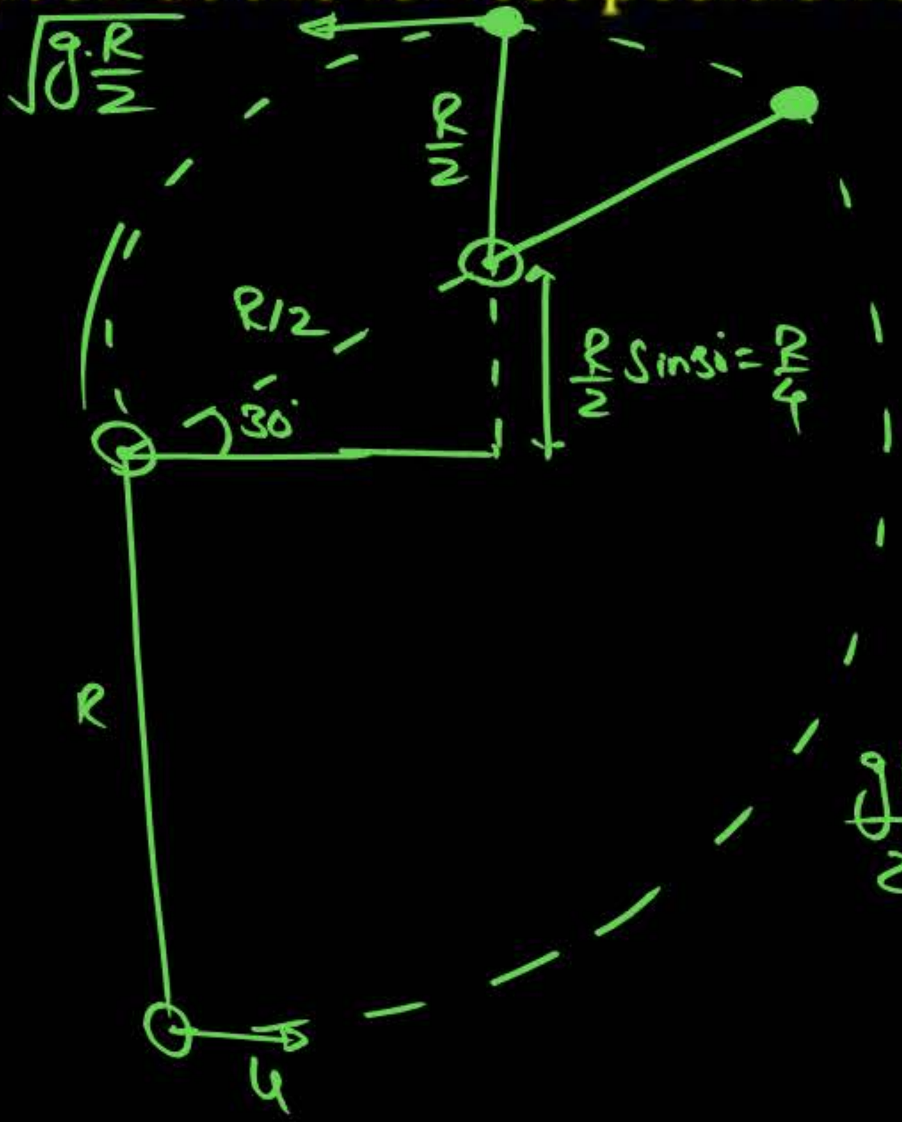
By Masti Sir

Topics to be covered

1. Center of mass
2.
3.
4.
5.

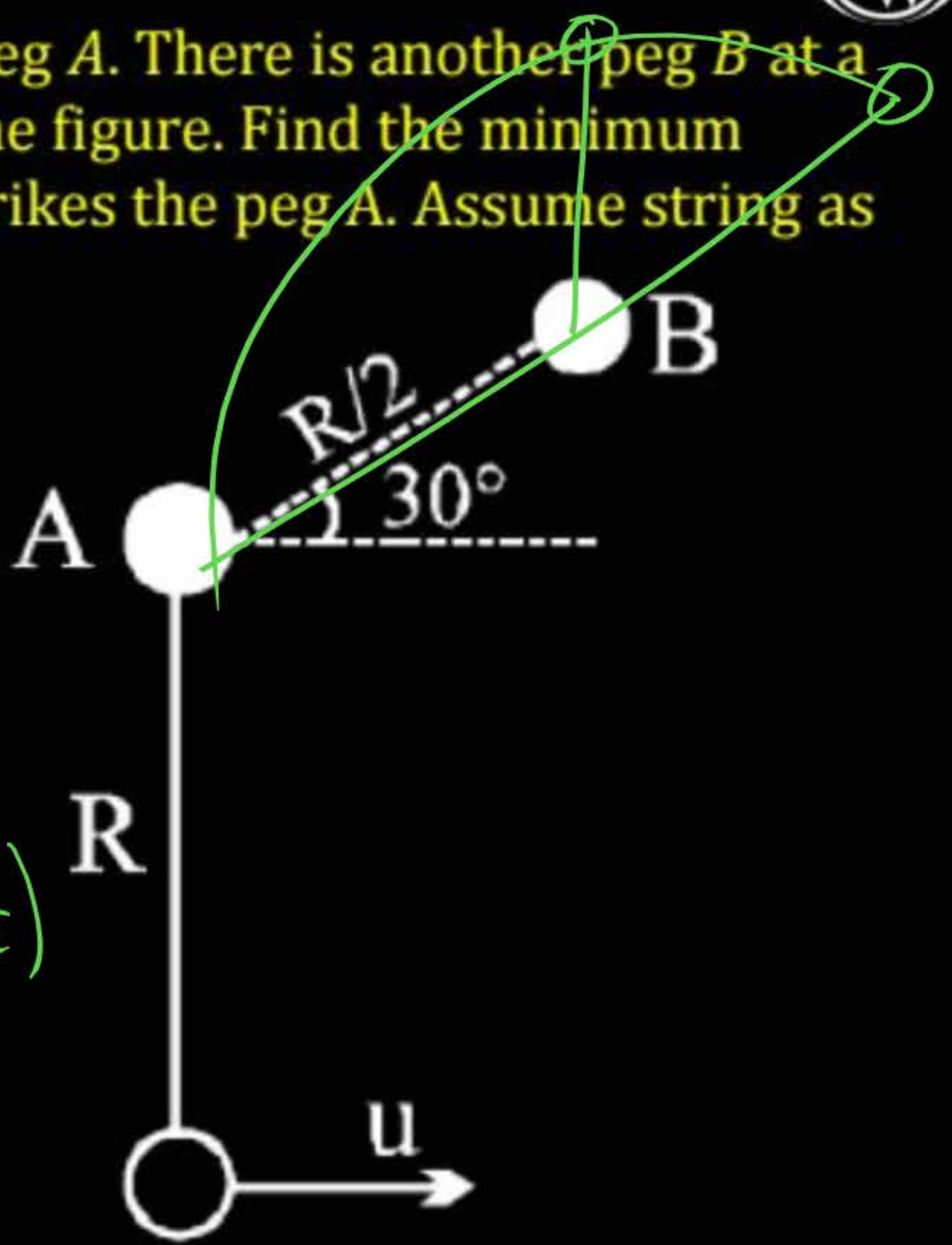
Question

A ball is hanging vertically by a string of length $R = 40cm$ form peg A. There is another peg B at a distance R/2 at an angle of 30° with the horizontal as shown in the figure. Find the minimum horizontal velocity given at the lowest position so that the ball strikes the peg A. Assume string as always taut.



$v^2 = u^2 - 2gh$

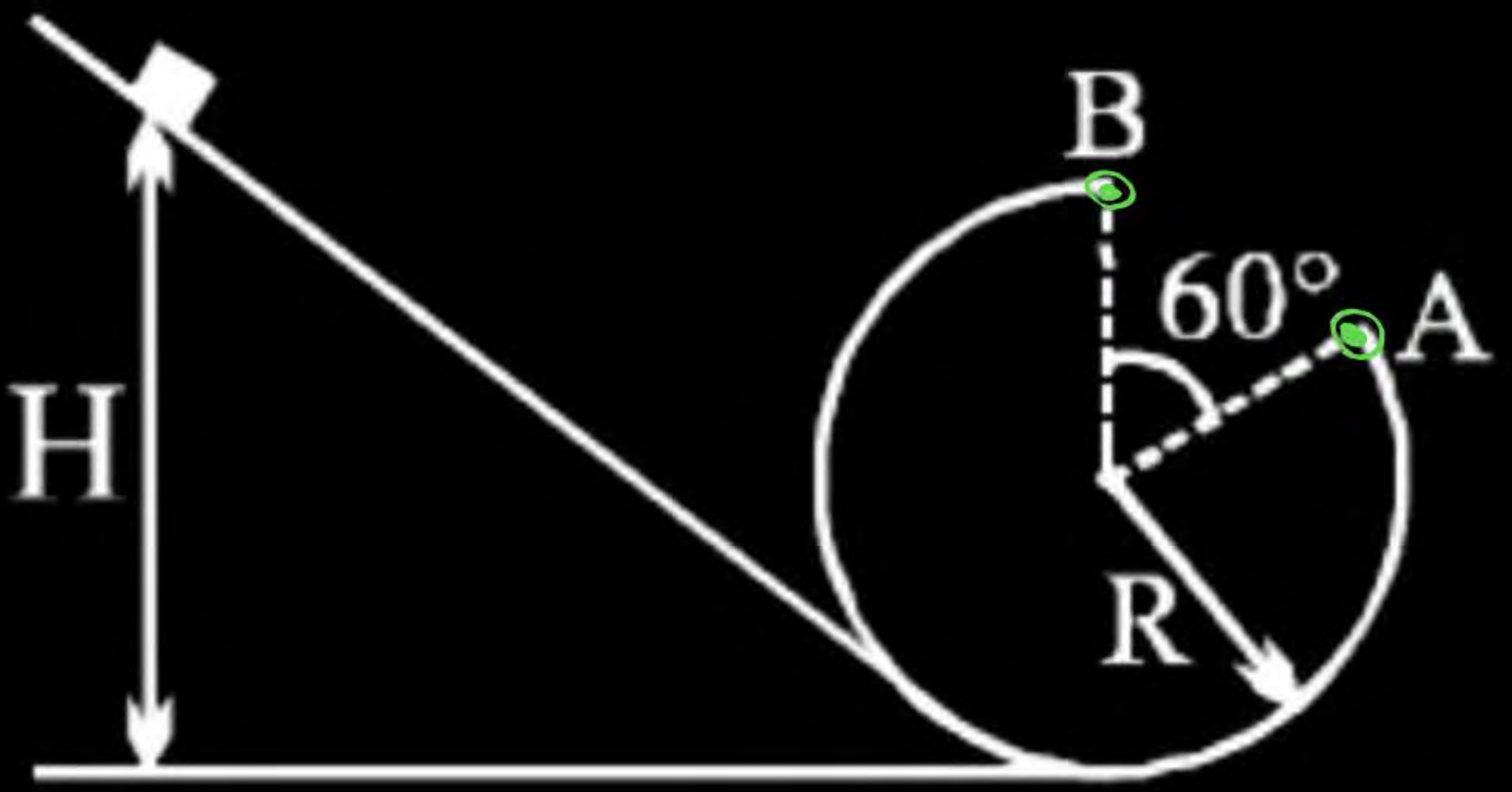
$\frac{gR}{2} = u^2 - 2g\left(R + \frac{R}{4} + \frac{R}{2}\right)$



Ans. 4

Question

A small particle slides from height $H = 45$ cm as shown and then loops inside the vertical loop of radius R from where a section of angle $\theta = 60^\circ$ has been removed. Find R (in cm) such that after losing contact at A and flying through the air, the particle will reach at the point B . Neglect friction everywhere.



Ans. 20

$$x = \frac{\sqrt{3}R}{2} = (v\cos 60^\circ)t \Rightarrow vt = \sqrt{3}R$$

$$y = \frac{R}{2} = (v\sin 60^\circ)t - \frac{1}{2}gt^2$$

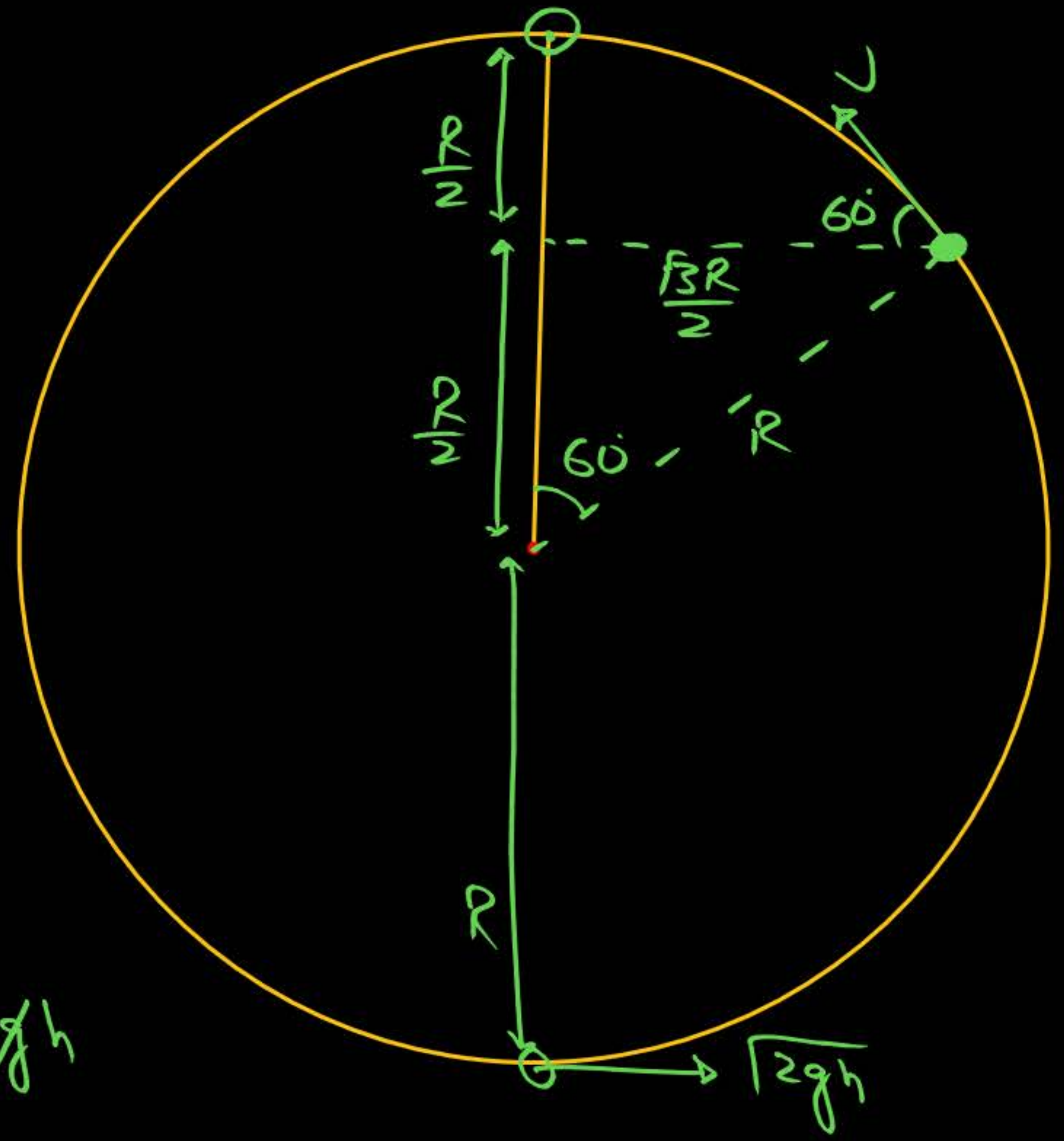
$$\frac{R}{2} = \frac{\sqrt{3}v}{2}\cdot\frac{\sqrt{3}R}{v} - \frac{1}{2}g\left(\frac{3R^2}{v^2}\right)$$

$$\cancel{R} = \frac{3R^{\cancel{2}}g}{2v^2} \Rightarrow v = \sqrt{\frac{3Rg}{2}}$$

