Ultimate kcet crash course 2026

Maths

DPP: 1

Differentiability

Q1 If $y = \sin(m \sin^{-1} x)$ then $\frac{dy}{dx}$ is

(A) $\frac{m\sqrt{1-x^2}}{\sqrt{1-y^2}}$

(B) $\frac{m\sqrt{1-y^2}}{\sqrt{1-x^2}}$

(C) $-\frac{m\sqrt{1-x^2}}{\sqrt{1-y^2}}$

(D) $-\frac{m\sqrt{1-y^2}}{\sqrt{1-x^2}}$

Q2 The function $f(x)=\begin{cases} x^2 & \text{if } x \geq 0 \\ x & \text{if } x < 0 \end{cases}$

(A) Differentiable at x = 0

(B) Differentiable for all $x \in R$

(C) Not differentiable for all $x \in R$

(D) Not differentiable at x = 0

Q3 If $y = e^{\{\sin^2 x + \sin^4 x + \sin^6 x + \ldots\}}$ then *dy/dx* =

(A) $e^{\tan^2 x}$

(B) $e^{2\tan x \cdot \sec^2 x}$

(C) $e^{\tan^2 x} \cdot 2\tan x \sec^2 x$

(D) $e^{\sec^2 x}$

Q4 $\frac{d}{dx}\left(2^x + x^{-2} + e^{-x}\right)$ is

(A) $x2^{x-1} - 2x^{-3} - xe^{-x-1}$

(B) $2^x - 2^{x-3} + e^{-x}$

(C) $x2^{x-1} + \frac{1}{2x} + \frac{1}{e^x}$

(D) $2^x \log_e 2 - \frac{2}{x^3} - e^{-x}$

Q5 If $xe^{xy} = y + \sin^2 x$, then at $x = 0$, dy/dx is equal to

(A) 1 (B) -1

(C) 2 (D) 0

Q6 If $y = \sin^{-1}\sqrt{\frac{1-x}{2}}$ then $\frac{dy}{dx} =$

(A) $\frac{-1}{2\sqrt{1-x^2}}$

(B) $\frac{1}{2\sqrt{1-x^2}}$

(C) $\frac{1}{\sqrt{1-x^2}}$

(D) $\frac{-1}{\sqrt{1-x^2}}$

Q7 If $y = \tan^{-1}(1-x)$ find $\frac{dy}{dx}$.

(A) $\frac{-1}{x^2+2x-2}$

(B) $\frac{1}{x^2-2x+2}$

(C) $\frac{-1}{x^2-2x+2}$

(D) $\frac{1}{x^2+2x-2}$

Q8 If $y = \sin^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{2}\right]$ then $\frac{dy}{dx}$ is

(A) $\frac{1}{\sqrt{1-x}}$

(B) $-\frac{1}{\sqrt{1-x^2}}$

(C) $\frac{1}{2}\cos^{-1} x$

(D) $\frac{1}{2\sqrt{1-x^2}}$

Q9 If $y = \log_7(\log x)$, then find $\frac{dy}{dx}$.

