ULTIMATE KCET

CRASH COURSE 2026

Mathematics | Lecture – 01

Integrals

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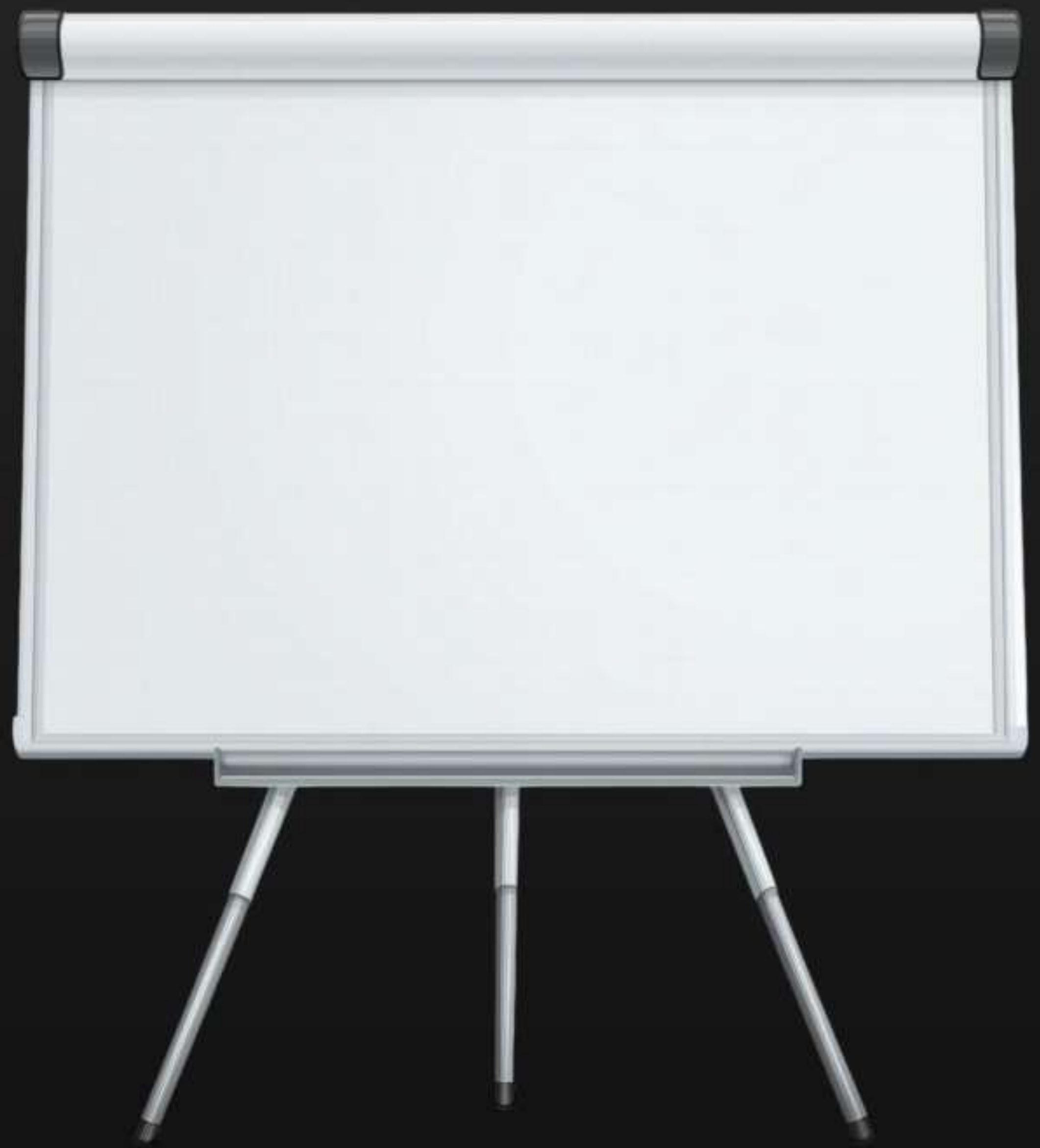
Recap *of previous lecture*

1. Probability
2.
3.
4.



Topics *to be covered*

1. Integrals — (1) Integration by Parts

   (2) Integration of standard func
2.
3.
4.

(A) 20-30

(B) 30-40

(C) 40-50

(D) 50-60

Integration by Parts:-

$$\int uv\,dx = u\int v\,dx - \int\left[\frac{d}{dx}u\int v\,dx\right]dx$$

Ex:

$$\int x\sin x\,dx = x(-\cos x) - \int (1)(-\cos x)\,dx$$

$$= -x\cos x + \sin x + C$$

⊛ Generalized form of Integration by Parts:-

⇓

This rule is helpful when the 1st func 'u' is a polynomial func

$$\int u\,v\,dx = +u\int v\,dx - u'\iint v\,dx\cdot dx + u''\iiint v\,dx\cdot dx\cdot dx - u'''\iiiint v\,dx\cdot dx\cdot dx\cdot dx$$

$+ \dots$ until the last term is zero

$$\int \underset{u}{x^3}\,\underset{v}{\sin x}\,dx = x^3(-\cos x) - (3x^2)(-\sin x) + 6x(+\cos x) - 6(\sin x) + 0$$

$$= -x^3\cos x + 3x^2\sin x + 6x\cos x - 6\sin x + c$$

$$\int x^3 e^{2x}\,dx = \frac{x^3 e^{2x}}{2} - \frac{3x^2 e^{2x}}{4} + \frac{6x e^{2x}}{8} - \frac{6e^{2x}}{16} + c$$

$$= e^{2x}\left[\frac{x^3}{2} - \frac{3}{4}x^2 + \frac{3}{4}x - \frac{3}{8}\right] + c$$

$$\int x^2 \cos 3x\,dx = \frac{x^2 \sin 3x}{3} - \frac{2x(-\cos 3x)}{9} + \frac{2(-\sin 3x)}{27} + c$$

$$= \frac{x^2 \sin 3x}{3} + \frac{2x \cos 3x}{9} - \frac{2\sin 3x}{27} + c$$

QUESTION

#Q. $\int 32x^3(\log x)^2 dx$ is equal to

(A) $8x^4(\log x)^2 + c$

(B) $x^4\{8(\log x)^2 - 4\log x + 1\} + c$

(C) $x^4\{8(\log x)^2 - 4\log x\} + c$

(D) $x^3\{(\log x)^2 + 2\log x\} + c$

Put $\log x = t$

$x = e^t \Rightarrow x^3 = e^{3t}$

$dx = e^t dt$

$\log_e x = t$

$x = e^t$

$\Downarrow$

$x^3 = (e^t)^3 = e^{3t}$

$x^4 = e^{4t}$

$I = 32\int e^{3t} t^2 (e^t dt)$

$= 32\int t^2 e^{4t} dt$

$= 32\left[t^2 \frac{e^{4t}}{4} - \frac{\cancel{2}t\, e^{4t}}{\cancel{16}\,8} + \frac{2e^{4t}}{64}\right] + c$

$= e^{4t}[8t^2 - 4t + 1] + c$

$= x^4[8(\log x)^2 - 4(\log x) + 1] + c$

QUESTION

$\int \sec^2\theta \, d\theta = \tan\theta + c$ | $\int \tan\theta \, d\theta = \log|\sec\theta| + c = -\log|\cos\theta| + c$