$$v^2 = u^2 - 2gh'$$

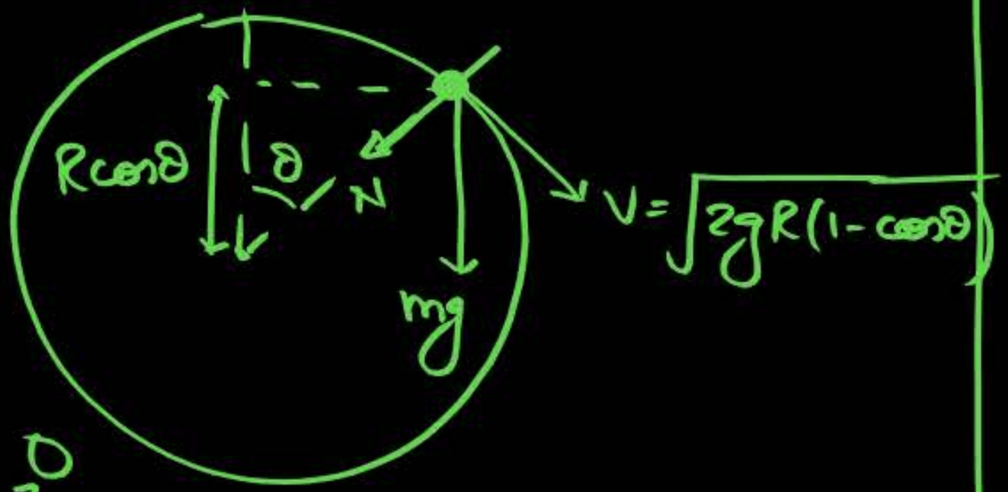
$$\frac{3Rg}{2} = 2gh - 2g\left(\frac{3R}{2}\right) \Rightarrow \frac{9R\cancel{g}}{2} = 2\cancel{g}h$$

$$R = \frac{4h}{9}$$



Question

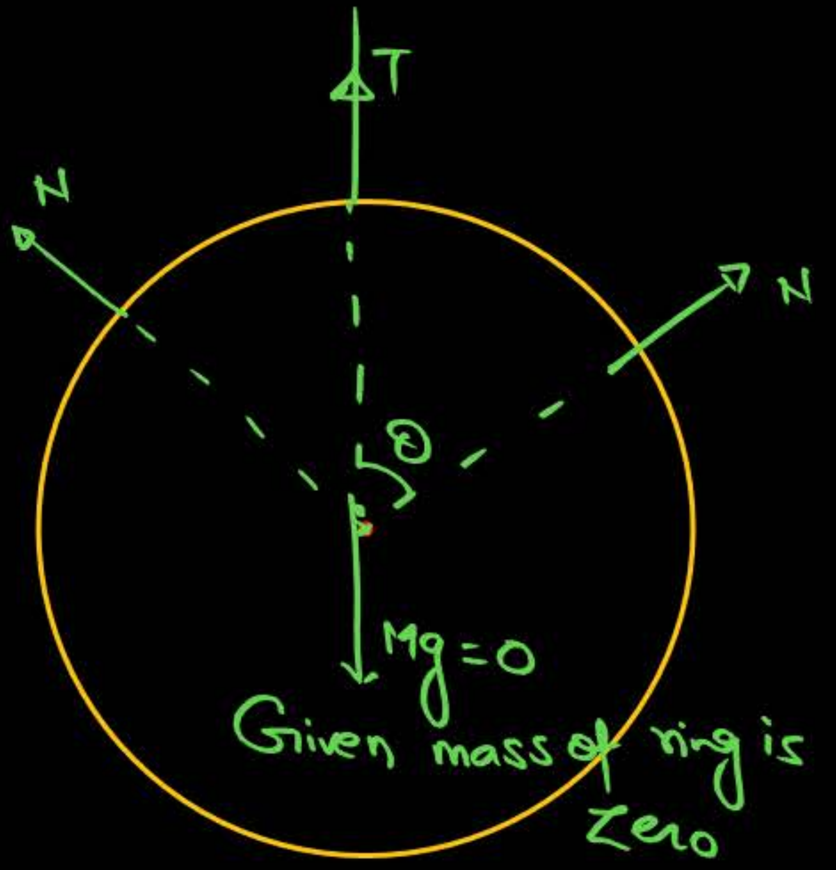
A massless ring hangs from a thread and two beads of mass m slide on it without friction. The beads are released simultaneously from the top of the ring and slide down opposite sides. If the angle from vertical at which tension in the thread becomes zero for the first time is θ. Find $\sec\theta$.



$$\overset{0}{\cancel{N}} + mg\cos\theta = \frac{mv^2}{R} = m(2g(1-\cos\theta))$$

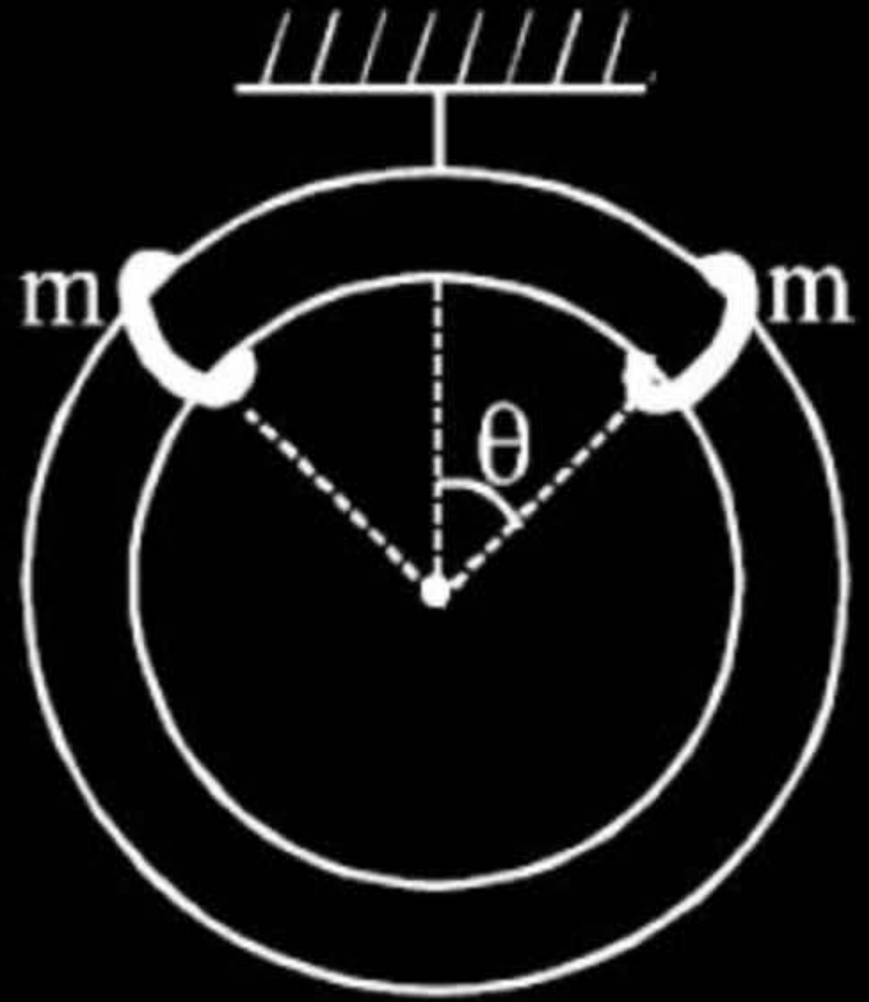
$$3\cos\theta = 2 \Rightarrow \cos\theta = \frac{2}{3}$$

$$\Rightarrow \sec\theta = 3/2 = 1.5$$



$$T + 2N\cos\theta = Mg = 0$$

$$\Rightarrow T = 0 \Rightarrow N = 0$$



Ans. 1.5

Question

Find center of mass of an object (Parabola) which is formed by a parabola $x = 4y^2$ as shown in figure (From x = 0 to x = h). Assume the object is of uniform density. (h = 15cm)

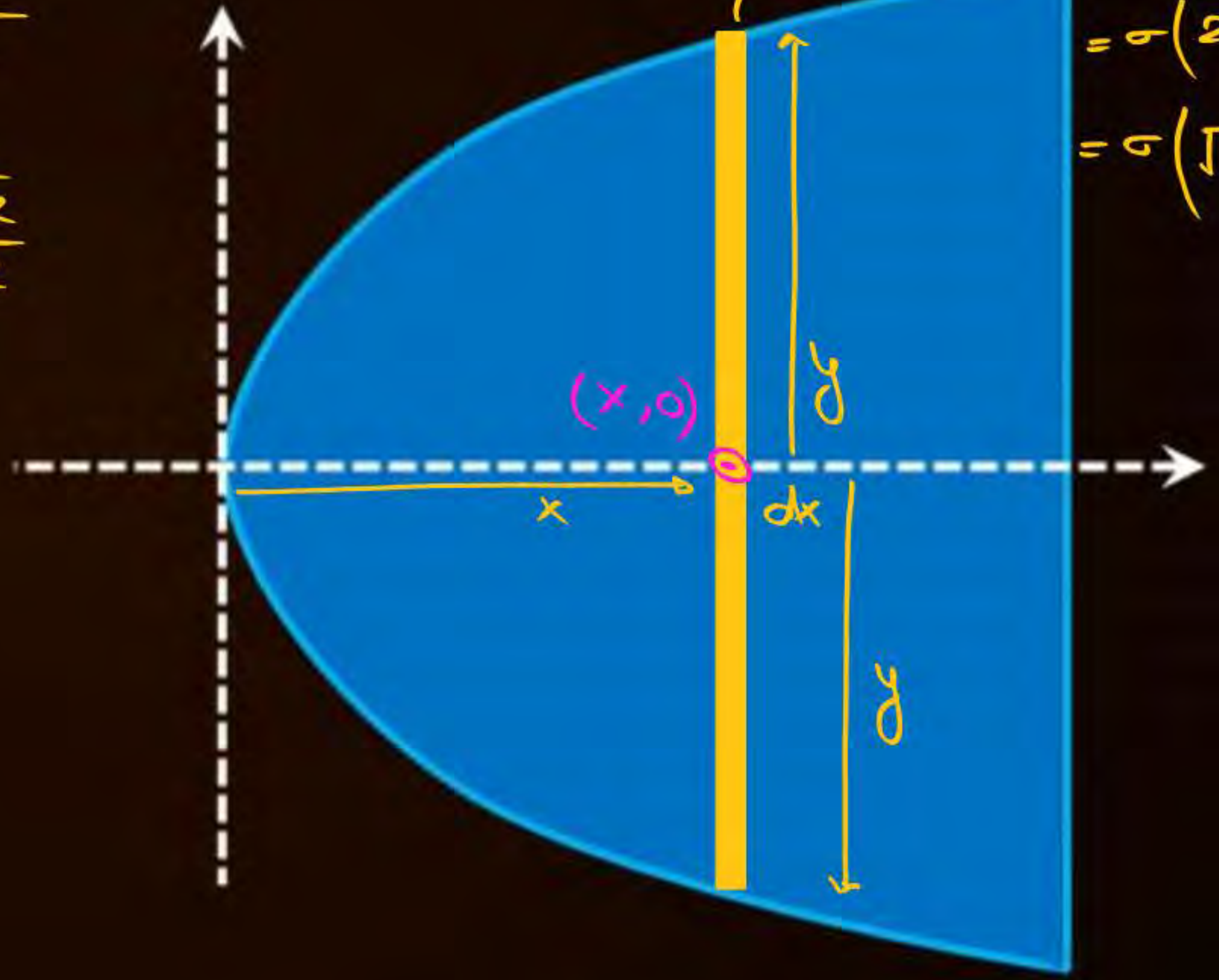
$$x = 4y^2$$

$$y = \pm\sqrt{\frac{x}{4}}$$

$$y = \pm\frac{\sqrt{x}}{2}$$

$$x_{cm} = \frac{\int x\,dm}{\int dm}$$

$$= \frac{\int_0^h \sigma(\sqrt{x})\,x\,dx}{\int_0^h \sigma\sqrt{x}\,dx}$$



$$dm = \sigma\,dA$$

$$= \sigma(2y\,dx)$$

$$= \sigma(\sqrt{x}\,dx)$$

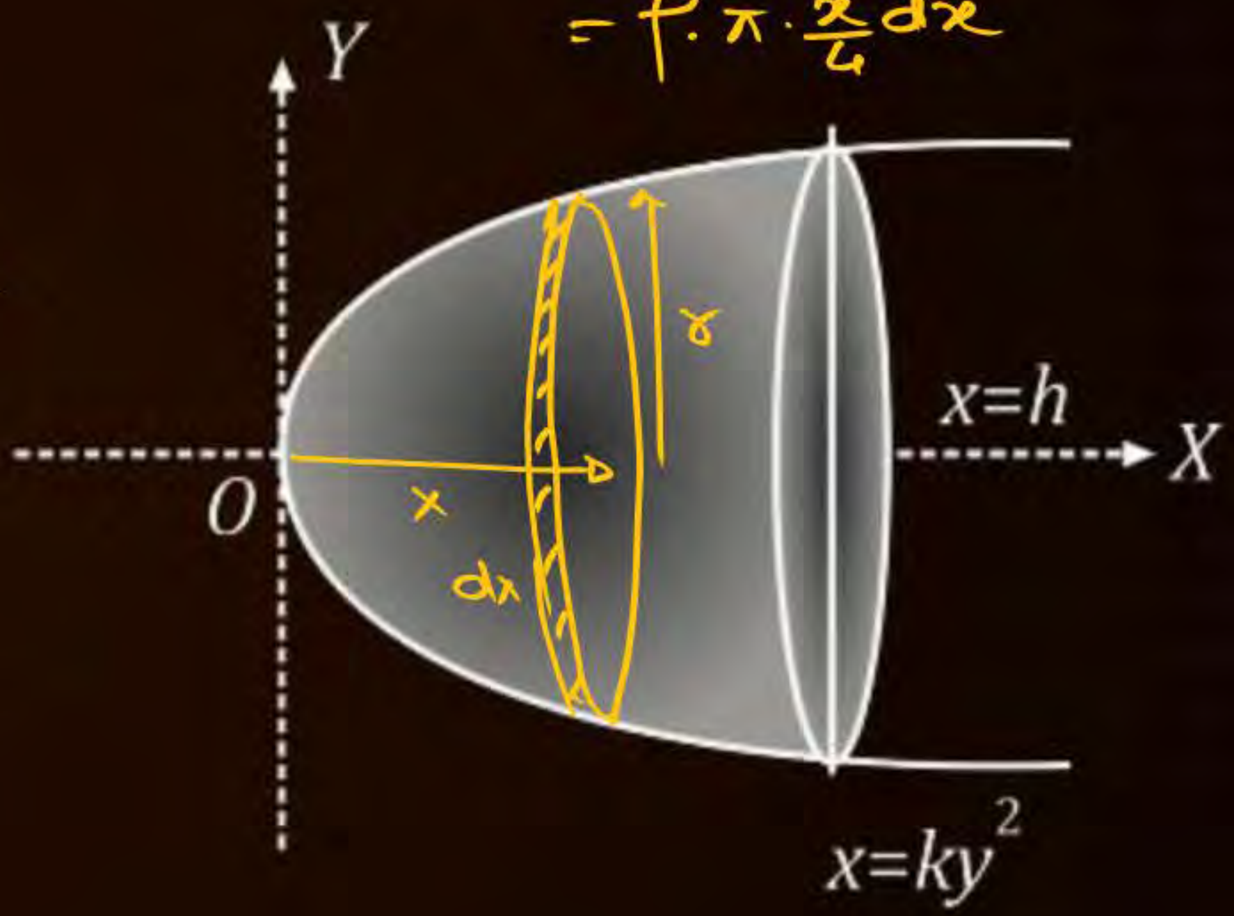
Question

Find center of mass of an object (Paraboloid) which is formed by rotating a parabola $x = 4y^2$ about x-axis as shown in figure (From x = 0 to x = h). Assume the object is of uniform density. (h = 15cm)

$$y = \pm\frac{\sqrt{x}}{2} \Rightarrow r = \frac{\sqrt{x}}{2}$$

$$dm = \rho\, dV = \rho \pi r^2 dx$$
$$= \rho \cdot \pi \cdot \frac{x}{4} dx$$

$$\frac{\int x\, dm}{\int dm} = \frac{\int_0^h x\left(\frac{\rho \pi x}{4} dx\right)}{\int_0^h \frac{\rho \pi x\, dx}{4}} = \frac{h^3/3}{h^2/2}$$