(A) $\frac{1}{x \log 7 \log x}$

(B) $\frac{1}{x \log x}$

(C) $\frac{x}{\log 7 \log x}$

(D) None of these

Q10 Let f(x) = e^x, g(x) = $\sin^{-1} x$ and h() = f(g(x)), then h'(x)/h(x) =

(A) $e^{\sin^{-1} x}$ (B) $1/\sqrt{1-x^2}$

(C) $\sin^{-1}x$ (D) $\sin^{-1}x$

Q11 Differentiate $\log_{e^3}(\log x)$ w.r.t. x. at x = e

(A) 1/3e (B) 1/4e

(C) 1/2e (D) 1/e

Q12 If $y = \tan^{-1}(\sec x + \tan x)$ then $\frac{dy}{dx} =$

(A) 1 (B) $\frac{1}{2}$

(C) –1 (D) 0

Q13 Differential coefficient of sec $(\tan^{-1} x)$ w.r.t. x is

(A) $\frac{x}{\sqrt{1+x^2}}$ (B) $\frac{x}{1+x^2}$

(C) $x\sqrt{1+x^2}$ (D) $\frac{1}{\sqrt{1+x^2}}$

Q14 $\frac{d}{dx}\{\log_{10}(\sin^{-1} x^2)\} =$

(A) $\frac{1}{\log 10.(\sin^{-1} x^2).\sqrt{1-x^4}}$

(B) $\frac{2x}{\log 10.(\sin^{-1} x^2).\sqrt{1-x^4}}$

(C) $\frac{-1}{\log 10.(\sin^{-1} x^2).\sqrt{1-x^4}}$

(D) $\frac{2}{\log 10.(\sin^{-1} x^2).\sqrt{1-x^4}}$

Q15 If $y = \log(\tan\sqrt{x})$, $\frac{dy}{dx} =$

(A) $\frac{1}{2\sqrt{x}\tan\sqrt{x}}$ (B) $\frac{\sec^2\sqrt{x}}{\sqrt{x}\sqrt{\tan x\sqrt{x}}}$

(C) $\frac{2\sec^2\sqrt{x}}{\sqrt{x}\tan\sqrt{x}}$ (D) $\frac{\sec^2\sqrt{x}}{2\sqrt{x}\tan\sqrt{x}}$

Q16 Differential coefficient of $\sqrt{\sec\sqrt{x}}$ is

(A) $\frac{1}{4\sqrt{x}}\sec\sqrt{x}\sin\sqrt{x}$

(B) $\frac{1}{4\sqrt{x}}(\sec\sqrt{x})^{3/2}\sin\sqrt{x}$

(C) $\frac{1}{2}\sqrt{x}\sec\sqrt{x}\sin\sqrt{x}$

(D) $\frac{1}{2}\sqrt{x}(\sec\sqrt{x})^{3/2}\sin\sqrt{x}$

Q17 If $y = \tan^{-1}\left(\frac{x^{\frac{1}{3}}+a^{\frac{1}{3}}}{1-x^{\frac{1}{3}}a^{\frac{1}{3}}}\right)$ then $\frac{dy}{dx}$

(A) $\frac{1}{3x^{\frac{2}{3}}\left(1+x^{\frac{2}{3}}\right)}$

(B) $\frac{-1}{3x^{\frac{2}{3}}\left(1+x^{\frac{2}{3}}\right)}$

(C) $\frac{1}{x^{\frac{2}{3}}\left(1+x^{\frac{2}{3}}\right)}$

(D) $\frac{-1}{x^{\frac{2}{3}}\left(1+x^{\frac{2}{3}}\right)}$

Q18 $\frac{d}{dx}\{x^a + a^x + x^x + a^a\} =$

(A) $x^{a-1} + ax + x^x \times \log x$

(B) $ax^{a-1} + a^x \log a + x^x \times \log x$

(C) $ax^{a-1} + a^x \log a + x^x \times \log ex$

(D) $x^a + a^x + x^x$

Q19 If $y = \log_{\tan x}\cot x \cdot \log_{\cot x}\tan x + \cos^{-1}\frac{1-x^2}{1+x^2}$ then y' at $x = \frac{\pi}{4}$ is

(A) $\frac{32}{x^2+16}$ (B) 1

(C) –1 (D) $\frac{32}{16+\pi^2}$

Q20 If $y = a\sin^2 x + b\cos^2 x$ then $\frac{dy}{dx}$ at $x = \frac{\pi}{6}$ is

(A) $\sqrt{2}(a-b)$ (B) 2(b – a)

(C) 0 (D) $(a-b)\frac{\sqrt{3}}{2}$

Q21 If $y = \text{cosec}(3\tan^{-1} x)$, then find dy/dx.

