ULTIMATE KCET

CRASH COURSE 2026

Mathematics | Lecture : 01

Limits and Continuity

By – Guru sir

Topics to be covered

1. Limits $\longrightarrow$ L-Hospital's rule
2. continuity
3.
4.

Limits

① Based on L'Hospital's rule

② $\lim_{x \to 0} \frac{\sin x}{x} = 1$ $\qquad$ $\lim_{x \to 0} \frac{x}{\sin x} = 1$

$\lim_{x \to 0} \frac{\tan x}{x} = 1$ $\qquad$ $\lim_{x \to 0} \frac{x}{\tan x} = 1$

$$\lim_{x \to a} \frac{x^n - a^n}{x - a} = n a^{n-1}$$

L-Hospital's rule

$\Downarrow$

This rule can be used only when

$\lim_{x \to a} \frac{f(x)}{g(x)}$ is of the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$

$\Downarrow$

Here if $\lim_{x \to a} \frac{f(x)}{g(x)} \simeq \frac{0}{0}$ or $\frac{\infty}{\infty}$

Then Differentiate N^r & D^r separately

$\Downarrow$

This procedure is continued until we get a finite value.

QUESTION

#Q. $\lim_{x \to \pi} \frac{\sin x}{x-\pi}$ is

A 1

B 2

C −1

D −2

$\sin\pi = 0$

$$\lim_{x \to \pi} \frac{\sin x}{(x-\pi)} \simeq \frac{0}{0}$$

$\pi - \pi = 0$

$$\lim_{x \to \pi} \frac{\cos x}{1-0}$$

$\cos\pi = -1$

$$\lim_{x\to\pi} \frac{\overset{\to 0}{\sin x}}{\underset{\downarrow 0}{(x-\pi)}\cos x} \neq 0$$

(A) ~~1~~

(B) -1

(C) 0

(D) $\frac{1}{2}$

$$= \lim_{x\to\pi} \frac{\overset{=0}{\sin x}}{\underset{\Downarrow 0}{x-\pi}} \times \lim_{x\to\pi} \frac{1}{\cos x}$$

$$= \lim_{x\to\pi} \frac{\cos x}{1-0} \times \left(\frac{1}{\cos \pi}\right)$$

$$= \frac{\cos\pi}{\cos\pi} = 1$$

$$\lim_{x\to a} f(x)\cdot g(x)$$

$$= \lim_{x\to a} f(x) \times \lim_{x\to a} g(x)$$

QUESTION

#Q. $\lim_{x\to 0} \frac{x^2 \cos x}{1-\cos x}$ is

(A) 2

(B) 3/2

(C) –3/2

(D) 1

$\lim_{x\to 0} \frac{x^2}{1-\cos x} \times \lim_{x\to 0} \cos x$

$\Downarrow$

$\lim_{x\to 0} \frac{2x}{\sin x} \times (1)$

$2 \lim_{x\to 0} \frac{x}{\sin x}$

$2(1) = 2$

$\lim_{x\to 0} \frac{x^2}{1-\cos x} \times \cos x$

$f(x) = \frac{0}{0}$ $\quad g(x) \neq 0$

$\lim_{x\to 0} \frac{x^2}{1-\cos x} \times \lim_{x\to 0} \cos x$

QUESTION

#Q. $\lim_{x\to 0} \frac{\operatorname{cosec} x - \cot x}{x}$ is $\quad$ N.D $\quad$ N.D $\quad \neq \frac{0}{0}$ $\frac{\infty}{\infty}$

A) $-1/2$

B) 1

C) $1/2$

D) -1

$$\lim_{x\to 0} \frac{\frac{1}{\sin x} - \frac{\cos x}{\sin x}}{x}$$

$$\lim_{x\to 0} \frac{1-\cos x}{x \sin x} \simeq \frac{1-1}{0} = \frac{0}{0}$$

$$\lim_{x\to 0} \frac{\sin x}{x\cos x + \sin x} \simeq \frac{0}{0}$$

$$\frac{d}{dx}(x\cos x + \sin x) = x(-\sin x) + \cos x(1) + \cos x$$

$$\lim_{x\to 0} \frac{\cos x}{\underbrace{x(-\sin x) + \cos x(1) + \cos x}}$$

$$= \frac{1}{0+1+1} = \frac{1}{2}$$

method 2

$$\lim_{x\to 0} \frac{\operatorname{cosec} x - \cot x}{x}$$

$$\lim_{x\to 0} \frac{\tan x/2}{x}$$

$$\lim_{x\to 0} \frac{\tan x/2}{\left(\frac{x}{2}\right)\times(2)}$$

$$\lim_{x\to 0} \frac{\tan\left(\frac{x}{2}\right)}{\left(\frac{x}{2}\right)} \quad \frac{1}{2}$$

1

$$(1)\left(\frac{1}{2}\right) = \frac{1}{2}$$

$\operatorname{cosec} x - \cot x$

$$= \frac{1}{\sin x} - \frac{\cos x}{\sin x} = \frac{1-\cos x}{\sin x}$$

$$= \frac{\cancel{2}\sin^{\cancel{2}} x/2}{\cancel{2}\cancel{\sin\frac{x}{2}}\cos\frac{x}{2}} = \frac{\sin x/2}{\cos x/2}$$

$$= \tan x/2$$

QUESTION

#Q. $\lim_{x\to 0} \frac{(1+x)^n-1}{x}$ is $= \frac{0}{0}$

$1-1$

A. n

B. 1

C. $-n$

D. 0

$$\lim_{x\to 0} \frac{n(1+x)^{n-1}-0}{1}$$

$$= n(1+0)^{n-1}$$

$$= n(1)$$

$$= n$$

method 2 (NCERT)

$$\lim_{x\to 0} \frac{(1+x)^n-1}{x}$$

$t = x+1$

Put $1+x=t$ | As $x\to 0$

$x = t-1$ | $t\to 1$

$$\lim_{t\to 1} \frac{t^n-1^n}{t-1} = n(1)^{n-1} = n$$

$a=1$

QUESTION

#Q. $\lim_{x\to 1} \frac{x^m-1}{x^n-1}$ is $\simeq \frac{1-1}{1-1} = \frac{0}{0}$

$$\lim_{x\to 1} \frac{mx^{m-1}}{nx^{n-1}} = \frac{m(1)}{n(1)} = \frac{m}{n}$$

(A) 1

(B) m/n

(C) $-m/n$

(D) m^2/n^2

QUESTION

#Q. If $\lim_{x\to5}\frac{x^k-5^k}{x-5}=500$, then the positive integral value of k is

(A) 3

(B) 4

(C) 5

(D) 6

$$\lim_{x\to5}\frac{kx^{k-1}-0}{1-0}=500$$

$$\lim_{x\to5}k(x)^{k-1}=500$$

$$k(5)^{k-1}=500$$

option verification

(1) $k=3$

$3(5)^2=75\neq500$

(2) $k=4$

$4(5)^3=4(125)=500$

QUESTION

#Q. $\lim_{\theta\to 0}\frac{1-\cos 4\theta}{1-\cos 6\theta}$ is $\simeq \frac{1-1}{1-1}=\frac{0}{0}$

$\Downarrow$

A 4/9

B 1/2

C −1/2

D −1

$$\lim_{\theta\to 0}\frac{0-4\sin 4\theta}{0-6\sin 6\theta}$$

$$\frac{4}{6}\lim_{\theta\to 0}\frac{\sin 4\theta}{\sin 6\theta}\simeq\frac{0}{0}$$

$$\frac{4}{6}\lim_{\theta\to 0}\frac{4\cos 4\theta}{6\cos 6\theta}=\frac{4}{6}\left(\frac{4}{6}\right)\frac{\cos 0}{\cos 0}$$

$$\frac{4}{6}\,\frac{4(1)}{6(1)}=\frac{2}{3}\,\frac{2}{3}=\frac{4}{9}$$

QUESTION

#Q. $\lim_{x\to 0}\frac{\sin x}{\sqrt{x+1}-\sqrt{1-x}}$ is $=\frac{0}{0}$

$$\frac{d}{dx}\sqrt{x+1}=\frac{1}{2\sqrt{x+1}}\frac{d}{dx}(x+1)=\frac{1}{2\sqrt{x+1}}$$

$$\frac{d}{dx}\sqrt{1-x}=\frac{1}{2\sqrt{1-x}}\frac{d}{dx}(1-x)=\frac{-1}{2\sqrt{1-x}}$$

(A) 2

(B) 0

(C) 1

(D) −1

$$\lim_{x\to 0}\frac{\cos x}{\frac{1}{2\sqrt{x+1}}-\left(\frac{-1}{2\sqrt{1-x}}\right)}$$

$$\frac{\cos 0}{\frac{1}{2\sqrt{0+1}}+\frac{1}{2\sqrt{1-0}}}=\frac{1}{\frac{1}{2}+\frac{1}{2}}=\frac{1}{1}=1$$

QUESTION

#Q. $\lim_{x\to\pi/4} \frac{\sec^2 x-2}{\tan x-1}$ is $= \frac{(\sqrt{2})^2-2}{1-1} = \frac{2-2}{1-1} = \frac{0}{0}$

(A) 3

(B) 1

(C) 0

(D) 2

$$\lim_{x\to\frac{\pi}{4}} \frac{2\sec^{\cancel{2}}x\tan x-0}{\cancel{\sec^2}x-0}$$

$= 2\tan\frac{\pi}{4}$

$= 2(1)$

$= 2$

$\frac{d}{dx}(\sec^2 x) = 2\sec x \frac{d}{dx}(\sec x)$

$= 2\sec x(\sec x\tan x)$

$= 2\sec^2 x\tan x$

QUESTION

#Q. $\lim_{x\to 1} \frac{(\overset{\to 0}{\sqrt{x}-1})(\overset{\to -1}{2x-3})}{\underbrace{2x^2+x-3}_{\to 0}}$ is $= \frac{0}{0}$

$$\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$$

(A) 1/10

(B) –1/10

(C) 1

(D) None of these

$$\lim_{x\to 1} \frac{\overset{\to 0}{\sqrt{x}-1}}{\underbrace{2x^2+x-3}_{\to 0}} \times \lim_{x\to 1} 2x-3$$

$$\lim_{x\to 1} \frac{\frac{1}{2\sqrt{x}}}{4x+1} \times (2-3)$$

$$= \frac{\frac{1}{2(1)}}{5}(-1) = -\frac{1}{10}$$

QUESTION

#Q. $\lim_{x\to 0} \frac{\overset{\to 0}{\tan 2x} - \overset{\to 0}{3x}}{\underset{\to 0}{3x} - \underset{\to 0}{\sin x}}$ is $\frac{0-0}{0-0} = \frac{0}{0}$

A 2

B 1/2

C −1/2

D 1/4

$\lim_{x\to 0} \frac{2\sec^2 2x - 3}{3 - \cos x}$

$\frac{2(1)-3}{3-1} = -\frac{1}{2}$

QUESTION

#Q. $\lim_{x\to 0} \frac{\overset{\to 0}{\sin x}}{\underset{\downarrow 0}{x}\underbrace{(1+\cos x)}_{\to 2}}$ is equal to

(A) 0

(B) 1/2

(C) 1

(D) –1

$$\lim_{x\to 0}\frac{\sin x}{x} \times \lim_{x\to 0}\frac{1}{1+\cos x}$$

$$= 1 \times \frac{1}{1+1}$$

$$\therefore = \frac{1}{2}$$

Sandwich theorem

$\Downarrow$

(1) $\lim_{x\to 0}\frac{\sin x}{x} = 1$

(2) $\lim_{x\to 0}\frac{\tan x}{x} = 1$

(3) $\lim_{x\to 0}\frac{x}{\sin x} = 1$

(4) $\lim_{x\to 0}\frac{x}{\tan x} = 1$

QUESTION

#Q. $\lim_{x \to \frac{\pi}{2}} \frac{1-\sin x}{\cos x}$ is equal to

$\rightarrow 0$

$\rightarrow 0$

(A) 0

$$\lim_{x \to \frac{\pi}{2}} \frac{-\cos x}{-\sin x} = \frac{\cos \pi/2}{\sin \pi/2} = \frac{0}{1} = 0$$

(B) −1

(C) 1

(D) does not exist

$$\lim_{x \to 3} [x]$$

$$[x] = \begin{cases} 3 & \text{if } x \in [3, 4) \quad x \geq 3 \\ 2 & \text{if } x \in (2, 3) \quad x < 3 \end{cases}$$

LHL

$\Downarrow$

$$\lim_{x \to 3^-} [x]$$

$$= 2$$

RHL

$\Downarrow$

$$\lim_{x \to 3^+} [x]$$

$$= 3$$

$$\underline{LHL \neq RHL}$$

Limit doesnot exist

when do we actually find LHL & RHL

$\Downarrow$

Ans when the given func is a split func at a given point

$x \to 3^-$

$\Downarrow$

The values of x will be

$x = 2.99$

$x = 2.999$

$x = 2.9999$

$\lim_{x \to 3^-} [x]$

$[2.99]$	$[2.999]$	$[2.9999]$
$= 2$	$= 2$	$= 2$

LHL = 2

$x \to 3^+$

$\Downarrow$

The values of x will be

$x = 3.01$

$x = 3.001$

$x = 3.0001$

$\lim_{x \to 3^+} [x]$

$[3.01]$	$[3.001]$	$[3.0001]$
$= 3$	$= 3$	$= 3$

RHL = 3

$$\lim_{x \to 3.6} [x]$$

$= 3$

$x \to 3.6^-$	$x \to 3.6^+$
$\Downarrow$	$\Downarrow$
$x = 3.59$	$x = 3.601$
$x = 3.599$	$x = 3.6001$
$x = 3.5999$	$x = 3.60001$

$\lim_{x \to 4} [x]$

$4 \in \mathbb{Z}$

LHL

$\lim_{x \to 4^-} [x]$

$= 3$

RHL

$\lim_{x \to 4^+} [x]$

$= 4$

$LHL \neq RHL$

limit doesnot exist

$\lim_{x \to 1.7} [x]$

$= \underline{1}$

$1.7 \in R - \mathbb{Z}$

$\Downarrow$

set of non integer values

QUESTION

#Q. $\lim_{x \to a}[x]$ = does not exist if ([x] is greatest integer function)

(A) a is integer

(B) a is a positive integer

(C) a is negative integer

(D) a is not a integer

$\lim_{x \to 0}[x]$

LHL $= -1$

RHL $= 0$

QUESTION

#Q. $\lim_{x\to 0}[x-1]$, where [.] is greatest integer function, is equal to

(A) 1

(B) 2

(C) 0

(D) does not exists

$\lim_{x\to 0}[x-1]$

Put $x-1=t$

$t=x-1$

$\therefore$ As $x\to 0$

$t\to -1$

$\lim_{t\to -1}[t]$

LHL $=-2$

RHL $=-1$

QUESTION

#Q. If $f(x) = \begin{cases} \frac{\sin[x]}{[x]}, & [x] \neq 0 \\ 0 & [x] = 0 \end{cases}$ where [.] denotes the greatest integer function, then $\lim_{x \to 0^-} f(x)$ is equal to

(A) 1

(B) 0

(C) –1

(D) None of these

$$\lim_{x \to 0^-} \frac{\sin[x]}{[x]}$$

$$\frac{\sin(-1)}{-1} = \frac{-\sin 1}{-1} = \sin 1$$

$$\lim_{x\to 0}\frac{\sin x}{x}=1$$

$$\lim_{x\to 0}\frac{\sin[x]}{[x]}$$

LHL

$$\lim_{x\to 0^-}\frac{\sin[x]}{[x]}$$

$$=\frac{\sin(-1)}{-1}$$

$$=\frac{-\sin 1}{-1}$$

$$=\sin 1$$

RHL

$$\lim_{x\to 0^+}\frac{\sin[x]}{[x]}$$

$$=\frac{\sin 0}{0}$$

$$=\frac{0}{0}$$

Doesnot exist

$$\lim_{x\to 0}[x]$$

LHL $=-1$

RHL $=0$

if $\quad f(x) = \begin{cases} \frac{|x|}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ Then $\lim_{x \to 0} f(x) =$

$x \neq 0$

$x > 0 \qquad x < 0$

$$f(x) = \begin{cases} \frac{x}{x} = 1 & x > 0 \quad \text{RHL} \\ \frac{-x}{x} = -1 & x < 0 \quad \text{LHL} \\ 0 & x = 0 \end{cases}$$

RHL	LHL
$= 1$	$= -1$

QUESTION

#Q. $\lim_{x \to 0} \frac{|x|}{x}$ is equal to

RHL = 1 & LHL = −1

(A) 0

(B) −1

(C) 1

(D) does not exist

QUESTION

#Q. $\lim_{x\to 0} \frac{|\sin x|}{x}$ is

(A) 1

(B) –1

(C) Does not exist

(D) None of these

$|\sin x| = \begin{cases} \sin x & \text{if } \sin x \geq 0 \quad x \in 1^{st} \,\&\, 2^{nd} \text{ Quad} \\ -\sin x & \text{if } \sin x < 0 \quad x \in 3^{rd} \,\&\, 4^{th} \text{ Quad} \end{cases}$

$0^+ \downarrow$ 1st Quad

$0^- \downarrow$ 4th Quad (–ve)

RHL

$\lim_{x\to 0^+} \frac{|\sin x|}{x}$

$= \lim_{x\to 0^+} \frac{\sin x}{x}$

$= 1$

LHL

$\lim_{x\to 0^-} \frac{|\sin x|}{x}$

$= \lim_{x\to 0^-} \frac{(-\sin x)}{x}$

$= -1$

LHL ≠ RHL

QUESTION

if α & β are the roots of a Quadratic eqn, Then The eqn is

$x^2-(\alpha+\beta)x+\alpha\beta=0$

#Q. Let $f(x)=\begin{cases}x^2-1, & 0<x<2 \\ 2x+3, & 2\le x<3\end{cases}$, the quadratic equation whose roots are $\lim_{x\to 2^-} f(x)$ and $\lim_{x\to 2^+} f(x)$ is

(A) $x^2-6x+9=0$

(B) $x^2-7x+8=0$

(C) $x^2-14x+49=0$

(D) $x^2-10x+21=0$

LHL

$\lim_{x\to 2^-} f(x)$

$\lim_{x\to 2^-} x^2-1 = 4-1$

$=3$

$=\alpha$

RHL

$\lim_{x\to 2^+} f(x)$

$\lim_{x\to 2^+} 2x+3 = 4+3 = 7$

$=\beta$

$x^2-(3+7)x+(3)(7)=0$

$x^2-10x+21=0$

$\frac{\pi}{2}^{+} \to 2^{nd}$ Quad $\to \sin x = +ve$

$\frac{\pi}{2}^{-} \to 1^{st}$ Quad $\to \sin x = +ve$

$$\lim_{x \to \frac{\pi}{2}} \frac{|\sin x|}{x}$$

LHL

$$\lim_{x \to \frac{\pi}{2}^-} \frac{|\sin x|}{x}$$

$$\lim_{x \to \frac{\pi}{2}^-} \frac{\sin x}{x}$$

$$\frac{\sin \pi/2}{\pi/2} = \frac{2}{\pi}$$

LHL

RHL

$$\lim_{x \to \frac{\pi}{2}^+} \frac{|\sin x|}{x}$$

$$\lim_{x \to \frac{\pi}{2}} \frac{\sin x}{x}$$

$$= \frac{\sin \pi/2}{\pi/2}$$

$$= \frac{2}{\pi} \text{ RHL}$$

$$\frac{\sin\frac{\pi}{2}}{\frac{\pi}{2}} = \frac{1}{\pi/2} = \frac{2}{\pi}$$

$$\lim_{x \to \pi} \frac{|\sin x|}{x}$$

$$\frac{|\sin \pi|}{\pi} = \frac{0}{\pi} = \underline{0}$$

$x \to 1$

$\Downarrow$

$x \neq 1$

but very near to one

$x = 0.99$

$= 0.999$

$x = 1.01$

$= 1.001$

$x \to \frac{\pi}{2}$

$\downarrow$

$x \to 90^\circ$

$x \neq 90$

LHL

$x = 89.9^\circ$

$x = 89.91^\circ$

$\Downarrow$

1^{st} Quad

$x = 90.1^\circ$

$x = 90.01^\circ$

$\Downarrow$

2^{nd} Quad

$$\frac{\sin \pi/2}{\pi/2} = \frac{1}{\pi/2} = \frac{2}{\pi}$$

$$\lim_{x \to \frac{\pi}{4}} \frac{\sin x}{x} = \frac{\sin \pi/4}{\pi/4}$$

$$= \frac{\frac{1}{\sqrt{2}}}{\pi/4}$$

$$= \frac{4}{\pi\sqrt{2}}$$

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

QUESTION

#Q. $\lim_{x\to 0} x \sin\frac{1}{x}$ is equal to

(A) 0 ✓

(B) 1

(C) 1/2

(D) does not exist

$\lim_{x\to 0} x \times \lim_{x\to 0} \sin\frac{1}{x}$

$0 \times$ {values b/w -1 & 1}

$= 0 \times [-1, 1]$

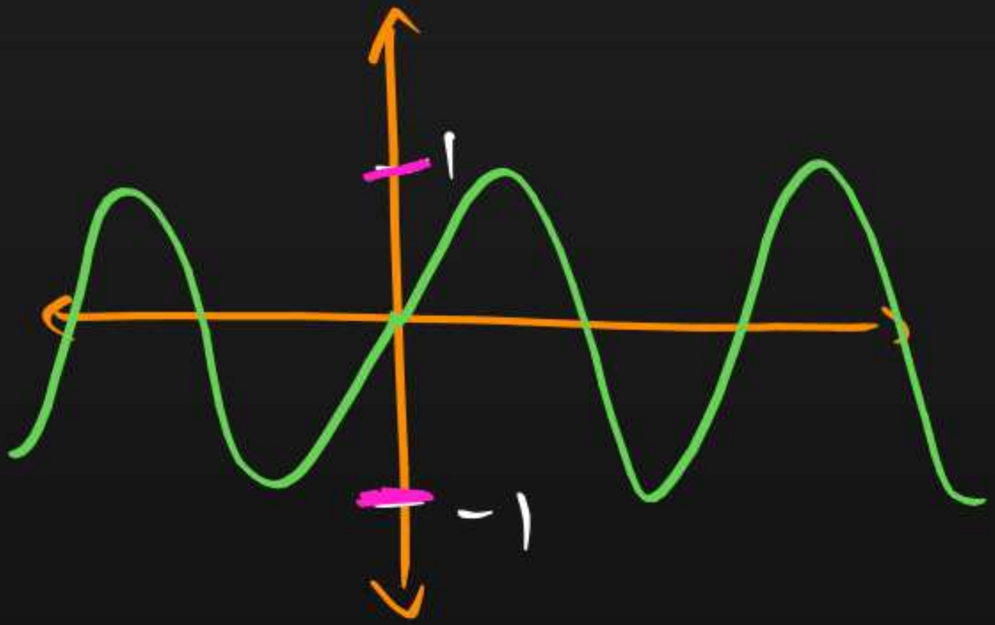
$= \underline{0}$

$\frac{1}{x} \to \infty$ as $x \to 0$

$f(x) = \sin\frac{1}{x}$

$x \to 0^-$ ⇓ $\frac{1}{x} \to -\infty$

$x \to 0^+$ ⇓ $\frac{1}{x} = +\infty$



Actual method

$$\lim_{x \to 0} \frac{\sin 2x}{x}$$

Put $2x = t$

$x = \frac{t}{2}$

$t = 2x$

$\because$ As $x \to 0$

$t \to 0$

$$\lim_{t \to 0} \frac{\sin(t)}{t/2}$$

$$\lim_{t \to 0} \frac{\sin t}{t} \times 2$$

$= 1 \times 2 = 2$

$$\lim_{x \to 0} \frac{\sin 2x}{x}$$ shortcut

$$\lim_{x \to 0} \frac{\sin 2x}{2x} \times 2$$

$$\lim_{x \to 0} \frac{\sin 2x}{2x} \times \lim_{x \to 0} 2$$

$(1) \times 2 = 2$

$$\lim_{x\to 0} x\sin(1/x)$$

$$\lim_{x\to 0} \frac{\sin(1/x)}{(1/x)}$$ → This is not in the std form $\lim_{\theta\to 0}\frac{\sin\theta}{\theta}$ → $\theta =$ Nr

Put $\frac{1}{x} = t$ → As $x \to 0$

$x = \frac{1}{t}$; $t \to \infty$

$$\lim_{t\to\infty} \frac{\sin t}{t} = [-1, 1]$$

QUESTION

#Q. $\lim_{n\to\infty} \frac{1+2+3+....+n}{n^2}$, $n \in Z$, is equal to

(A) 0

(B) 1

(C) 1/2

(D) 1/4

$$\lim_{n\to\infty} \frac{n(n+1)}{2n^2}$$

$$\frac{1}{2}\lim_{n\to\infty} \frac{n^2+n}{n^2}$$

$$\frac{1}{2}\lim_{n\to\infty} 1+\frac{1}{n}$$

$$\frac{1}{2}\cdot(1+0) = \frac{1}{2}$$

$$1+2+3+\dots+n = \frac{n(n+1)}{2}$$

As $n \to \infty$

$$\frac{1}{n} \to 0$$

$$\lim_{n\to\infty} \frac{1^2+2^2+3^2+\dots+n^2}{n^3}$$

$1^2+2^2+3^2+\dots+n^2$
$= \frac{n(n+1)(2n+1)}{6}$

(A) 1

(B) $\frac{1}{2}$

(C) $\frac{1}{6}$

(D) $\frac{1}{3}$

$$\lim_{n\to\infty} \frac{n(n+1)(2n+1)}{6n^3}$$

$$\frac{1}{6}\lim_{n\to\infty} \frac{\cancel{n}(n+1)\;(2n+1)}{\cancel{n}\;n\;\;n}$$

$$\left(\frac{n+1}{n}\right)\left(\frac{2n+1}{n}\right)$$

$$\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)$$

$$\frac{1}{6}\lim_{n\to\infty}\left(1+\frac{1}{n}\right)\left(2+\frac{2}{n}\right)$$

$$\frac{1}{6}(1+0)(2+0) = \frac{2}{6} = \frac{1}{3}$$

$$\lim_{n\to\infty} \frac{1^3+2^3+3^3+\dots+n^3}{n^4}$$

$1^3+2^3+3^3+\dots+n^3$

$$= \left[\frac{n(n+1)}{2}\right]^2$$

$$= \frac{n^2(n+1)^2}{4}$$

(A) 1

(B) $\frac{1}{4}$

(C) $\frac{1}{2}$

(D) $\frac{1}{8}$

$$\lim_{n\to\infty} \frac{\cancel{n^2}(n+1)^2}{4\,\cancel{n^2}\cdot n^2} = \lim_{n\to\infty} \frac{1}{4}\,\frac{(n+1)^2}{n^2}$$

$$\frac{1}{4}\lim_{n\to\infty}\left(\frac{n+1}{n}\right)^2$$

$$\frac{1}{4}\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^2$$

$$\frac{1}{4}(1+0) = \frac{1}{4}$$

QUESTION

$$\sqrt{x^2} = |x|$$

#Q. $\lim\limits_{x \to 0} \dfrac{\sin x}{\sqrt{x^2}} =$

$$\lim_{x \to 0} \frac{\sin x}{|x|}$$

LHL $= -1$

RHL $= +1$

(A) 1

(B) –1

(C) 0

(D) doesn't exist

LHL $\neq$ RHL

QUESTION

#Q. $\lim_{x\to\frac{\pi}{2}} \frac{\left(\frac{\pi}{2}-x\right)\sec x}{\operatorname{cosec} x} =$ $\frac{0\times\infty}{1}$

(A) 1

(B) 0

(C) –1

(D) $1/\pi$

$$\lim_{x\to\frac{\pi}{2}} \frac{\left(\frac{\pi}{2}-x\right)\sin x}{\cos x} \simeq \frac{0}{0}$$

$$\lim_{x\to\frac{\pi}{2}} \frac{\frac{\pi}{2}-x}{\cos x} \times \lim_{x\to\frac{\pi}{2}} \sin x$$

$$\lim_{x\to\frac{\pi}{2}} \frac{0-1}{-\sin x} \times \left(\sin\frac{\pi}{2}\right)$$

$$\frac{-1}{-\sin\frac{\pi}{2}}(1)$$

$$= \frac{1}{1}(1)$$

$$= 1$$

QUESTION

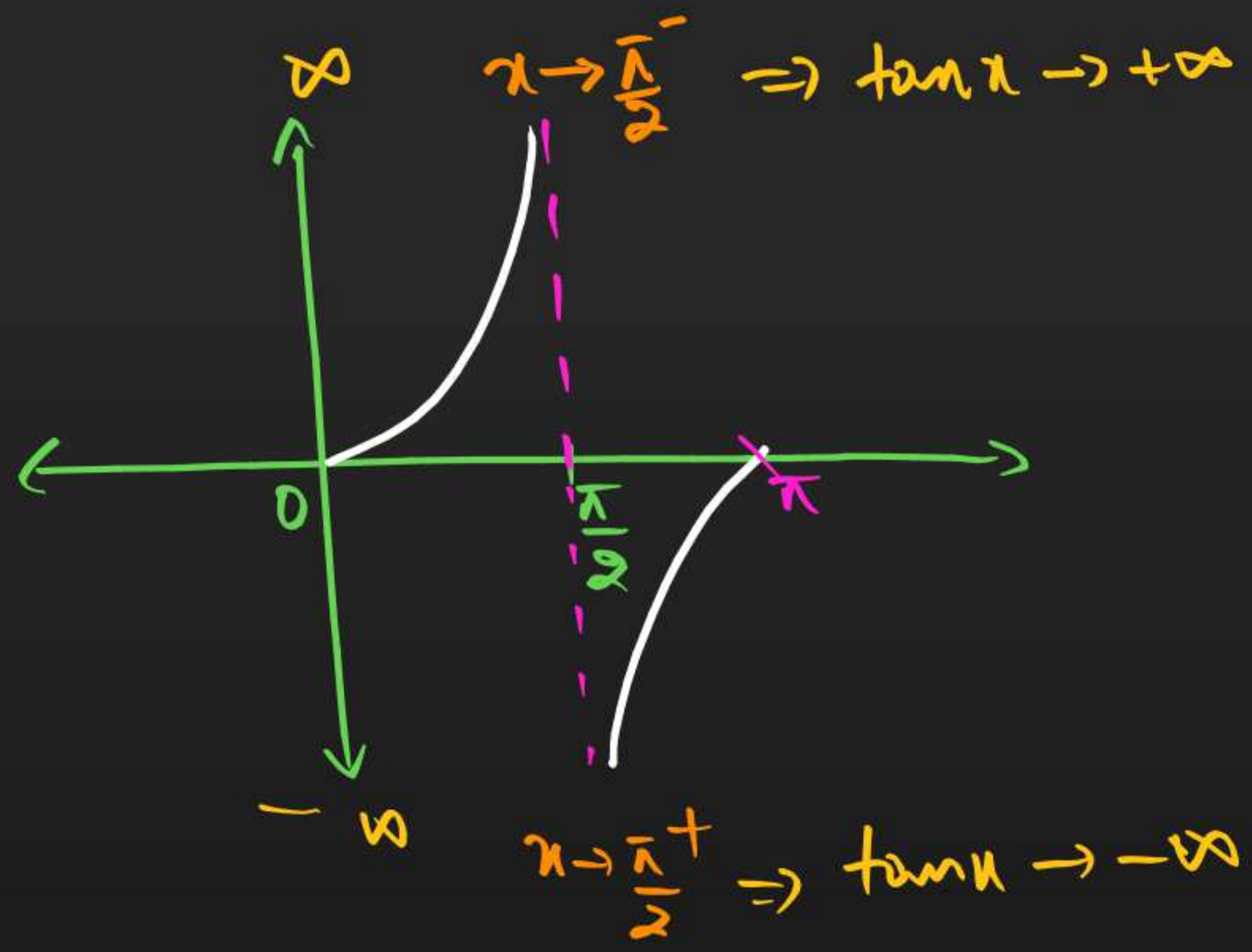
#Q. $\lim_{x \to \frac{\pi}{2}} \tan x =$

(A) 1

(B) 0

(C) $1/\pi$

(D) doesn't exist



QUESTION

#Q. $\lim_{x\to\frac{\pi}{2}} \dfrac{\overset{\to 0}{\cot x} - \overset{\to 0}{\cos x}}{\underbrace{\left(\frac{\pi}{2}-x\right)^3}_{\to 0}} = \dfrac{0}{0}$

(A) $-1/2$

(B) $\cancel{1/2}$

(C) 2

(D) -2

$$\lim_{x\to\frac{\pi}{2}} \frac{-\operatorname{cosec}^2 x+\sin x}{-3\left(\frac{\pi}{2}-x\right)^2} = -\frac{1+1}{0}$$

$$\lim_{x\to\frac{\pi}{2}} \frac{+2\operatorname{cosec}^2 x\cot x+\cos x}{+6\left(\frac{\pi}{2}-x\right)}$$

Put $\frac{\pi}{2}-x=t$ | As $x\to\frac{\pi}{2}$

$x=\frac{\pi}{2}-t$ | $\Downarrow$

$t\to 0$

$$\lim_{t\to 0} \frac{2\sec^2 t\,\overset{\to 1}{\tan t}}{6\,t}+\frac{\overset{\to 1}{\sin t}}{6t}$$

since

$\operatorname{cosec} x\to \operatorname{cosec}\left(\frac{\pi}{2}-t\right)=\sec t$

$\cot x\to \cot\left(\frac{\pi}{2}-t\right)=\tan t$

$\cos x\to \cos\left(\frac{\pi}{2}-t\right)=\sin t$

$$\frac{2}{6}(1)+\frac{1}{6}=\frac{3}{6}=\frac{1}{2}$$

QUESTION

#Q. $\lim_{x\to 0}\left(\sin \overset{\to 0}{mx}\, cot \overset{\to N.D}{\frac{x}{\sqrt{3}}}\right) = 2$, then $m =$

$$\lim_{x\to 0} \frac{\sin \overset{\to 0}{mx}}{\underset{\downarrow 0}{\tan \frac{x}{\sqrt{3}}}} = 2$$

$$\lim_{x\to 0} \frac{m\cos mx}{\sec^2\left(\frac{x}{\sqrt{3}}\right)\left(\frac{1}{\sqrt{3}}\right)} = 2$$

$$\frac{m(1)}{(1)\frac{1}{\sqrt{3}}} = 2$$

$m = \frac{2}{\sqrt{3}}$

QUESTION

#Q. Evaluate $\lim_{x \to \frac{\pi}{6}} \frac{2\sin^2 x + \sin x - 1}{2\sin^2 x - 3\sin x + 1}$

Hint Put $\sin x = t$

$\therefore$ As $x \to \frac{\pi}{6}$ | $t \to \sin\frac{\pi}{6} = \frac{1}{2}$

$$\lim_{t \to \frac{1}{2}} \frac{2t^2 + t - 1}{2t^2 - 3t + 1} \quad \to 0$$

$$= \lim_{t \to 1/2} \frac{4t + 1}{4t - 3} = \frac{4/2 + 1}{4/2 - 3} = \frac{3}{-1} = -3$$

$$\lim_{x\to 3^-} \frac{x}{[x]}$$

(A) 1

(B) $\frac{2}{3}$

(C) $\frac{3}{2}$

(D) $\frac{1}{3}$

$$\frac{\lim\limits_{x\to 3^-} x}{\lim\limits_{x\to 3^-} [x]} = \frac{3}{2}$$

$$\lim_{x\to 3^-} [x] = 2$$

$$\lim_{x\to 3^+} [x] = 3$$

QUESTION

#Q. $\lim_{x \to 3^+} \frac{x}{[x]} =$ $\frac{3}{3} = 1$

(A) 1

(B) $\frac{3}{2}$

(C) $\frac{2}{3}$

(D) $\frac{1}{3}$

QUESTION

#Q. Let $f(x)=\begin{cases}\frac{k\cos x}{\pi-2x} & \text{when } x\neq\frac{\pi}{2}\\ 3 & x=\frac{\pi}{2}\end{cases}$ and if $\lim_{x\to\frac{\pi}{2}} f(x)=f\left(\frac{\pi}{2}\right)$ find the value of k.

$\Downarrow$

$$\lim_{x\to\frac{\pi}{2}}\frac{k\cos x}{\pi-2x}=3$$

$$\lim_{x\to\frac{\pi}{2}}\frac{k(-\sin x)}{0-2}=3$$

$$\frac{-k(1)}{-2}=3$$

$$\frac{k}{2}=3$$

$$k=6$$

QUESTION

#Q. Let $f(x) = \begin{cases} x+2 & x \leq -1 \\ cx^2 & x > -1 \end{cases}$, find $'c'$ if $\lim\limits_{x \to -1} f(x)$ exists.

LHL = RHL

$$\lim_{x \to -1^-} x+2 = \lim_{x \to -1^+} cx^2$$

$$-1+2 = c(-1)^2$$

$$1 = c$$

Types of Problems in continuity

① To find value of K (L-Hospitals rule)

② Points of Discontinuity

③ Based on ① "every differentiable func is continuous"

② "every discontinuous func is not Differentiable"

③ "every continuous func need not be differentiable"

Continuity

A func $f(x)$ is said to be continuous at $x = a$

if $\lim_{x \to a} f(x) = f(a)$

$\Downarrow$

limiting value = Functional value

$$\lim_{x \to 0} \frac{\tan^2 3x}{x^2}$$

$$\lim_{x \to 0} \left(\frac{\tan 3x}{x}\right)^2$$

$$= \lim_{x \to 0} \left(\frac{\tan 3x}{3x} \times 3\right)^2$$

$$= 9 \lim_{x \to 0} \left(\frac{\tan 3x}{3x}\right)^2 \longrightarrow \text{Sandwich Theorem}$$

$$= 9(1) = 9$$

QUESTION

#Q. If $f(x) = \begin{cases} \dfrac{\tan^2\left(\frac{3x}{2}\right)}{x^2} & \text{for } x \neq 0 \\ k+2 & \text{for } x = 0 \end{cases}$ for $x \neq 0$ is $\boxed{\text{continuous at } x = 0}$ then k is

(A) 9/4

(B) 4/9

(C) 1/4

(D) 4

$$\lim_{x\to 0} f(x) = f(0)$$

$$\lim_{x\to 0} \frac{\tan^2 \frac{3x}{2}}{x^2} = k+2$$

$$\lim_{x\to 0}\left(\frac{\tan\frac{3x}{2}}{x}\right)^2 = k+2$$

$$\lim_{x\to 0}\left(\frac{\tan\frac{3x}{2}}{\frac{3x}{2}} \times \frac{3}{2}\right)^2 = k+2$$

$$\frac{9}{4}\lim_{x\to 0}\left(\frac{\tan\frac{3x}{2}}{\frac{3x}{2}}\right)^2 = k+2$$

$$\frac{9}{4}\times 1 = k+2$$

$$k = \frac{9}{4} - 2 = \frac{1}{4}$$

QUESTION

#Q. The value of k for which the function $f(x) = \begin{cases} \frac{x^2+3x-4}{x-1} & \text{for } x \neq 1 \\ k^2+1 & \text{for } x = 1 \end{cases}$ is continuous at $x = 1$ is

A. 2

B. –1

C. ±2

D. 5

$$\lim_{x \to 1} \frac{x^2+3x-4}{x-1} = k^2+1$$

$$\lim_{x \to 1} 2x+3 = k^2+1$$

$5 = k^2+1$

$k^2 = 4$

$k = \pm 2$

QUESTION

$$\frac{d}{dx}\log 1+2x = \frac{1}{1+2x}\frac{d}{dx}(1+2x) = \frac{2}{1+2x}$$

$$\frac{d}{dx}\log 1-2x = \frac{1}{1-2x}\frac{d}{dx}(1-2x) = \frac{-2}{1-2x}$$

#Q. If $f(x) = \begin{cases} \frac{\log_e(1+2x)-\log_e(1-2x)}{x} & \text{for } x \neq 0 \\ e^k & \text{for } x \neq 0 \end{cases}$ is continuous at $x = 0$, then k is

(A) 4

(B) e^4

(C) $\log_e 4$

(D) $\log_4 e$

$$\lim_{x\to 0} \frac{\log(1+\overset{\to 0}{2x}) - \log(1-2x)}{\underset{x\to 0}{x}} = e^k$$

Diff

$$\lim_{x\to 0} \frac{2}{1+2x} - \frac{-2}{1-2x} = e^k$$

$$2+2 = e^k$$

$$e^k = 4$$

$$e^k = 4$$

Take $\log_e$ on B.S

$$\log_e e^k = \log_e 4$$

$$k(\log_e e) = \log_e 4$$

$$k = \log_e 4$$

QUESTION

#Q. If $f(x)=\begin{cases}\frac{1-\cos 4x}{x^2} & \text{if } x<0\\ k & \text{if } x=0\\ \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4} & \text{if } x>0\end{cases}$ is $\boxed{\text{continuous at } x=0}$, then k is

$\Downarrow$

LHL = RHL = f(0)

$\lim_{x\to 0^-}\frac{1-\cos 4x}{x^2} = k$

$\lim_{x\to 0^-}\frac{\cancel{4}^{2}\sin 4x}{\cancel{2}x} = k$

$2\lim_{x\to 0^-}\frac{\sin 4x}{4x}\times 4 = k$

$2(1)4 = k$

$k=8$

(A) 3

(B) 4

(C) 5

(D) 8

QUESTION

#Q. Let $f(x) = \begin{cases} \dfrac{\sin ax^2}{x^2}, & x \neq 0 \\ \dfrac{3}{4} + \dfrac{1}{4a}, & x = 0 \end{cases}$

For what values of a is $f(x)$ continuous at $x = 0$?

-4 $-4+1$

$\sin(x^2) \neq (\sin x)^2$

$$\lim_{x \to 0} \frac{\sin a(x^2)}{(x^2)} = \frac{3}{4} + \frac{1}{4a}$$

$$\lim_{x \to 0} \underbrace{\frac{\sin a(x^2)}{a(x^2)}}_{\to 1} \times a = \frac{3}{4} + \frac{1}{4a}$$

$$a = \frac{3}{4} + \frac{1}{4a}$$

$$a - \frac{1}{4a} = \frac{3}{4}$$

$$\frac{4a^2 - 1}{\cancel{4}a} = \frac{3}{\cancel{4}}$$

$$4a^2 - 1 = 3a$$

$$4a^2 - 3a - 1 = 0$$

$$4a^2 - 4a + a - 1 = 0$$

$$4a(a-1) + 1(a-1) = 0$$

$$(4a+1)(a-1) = 0$$

$a = -\frac{1}{4}$

or

$a = 1$

Thank

You