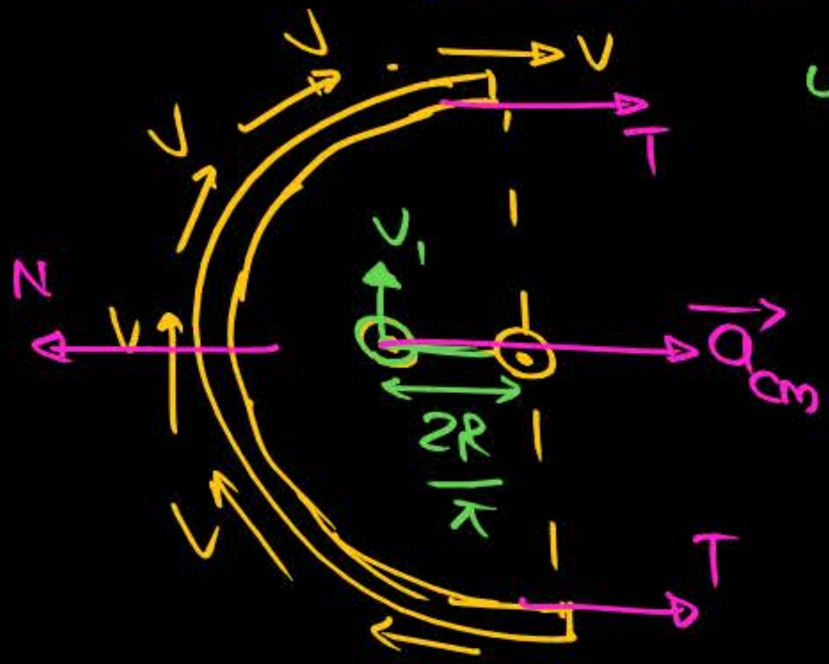


Question

$m = \lambda \pi R$

A flexible drive belt runs over a frictionless flywheel (see Figure). The mass per unit length of the drive belt is 1 kg/m, and the tension in the drive belt is 10 N . The speed of the drive belt is 2 m/s. The whole system is located on a horizontal plane. Find the normal force (in N) exerted by the belt on the flywheel.

$$\boxed{\vec{F}_{ext.} = M\vec{a}_{cm}}$$

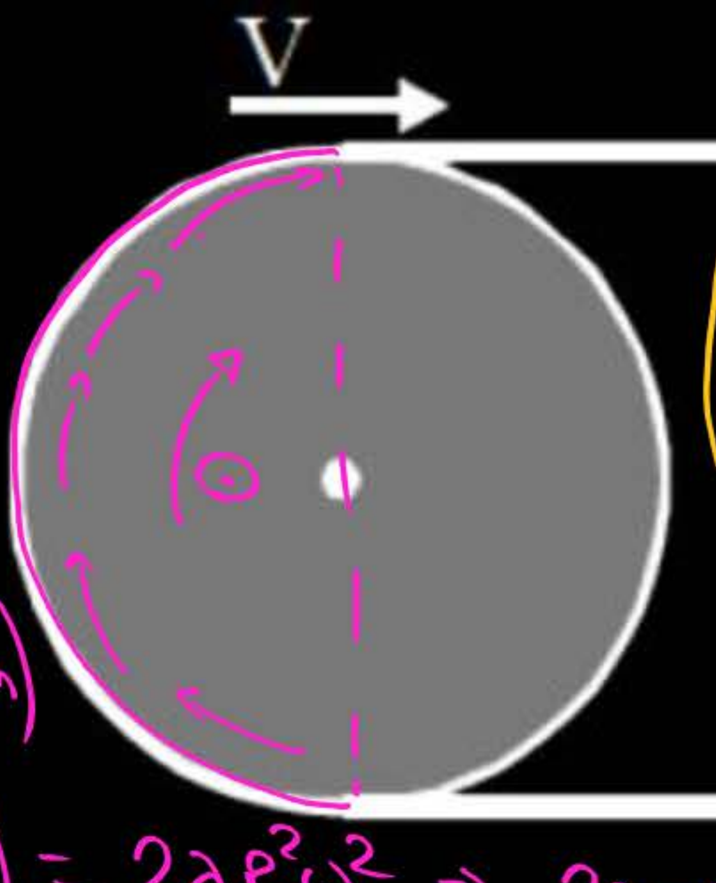


$$\omega = \frac{v}{R} = \frac{v_1}{2R/\pi} \Rightarrow v_1 = \frac{2v}{\pi}$$

$$2T - N = m a_{cm}$$

$$2T - N = (\lambda \pi R)(\omega^2 \cdot r_{cm})$$

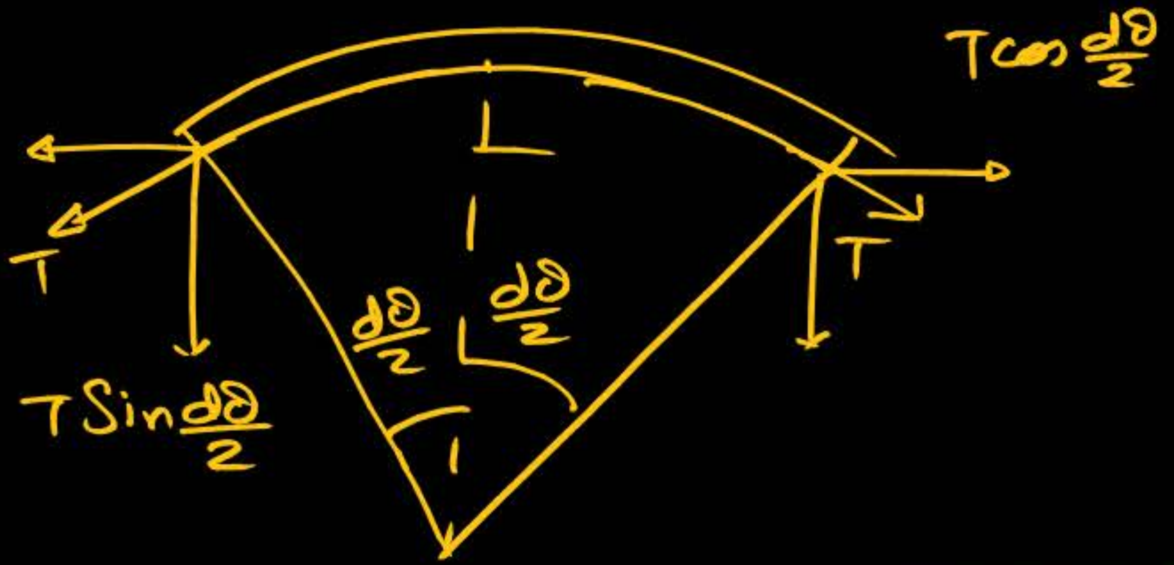
$$2T - N = (\lambda \pi R)\left(\omega^2 \cdot \frac{2R}{\pi}\right) = 2\lambda \underbrace{R^2\omega^2}_{v^2} \Rightarrow 20 - N = 2(1)(2)^2 \Rightarrow N = 12N$$

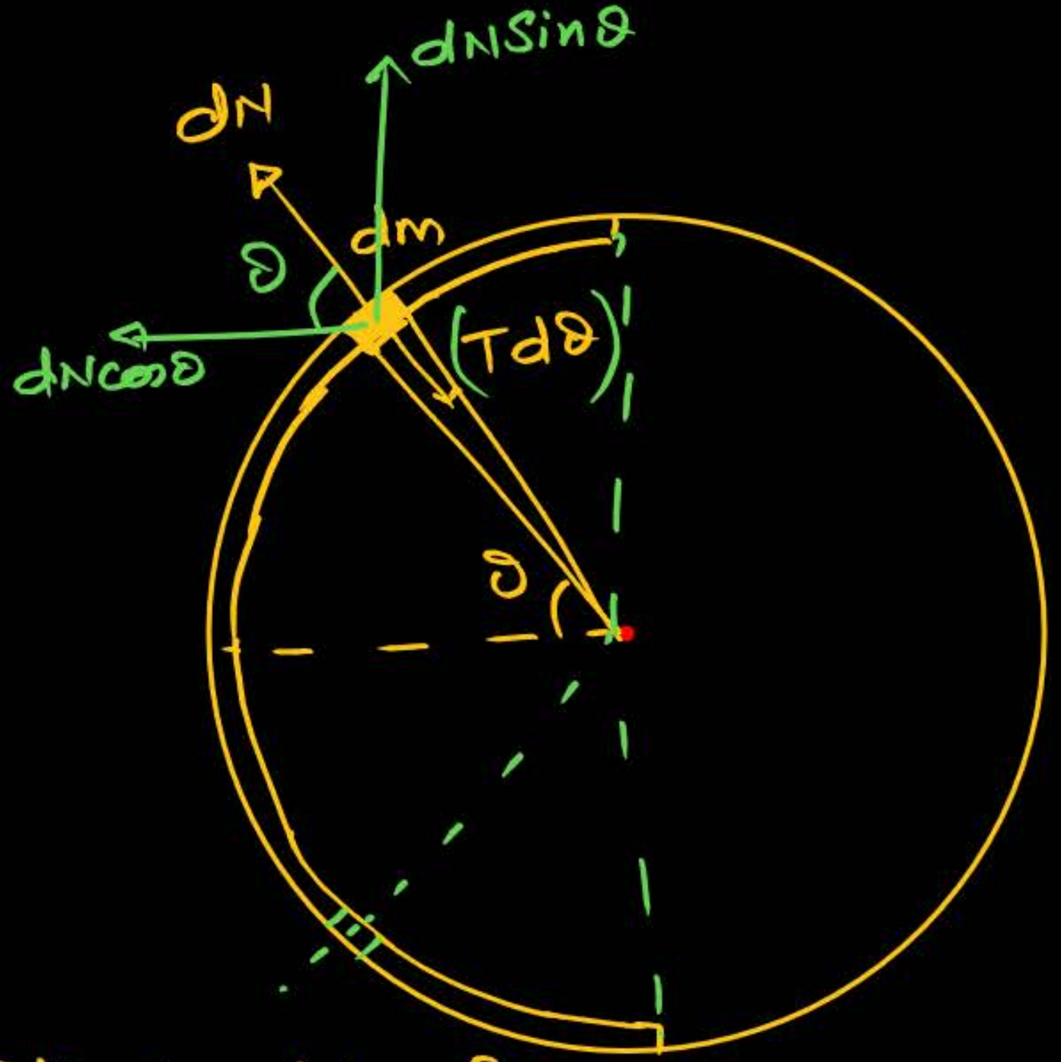


$P = m v_{cm}$

$P = (\lambda \pi R)\left(\frac{2v}{\pi}\right)$

Ans. 12





$$dN = 10\,d\theta - \lambda(R\,d\theta)\omega^2 R$$

$$dN = 10\,d\theta - \lambda\omega^2 R^2\,d\theta$$

$$dN = 10\,d\theta - (1)(2)^2\,d\theta = \underline{6\,d\theta}$$

$$\int_{-\pi/2}^{\pi/2} dN\sin\theta = 6\int_{-\pi/2}^{\pi/2}\sin\theta\,d\theta = 0$$

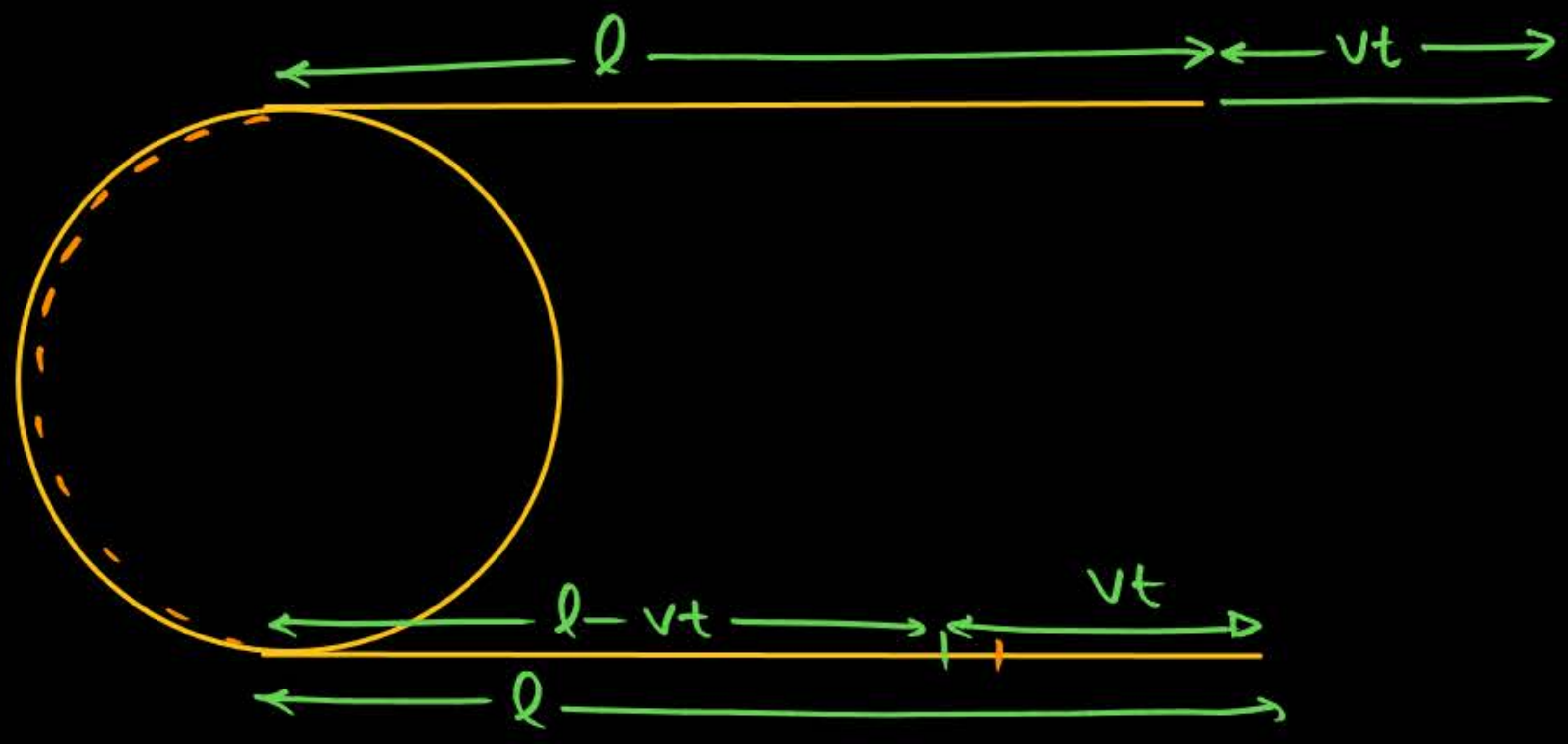
$$\int_{-\pi/2}^{\pi/2} dN\cos\theta = 6\int_{-\pi/2}^{\pi/2}\cos\theta\,d\theta = 6(2) = 12\text{ N}$$