(A) $\frac{-3\ \text{cosec}(\tan^{-1} x)\cot(\tan^{-1} x)}{1+x^2}$

(B) $\frac{3\text{cosec}\ (3\tan^{-1} x)\cot(3\tan^{-1} x)}{1-x^2}$

(C) $\frac{3\ \text{cosec}\ (\tan^{-1} x)\cot(3\tan^{-1} x)}{1+x^2}$

(D) $\frac{-3\ \text{cosec}\ (3\tan^{-1} x)\cot(3\tan^{-1} x)}{1+x^2}$

Q22 If $f(x^5) = 5x^3$, then $f'(x) =$

(A) 3/x (B) $\sqrt[5]{x}$

(C) $\frac{3}{\sqrt[5]{x^2}}$ (D) $\frac{3}{\sqrt[5]{x}}$

Q23 $\frac{d}{dx}\left\{5^{\log x}\right\}=$

(A) $5^{\log x}$

(B) $5^{\log x}\cdot\log 5$

(C) $x\cdot 5^{\log x}\cdot\log^5$

(D) $\frac{5^{\log x}\cdot\log 5}{5}$

Q24 $\frac{d}{dx}\left[\sin^{-1}\left(\frac{3+4x}{5\sqrt{1+x^2}}\right)\right]=$

(A) $\frac{1}{1+x^2}$ (B) $-\frac{1}{1+x^2}$

(C) $\frac{1}{\sqrt{1-x^2}}$ (D) $\frac{1}{\sqrt{1+x^2}}$

Q25 If $y=\log_{\cos x}\sin x, then \frac{dy}{dx}$ is equal to

(A) $\frac{\cot x\log\cos x+\tan x\log\sin x}{(\log\cos x)^2}$

(B) $\frac{\tan x\log\cos x+\cot x\log\sin x}{(\log\cos x)^2}$

(C) $\frac{\cot x\log\cos x+\tan x\log\sin x}{(\log\sin x)^2}$

(D) None of these

Q26 $\frac{d}{dx}\left[\left(\log_e x\right)\left(\log_a x\right)\right]=$

(A) $\frac{\log_a x}{x}$ (B) $\frac{\log_x x}{x}$

(C) $\frac{2\log x}{x}$ (D) $\frac{2\log_a x}{x}$

Q27 If $f(x)=x\cot^{-1}x,$ then $f'(1)=$

(A) $\frac{\pi}{4}-\frac{1}{2}$

(B) $\frac{\pi}{4}+\frac{1}{2}$

(C) $\frac{1}{2}$

(D) $-\frac{1}{2}$

Q28 Differentiate $\log_{e^3}(\log x)$ w.r.t. x. at x = e

(A) 1/3e (B) 1/4e

(C) 1/2e (D) 1/e

Q29 $\frac{d}{dx}\left\{4\cos^3 x^o-3\cos x^o\right\}$

(A) $\frac{-\pi}{60}\sin 3x^\circ$

(B) cos 3x°

(C) tan 3x°

(D) $\frac{\pi}{60}\sin 3x^\circ$

Q30 If $3^x+3^y=3^{x+y}$, then dy/dx=

(A) $\frac{3^x+3^y}{3^x-3^y}$

(B) $\frac{3^x-3^y}{3^x+3^y}$

(C) $3^{x-y}\left(\frac{3^y-1}{1-3^x}\right)$

(D) $\frac{3^x+3^y+y}{1+3^x+y}$

Answer Key

Q1	B
Q2	D
Q3	C
Q4	D
Q5	A
Q6	A
Q7	C
Q8	D
Q9	A
Q10	B
Q11	A
Q12	B
Q13	A
Q14	B
Q15	D
Q16	B
Q17	A
Q18	C
Q19	D
Q20	D
Q21	D
Q22	C
Q23	D
Q24	A
Q25	A
Q26	D
Q27	A
Q28	A
Q29	A
Q30	C

Hints & Solutions

Note: scan the QR code to watch video solution

Q1 Text Solution:

Given $y = \sin(m \sin^{-1} x)$

$\Rightarrow \sin^{-1} y = m \sin^{-1} x$

Differentiating both sides w.r.t. x

$$\frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = m \cdot \frac{1}{\sqrt{1-x^2}}$$

$$\therefore \frac{dy}{dx} = \frac{m \cdot \sqrt{1-y^2}}{\sqrt{1-x^2}}$$

Video Solution:

Q2 Text Solution:

LHD = 1 at x = 0

RDH = 2x at x = 0

= 2(0) = 0

LHD ≠ RHD

∴ f(x) is not differentiable at x = 0.

Video Solution:

Q3 Text Solution:

$$y = e\left[\sin^2 x + \sin^4 x + \sin^6 x + ...\right]$$

$$y = e^{\frac{\sin^2 x}{1-\sin^2 x}} = e^{\tan^2 x}$$

$$\frac{dy}{dx} = e^{\tan^2 x}[2 \tan x]\sec^2 x$$

Video Solution:

Q4 Text Solution:

The required differentiation is $2^x \log_e 2 - \frac{2}{x^3} - e^{-x}$

Video Solution:

Q5 Text Solution:

$$e^{xy} + xe^{xy}\left[xy' + y\right] = y' + 2 \sin x \cos x$$

Putting x = 0, y = 0

$$e^0 + 0 \cdot e^0\left[0 \times \frac{dy}{dx} + 0\right] = \left.\frac{dy}{dx}\right|_{x=0} + 0$$

$$\therefore \left.\frac{dy}{dx}\right|_{x=0} = 1$$

Video Solution:

Q6 Text Solution:

$y = \sin^{-1}\sqrt{\frac{1-x}{2}}$

Use substitution $x = \cos\theta \Rightarrow \theta = \cos^{-1} x$

$y = \theta/2 = \frac{\cos^{-1} x}{2}$

$\frac{dy}{dx} = \frac{-1}{2\sqrt{1-x^2}}$

Video Solution:

Q7 Text Solution:

$y = \tan^{-1}(1-x)$

$\frac{dy}{dx} = \frac{1}{1+(1-x)^2} \times \frac{d}{dx}(1-x)$

$= \frac{1}{1+(1-x)^2} \times(-1)$

$= \frac{-1}{1+1+x^2-2x} = \frac{-1}{x^2-2x+2}$

Video Solution:

Q8 Text Solution:

$y = \sin^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{2}\right] = \frac{1}{2\sqrt{1-x^2}}$

Video Solution:

Q9 Text Solution:

(A)

We have, $y = \log_7(\log x) = \frac{\log(\log x)}{\log 7}$

$\therefore \frac{dy}{dx} = \frac{1}{\log 7} \cdot \frac{1}{\log x} \cdot \frac{1}{x}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{x \log 7 \log x}$

Video Solution:

Q10 Text Solution:

$f(x) = e^x$ and $g(x) = \sin^{-1} x$

Now, h(x) = f(g(x)) = $f(\sin^{-1} x) = e^{\sin^{-1} x}$

$h'(x) = e^{\sin^{-1} x}\frac{1}{\sqrt{1-x^2}}$

$\therefore \frac{h'(x)}{h(x)} = \frac{1}{\sqrt{1-x^2}}$

Video Solution:

Q11 Text Solution:

$y = \log_{e^3}(\log x)$

$= \frac{\log(\log x)}{\log e^3} = \frac{\log(\log x)}{3\log_e e} = \frac{1}{3}\log(\log x)$

$\frac{dy}{dx} = \frac{1}{3}\frac{1}{\log x} \times \frac{1}{x}$

$= \frac{1}{3x \log x}$

$\left.\frac{dy}{dx}\right]_{x=e} = \frac{1}{3e \log e} = \frac{1}{3e}$

Video Solution:

Q12 **Text Solution:**

$$y = \tan^{-1}\left(\sec x + \tan x\right)$$

$$= \tan^{-1}\left(\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right) = \frac{\pi}{4} + \frac{x}{2}$$

$$\frac{dy}{dx} = \frac{1}{2}$$

Video Solution:

Q13 **Text Solution:**

$$\frac{d}{dx}\left[\sec\left(\tan^{-1} x\right)\right]$$

$$= \frac{d}{dx}\left[\sec\left(\sec^{-1}\sqrt{1+x^2}\right)\right]$$

$$= \frac{d}{dx}\left(\sqrt{1+x^2}\right) = \frac{x}{\sqrt{1+x^2}}$$

Video Solution:

Q14 **Text Solution:**

$$\frac{d}{dx}\left\{\log_{10}\left(\sin^{-1} x^2\right)\right\} = \frac{2x}{\log 10.\left(\sin^{-1} x^2\right).\sqrt{1-x^4}}$$

Video Solution:

Q15 **Text Solution:**

$$y = \log\left(\tan\sqrt{x}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\tan\sqrt{x}} \cdot \sec^2\sqrt{x} \cdot \frac{1}{2\sqrt{x}}$$

$$= \frac{\sec^2\sqrt{x}}{2\sqrt{x}\tan\sqrt{x}}$$

Video Solution:

Q16 **Text Solution:**

$$\frac{d}{dx}\left(\sqrt{\sec\sqrt{x}}\right)$$

$$\frac{1}{2\sqrt{\sec\sqrt{x}}} \times \sec\sqrt{x}\tan\sqrt{x} \times \frac{1}{2\sqrt{x}}$$

$$\frac{1}{4\sqrt{x}\sqrt{\sec\sqrt{x}}}\sec\sqrt{x}\frac{\sin\sqrt{x}}{\cos\sqrt{x}}$$

$$\frac{1}{4\sqrt{x}}\left(\sec\sqrt{x}\right)^{1/2}\sec\sqrt{x}\sin\sqrt{x}$$

$$\frac{1}{4\sqrt{x}}\left(\sec\sqrt{x}\right)^{3/2}\sin\sqrt{x}$$

Video Solution:

Q17 **Text Solution:**

$$\frac{dy}{dx} = \frac{1}{3x^{\frac{2}{3}}\left(1+x^{\frac{2}{3}}\right)}$$

Video Solution:

Q18 **Text Solution:**

$\frac{d}{dx}\{x^a + a^x + x^x + a^a\} = ax^{a-1} + a^x \log a + x^x \times \log ex$

Video Solution:

Q19 **Text Solution:**

By change of base property

$$y = \frac{\log\cot x}{\log\tan x} \cdot \frac{\log\tan x}{\log\cot x} + \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$

$$\therefore y = 1 + \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$

$$\text{Let } x = \tan\theta \Rightarrow \theta = \tan^{-1} x$$

$$\therefore y = 1 + \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$$

$$\therefore y = 1 + \cos^{-1}(\cos 2\theta)$$

$$\therefore y = 1 + 2\theta = 1 + 2\tan^{-1} x$$

$$\therefore \frac{dy}{dx} = \frac{2}{1+x^2}$$

$$\left.\frac{dy}{dx}\right|_{x=\frac{\pi}{4}} = \frac{2}{1+\left(\frac{\pi}{4}\right)^2} = \frac{32}{16+\pi^2}$$

Video Solution:

Q20 **Text Solution:**

$$\frac{dy}{dx} = \left(a-b\right)\sin 2x$$

$$when\ x = \frac{\pi}{6}$$

$$\frac{dy}{dx} = \left(a-b\right)\frac{\sqrt{3}}{2}$$

Video Solution:

Q21 **Text Solution:**

$$\text{Let } y = \cos ec\left(3\tan^{-1} x\right)$$

$$\therefore \frac{dy}{dx} = -\text{cosec}\left(3\tan^{-1} x\right)\cdot\cot\left(3\tan^{-1} x\right) \times \frac{3}{1+x^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-3\ \text{cosec}\left(3\tan^{-1} x\right)\cot\left(3\tan^{-1} x\right)}{1+x^2}$$