#Q. $\int x \sec^2 2x \, dx$ is equal to

$= x\frac{\tan 2x}{2} - \frac{\log(\sec 2x)}{4} + c$

$= \frac{x}{2}\tan 2x + \frac{\log \cos 2x}{4} + c$

$\log\frac{A}{B} = -\log\frac{B}{A}$

$\log \sec x = -\log \cos x$

A. $\frac{x}{2}\tan 2x + \frac{1}{4}\ln \sin 2x + c$

B. ✓ $\frac{x}{2}\tan 2x + \frac{1}{4}\ln \cos 2x + c$

C. $\frac{x}{2}\sec 2x + \frac{1}{4}\ln \sin 2x + c$

D. $\frac{x}{2}\sec 2x + \frac{1}{4}\ln \cos 2x + c$

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Type 2 of Integration by Parts

$$\int e^x [f(x) + f'(x)]\, dx$$

$$= e^x \underline{f(x)} + C$$

QUESTION

#Q. $\int \frac{\cos x-1}{\sin x+1} e^x dx$ is equal to

A $\frac{e^x \cos x}{1+\sin x}+c$

B $-\frac{e^x \sin x}{1+\sin x}+c$

C $\frac{e^x}{x+4}+c$

D $-\frac{e^x \cos x}{1+\sin x}+c$

$$\int e^x\left[\underbrace{\frac{\cos x}{1+\sin x}}_{f(x)}+\underbrace{\frac{-1}{1+\sin x}}_{f'(x)}\right]dx = e^x f(x)+c = \frac{e^x \cos x}{1+\sin x}+c$$

$$f(x)=\frac{\cos x}{1+\sin x}$$

$$f'(x)=\frac{(1+\sin x)(-\sin x)-\cos x(\cos x)}{(1+\sin x)^2}$$

$$=\frac{-\sin x-\sin^2 x-\cos^2 x}{(1+\sin x)^2}=\frac{-(\sin x+1)}{(1+\sin x)^2}=\frac{-1}{1+\sin x}$$

(*) If $\int \frac{\log x}{x^3} dx = \frac{1}{x^2}(a + b\log x) + c$ Then $a+b =$

(A) $-\frac{1}{4}$

(B) $-\frac{1}{2}$

(C) $\frac{-3}{4}$

(D) -1

$\Downarrow$

Put $\log x = t$

$x = e^t$

$dx = e^t dt$

$I = \int t e^{-3t} e^t dt$

$= \int t e^{-2t} dt$

$= \frac{t e^{-2t}}{-2} - (1)\frac{e^{-2t}}{4} + c$

$I = e^{-2t}\left[-\frac{1}{2}t - \frac{1}{4}\right] + c$

$= \frac{1}{x^2}\left[\underbrace{-\frac{1}{2}}_{b}\log x - \underbrace{\frac{1}{4}}_{a}\right] + c$

$a + b = -\frac{1}{4} - \frac{1}{2}$

$= -\frac{3}{4}$

QUESTION

#Q. Find $\int x \sin^2 x\, dx$. $= \int x\left(\frac{1-\cos 2x}{2}\right)dx = \frac{1}{2}\int x - x\cos 2x\, dx$

$= \frac{1}{2}\left[\frac{x^2}{2} - \left[x\frac{\sin 2x}{2} - (1)\left(-\frac{\cos 2x}{4}\right)\right]\right] + c$

$= \frac{x^2}{4} - \frac{x\sin 2x}{4} - \frac{\cos 2x}{8} + c$

A) $\frac{x^2}{4} + \frac{x\sin 2x}{4} + \frac{\cos 2x}{8} + c$

B) $\frac{x^2}{4} + \frac{x\sin 2x}{4} - \frac{\cos 2x}{8} + c$

C) $\frac{x^2}{4} - \frac{x\sin 2x}{4} + \frac{\cos 2x}{8} + c$

D) $\frac{x^2}{4} - \frac{x\sin 2x}{4} - \frac{\cos 2x}{8} + c$

$$\int \sec^2\theta\, d\theta = \tan\theta + C$$

$$\int \operatorname{cosec}^2\theta\, d\theta = -\cot\theta + C$$

$$\int \sin^2\theta\, d\theta = \int \frac{1-\cos 2\theta}{2}\, d\theta = \frac{1}{2}\left[\theta - \frac{\sin 2\theta}{2}\right] + C$$

$$\int \cos^2\theta\, d\theta = \int \frac{1+\cos 2\theta}{2} = \frac{1}{2}\left[\theta + \frac{\sin 2\theta}{2}\right] + C$$

$$\int \tan^2\theta\, d\theta = \int \sec^2\theta - 1\, d\theta = \tan\theta - \theta + C$$

$$\int \cot^2\theta\, d\theta = \int \operatorname{cosec}^2\theta - 1\, d\theta = -\cot\theta - \theta + C$$

QUESTION

#Q. $\int x^2 e^{3x} dx$ $= x^2 \frac{e^{3x}}{3} - 2x \frac{e^{3x}}{9} + 2\frac{e^{3x}}{27} + c = \frac{\quad}{27}$

(A) $\frac{e^{3x}}{9}(9x^2 + 6x + 2) + c$ $= \frac{e^{3x}}{27}\left[9x^2 - 6x + 2\right] + c$

(B) $\frac{e^{3x}}{9}(9x^2 - 6x + 2) + c$

(C) $\frac{e^{3x}}{27}(9x^2 + 6x + 2) + c$

(D) $\frac{e^{3x}}{27}(9x^2 - 6x + 2) + c$

QUESTION

$\sin A \sin B = -\frac{1}{2}[\cos(A+B) - \cos(A-B)]$

#Q. $\int x \sin x \sin 2x \, dx$ is equal to $= -\frac{1}{2}\int x(\cos 3x - \cos x)\,dx$

A $\frac{1}{2}\left[x\sin x + \cos x - \frac{x\sin 3x}{3} - \frac{\cos 3x}{9}\right] + c$ $= -\frac{1}{2}\left[x\left(\frac{\sin 3x}{3} - \sin x\right) - 1\left(-\frac{\cos 3x}{9} + \cos x\right) + c\right]$

B $x\sin x + \cos x - \frac{x\sin 3x}{3} - \frac{\cos 3x}{9} + c$ $= \frac{1}{2}\left[-\frac{x\sin 3x}{3} + x\sin x - \frac{\cos 3x}{9} + \cos x\right) + c$

C $\frac{1}{2}\left[\cos x - \frac{\cos 3x}{9}\right] + c$

D $\frac{1}{2}\left[x\sin x - \frac{x\sin 3x}{3}\right] + c$

$$\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}\left[a\cos bx + b\sin bx\right] + c$$

$$\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}\left[a\sin bx - b\cos bx\right] + c$$

QUESTION

#Q. $\int e^{ax} \sin bx \, dx$ is equal to

(A) $\dfrac{e^{ax}}{a^2+b^2}(a \sin bx - b \cos bx) + c$

(B) $e^{ax}(b\sin bx - a \cos bx) + c$

(C) $\dfrac{e^{ax}}{a^2+b^2}(a \sin bx + b \cos bx) + c$

(D) $\dfrac{a^2+b^2}{e^{ax}}(b \cos bx - a \sin bx) + c$

QUESTION

#Q. $\int \cos(\log_e x)dx$ is equal to

(A) $\frac{1}{2}x[\cos(\log_e x) + \sin(\log_e x)]$

(B) $x[\cos(\log_e x) + \sin(\log_e x)]$

(C) $\frac{1}{2}x[\cos(\log_e x) - \sin(\log_e x)]$

(D) $x[\cos(\log_e x) - \sin(\log_e x)]$

Put $\log x = t$

$x = e^t$

$dx = e^t dt$

$\int e^{at} \cos bt \, dt$

$= \frac{e^{at}}{a^2+b^2}[a\cos bt + b\sin bt] + c$

$I = \int e^t \cos t \, dt$

$= \frac{e^t}{1+1}[\cos t + \sin t] + c$

$= \frac{x}{2}[\cos(\log x) + \sin(\log x)] + c$

QUESTION

#Q. Find $\int \tan^{-1}\sqrt{\frac{1-x}{1+x}}\,dx$.

(A) $\frac{x}{2}\cos^{-1}x-\frac{1}{2}\sqrt{1-x^2}+c$

(B) $\frac{-x}{2}\cos^{-1}x+\frac{1}{2}\sqrt{1+x^2}+c$

(C) $\frac{-x}{2}\sin^{-1}x+\frac{1}{2}\sqrt{1-x^2}+c$

(D) $x\sin^{-1}x+\frac{1}{2}\sqrt{1+x^2}+c$

$\sin\theta=\sqrt{1-x^2}$

Put $x=\cos\theta$

$dx=-\sin\theta\,d\theta$

$\sqrt{\frac{1-x}{1+x}}=\tan\frac{\theta}{2}$

$$I=\int\tan^{-1}\left(\tan\frac{\theta}{2}\right)(-\sin\theta)\,d\theta$$

$$=-\int\frac{\theta}{2}\sin\theta\,d\theta$$

$$=-\frac{1}{2}\left[\theta(-\cos\theta)-(1)(-\sin\theta)\right]+c$$

$$=\frac{\theta}{2}\cos\theta-\frac{1}{2}\sin\theta+c$$

$$=\frac{x}{2}\cos^{-1}x-\frac{1}{2}\sqrt{1-x^2}+c$$

QUESTION

#Q. $\int e^x\left(\frac{1-\sin x}{1-\cos x}\right)dx$ is equal to

(A) $-e^x\tan\left(\frac{x}{2}\right)+c$

(B) $-e^x\cot\left(\frac{x}{2}\right)+c$

(C) $-\frac{1}{2}e^x\tan\left(\frac{x}{2}\right)+c$

(D) $\frac{1}{2}e^x\cot\left(\frac{x}{2}\right)+c$

$$\int e^x\left[\frac{1}{1-\cos x}+\frac{-\sin x}{1-\cos x}\right]dx$$

$$\int e^x\left[\frac{1}{2\sin^2 x/2}+\frac{-2\sin\frac{x}{2}\cos\frac{x}{2}}{2\sin^2 x/2}\right]dx$$

$$I=\int e^x\left[\frac{1}{2}\operatorname{cosec}^2\frac{x}{2}-\cot\frac{x}{2}\right]dx$$

$$=-\int e^x\Big[\underbrace{\cot\frac{x}{2}}_{f(x)}+\underbrace{\left(-\frac{1}{2}\operatorname{cosec}^2\frac{x}{2}\right)}_{f'(x)}\Big]dx=-e^x\cot\frac{x}{2}+c$$

QUESTION

#Q. Find $\int e^{\cos x}(\sin x\cos x+\sin x)dx$

$\int e^{\cos x}(\cos x+1)\sin x\,dx$

(A) $e^{\cos x}\cos x+c$

Put $\cos x=t$

$\sin x\,dx=-dt$

(B) $-e^{\cos x}\cos x+c$

$-\int e^{t}[t+1]dt$

$\quad\; f(t) \quad f'(t)$

(C) $e^{\cos x}(1+\cos x)+c$

(D) $-e^{\cos x}(1+\cos x)+c$

$=-e^{t}f(t)+c=-e^{t}(t)+c=-e^{\cos x}\cos x+c$

QUESTION

#Q. $\int \frac{\log(x/e)}{(\log x)^2}\,dx$ is equal to

A $\frac{\log x}{x} + c$

B $\frac{x}{\log x} + c$

C $\frac{x}{(\log x)^2} + c$

D None of these

$\int \frac{\log x - \log e}{(\log x)^2}\,dx$

$\int \frac{\log x}{(\log x)^2} - \frac{1}{(\log x)^2}\,dx$

$\int \frac{1}{\log x} - \frac{1}{(\log x)^2}\,dx$

Put $\log x = t$

$x = e^t$

$dx = e^t\,dt$

$I = \int \left(\frac{1}{t} - \frac{1}{t^2}\right) e^t\,dt$

$= \int e^t \left[\underbrace{\frac{1}{t}}_{f(t)} + \underbrace{\left(\frac{-1}{t^2}\right)}_{f'(t)}\right] dt$

$= e^t\left(\frac{1}{t}\right) + c$

$= \frac{x}{\log x} + c$

QUESTION

$$f(x)=\frac{1}{x} \qquad f'(x)=\frac{-1}{x^2}$$

$$f(x)=\frac{1}{(x+1)^2} \qquad f'(x)=-\frac{1}{(x+1)^4}2(x+1)=\frac{-2}{(x+1)^3}$$

PYQ

#Q. $\int \frac{(x-1)e^x}{(x+1)^3}dx$ is equal to

(A) $\frac{e^x}{x+1}+c$

(B) $e^x\left(\frac{x}{x+1}\right)+c$

(C) $\frac{e^x(x-1)}{(x+1)^2}+c$

(D) $\frac{e^x}{(x+1)^2}+c$ ✓

$$\int e^x\left[\frac{x+1}{(x+1)^3}-\frac{2}{(x+1)^3}\right]dx$$

$$\int e^x\left[\underbrace{\frac{1}{(x+1)^2}}_{f(x)}+\underbrace{\frac{-2}{(x+1)^3}}_{f'(x)}\right]dx=\frac{e^x}{(x+1)^2}+c$$

$$f(x)=\frac{1}{(x+1)^2}$$

$$f'(x)=-\frac{1}{(x+1)^4}2(x+1)=\frac{-2}{(x+1)^3}$$

$$\frac{x-1}{(x+1)^3}$$

$$=\frac{x-1+1-1}{(x+1)^3}$$

$$=\frac{(x+1)-2}{(x+1)^3}$$

QUESTION

$\frac{(x-1)^2}{(x^2+1)^2}$

#Q. Find $\int e^x\left(\frac{1-x}{x^2+1}\right)^2 dx$ $= \int e^x\left[\frac{x^2+1-2x}{(x^2+1)^2}\right]dx$

(A) $\frac{e^x}{1+x^2}+c$

$= \int e^x\left[\frac{x^2+1}{(x^2+1)^2} + \frac{-2x}{(x^2+1)^2}\right]dx$

(B) $-\frac{e^x}{1+x^2}+c$

$= \int e^x\left[\frac{1}{(x^2+1)} + \frac{-2x}{(x^2+1)^2}\right]dx$

$f(x)$ $\quad$ $f'(x)$

(C) $\frac{e^x}{(1+x^2)^2}+c$

(D) $-\frac{e^x}{(1+x^2)^2}+c$

$= \frac{e^x}{x^2+1}+c$

QUESTION

#Q. $\int \frac{e^x}{x}(x(\log x)^2 + 2\log x)dx$ is equal to

$\int e^x \left[(\log x)^2 + \frac{2}{x}\log x\right] dx$

$(\log x)^2 \rightarrow f(x)$, $\frac{2}{x}\log x \rightarrow f'(x)$

$= e^x (\log x)^2 + c$

A) $2e^x \log x + c$

B) $-2e^x \log x + c$

C) $e^x(\log x)^2 + c$

D) $-e^x(\log x)^2 + c$

QUESTION

#Q. $\int e^x\left(\frac{2}{x}-\frac{2}{x^2}\right)dx$ is equal to

$$2\int e^x\left(\underbrace{\frac{1}{x}}_{f(x)}+\underbrace{\left(-\frac{1}{x^2}\right)}_{f'(x)}\right)dx$$

(A) $\frac{e^x}{x}+c$

(B) $\frac{e^x}{2x^2}+c$

(C) $\frac{2e^x}{x}+c$

(D) $\frac{2e^x}{x^2}+c$

$$=\frac{2e^x}{x}+c$$

QUESTION

#Q. $\int \frac{(x+3)e^x}{(x+4)^2}\,dx$ is equal to

(A) $\frac{1}{(x+4)^2}+c$

(B) $\frac{e^x}{(x+4)^2}+c$

(C) $\frac{e^x}{x+4}+c$ ✓

(D) $\frac{e^x}{x+3}+c$

$\int e^x\left[\frac{x+3+1-1}{(x+4)^2}\right]dx$

$\int e^x\left[\frac{x+4}{(x+4)^2}-\frac{1}{(x+4)^2}\right]dx$

$\int e^x\left[\underbrace{\frac{1}{x+4}}_{f(x)}+\underbrace{\frac{-1}{(x+4)^2}}_{f'(x)}\right]dx = \frac{e^x}{x+4}+c$

QUESTION

$\ln x = \log_e x$

#Q. The value of $\int x^n \ln x \, dx$ is

$\int \log x \; x^n dx = \log x \dfrac{x^{n+1}}{n+1} - \int \dfrac{1}{x} \dfrac{x^{n+1}}{n+1} dx$

$= \dfrac{\log x \cdot x^{n+1}}{n+1} - \dfrac{1}{n+1} \int x^n dx$

$= \dfrac{x^{n+1} \log x}{n+1} - \dfrac{x^{n+1}}{(n+1)^2} + c$

$= \dfrac{x^{n+1}}{(n+1)^2} \left[(n+1) \log x - 1 \right] + c$

A) $\dfrac{x^{n+1}}{n+1} + c$

B) $\dfrac{(\ln x)^n}{n} + c$

C) $\dfrac{x^{n+1}}{(n+1)^2} [(n+1)\ln x - 1]$

D) $\dfrac{x^{n+1}}{n+2} + c$

QUESTION

#Q. The value of $\int e^{\tan^{-1}x}\left(\frac{1+x+x^2}{1+x^2}\right)dx$ is

(A) $e^{\tan^{-1}x}+c$

(B) $\tan^{-1}(e^x)+c$

(C) $e^{\tan^{-1}x}x+c$

(D) $e^{\tan^{-1}x}+x+c$

$\int e^{\tan^{-1}x}\left[x+(1+x^2)\right]\frac{dx}{1+x^2}$

Put $\tan^{-1}x = t \Rightarrow x = \tan t$

$\frac{dx}{1+x^2} = dt$

& $1+x^2 = 1+\tan^2 t = \sec^2 t$

$I = \int e^t\left[\underset{f(t)}{\tan t} + \underset{f'(t)}{\sec^2 t}\right]dt$

$= e^t \tan t + c$

$= e^{\tan^{-1}x}\cdot x + c$

QUESTION

#Q. The value of $\int e^{\sqrt{x}} dx$ is

(A) $2e^{\sqrt{x}}(\sqrt{x}+1)+c$

(B) $2e^{\sqrt{x}}(\sqrt{x}-1)+c$

(C) $e^{\sqrt{x}}(x-1)+c$

(D) $\frac{1}{2}e^{\sqrt{x}}+c$

Put

$\sqrt{x}=t$

$x=t^2$

$dx=2t\,dt$

$I=\int e^t(2t)\,dt$

$=2\int t e^t\,dt$

$I=2[te^t-e^t]+c$

$=2e^t(t-1)+c$

$=2e^{\sqrt{x}}(\sqrt{x}-1)+c$

QUESTION

#Q. The value of $\int e^{3x} \cos 4x \, dx$ is

$$\int e^{ax} \cos bx \, dx$$

$$= \frac{e^{ax}}{a^2+b^2}[a \cos bx + b \sin bx]$$

(A) $\frac{e^{3x}}{25}[3\sin 4x + 4\cos 4x] + c$

(B) $\frac{e^{3x}}{25}[3 \sin 4x - 4 \cos 4x] + c$

(C) $\frac{e^{3x}}{25}[3 \cos 4x - 4 \sin 4x] + c$

(D) $\frac{e^{3x}}{25}[3\cos 4x + 4\sin 4x] + c$

QUESTION

$\cos^{-1}x = \frac{\pi}{2} - \sin^{-1}x$

#Q. The value of $\int \frac{\sin^{-1}(x)-\cos^{-1}(x)}{\sin^{-1}(x)+\cos^{-1}(x)}dx$ is

(A) $\frac{4}{\pi}\left[x\sin^{-1}(x)+\sqrt{1-x^2}\right]-x+c$

(B) $\ln\left[\sin^{-1}(x)+\cos^{-1}(x)\right]+c$

(C) $\frac{4}{\pi}\left[x\sin^{-1}(x)+\sqrt{1-x^2}\right]+c$

(D) $\ln\left[\sin^{-1}(x)-\cos^{-1}(x)\right]+c$

$\Rightarrow I = \int \frac{2\sin^{-1}x - \frac{\pi}{2}}{\frac{\pi}{2}}dx$

$= \frac{2}{\pi}\int 2\sin^{-1}x - \frac{\pi}{2}\,dx$

$= \frac{2}{\pi}\left[2\int \sin^{-1}x \cdot 1\,dx - \frac{\pi}{2}x\right]+c$

$= \frac{2}{\pi}\left[2\left(\sin^{-1}x(x) - \int \frac{x}{\sqrt{1-x^2}}dx\right) - \frac{\pi}{2}x\right]+c$

$\longrightarrow$ Put $1-x^2 = t$

$x\,dx = -\frac{dt}{2}$

$= \frac{2}{\pi}\left[2\left(x\sin^{-1}x + \frac{1}{2}\int \frac{1}{\sqrt{t}}dt\right) - \frac{\pi}{2}x\right]+c$

$= \frac{2}{\pi}\left[2x\sin^{-1}x + 2\sqrt{t} - \frac{\pi}{2}x\right]+c$

$$= \frac{2}{\pi}\left[2x\sin^{-1}x + 2\sqrt{x} - \frac{\pi}{2}x\right] + c$$

$$= \frac{4}{\pi}x\sin^{-1}x + \frac{4}{\pi}\sqrt{1-x^2} - x + c$$

$$\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$$

$$\int \frac{1}{2\sqrt{x}}\,dx = \sqrt{x} + c$$

QUESTION

#Q. The value of $\int \tan^{-1}x\,dx$ is

(A) $\dfrac{1}{x^2+1}+c$

(B) $-\dfrac{1}{x^2+1}+c$

(C) $x\tan^{-1}x-\dfrac{1}{2}\ln(1+x)+c$

(D) $x\tan^{-1}x-\dfrac{1}{2}\ln(1+x^2)+c$

$I=\int \tan^{-1}x\,(1)\,dx$

$=\tan^{-1}x\,(x)-\int \frac{x}{1+x^2}\,dx$

$=x\tan^{-1}x-\frac{1}{2}\int \frac{2x}{1+x^2}\,dx$

$=x\tan^{-1}x-\underline{\frac{1}{2}\log}\,(1+x^2)+c$

QUESTION

#Q. If $I_n = \int x^n e^x dx$, where $n \in$ N then $I_n + nI_{n-1}$ is

(A) $x^n e^x + c$ $\quad I_{n-1} = \int x^{n-1} e^x dx$

(B) $x^{n-1} e^x + c$ $\quad I_n + nI_{n-1} = \int e^x [\underbrace{x^n}_{f(x)} + \underbrace{nx^{n-1}}_{f'(x)}] dx$

(C) $\frac{1}{n} x^n e^x + c$ $\quad = \underline{e^x x^n + c}$

(D) $x^{n+1} e^x + c$

QUESTION

#Q. The value of $\int e^x\left[\frac{1+\sin x\cos x}{1+\cos 2x}\right]dx$ is

(A) $e^x\tan x + c$

(B) $\frac{1}{2}e^x\cot x + c$

(C) $2.\,e^x\tan x + c$

(D) $\frac{1}{2}e^x\tan x + c$

$I = \int e^x\left[\frac{1}{1+\cos 2x} + \frac{\sin x\cos x}{1+\cos 2x}\right]dx$

$= \int e^x\left[\frac{1}{2\cos^2 x} + \frac{\sin x\,\cancel{\cos x}}{2\cos^{\cancel{2}} x}\right]dx$

$= \int e^x\left[\frac{1}{2}\sec^2 x + \frac{1}{2}\tan x\right]dx$

$= \frac{1}{2}\int e^x[\underbrace{\tan x}_{f(x)} + \underbrace{\sec^2 x}_{f'(x)}]dx = \frac{e^x\tan x}{2} + c$

QUESTION

#Q. The value of $\int \frac{\sin 4x-2}{1-\cos 4x} e^{2x} dx$ is

(A) $\frac{1}{2}e^{2x}\cot 2x + c$

(B) $e^{2x}\cot 2x + c$

(C) $e^{2x}\text{cosec}\, 2x + c$

(D) $e^{2x}\tan 2x + c$

Put $2x = t$

$dx = \frac{dt}{2}$

$I = \int e^t \left[\frac{\sin 2t - 2}{1-\cos 2t}\right]\frac{dt}{2}$

$= \int e^t \left[\frac{\sin 2t}{1-\cos 2t} + \frac{-2}{1-\cos 2t}\right]\frac{dt}{2}$

$= \frac{1}{2}\int e^t \left[\frac{\cancel{2}\sin \cancel{t} \cos t}{\cancel{2}\sin^{\cancel{2}} t} + \frac{-\cancel{2}}{\cancel{2}\sin^2 t}\right] dt$

$I = \frac{1}{2}\int e^t \left[\underbrace{\cot t}_{f(t)} + \underbrace{(-\text{cosec}^2 t)}_{f'(t)}\right] dt$

$I = \frac{1}{2} e^t \cot t + c$

$I = \frac{1}{2} e^{2x} \cot 2x + c$

QUESTION

NCERT

#Q. The value of $\int \frac{e^x(x^2+1)}{(1+x)^2}\,dx$ is

(A) $e^x\left(\frac{x+1}{x-1}\right)+c$

(B) $\frac{e^x(x-2)}{x+1}+c$

(C) $\frac{e^x(x-1)}{x+1}+c$

(D) $\frac{e^x}{x+1}+c$

$$I=\int e^x\left[\frac{x^2+1-1+1}{(x+1)^2}\right]dx$$

$$=\int e^x\left[\frac{(x^2-1)}{(x+1)^2}+\frac{2}{(x+1)^2}\right]dx$$

$$=\int e^x\left[\frac{\cancel{(x+1)}(x-1)}{(x+1)^{\cancel{2}}}+\frac{2}{(x+1)^2}\right]dx$$

$$=\int e^x\left[\underbrace{\frac{x-1}{x+1}}_{f(x)}+\underbrace{\frac{2}{(x+1)^2}}_{f'(x)}\right]dx$$

$$I=e^x\left(\frac{x-1}{x+1}\right)+c$$

① $\int \frac{1}{x^2+1}\,dx$

$= \tan^{-1}x + C$

② $\int \frac{x}{x^2+1}\,dx$

$= \frac{1}{2}\int \frac{2x \rightarrow f'(x)}{x^2+1 \rightarrow f(x)}\,dx$

$= \frac{1}{2}\log(x^2+1) + C$

③ $\int \frac{x^2}{x^2+1}\,dx$

$\int \frac{x^2+1-1}{x^2+1}\,dx$

$= \int 1 - \frac{1}{x^2+1}\,dx$

$= x - \tan^{-1}x + C$

① $I = \int \frac{x}{1-3x^2}\,dx$

$= -\frac{1}{6}\int \frac{-6x}{1-3x^2}\,dx$ $\quad -6x \to f'(x)$, $\quad 1-3x^2 \to f(x)$

$= -\frac{1}{6}\log(1-3x^2)+c$

② $\int \frac{1}{1-3x^2}\,dx$

$I = \int \frac{1}{1-(\sqrt{3}x)^2}\,dx$

$= \dfrac{\frac{1}{2(1)}\log\left|\frac{1+\sqrt{3}x}{1-\sqrt{3}x}\right|}{\sqrt{3}}+c$

$= \frac{1}{2\sqrt{3}}\log\left|\frac{1+\sqrt{3}x}{1-\sqrt{3}x}\right|+c$

③ $\int \frac{x^2}{1-3x^2}\,dx$

$I = -\frac{1}{3}\int \frac{-3x^2}{1-3x^2}$

$= -\frac{1}{3}\int \frac{(-3x^2+1)-1}{1-3x^2}\,dx$

$= -\frac{1}{3}\int 1-\frac{1}{1-(\sqrt{3}x)^2}\,dx$

$= -\frac{1}{3}\left[x-\frac{1}{2\sqrt{3}}\log\left|\frac{1+\sqrt{3}x}{1-\sqrt{3}x}\right|\right]+c$

$$\int \frac{1}{x^2-a^2}\,dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + c$$

$$\int \sin 2x\,dx = -\frac{\cos 2x}{2} + c$$

$$\int \frac{1}{a^2-x^2}\,dx = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + c$$

$$\int \frac{1}{1-3x^2}\,dx = \int \frac{1}{1-(\sqrt{3}x)^2}\,dx = \frac{\frac{1}{2(1)}\log\left|\frac{1-\sqrt{3}x}{1+\sqrt{3}x}\right| + c}{\sqrt{3}}$$

$a=1$

$\sqrt{3}$ ↳ coefficient of x

$$= \frac{1}{2\sqrt{3}}\log\left|\frac{1-\sqrt{3}x}{1+\sqrt{3}x}\right| + c$$

QUESTION

#Q. The value of $\int \frac{dx}{\sqrt{x^2+8}}$ is

(A) $\ln\left|x+\sqrt{x^2+8}\right|+c$

(B) $\ln\left|x-\sqrt{x^2+8}\right|+c$

(C) $\tan^{-1}\left(\frac{1}{2\sqrt{2}}\right)+c$

(D) $\sin^{-1}\left(\frac{x}{2\sqrt{2}}\right)+c$

① $I = \int \frac{x}{\sqrt{x^2+8}} dx$

$\Downarrow$

Put $x^2+8 = t$

$x\,dx = \frac{dt}{2}$

$I = \frac{1}{2}\int \frac{1}{\sqrt{t}} dt$

$= \sqrt{t} + c$

$= \sqrt{x^2+8} + c$

② $\int \frac{x^2}{\sqrt{2x^2+8}} dx$

$I = \frac{1}{2}\int \frac{2x^2}{\sqrt{2x^2+8}} dx$

$= \frac{1}{2}\int \frac{2x^2+8}{\sqrt{2x^2+8}} - \frac{8}{\sqrt{2x^2+8}} dx$

$= \frac{1}{2}\int \sqrt{(\sqrt{2}x)^2+8} - \frac{8}{\sqrt{(\sqrt{2}x)^2+8}} dx$

$= \frac{1}{2}\left[\frac{\sqrt{2}x}{2}\sqrt{2x^2+8} + \frac{8}{2}\log\left|\sqrt{2}x + \sqrt{2x^2+8}\right|\right] - 8\log\left|\sqrt{2}x + \sqrt{2x^2+8}\right| + c$

$= \frac{x}{2\sqrt{2}}\sqrt{2x^2+8} - 6\log\left|\sqrt{2}x + \sqrt{2x^2+8}\right| + c$

QUESTION

#Q. The value of $\int \frac{dx}{6-2x+x^2}$ is

x^2-2x+6

$x^2-2x+1-1+6$

$(x-1)^2+(\sqrt{5})^2$

$\int \frac{1}{a^2+x^2}dx$

$= \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c$

(A) $\tan^{-1}(x-1)+c$ F

(B) F $\tan^{-1}\left(\frac{x-1}{\sqrt{5}}\right)+c$

(C) ✓ $\frac{1}{\sqrt{5}}\tan^{-1}\left(\frac{x-1}{\sqrt{5}}\right)+c$

(D) $\frac{1}{2\sqrt{5}}\tan^{-1}\left(\frac{x-1}{\sqrt{5}}\right)+c$

$I = \int \frac{1}{(\sqrt{5})^2+(x-1)^2}$

$= \frac{1}{\sqrt{5}}\tan^{-1}\left(\frac{x-1}{\sqrt{5}}\right)+c$

QUESTION

#Q. The value of $\int \frac{2dx}{\sqrt{1-4x^2}}$ is

$$I = 2\int \frac{1}{\sqrt{1-(2x)^2}}dx$$

$$= \frac{\cancel{2}\left[\sin^{-1}\left(\frac{2x}{1}\right)\right]}{\cancel{2}} + c$$

$$= \sin^{-1}2x + c$$

or

$$= -\cos^{-1}(2x) + c$$

$$\sin^{-1}2x + c$$

$$\Downarrow$$

$$\frac{\pi}{2} - \cos^{-1}2x + c$$

$$= -\cos^{-1}2x + \left(c + \frac{\pi}{2}\right)$$

$$= -\cos^{-1}2x + c_1$$

QUESTION

#Q. The value of $\int \frac{dx}{4-9x^2}$ is

(A) $\frac{1}{12}\ln\left(\frac{2+3x}{2-3x}\right)+c$

(B) $\frac{1}{3}\ln\left(\frac{2+3x}{2-3x}\right)+c$

(C) $\frac{1}{2}\ln\left(\frac{2+3x}{2-3x}\right)+c$

(D) $\frac{1}{12}\ln\left(\frac{3x+2}{3x-2}\right)+c$

$$= \int \frac{1}{2^2-(3x)^2}\,dx$$

$$= \frac{\frac{1}{2(2)}\log\left|\frac{2+3x}{2-3x}\right|}{3}+c$$

$$= \frac{1}{12}\log\left|\frac{2+3x}{2-3x}\right|+c$$

QUESTION

#Q. The value of $\int \frac{\sin x}{9-\cos^2 x} dx$ is

(A) $\frac{1}{6} \ln\left(\frac{3+\cos x}{3-\cos x}\right) + c$

(B) $\frac{1}{6} \ln\left(\frac{3-\cos x}{3+\cos x}\right) + c$

(C) $\frac{1}{2} \ln\left(\frac{3+\cos x}{3-\cos x}\right) + c$

(D) $\frac{1}{2} \ln\left(\frac{3-\cos x}{3+\cos x}\right) + c$

Put $\cos x = t$

$\sin x \, dx = -dt$

$$I = -\int \frac{dt}{9-t^2}$$

$$= -\frac{1}{2(3)} \log\left|\frac{3+t}{3-t}\right| + c$$

$$= -\frac{1}{6} \log\left|\frac{3+\cos x}{3-\cos x}\right| + c = \frac{1}{6} \log\left|\frac{3-\cos x}{3+\cos x}\right| + c$$

$$I = \int \frac{1}{9-\cos^2 x}\,dx$$

$\div$ N^r & D^r by $\cos^2 x$

$$= \int \frac{\sec^2 x}{9\sec^2 x - 1}\,dx$$

$$= \int \frac{\sec^2 x}{9(1+\tan^2 x)-1}\,dx$$

$$= \int \frac{\sec^2 x}{8+9\tan^2 x}\,dx$$

Put $\tan x = t$

$$I = \int \frac{dt}{(2\sqrt{2})^2+(3t)^2}$$

$$= \frac{\frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{3t}{2\sqrt{2}}\right)}{3} + C$$

$$= \frac{1}{6\sqrt{2}}\tan^{-1}\left(\frac{3\tan x}{2\sqrt{2}}\right) + C$$

$$I = \int \frac{1}{4+\sin^2 x}\,dx$$

$\div$ by $\sin^2 x$

$$= \int \frac{\operatorname{cosec}^2 x}{4\operatorname{cosec}^2 x + 1}\,dx$$

$$= \int \frac{\operatorname{cosec}^2 x}{4(1+\cot^2 x)+1}\,dx$$

$$= \int \frac{\operatorname{cosec}^2 x}{5+4\cot^2 x}\,dx$$

Put $\cot x = t$

$$I = -\int \frac{dt}{(\sqrt{5})^2 + (2t)^2}$$

$$= \frac{\frac{-1}{\sqrt{5}} \tan^{-1}\left(\frac{2t}{\sqrt{5}}\right)}{2} + c$$

$$= \frac{-1}{2\sqrt{5}} \tan^{-1}\left(\frac{2\cot x}{\sqrt{5}}\right) + c$$

$$\int \sin 2x\,dx = -\frac{\cos 2x}{2} + c$$

$$I = \int \frac{\cos^2 x}{1+\sin x}\,dx$$

$$= \int \frac{1-\sin^2 x}{1+\sin x}\,dx$$

$$= \int \frac{(1+\sin x)(1-\sin x)}{1+\sin x}\,dx$$

$$= \int 1-\sin x\,dx$$

$$= x+\cos x+c$$

$$I = \int \frac{1+\sin x}{\cos^2 x}\,dx$$

$$= \int \frac{1}{\cos^2 x} + \frac{\sin x}{\cos^2 x}\,dx$$

$$= \int \sec^2 x + \sec x \tan x\,dx$$

$$= \tan x + \sec x + c$$

QUESTION

#Q. The value of $\int \frac{x^2}{x^6+2x^3+2} dx$ is $= \int \frac{x^2}{(x^3)^2+2x^3+2} dx$

Put $x^3 = t$

$x^2 dx = \frac{dt}{3}$

(A) $\tan^{-1}(x^3 + 1) + c$

(B) $1/3 \tan^{-1}(x^3 + 1) + c$

$= \frac{1}{3}\int \frac{dt}{t^2+2t+1+1}$

(C) $1/3 \cot^{-1} (x^3 + 1) + c$

(D) $\cot^{-1} (x^3 + 1) + c$

$= \frac{1}{3}\int \frac{dt}{(t+1)^2+1}$

$= \frac{1}{3}\tan^{-1}(t+1) + c$

$= \frac{1}{3}\tan^{-1}(x^3+1) + c$

QUESTION

#Q. The value of $\int \frac{\cos x \, dx}{\sqrt{1+\cos^2 x}}$ is

(A) $\frac{1}{2}\ln\left|\cos x+\sqrt{1+\cos^2 x}\right|+c$

(B) $\ln\left|\sin x+\sqrt{4+\sin^2 x}\right|+c$

(C) $\sin^{-1}\left(\frac{\cos x}{2}\right)+c$

(D) $\sin^{-1}\left(\frac{\sin x}{\sqrt{2}}\right)+c$

$= \int \frac{\cos x}{\sqrt{2-\sin^2 x}} dx$

Put $\sin x = t$

$I = \int \frac{dt}{\sqrt{(\sqrt{2})^2 - t^2}}$

$= \sin^{-1}\frac{t}{\sqrt{2}} + c$

$= \sin^{-1}\left(\frac{\sin x}{\sqrt{2}}\right) + c$

$$I = \int \frac{1}{4\sin^2 x + 9\cos^2 x}\,dx$$

$\div$ by $\cos^2 x$

$$I = \int \frac{\sec^2 x}{4\tan^2 x + 9}\,dx$$

Put $\tan x = t$

$$I = \int \frac{dt}{3^2 + (2t)^2}$$

$$= \frac{\frac{1}{3}\tan^{-1}\left(\frac{2t}{3}\right)}{2} + C$$

$$= \frac{1}{6}\tan^{-1}\left(\frac{2\tan x}{3}\right) + C$$

QUESTION

#Q. The value of $\int \frac{1}{\sqrt{1-2x-x^2}} dx$ is

(A) $\sin^{-1}\left(\frac{x-1}{\sqrt{2}}\right)+c$

(B) $\sin^{-1}\left(\frac{x+1}{\sqrt{2}}\right)+c$

(C) $\ln\left|\frac{x+2}{\sqrt{2}}+\sqrt{1-2x-x^2}\right|+c$

(D) $\ln\left|\frac{x+1}{\sqrt{2}}+\sqrt{1+\left(\frac{x-1}{\sqrt{2}}\right)^2}\right|+c$

$-(x^2+2x-1)$

$=-[x^2+2x+1-1-1]$

$=-[(x+1)^2-2]$

$=2-(x+1)^2$

$I=\int \frac{1}{\sqrt{(\sqrt{2})^2-(x+1)^2}} dt \rightarrow \int \frac{1}{\sqrt{a^2-x^2}} dt$

$=\sin^{-1}\left(\frac{x+1}{\sqrt{2}}\right)+c$

QUESTION

#Q. The value of $\int e^{x}\sqrt{e^{2x}+1}\,dx$ is

$I = \int e^{x}\sqrt{(e^{x})^{2}+1}\,dx$

Put $e^{x} = t$

(A) $\frac{e^{x}}{2}\sqrt{e^{2x}+1}+\frac{1}{2}\ln\left|e^{x}+\sqrt{e^{2x}+1}\right|$

(B) $\frac{e^{x}}{2}\sqrt{e^{2x}+1}-\frac{1}{2}\ln\left|e^{x}+\sqrt{e^{2x}-1}\right|+c$

(C) $\frac{e^{x}}{2}\sqrt{e^{2x}+1}-\frac{1}{2}\sin^{-1}(e^{x})+c$

(D) $\frac{e^{x}}{2}\sqrt{e^{2x}+1}+\frac{1}{2}\sin^{-1}(e^{x})+c$

$I = \int \sqrt{t^{2}+1}\,dt$

$I = \frac{t}{2}\sqrt{t^{2}+1}+\frac{1}{2}\log\left|t+\sqrt{t^{2}+1}\right|+c$

$= \frac{e^{x}}{2}\sqrt{(e^{x})^{2}+1}+\frac{1}{2}\log\left|e^{x}+\sqrt{e^{2x}+1}\right|+c$

QUESTION

#Q. The value of $\int \frac{\sin x\, dx}{\sqrt{8+\sin^2 x}}$ is $= \int \frac{\sin x}{\sqrt{9-\cos^2 x}} dx$

Put $\cos x = t$

(A) $-\cos^{-1}\left(\frac{\cos x}{3}\right) + c$

$I = -\int \frac{dt}{\sqrt{3^2 - t^2}}$

(B) $-\sin^{-1}\left(\frac{\cos x}{3}\right) + c$

$I = -\sin^{-1}\left(\frac{t}{3}\right) + c$

(C) $-\sin^{-1}\left(\frac{\sin x}{3}\right) + c$

$= -\sin^{-1}\left(\frac{\cos x}{3}\right) + c = \cos^{-1}\left(\frac{\cos x}{3}\right) + c$

(D) $-\frac{1}{3}\sin^{-1}\left(\frac{\cos x}{3}\right) + c$

QUESTION

$a\cos^2 x + b\sin^2 x$
$= a + (b-a)\sin^2 x$

$\to 2\sin x\cos x$

#Q. The value of $\int \frac{\sin 2x}{a\cos^2 x + b\sin^2 x}\,dx$ is

(A) $\frac{1}{b-a}\ln(a\cos^2 x + b\sin^2 x) + c$

(B) $\frac{1}{a-b}\ln(a\cos^2 x + b\sin^2 x) + c$

(C) $(b-a)(a\cos^2 x + b\sin^2 x) + c$

(D) $\frac{\tan^2 x}{b-a} + c$

$I = \int \frac{2\sin x\cos x}{a(1-\sin^2 x) + b\sin^2 x}\,dx$

$= \int \frac{2\sin x\cos x}{a + (b-a)\sin^2 x}\,dx$

Put $a + (b-a)\sin^2 x = t$

$2\sin x\cos x\,dx = \frac{dt}{b-a}$

$I = \frac{1}{b-a}\int \frac{1}{t}\,dt$

$= \frac{1}{b-a}\log\left(a + (b-a)\sin^2 x\right) + c$

$= \frac{1}{b-a}\log\left[a\cos^2 x + b\sin^2 x\right] + c$

QUESTION

#Q. The value of $\int \frac{\sin x \cos x}{4\cos^2 x + 9\sin^2 x} dx$ is $= \int \frac{\sin x \cos x}{4+5\sin^2 x} dx$

$4(1-\sin^2 x)$

Put $4+5\sin^2 x = t$

$\sin x \cos x \, dx = \frac{dt}{10}$

$I = \frac{1}{10}\int \frac{1}{t} dt$

$= \frac{1}{10}\log(4+5\sin^2 x) + c$

$= \frac{1}{10}\log(4\cos^2 x + 9\sin^2 x) + c$

(A) $\frac{1}{10}\ln(4\cos^2 x + 9\sin^2 x) + c$

(B) $\frac{1}{5}\ln(4\cos^2 x + 9\sin^2 x) + c$

(C) $\ln(4\cos^2 x + 9\sin^2 x) + c$

(D) $\ln(13\sin 2x) + c$

QUESTION

#Q. The value of $\int \frac{dx}{4\cos^2 x - 9\sin^2 x}$ is

$\div$ by $\cos^2 x$

$$I = \int \frac{\sec^2 x}{4 - 9\tan^2 x}\,dx$$

Put $\tan x = t$

$$I = \int \frac{dt}{2^2 - (3t)^2}$$

$$= \frac{1}{2(2)} \frac{\log\left|\frac{2+3t}{2-3t}\right|}{3} + c = \frac{1}{12}\log\left|\frac{2+3\tan x}{2-3\tan x}\right| + c$$

(A) $\ln(4\cos^2 x - 9\sin^2 x) + c$

(B) $\frac{1}{12}\ln\left(\frac{2-3\tan x}{2+3\tan x}\right) + c$

(C) $\frac{1}{12}\ln\left(\frac{2+3\tan x}{2-3\tan x}\right) + c$

(D) $\frac{1}{12}\ln\tan x + c$

QUESTION

#Q. The value of $\int \frac{dx}{4\cos^2 x+9\sin^2 x+3}$ is $\div$ by $\cos^2 x$

(A) $\tan^{-1}\left(\frac{\tan x}{7}\right)+c$

(B) $\frac{1}{\sqrt{84}}\tan^{-1}\left(\frac{\sqrt{12}\tan x}{\sqrt{7}}\right)+c$

(C) $\frac{1}{\sqrt{84}}\tan^{-1}\left(\frac{\tan x}{\sqrt{7}}+c\right)$

(D) $\frac{1}{\sqrt{7}}\tan^{-1}\left(\frac{\tan x}{3}\right)+c$

$I=\int \frac{\sec^2 x \, dx}{4+9\tan^2 x+3\sec^2 x}$ $\longrightarrow 3(1+\tan^2 x)$

$=\int \frac{\sec^2 x \, dx}{7+12\tan^2 x}$

Put $\tan x = t$

$=\int \frac{dt}{(\sqrt{7})^2+(\sqrt{12}t)^2} = \frac{\frac{1}{\sqrt{7}}\tan^{-1}\left(\frac{\sqrt{12}t}{\sqrt{7}}\right)}{\sqrt{12}}+c$

$=\frac{1}{\sqrt{84}}\tan^{-1}\left(\frac{\sqrt{12}\tan x}{\sqrt{7}}\right)+c$

QUESTION

#Q. The value of $\int \sin(2\ln x)dx$ is

→ Put $\log x = t$

$x = e^t$

$dx = e^t dt$

$\int e^{ax}\sin bx\,dx$

$= \frac{e^{ax}}{a^2+b^2}[a\sin bx - b\cos bx] + c$

(A) $\frac{x}{5}[\sin(2\ln x) - 2\cos(2\ln x)] + c$

(B) $\frac{-\cos(2\ln x)}{2} + c$

(C) $\frac{x\cos(2\ln x)}{2} + c$

(D) $2\cos(2\ln x) + c$

$I = \int e^t \sin 2t\, dt$

$= \frac{e^t}{1^2+2^2}[1\sin 2t - 2\cos 2t] + c$

$= \frac{x}{5}[\sin(2\log_e x) - 2\cos(\log_e x)] + c$

QUESTION

#Q. The value of $\int \frac{x^3 dx}{x^8+4x^4+5}$ is $= \int \frac{x^3 dx}{(x^4)^2+4x^4+5}$

(A) $\frac{1}{4}\tan^{-1}\left(\frac{x^4+2}{\sqrt{5}}\right)+c$

(B) ✓ $\frac{1}{4}\tan^{-1}\left(\frac{x^4+2}{1}\right)+c$

(C) $\frac{1}{4}\tan^{-1}\left(\frac{x^4+2}{5}\right)+c$

(D) $\frac{1}{4}\sin^{-1}\left(\frac{x^4+2}{5}\right)+c$

Put $x^4 = t$

$x^3 dx = \frac{dt}{4}$

$I = \frac{1}{4}\int \frac{dt}{t^2+4t+2^2-2^2+5}$

$= \frac{1}{4}\int \frac{dt}{(t+2)^2+1}$

$= \frac{1}{4}\left[\frac{1}{1}\tan^{-1}\left(\frac{t+2}{1}\right)\right]+c$

$= \frac{1}{4}\tan^{-1}(x^4+2)+c$

QUESTION

#Q. The value of $\int \frac{\sin x\,dx}{\sqrt{\cos^4 x+\cos^2 x}}$ is $= \int \frac{\sin x}{\cos^2 x\sqrt{1+\sec^2 x}}\,dx$

(A) $\ln\left|\sec x+\sqrt{\sec^2 x-1}\right|+c$ $\quad = \int \frac{\sec x\tan x}{\sqrt{1+\sec^2 x}}\,dx$

(B) $\ln\left|\sec x+\sqrt{\sec^2 x+1}\right|+c$ $\quad$ Put $\sec x = t$

(C) $\ln\left|\cos x+\sqrt{\cos^2 x-1}\right|+c$ $\quad \int \frac{dt}{\sqrt{1+t^2}}$

$= \log\left|t+\sqrt{1+t^2}\right|+c$

(D) $\ln\left|\cos x+\sqrt{\cos^2 x+1}\right|+c$ $\quad = \log\left|\sec x+\sqrt{\sec^2 x+1}\right|+c$

QUESTION

$3+\cos x = 2+(1+\cos x) = 2+2\cos^2\frac{x}{2}$

#Q. The value of $\int \frac{dx}{3+2\sin x+\cos x}$ is

(A) $\tan^{-1}(\tan x)+c$

(B) $\tan^{-1}\left(\frac{\tan x}{3}\right)+c$

(C) $\tan^{-1}\left(\frac{\tan x}{2}\right)+c$

(D) $\tan^{-1}\left(\tan\frac{x}{2}+1\right)+c$

$$\int \frac{dx}{3+\cos x+2\sin x}$$

$$= \int \frac{dx}{2+2\cos^2\frac{x}{2}+4\sin\frac{x}{2}\cos\frac{x}{2}}$$

$\div$ by $\cos^2\frac{x}{2}$

$$I = \int \frac{\sec^2 x/2}{2\sec^2\frac{x}{2}+2+4\tan\frac{x}{2}}$$

$$I = \int \frac{\sec^2 x/2}{4+2\tan^2 x/2+4\tan x/2}$$

$$I = \int \frac{\sec^2 x/2}{2\tan^2\frac{x}{2} + 4\tan\frac{x}{2} + 4}$$

Put $\tan\frac{x}{2} = t$

$$\frac{1}{2}\sec^2\frac{x}{2} = dt$$

$$\sec^2\frac{x}{2}\,dx = 2\,dt$$

$$I = \int \frac{2\,dt}{2t^2 + 4t + 4}$$

$$= \frac{1}{2}\int \frac{2\,dt}{t^2 + 2t + 2}$$

$$I = \int \frac{dt}{\underbrace{t^2 + 2t + 1}-1+2}$$

$$= \int \frac{1}{(t+1)^2 + 1}$$

$$= \tan^{-1}(t+1) + C$$

$$= \tan^{-1}\left(\tan\frac{x}{2} + 1\right) + C$$

Thank You