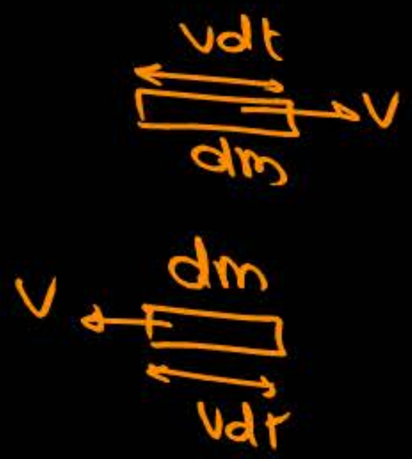
$$T\,d\theta - dN = (dm)\omega^2 R$$

$$F = \frac{dP}{dt}$$

$$2T - N = \frac{2\,dm\,V}{dt} = \frac{2V(\lambda V \cancel{dt})}{\cancel{dt}}$$

$$20 - N = 2(2)(1)(2)$$





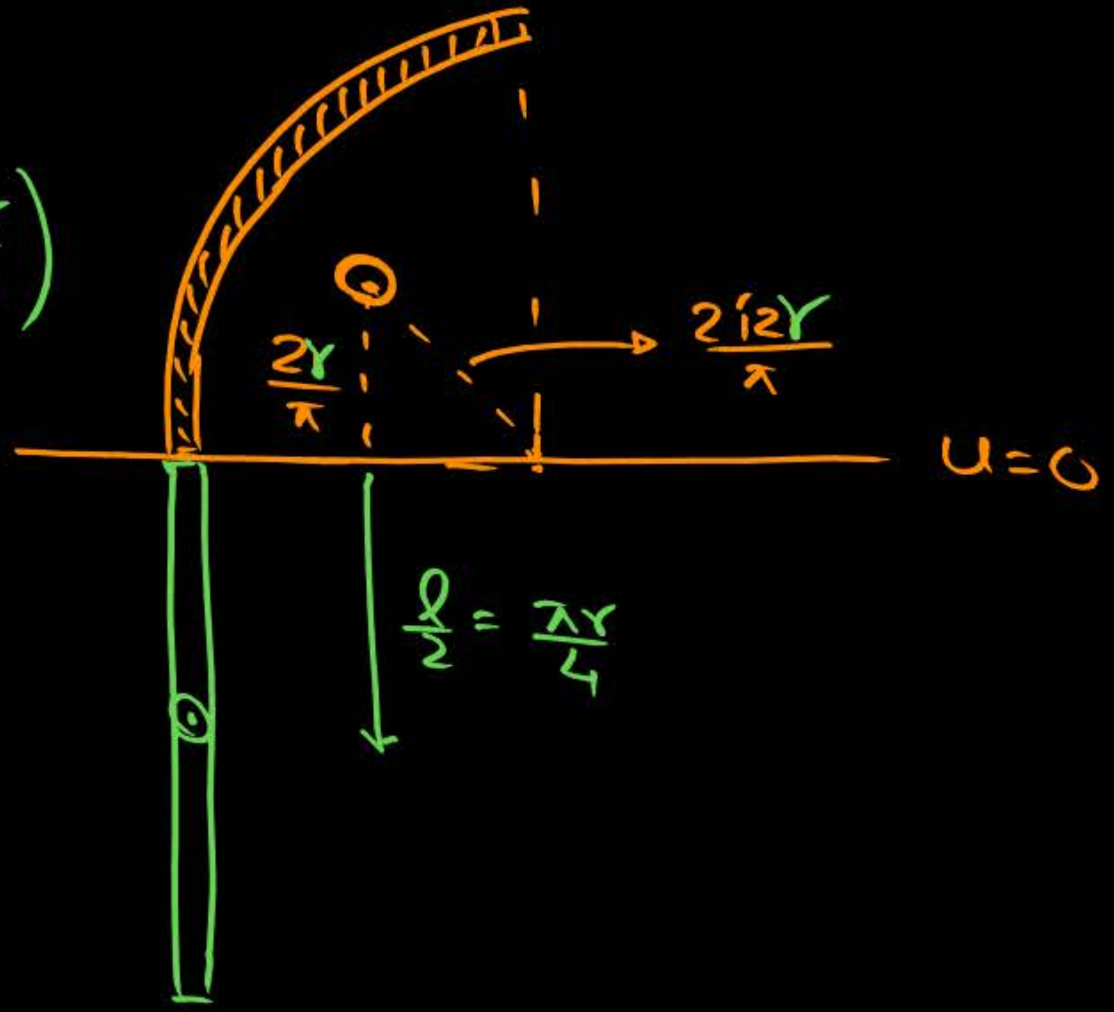
Question

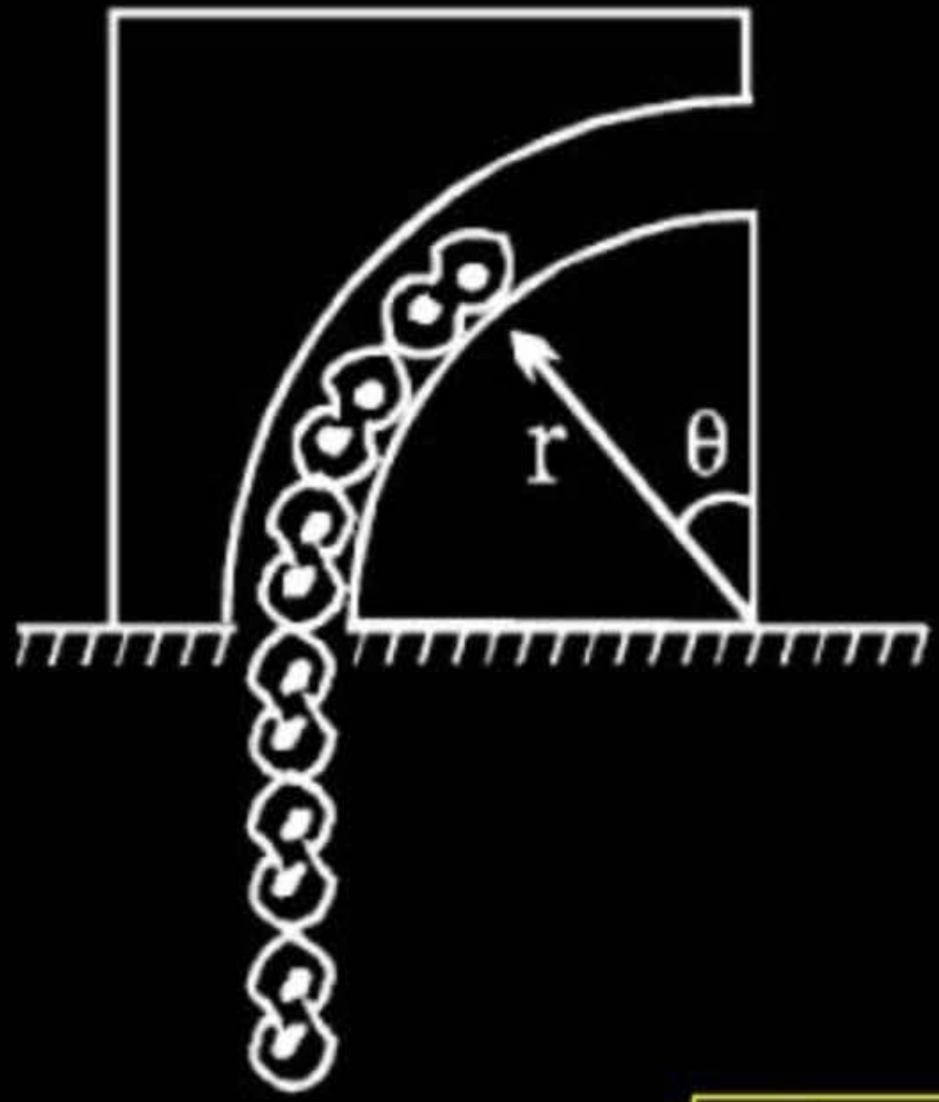
$U = Mgh_{cm}$

The flexible bicycle type chain of length $\frac{\pi r}{2}$ and mass per unit length r is released from rest with $\theta = 0°$ in the smooth circular channel and falls through the hole in the supporting surface. Determine the velocity v of the chain as the last link leaves the slot.

$K_1 + U_1 = K_2 + U_2$

$0 + mg\left(\frac{2r}{\pi}\right) = \frac{1}{2}mv^2 - mg\left(\frac{\pi r}{4}\right)$





Ans. $\sqrt{gr\left(\frac{\pi}{2}+\frac{4}{\pi}\right)}$

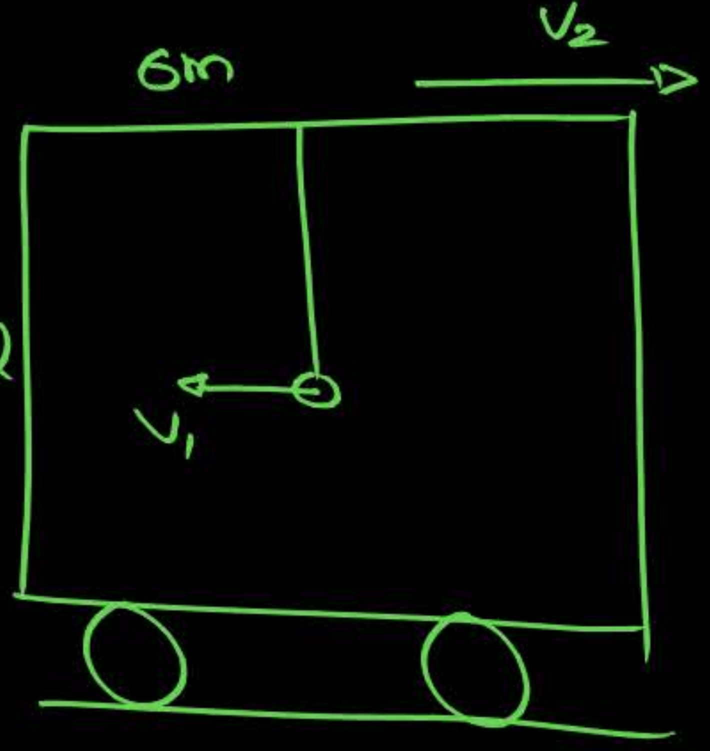
Question

$k.E = \underline{\frac{1}{2} M v_{cm}^2} + \underline{\frac{1}{2} \mu v_{rel.}^2}$

The frame of mass 6 m is initially at rest. A particle of mass m is attached to the end of the light rod, which pivots freely at A . If the rod is released from rest in the horizontal position shown, determine the velocity v_{rel} (in m/s) of the particle with respect to the frame when the rod is vertical. (Length of the rod is 2.1 m)

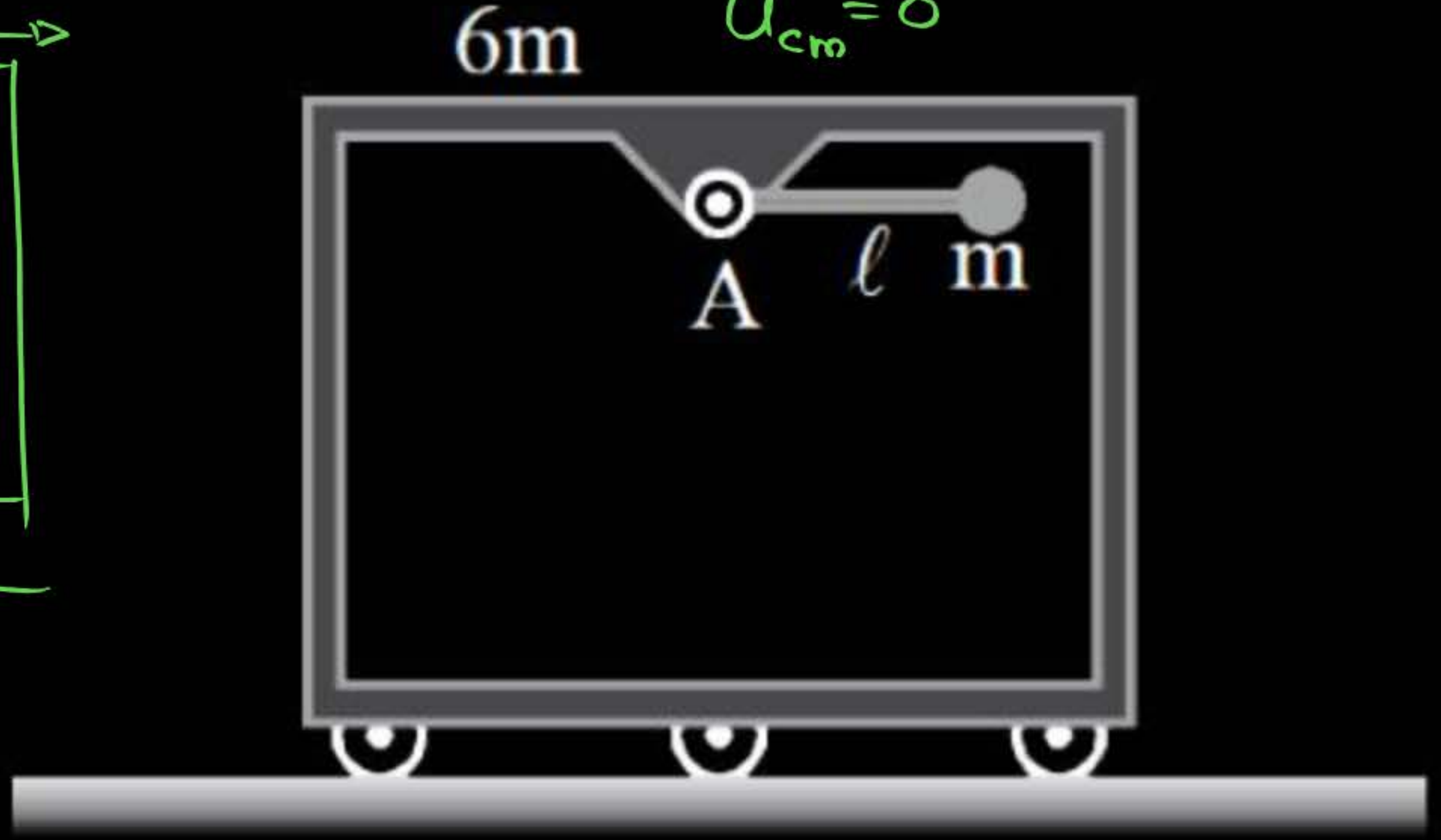
$V_{cm} = 0$

$u_{cm} = 0$



$\frac{1}{2}\mu(0)^2 + 0 = \frac{1}{2}\mu v_{rel.}^2 - mgl$

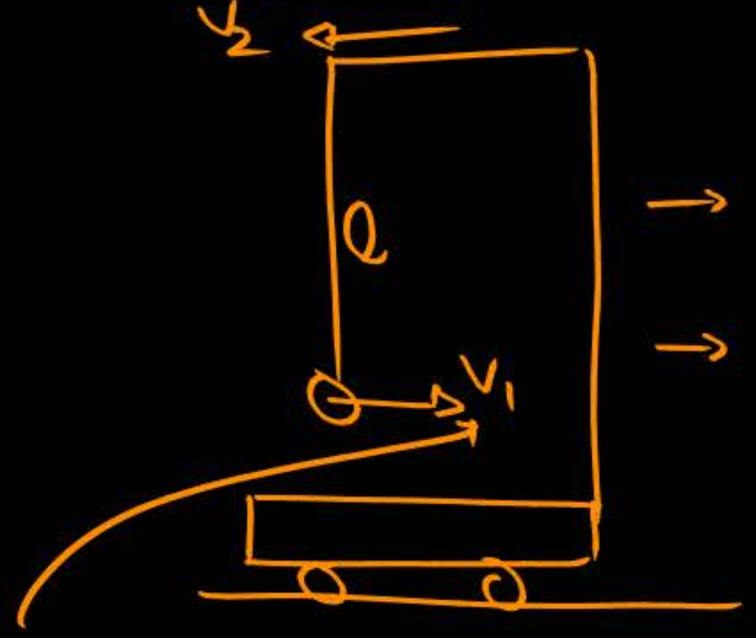
$\Rightarrow \frac{1}{2}\left(\frac{6m \times m}{7m}\right) v_{rel.}^2 = mgl$



Ans. 7

Question

A pendulum bob of length 1 and mass m(500 g) is suspended on a smoothly running trolley of mass M(50 kg). The pendulum bob is given an impulse so as to impart it a horizontal velocity v_0. Maximum velocity attained by trolley in subsequent motion is $\frac{K}{101}v_0$. Assume that pendulum bob oscillates subsequently find the value of K^2.

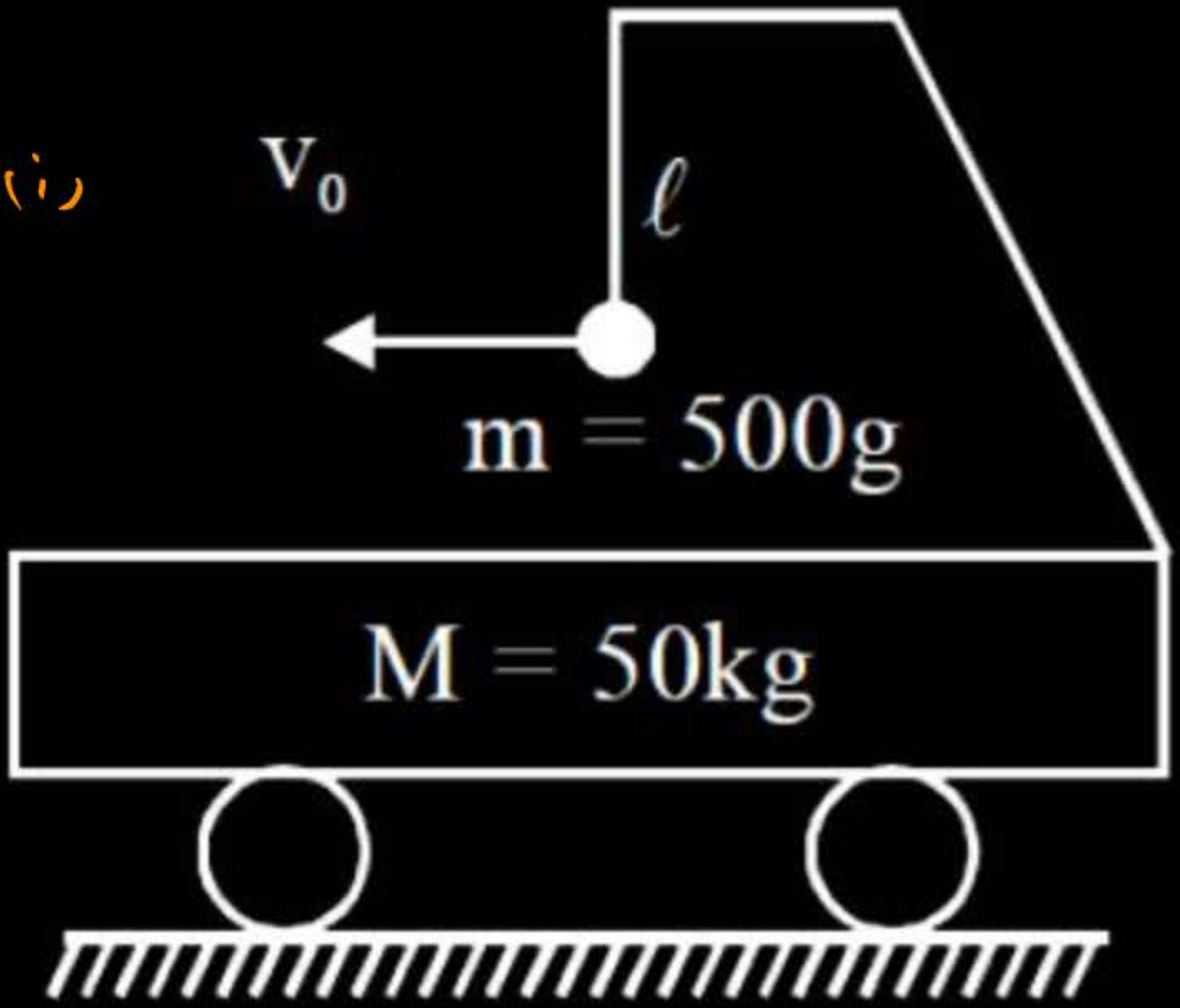


$V_2 \rightarrow$ max.

$\rightarrow \frac{1}{2}mv_1^2 + \frac{1}{2}Mv_2^2 = \frac{1}{2}mv_0^2$ — (i)

$\rightarrow mv_0 = Mv_2 - mv_1$ — (ii)

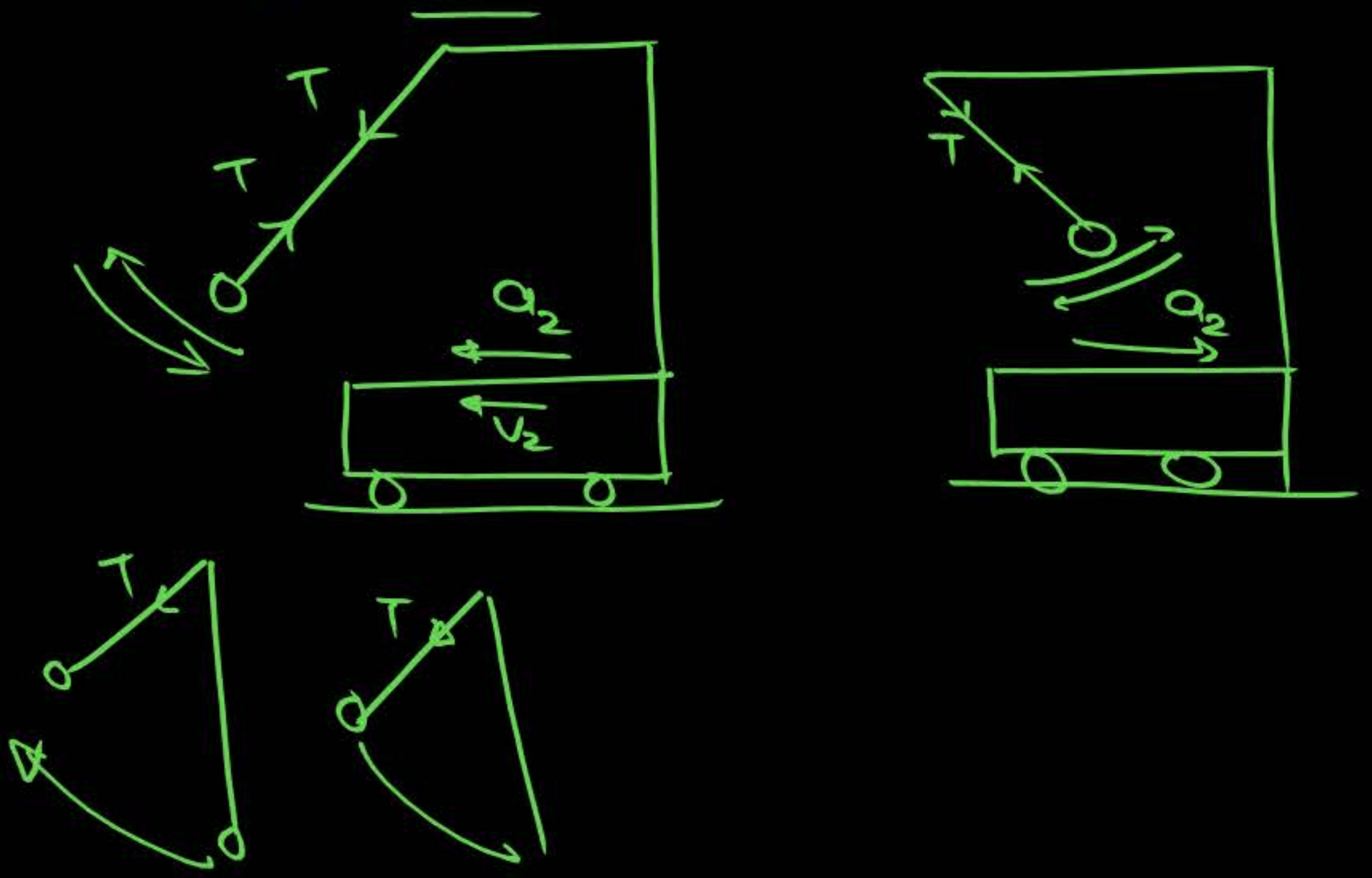
w.r.t Ground



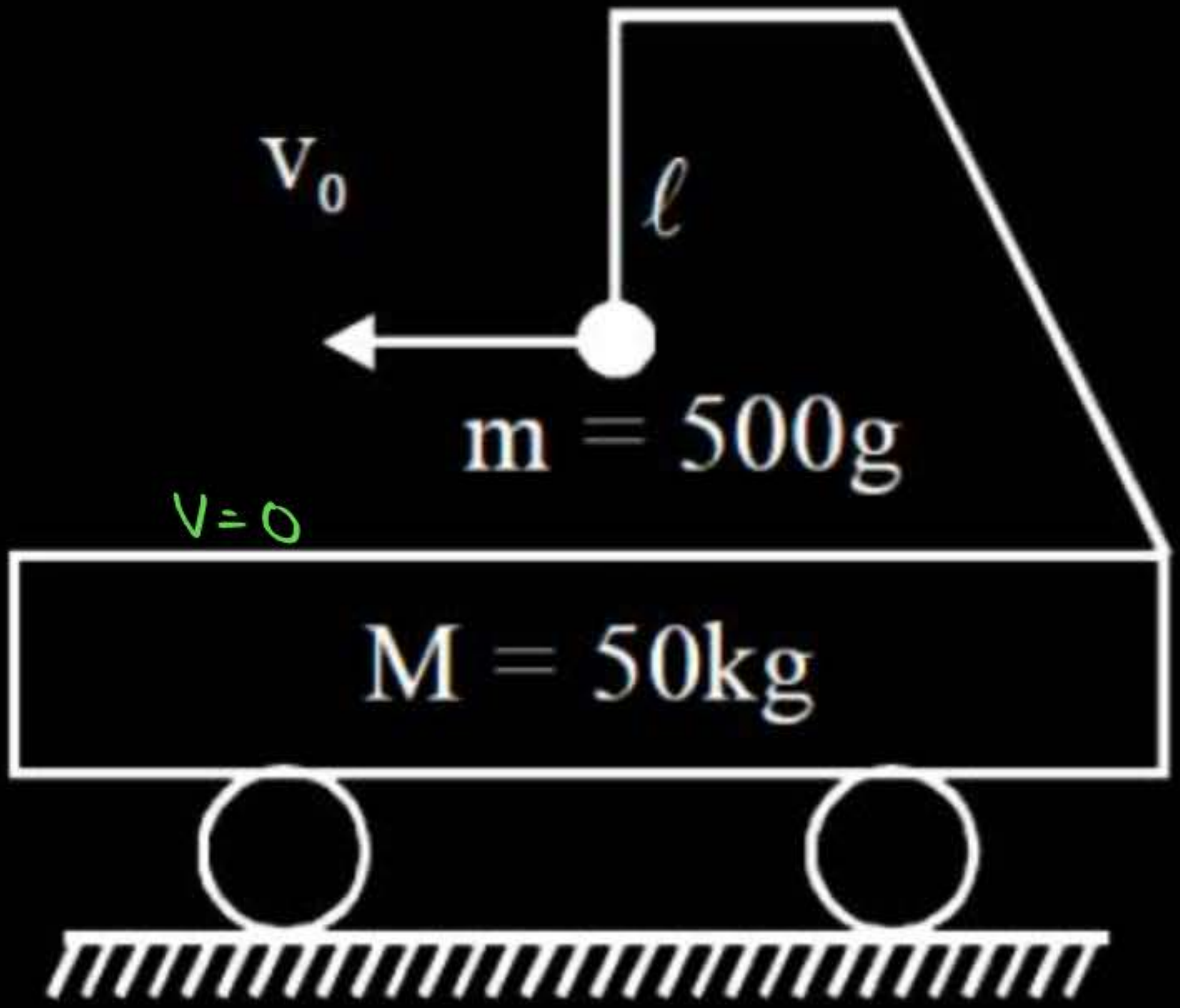
Ans. 4

Question

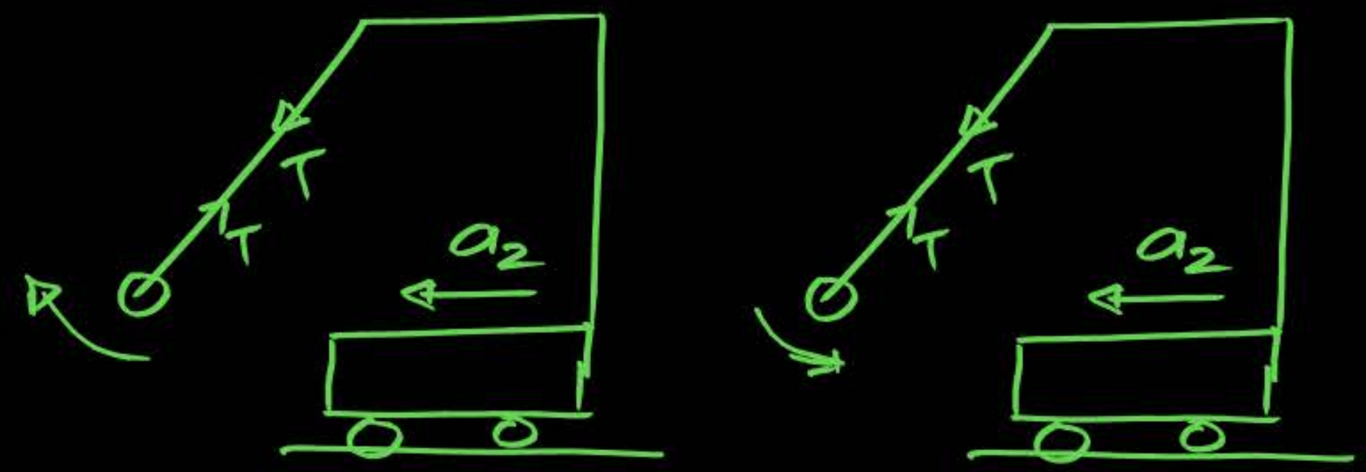
A pendulum bob of length 1 and mass m(500 g) is suspended on a smoothly running trolley of mass M(50 kg). The pendulum bob is given an impulse so as to impart it a horizontal velocity v_0. Maximum velocity attained by trolley in subsequent motion is $\frac{K}{101}v_0$. Assume that pendulum bob oscillates subsequently find the value of K^2.



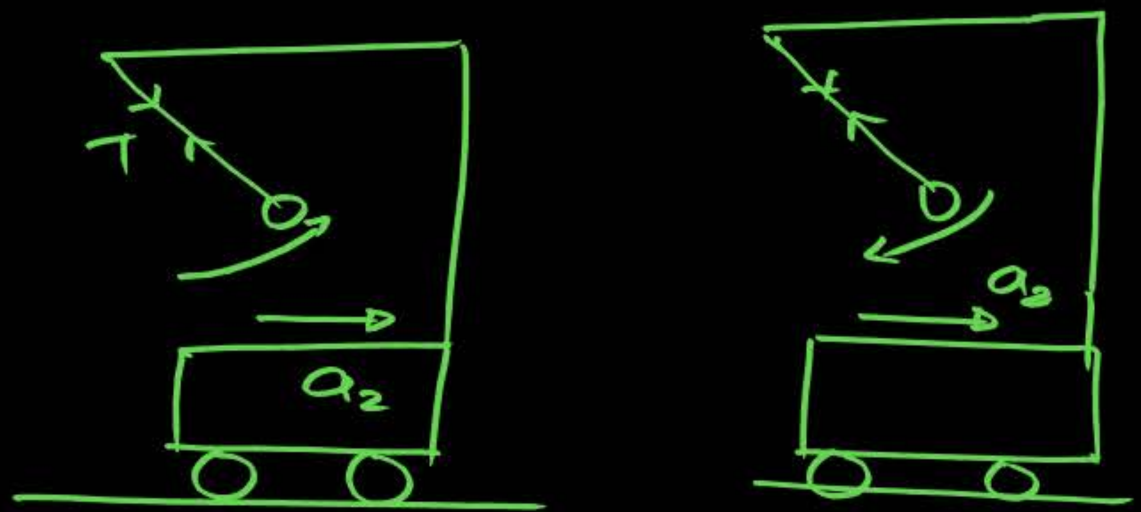
acc. of cart is in left dirn



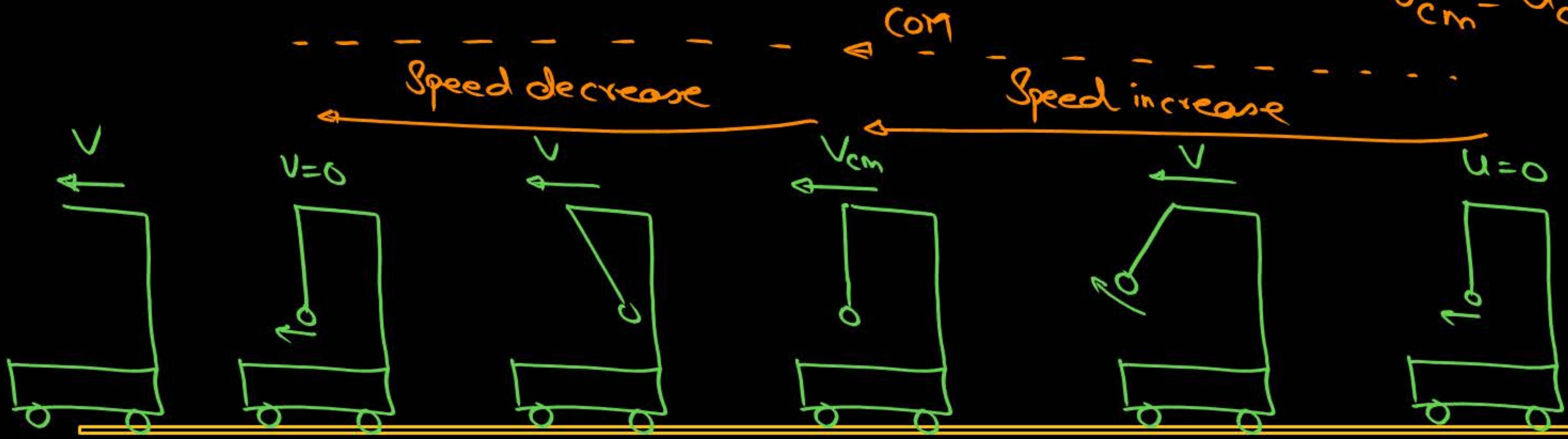
Ans. 4



Speed of Cart increasing



$$V_{cm} = U_{cm} = \frac{mv_0 + M(0)}{m+M}$$
$$= \text{Const.}$$



Question

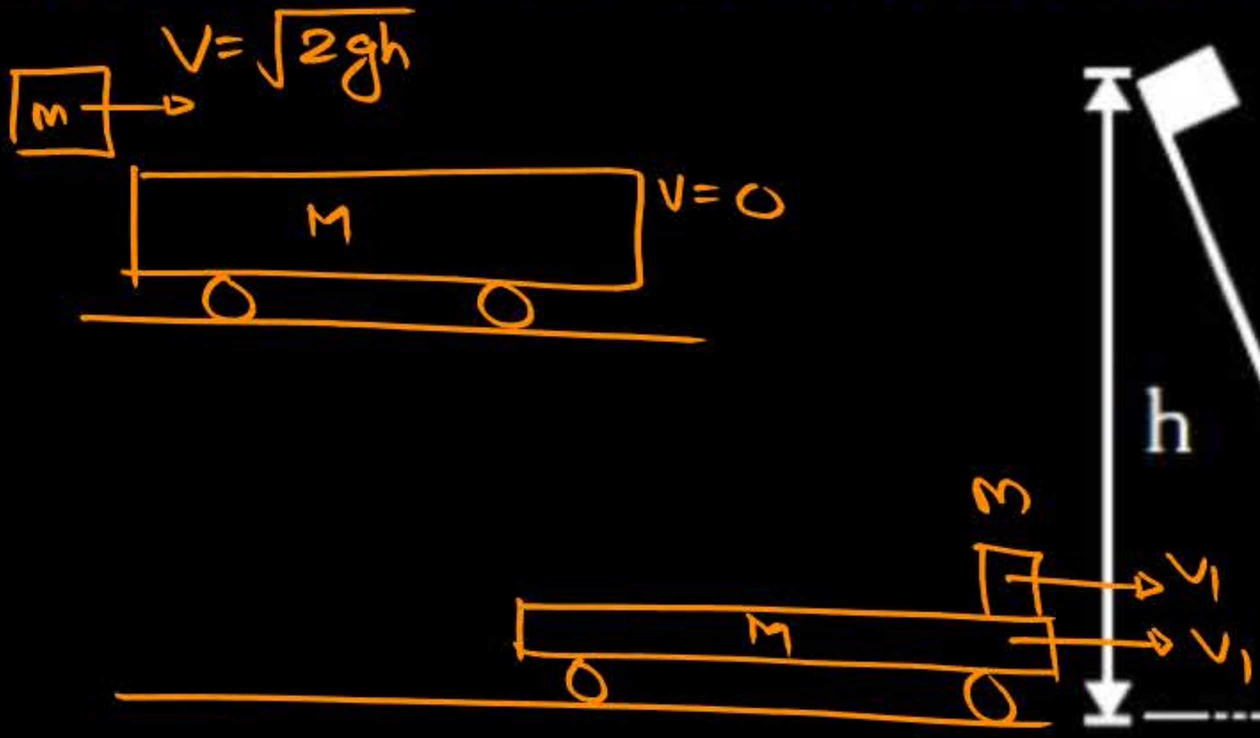
A carriage of mass M and length ℓ is joined to the end of a slope as shown in the figure. A block of mass m is released from the slope from height h . It slides till end of the carriage. The friction between the mass and the slope and also friction between carriage and horizontal floor is negligible. Coefficient of friction between block and carriage is μ. Find h in the given terms.

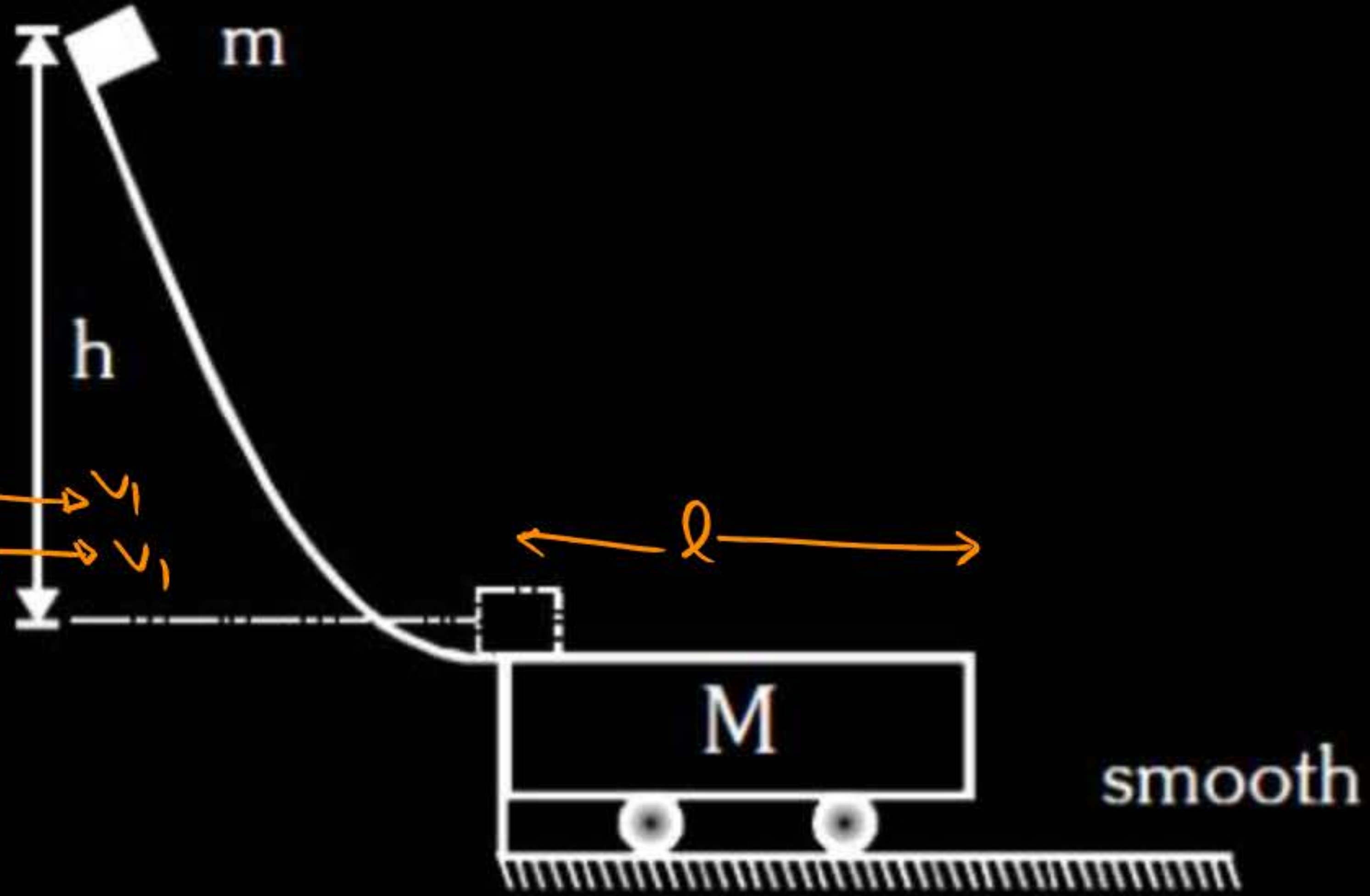
(A) $\mu\left(1+\frac{M}{m}\right)\ell$

(B) $2\mu\left(1+\frac{M}{m}\right)\ell$

(C) $\mu\left(2+\frac{m}{M}\right)\ell$

(D) $\mu\left(1+\frac{m}{M}\right)\ell$





$P_i = P_f = m\sqrt{2gh} = mv_1 + Mv_1$

Ans. D

friction → kinetic

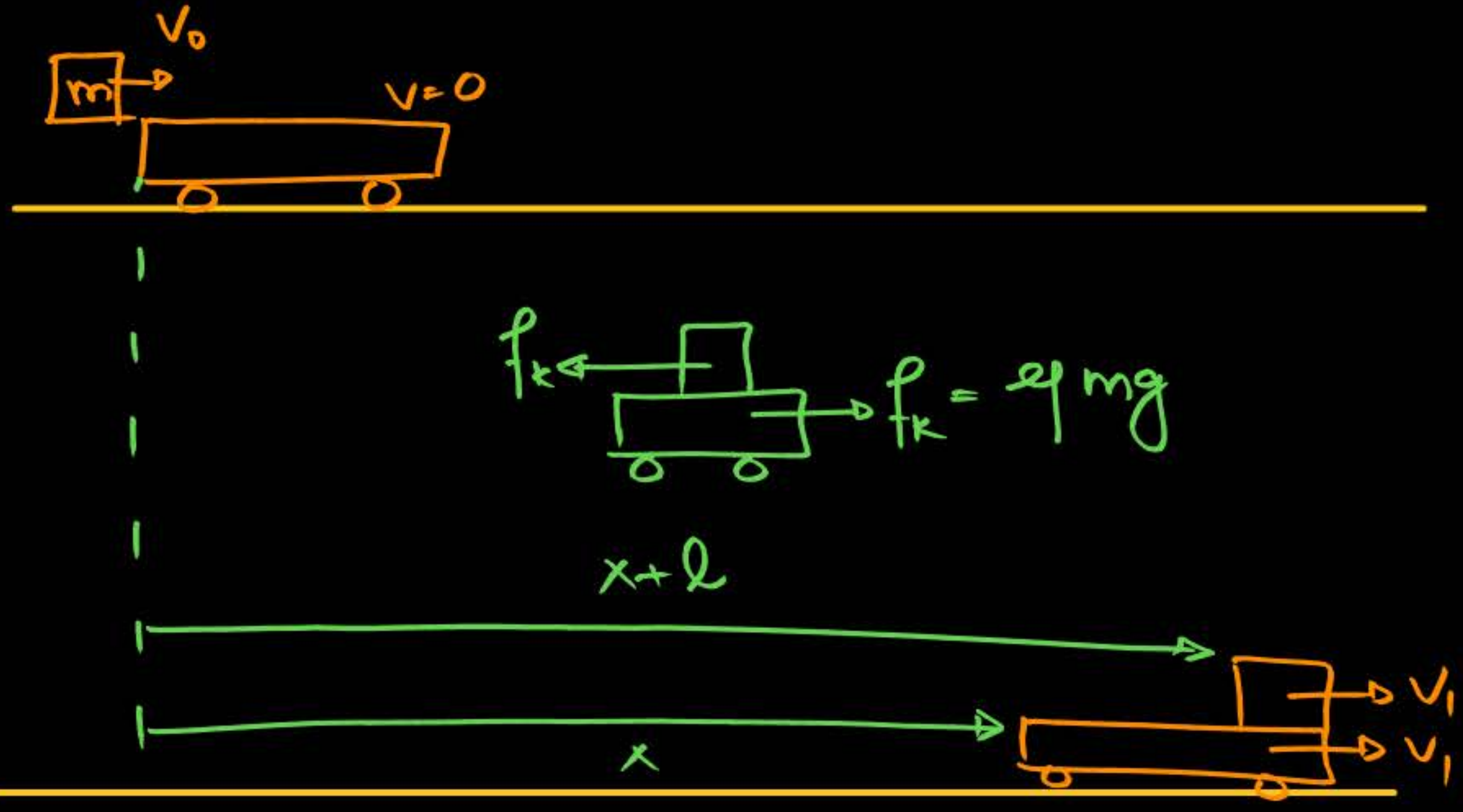
$W_{net} = \Delta K.E$ (w.r.t Ground)

$W_{fr}\big|_{block} + W_{fr}\big|_{cart} = \Delta K.E.$

$-\mu mg(x+l) + \mu mg x$

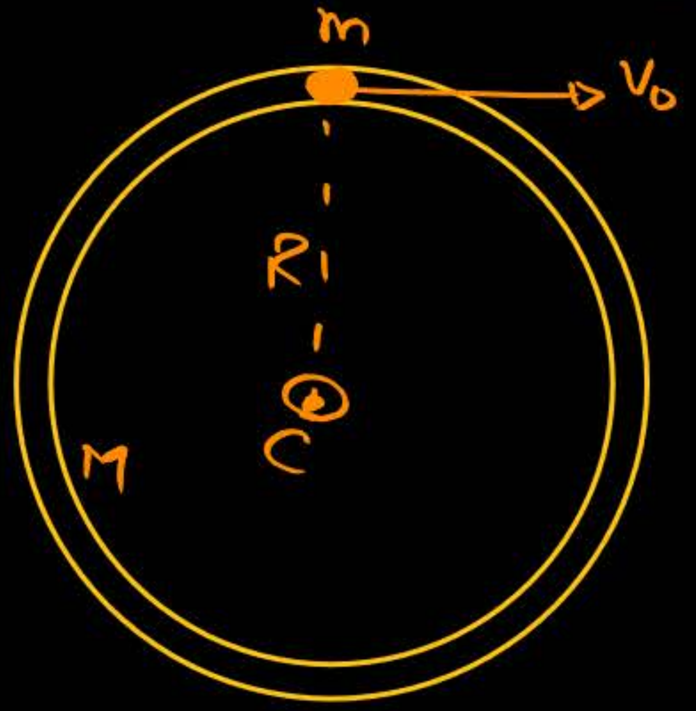
$= \frac{1}{2} M v_1^2 + \frac{1}{2} m v_1^2 - \frac{1}{2} m v_0^2$

$-\mu mg l = \frac{1}{2}(M+m)\left(\frac{m v_0}{m+M}\right)^2 - \frac{1}{2} m v_0^2$



Question

A small ball of mass m is placed in a circular tube of negligible width, mass M and radius R which is kept on a horizontal plane in gravity free space. Friction is absent between tube and ball. Ball is given a velocity v_0 as shown. Then,



(A) Path of ball from center of mass of system (ball + tube) will be circular.

(B) Path of ball from center of mass of system (ball + tube) will be elliptical

(C) Radius of curvature of ball at the time of projection of ball is $\frac{MR}{m+M}$

(D) Normal force between tube and ball if $M = 2m$, at the time of projection of ball is $\frac{2mV_0^2}{3R}$

Ans. AD

Question

Energy $\frac{1}{2}mv_0^2 = \frac{1}{2}m(\cancel{v_2 - v_1})^2 + \frac{1}{2}Mv_2^2 + mg(2R)$

$\Rightarrow \frac{1}{2}mv_0^2 - \frac{1}{2}M\left(\frac{mv_0}{M}\right)^2 = 2mgR \Rightarrow v_0^2\left(1-\frac{m}{M}\right) = 4gR$

$\Rightarrow 2gh\left(1-\frac{m}{M}\right) = 4gR$

Paragraph for Next 3 Questions

A small object of mass $m = 1$ kg is released from rest at the top of an inclined plane that connects to a horizontal plane without an edge. it slides onto a cart of mass M that has a semi-cylindrical surface of radius $R = 0.36$ m fixed to the middle of it, as shown in the figure.

The small object reaches the topmost point of the semi-cylinder and stops there. It continues to move vertically with free fall and hits the cart exactly at the edge. All friction can be neglected.

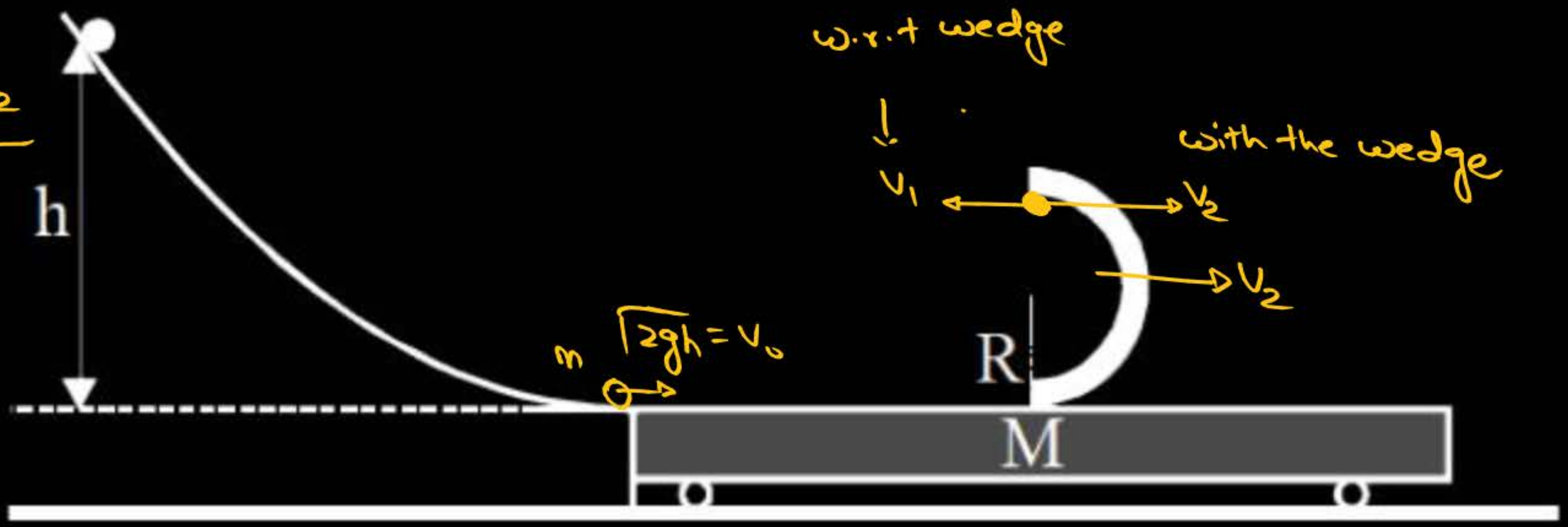
Given

$\rightarrow V_{ball/G} = 0 = v_1 - v_2 \Rightarrow v_1 = v_2$

$\rightarrow P_i = P_f$

$mv_0 = m(\cancel{v_2 - v_1}) + Mv_2$

$v_2 = \frac{m}{M}v_0$



Ans. 4

Question

$$\boxed{V_2=\frac{m}{M}\sqrt{2gh}\ ,\ V_1=V_2\ ,\ h\left(1-\frac{m}{M}\right)=2R}$$

What is the minimum possible length of the cart?

(A) R

(B) 2 R

(C) 3R

(D) 4 R

$$T=\sqrt{\frac{2(2R)}{g}}$$

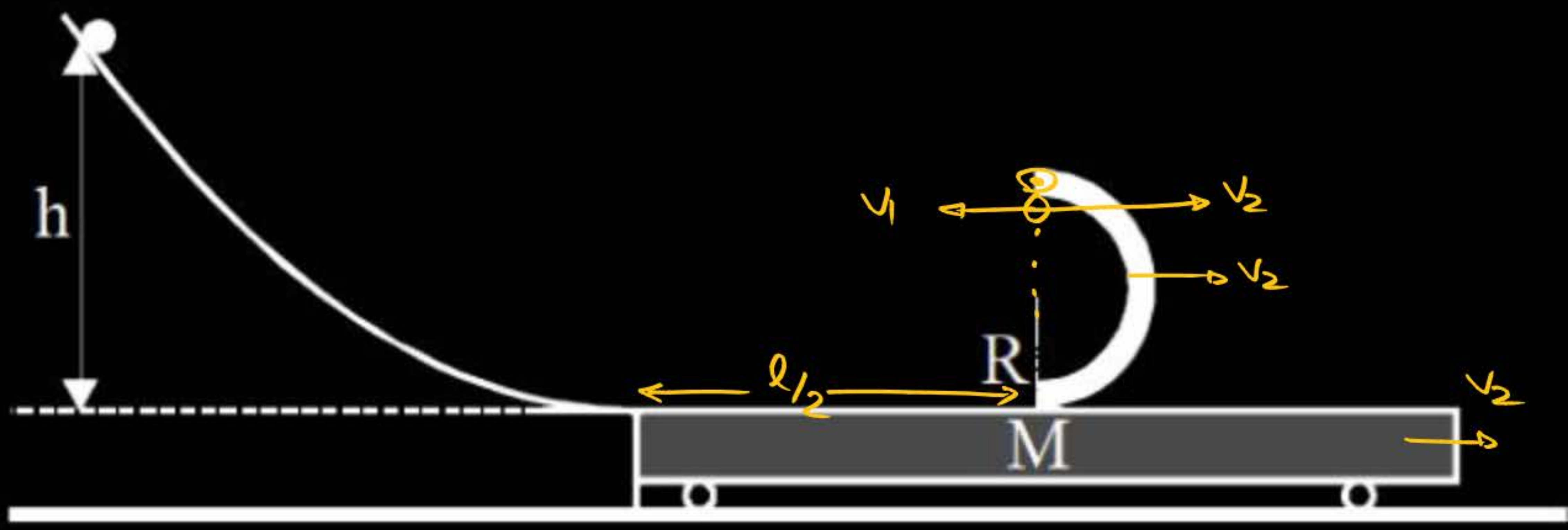
$$\frac{l}{2}=V_2T$$

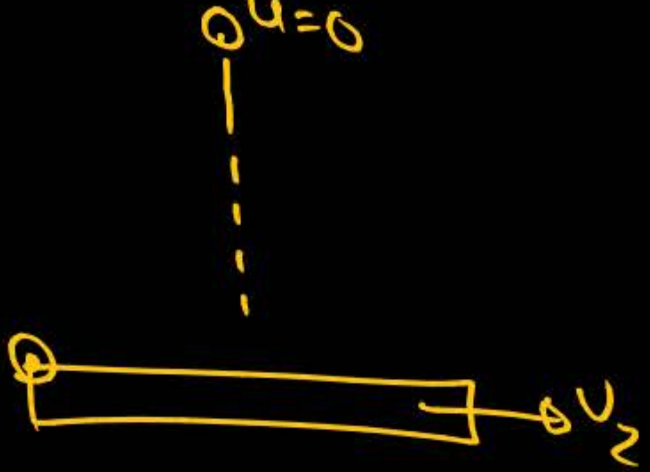
$$l=2V_2\sqrt{\frac{4R}{g}}$$

Now min^m Condition

$$V_1\geq\sqrt{gR}\Rightarrow V_2\geq\sqrt{gR}$$

$$l\geq 2\sqrt{\frac{4R}{g}}\cdot\sqrt{gR}\Rightarrow \underline{l\geq 4R}$$





Ans. D

Question

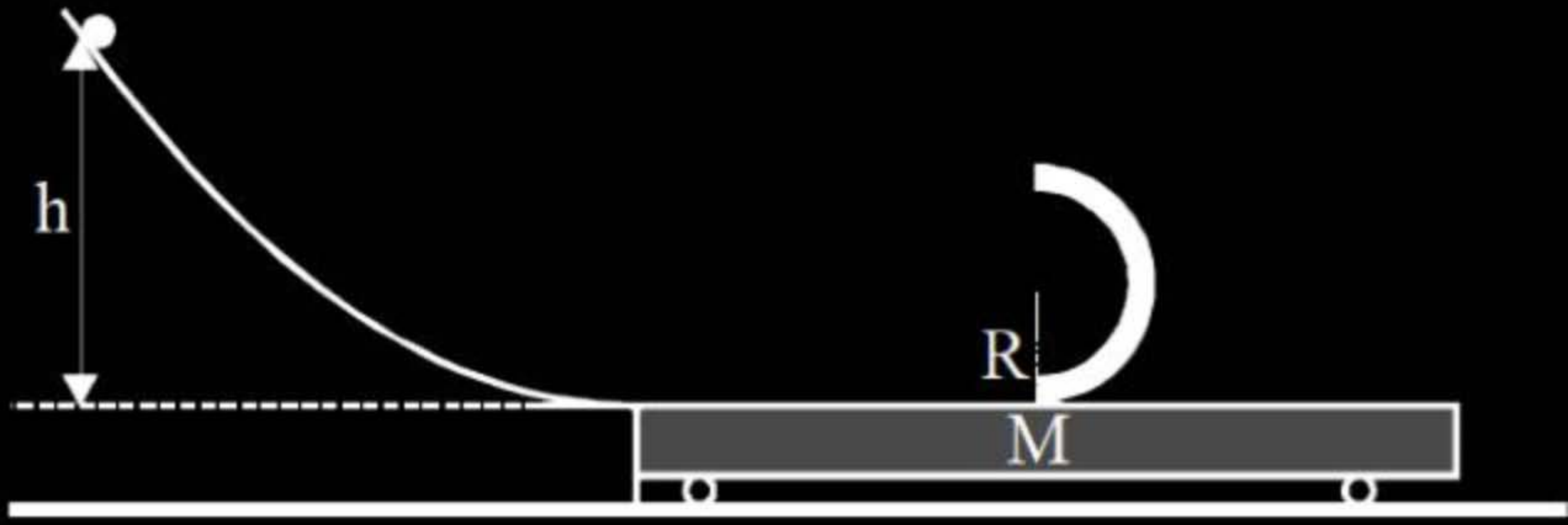
At what height h was the small object released ? (for Condition of question No. 1)

(A) $h = \frac{1}{2}\left(\frac{m}{M}\right)^2 R$

(B) $h = \frac{1}{2}\left(\frac{M}{m}\right)^2 R$ ✓

(C) $h = \frac{1}{2}\left(\frac{m}{M}\right) R$

(D) $h = \left(\frac{M}{m}\right)^2 R$



$$\boxed{\begin{aligned} v_1 = v_2 &= \frac{m}{M}\sqrt{2gh} \\ h\left(1-\frac{m}{M}\right) &= 2R \end{aligned}}$$

$v_1 = \sqrt{gR} \Rightarrow \sqrt{gR} = \frac{m}{M}\sqrt{2gh} \Rightarrow R = \frac{2m^2}{M^2} \cdot h$

$\rightarrow \frac{M^2}{2m^2}\cancel{R}\left(1-\frac{m}{M}\right) = 2\cancel{R}$

Ans. B

Question

What is the ratio of mass of cart and mass of the particle ? (for Condition of 9.N. 1)

(A) $k = \frac{1+\sqrt{17}}{2}$

(B) $k = \frac{1+\sqrt{14}}{2}$

(C) $k = \frac{1+\sqrt{16}}{2}$

(D) $k = \frac{1+\sqrt{18}}{2}$

Ans. A

Question

$\vec{P}_i = \vec{P}_f \Rightarrow 2mv_0 = 3mv_2$

$v_2 = \frac{2v_0}{3}$

$\left(\frac{1}{2}mv_0^2\right)2 = \frac{1}{2}3m(v_2)^2 + \frac{1}{2}mv_1^2 \times 2$

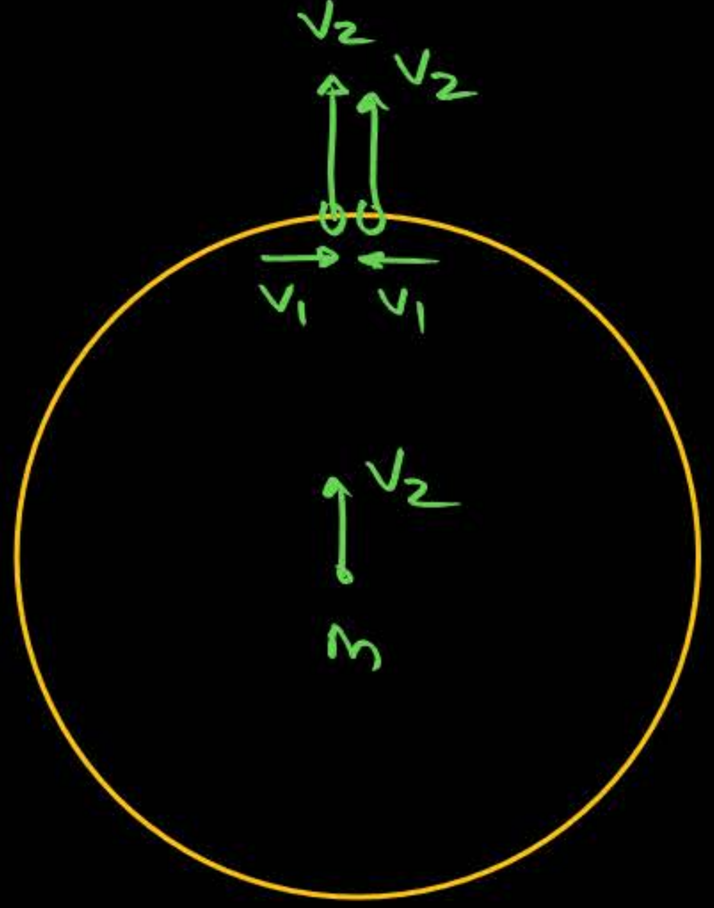
Two paticles of mass m , constrained to move along the circumference of a smooth circular hoop of equal mass m , are initially located at opposite ends of a diameter and given equal velocities v_0 shown in the figure. The entire arrangement is located in gravity free space. Their relative velocity just before collision is:

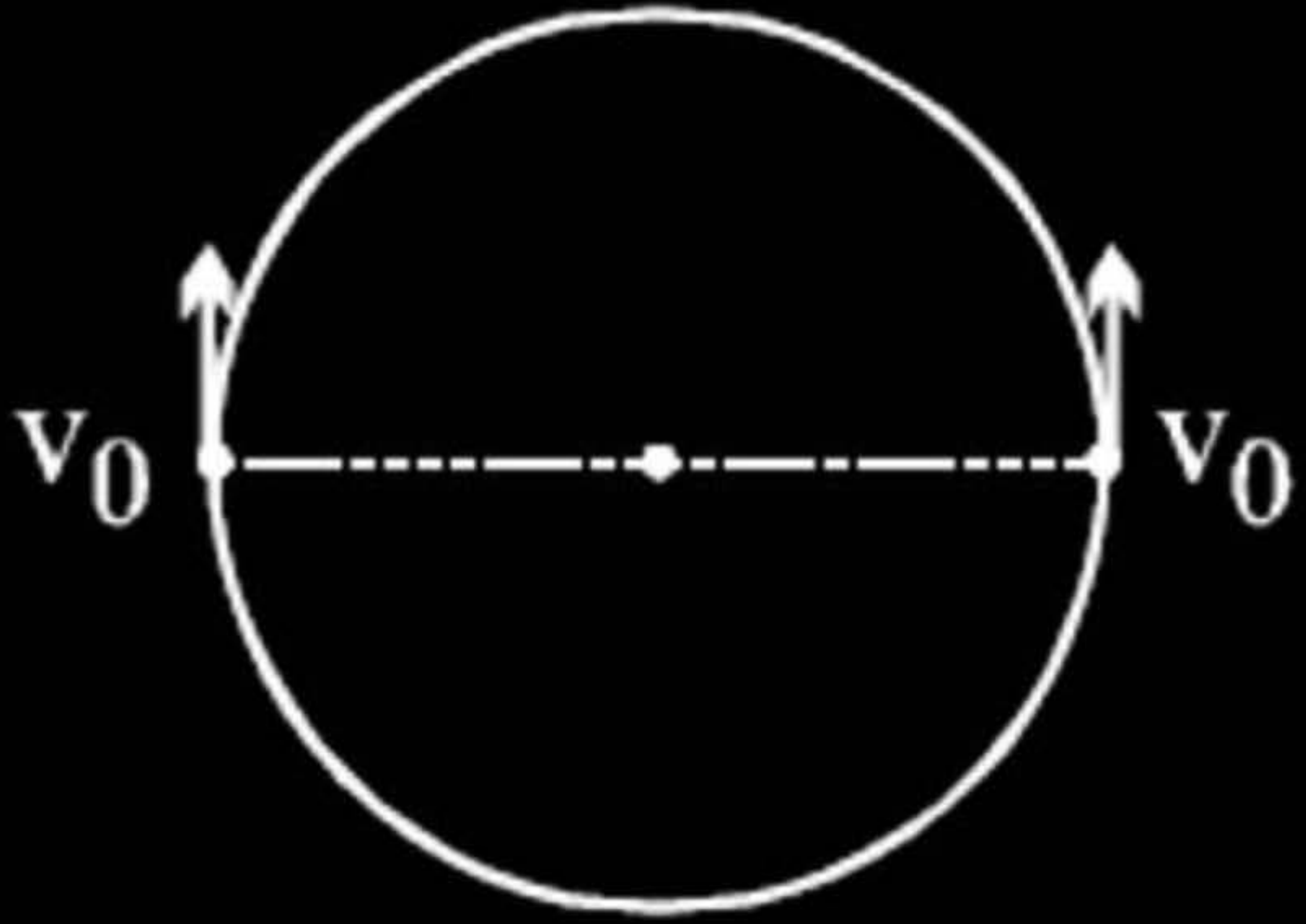
(A) $\frac{1}{\sqrt{3}}v_0$

(B) $\frac{\sqrt{3}}{2}v_0$

(C) $\frac{2}{\sqrt{3}}v_0$

(D) none of these



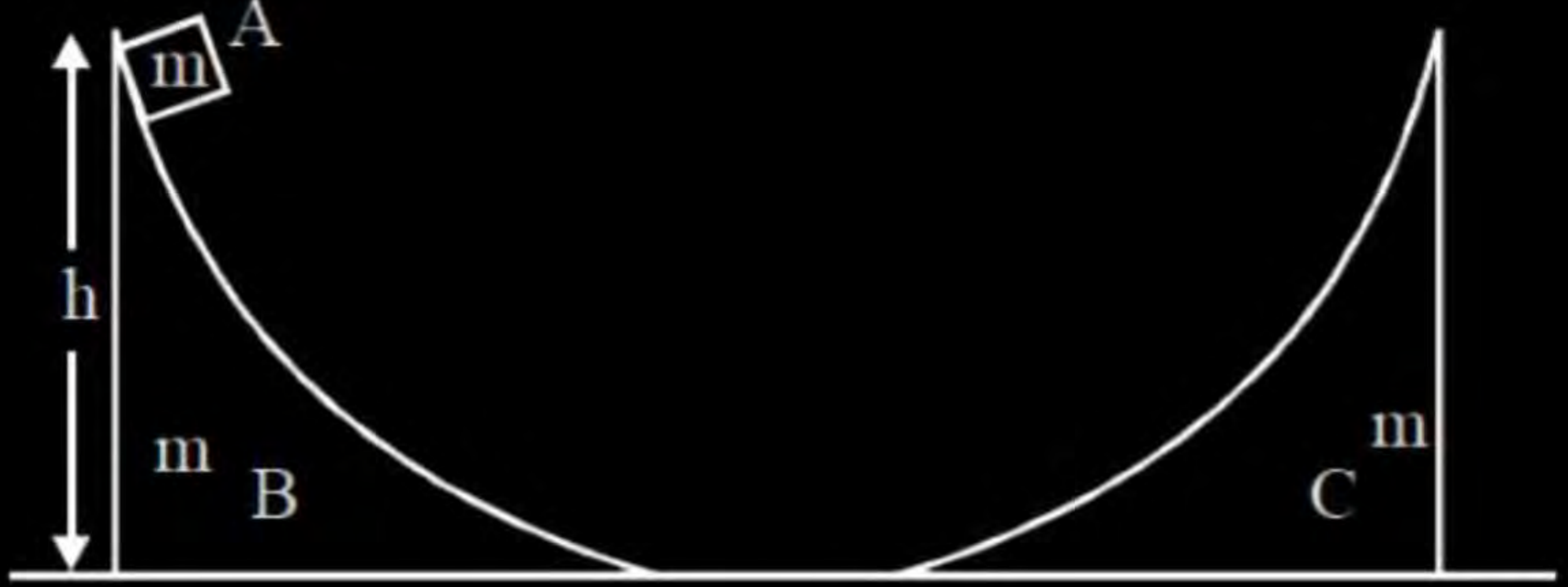


Ans. C

Question

Paragraph for next 3 Questions

Block A is placed on wedge B at a height h above ground. Block and the two wedges are all of same mass m . Neglect friction every where :



Question

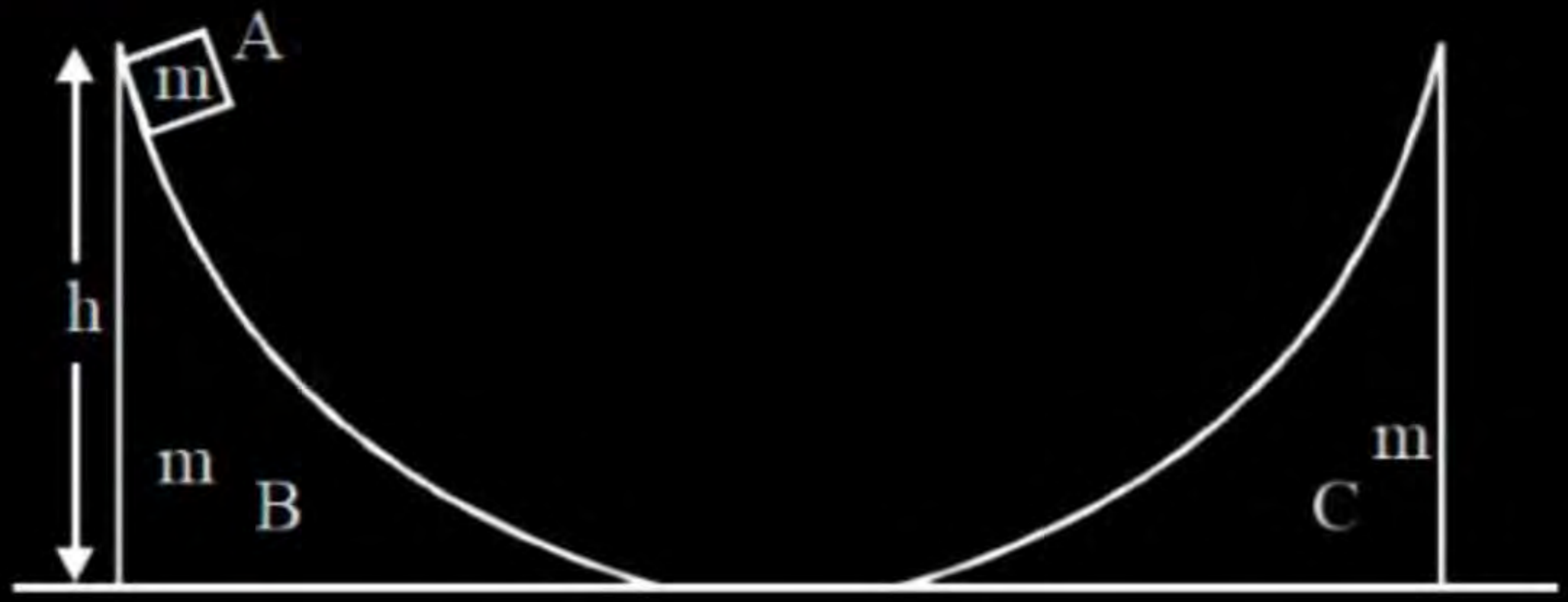
Find velocity of B when A has slid down from it :

(A) $\sqrt{gh}$

(B) $\sqrt{\frac{gh}{2}}$

(C) $\frac{\sqrt{gh}}{2}$

(D) None of these



Ans. A

Question

Find maximum height upto which block A rises on wedge C :

(A) h

(B) $\frac{h}{2}$

(C) $\frac{h}{4}$

(D) None of these

Ans. C

Question

Find velocity of A when it has slid down to ground from wedge C :

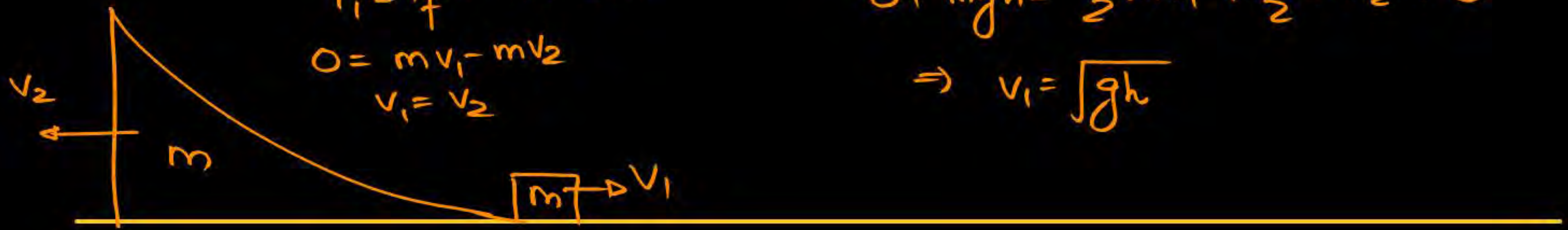
(A) 0

(B) $\sqrt{\frac{gh}{2}}$

(C) $\sqrt{\frac{gh}{4}}$

(D) None of these

Ans. A



$P_i = P_f$ - in x-dir^n

$0 = mv_1 - mv_2$

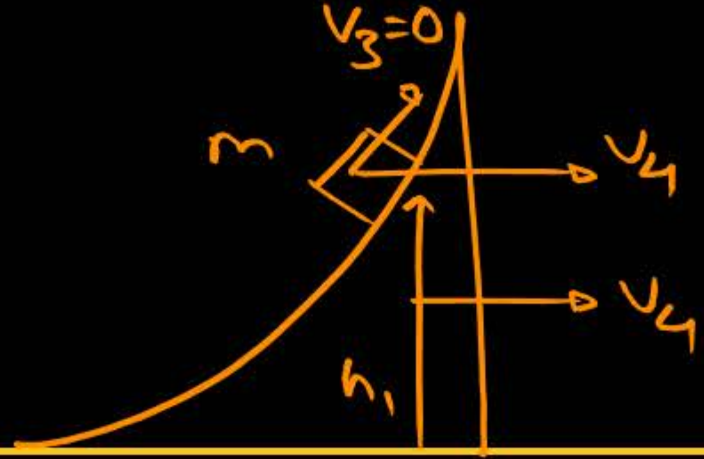
$v_1 = v_2$

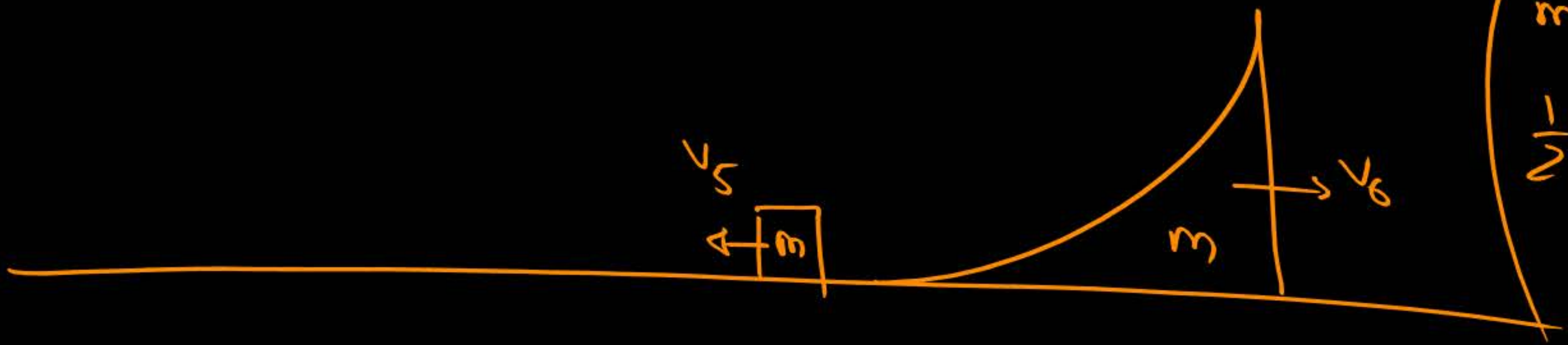
$0 + mgh = \frac{1}{2} m v_1^2 + \frac{1}{2} m v_2^2 + 0$

$\Rightarrow \quad v_1 = \sqrt{gh}$

$$m\sqrt{gh} = (m+m)v_4 \Rightarrow v_4 = \frac{\sqrt{gh}}{2}$$

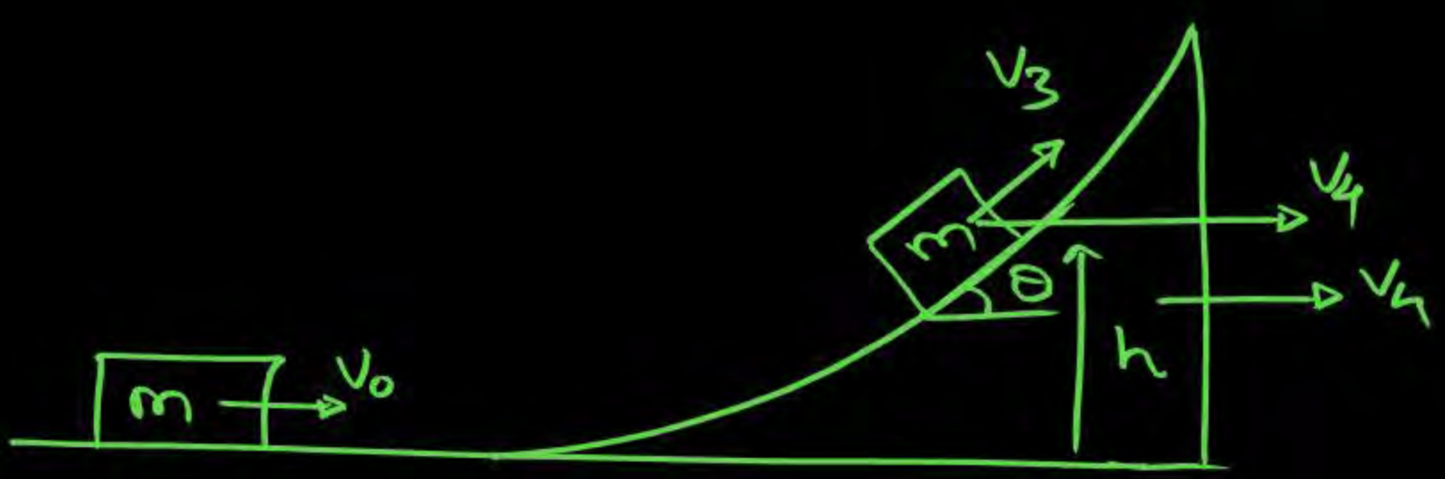
$$\frac{1}{2}m(gh) = \frac{1}{2}2m v_4^2 + mgh_1$$





$$m v_6 - m v_5 = m\sqrt{gh}$$

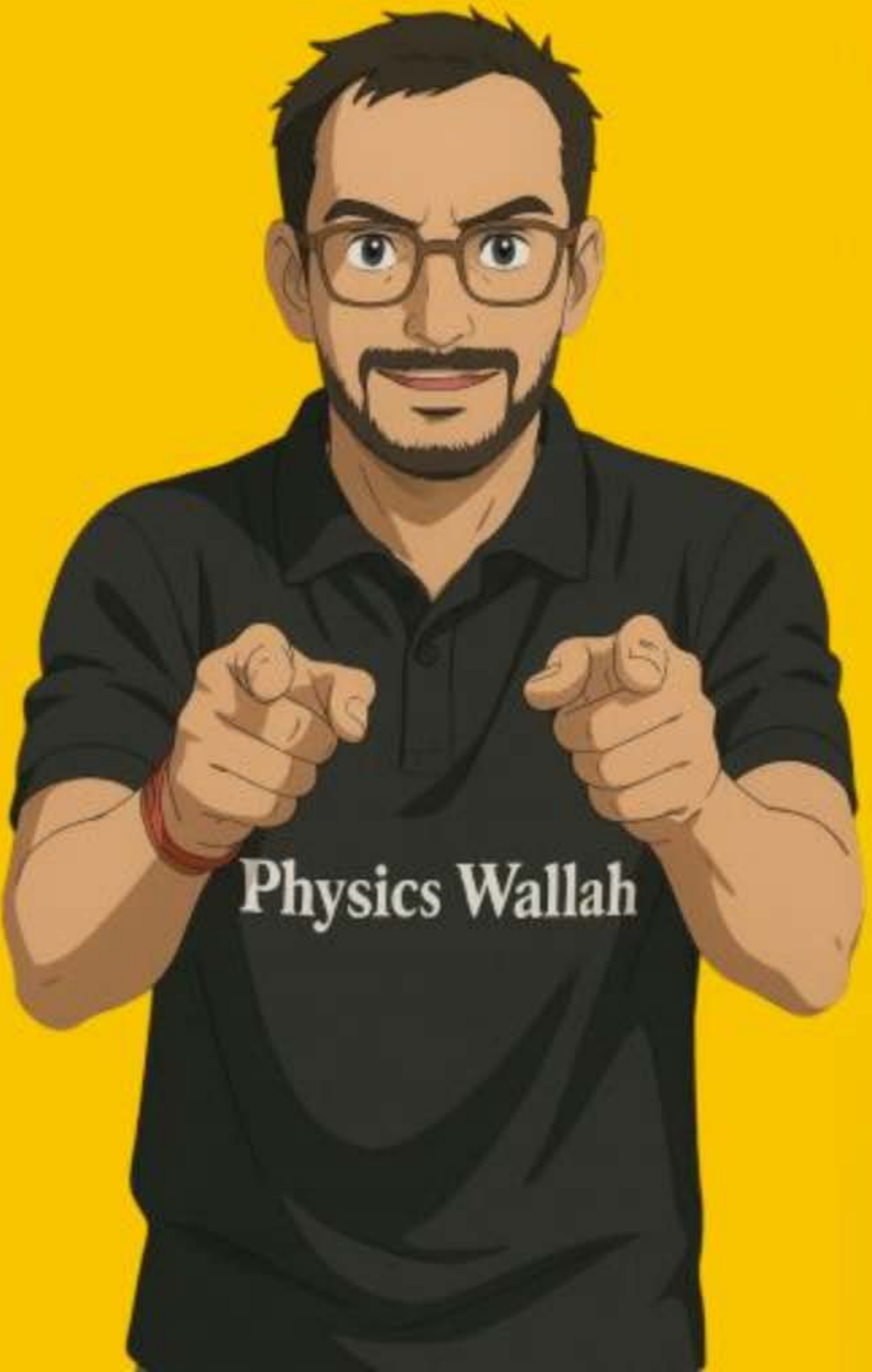
$$\frac{1}{2}m(\sqrt{gh})^2 = \frac{1}{2}m v_5^2 + \frac{1}{2}m v_6^2$$



$$mv_0 = m(v_4 + v_3\cos\theta) + mv_4$$

$$\frac{1}{2}mv_0^2 = \frac{1}{2}m\left((v_4 + v_3\cos\theta)^2 + (v_3\sin\theta)^2\right)$$

$$+\frac{1}{2}mv_4^2$$



THANK YOU BAWAL BACCHA PARTY