Video Solution:

Q22 Text Solution:

Given : $f(x^5) = 5x^3$

Let $x^5 = t \Rightarrow x = t^{1/5}$

Also : $x^3 = t^{3/5}$

Now, $f(t) = 5t^{3/5}$

Differentiating both sides w.r.t. 't'

$$f'('t) = 5 \cdot \frac{3}{5} t^{-2/5}$$

$$f'('t) = \frac{3}{\sqrt[5]{t^2}}$$

Replacing t by x (dummy variable)

$$f'(x) = \frac{3}{\sqrt[5]{x^2}}$$

Video Solution:

Q23 Text Solution:

$$\frac{d}{dx}\left\{5^{\log x}\right\} = \frac{5^{\log x} \cdot \log 5}{x}$$

Video Solution:

Q24 Text Solution:

$$\frac{d}{dx}\left[\sin^{-1}\left(\frac{3+4x}{5\sqrt{1+x^2}}\right)\right] = \frac{1}{1+x^2}$$

Video Solution:

Q25 Text Solution:

We have $y = \log_{\cos x} \sin x = \frac{\log \sin x}{\log \cos x}$

$$\therefore \frac{dy}{dx} = \frac{\cot x . \log\cos x + (\log \sin x)\tan x}{(\log \cos x)^2}$$

Video Solution:

Q26 Text Solution:

$$\frac{d}{dx}\left\{\frac{\log x}{\log e} \cdot \frac{\log x}{\log a}\right\} = \frac{1}{\log a}\frac{d}{dx}\left\{(\log x)^2\right\}$$

$$= \frac{1}{\log a} 2 \log x \frac{1}{x} = \frac{2 \log_a x}{x}$$

Video Solution:

Q27 Text Solution:

$$f(x) = x \cot^{-1} x$$

$$f'(x) = x \times \frac{-1}{1+x^2} + \cot^{-1} x$$

$$f'(x) = \frac{-x}{1+x^2} + \cot^{-1} x$$

$$f'(1) = \frac{-1}{1+1} + \cot^{-1}(1)$$

$$= -\frac{1}{2} + \frac{\pi}{4}$$

Video Solution:

Q28 **Text Solution:**

$y = \log_{e^3}(\log x)$

$= \frac{\log(\log x)}{\log e^3} = \frac{\log(\log x)}{3\log_e e} = \frac{1}{3}\log(\log x)$

$\frac{dy}{dx} = \frac{1}{3}\ \frac{1}{\log x} \times \frac{1}{x}$

$= \frac{1}{3x \log x}$

$\left.\frac{dy}{dx}\right]_{x=e} = \frac{1}{3e \log e} = \frac{1}{3e}$

Video Solution:

Q29 **Text Solution:**

$\frac{d}{dx}\{4\cos^3 x^o - 3\cos x^o\} = \frac{d}{dx}\{\cos 3x^o\}$

$= \frac{d}{dx}\left\{\cos\left(\frac{3\pi x}{180}\right)\right.$

$\Rightarrow -\frac{\pi}{60}\sin 3x^o$

Video Solution:

Q30 **Text Solution:**

$3^x + 3^y = 3^{x+y} \Rightarrow 3^x \cdot \log 3 + 3^y \log 3\frac{dy}{dx}$

$= 3^{x+y}\log 3\left(1 + \frac{dy}{dx}\right)$

$\Rightarrow \left(3^y - 3^{x+y}\right)\frac{dy}{dx} = 3^{x+y} - 3^x$

$\Rightarrow \frac{dy}{dx} = \frac{3^{x+y}-3^x}{3^y-3^{x+y}} = \frac{3^x(3^y-1)}{3^y(1-3^x)} = 3^{x-y}\frac{(3^y-1)}{(1-3^x)}$

Video Solution: