GOVERNMENT OF KARNATAKA

DEPARTMENT OF SCHOOL EDUCATION (PRE-UNIVERSITY)

18TH CROSS, MALLESHWARAM, BENGALURU – 560 012

CHAPTERWISE MULTIPLE CHOICE QUESTIONS FOR COMPETATIVE EXAM

SUBJECT:MATHEMATICS – I PUC

NAME OF THE CHAPTER: RELATIONS AND FUNCTIONS

RECAP

Cartesian Products of Sets:

Let A and B be any two non-empty sets. The set of all ordered pairs (a, b) such that $a \in A$ and $b \in B$ is called the cartesian product of the sets A and B and is denoted by $A \times B$.

Thus, $A \times B = [(a, b): a \in A \text{ and } b \in B]$

If $A = \phi$ or $B = \phi$, then we define $A \times B = \phi$.

***Example*:** Let $A = \{a, b, c\}$ and $B = \{p, q\}$.

Then $A \times B = \{(a, p), (a, q), (b, p), (b, q), (c, p), (c, q)\}$

Also$B \times A = \{(p, a), (p, b), (p, c), (q, a), (q, b), (q, c)\}$

✓ For any three sets A, B, C

- $A \times (B \cup C) = (A \times B) \cup (A \times C)$
- $A \times (B \cap C) = (A \times B) \cap (A \times C)$
- $A \times (B - C) = (A \times B) - (A \times C)$

✓ If A and B are any two non-empty sets, then

- $A \times B = B \times A \Leftrightarrow A = B$

✓ If $A \subseteq B$, then $A \times A \subseteq (A \times B) \cap (B \times A)$

✓ If $A \subseteq B$, then $A \times C \subseteq B \times C$ for any set C.

✓ If $A \subseteq B$ and $C \subseteq D$, then $A \times C \subseteq B \times D$

✓ For any sets A, B, C, D

- $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$

✓ For any three sets A, B, C

- $A \times (B' \cup C')' = (A \times B) \cap (A \times C)$
- $A \times (B' \cap C')' = (A \times B) \cup (A \times C)$

Relations

- A relation R from a non-empty set A to a non-empty set B is a subset of the cartesian product A × B. The subset is derived by describing a relationship between the first element and the second element of the ordered pairs in A × B. The second element is called the image of the first element
- Let $R \subseteq A \times B$ and $(a, b) \in R$. Then we say that a is related to b by the relation R and write it as $a \, R \, b$.

  If $(a, b) \in R$, we write it as $a R b$.

Remarks:

(i) A relation may be represented algebraically either by the Roster method or by the Set-builder method.

(ii) An arrow diagram is a visual representation of a relation.

Note:

The total number of relations that can be defined from a set A to a set B is the number of possible subsets of A × B. If n(A) = p and n(B) = q, then n (A × B) = pq and the total number of relations is $\mathbf{2^{pq}}$

Domain and range of a relation

- The set of all first elements of the ordered pairs in a relation R from a set A to a set B is called the domain of the relation R
- The set of all second elements in a relation R from a set A to a set B is called the range of the relation R. The whole set B is called the codomain of the relation R. Note that range ⊂ codomain
- Dom $(R) = \{a: (a, b) \in R\}$ and Range $(R) = \{b: (a, b) \in R\}$.

Inverse relation

Let A, B be two sets and let R be a relation from a set A to a set B. Then the inverse of R, denoted by R^{-1}, is a relation from B to A and is defined by $R^{-1} = \{(b, a) : (a, b) \in R\}$

Clearly $(a, b) \in R \Leftrightarrow (b, a) \in R^{-1}$. Also, Dom (R) = Range (R^{-1}) and Range (R) = Dom(R^{-1})

Example: Let $A = \{a, b, c\}$, $B = \{1, 2, 3\}$ and $R = \{(a, 1), (a, 3), (b, 3), (c, 3)\}$.

Then, (i) $R^{-1} = \{(1, a), (3, a), (3, b), (3, c)\}$

(ii) Dom $(R) = \{a, b, c\}$ = Range(R^{-1})

(iii) Range $(R) = \{1, 3\}$ = Dom(R^{-1})

Functions

- A relation f from a set A to a set B is said to be a function if every element of set A has one and only one image in set B.
- In other words, a function f is a relation from a non-empty set A to a non-empty set B such that the domain of f is A and no two distinct ordered pairs in f have the same first element.
- If f is a function from A to B and (a, b) ∈ f, then f (a) = b, where b is called the image of a under f and a is called the preimage of b under f.

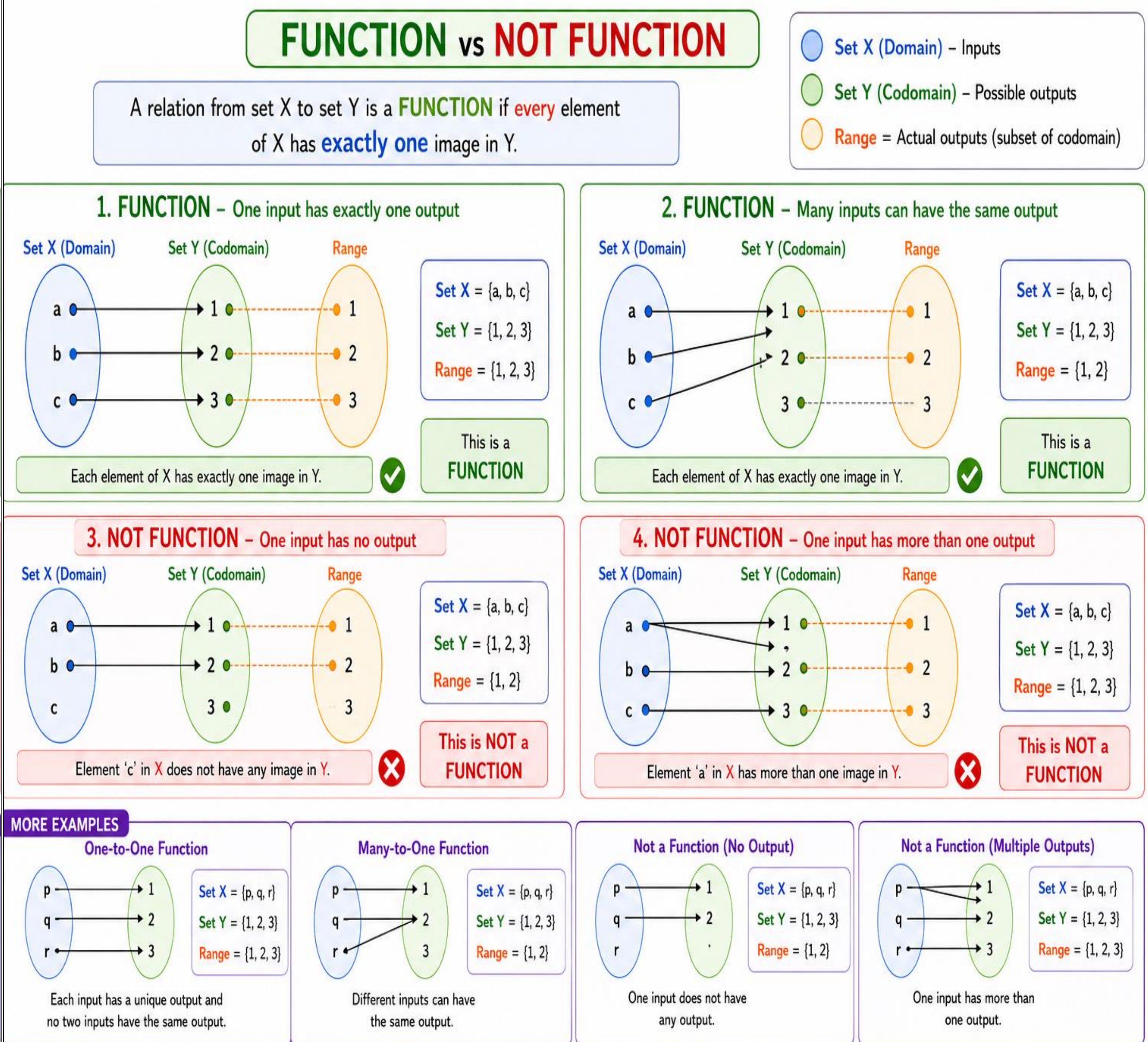
- The function f from A to B is denoted by f: A $\to$ B.

Two things should always be kept in mind:

(i) A mapping $f : X \to Y$ is said to be a function if each element in the set X hasits image in set Y. It is also possible that there are few elements in set Y which are not the images of any element in set X.

(ii) Every element in set X should have one and only one image. That means it is impossible to have more than one image for a specific element in set X. Functions can not be multi-valued

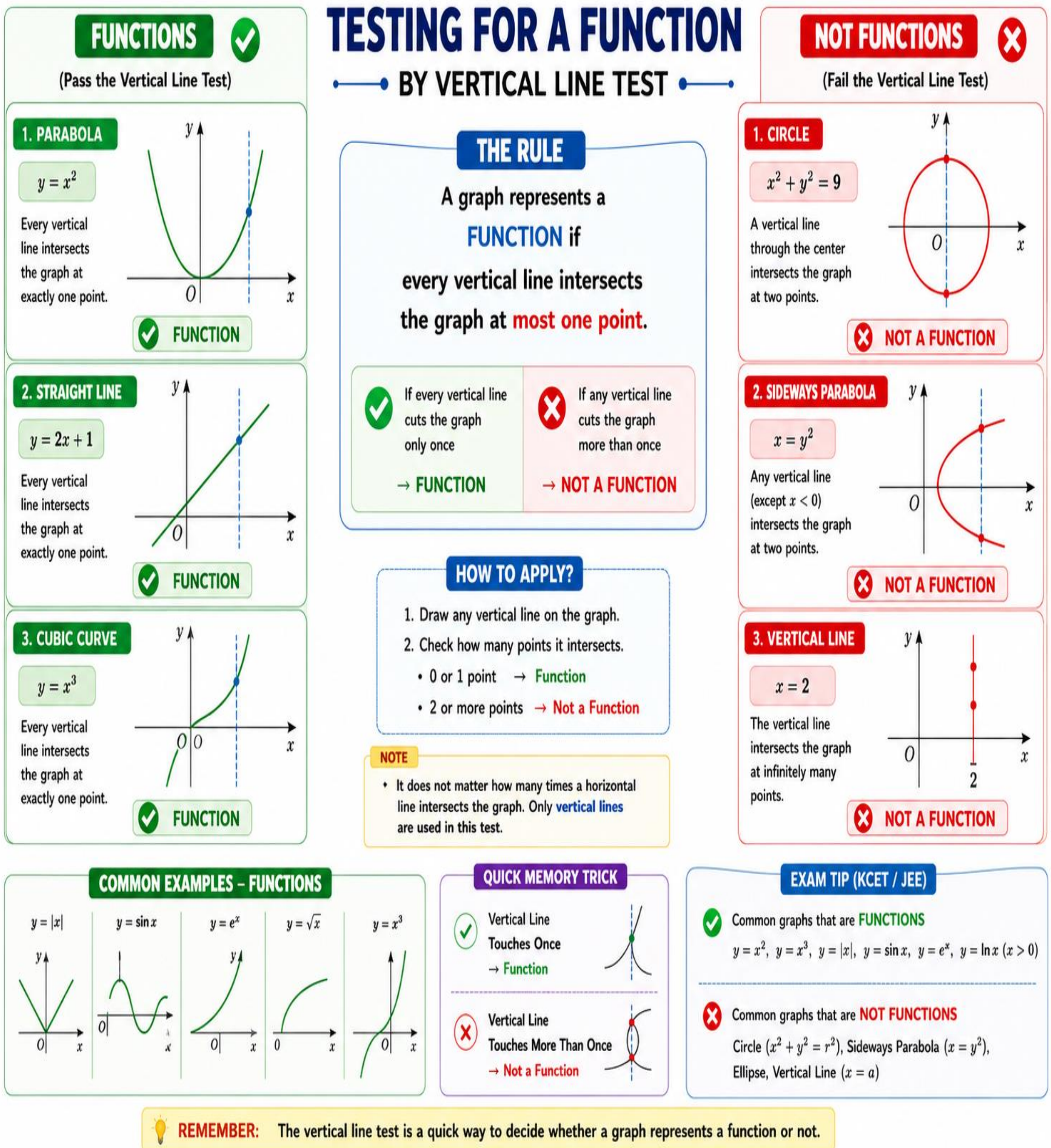
(A mapping that is multi-valued is called a relation from X and Y) *e.g.*



Testing for a function by vertical line test:

A relation $f : A \to B$ is a function or not it can be checked by a graph of the relation.

If it is possible to draw a vertical line which cuts the given curve at more than one point then the given relation is not a function and when this vertical line means line parallel to Y-axis cuts the curve at only one point then it is a function.



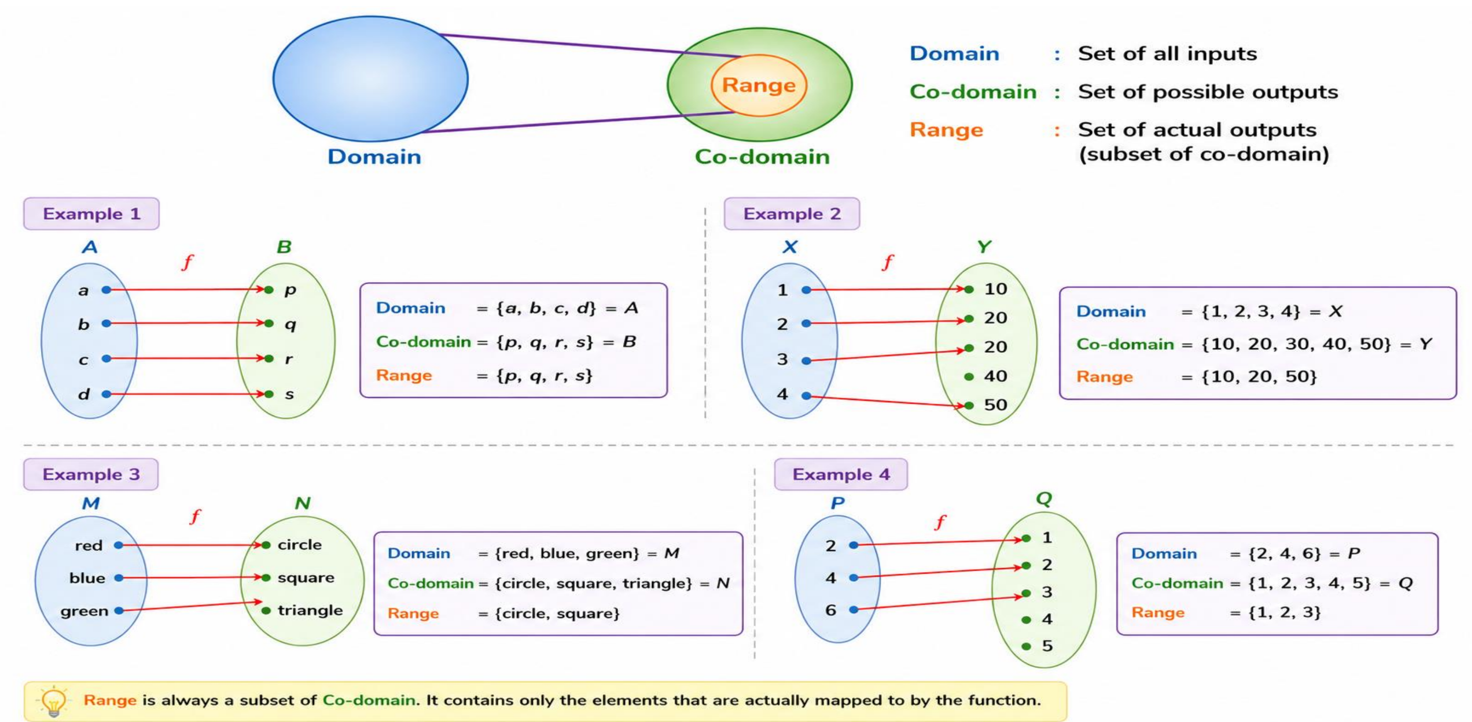
Domain, co-domain and range of function

If a function f is defined from a set A to set B then for $f : A \to B$ set A is called the domain of function f and set B is called the co-domain of function f.

The set of all f-images of the elements of A is called the range of function f.

In other words, we can say: Domain = All possible values of x for which $f(x)$ exists.

Range = For all values of x, all possible values of $f(x)$.



Methods for finding domain and range of function

(i) **Domain**

(a) Expression under even root (*i.e.*, square root, fourth root etc.) ≥ 0. Denominator $\neq 0$.

If domain of $y = f(x)$ and $y = g(x)$ are D_1 *and* D_2 respectively then the domain of $f(x) \pm g(x)$ or $f(x).g(x)$

is $D_1 \cap D_2$

Domain of $\dfrac{f(x)}{g(x)}$ is $D_1 \cap D_2 - \{g(x) = 0\}$ Domain of $\sqrt{f(x)} = D_1 \cap \{x : f(x) \geq 0\}$

(ii) **Range:** Range of $y = f(x)$ is collection of all outputs $f(x)$ corresponding to each real number in the domain.

(a) If domain $\in$ finite number of points $\Rightarrow$ range $\in$ set of corresponding $f(x)$ values.

(b) If domain $\in R$ or R – [some finite points]. Then express x in terms of y.

From this find y for x to be defined (*i.e.*, find the values of y for which x exists).

(c) If domain $\in$ a finite interval, find the least and greatest value for range using monotonicity.

Domain and Range of Some Standard Functions

Function	Domain	Range
Polynomial function	R	R
Identity function x	R	R
Constant function K	R	$\{K\}$
Reciprocal function $\frac{1}{x}$	R_0	R_0
x^2, $\lvert x \rvert$	R	$R^+ \cup \{0\}$
x^3, $x\lvert x \rvert$	R	R
Signum function	R	$\{-1, 0, 1\}$
$x + \lvert x \rvert$	R	$R^+ \cup \{0\}$
$x - \lvert x \rvert$	R	$R^- \cup \{0\}$
$[x]$ (Greatest Integer Function)	R	I (Integers)
$x - [x]$ (Fractional Part)	R	$[0, 1)$
$\sqrt{x}$	$[0, \infty)$	$[0, \infty)$
x^n $(n \in N)$	R	R (n odd), $[0, \infty)$ (n even)
$\sqrt[n]{x}$ $(n \in N)$	R	R (n odd) ; $[0, \infty)$ (n even)
a^x $(a > 0,\ a \neq 1)$	R	R^+
$\log_a x$ $(a > 0,\ a \neq 1)$	R^+	R
e^x	R	$(0, \infty)$
$\ln x$	$(0, \infty)$	R
$\sin x$	R	$[-1, 1]$
$\cos x$	R	$[-1, 1]$
$\tan x$	$R - \left\{ \frac{\pi}{2} + k\pi \mid k \in Z \right\}$	R
$\cot x$	$R - \{ k\pi \mid k \in Z \}$	R
$\sec x$	$R - \left\{ \frac{\pi}{2} + k\pi \mid k \in Z \right\}$	$R - \{-1, 1\}$
$\text{cosec}\, x$	$R - \{ k\pi \mid k \in Z \}$	$R - \{-1, 1\}$
$\sin^{-1} x$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$\cos^{-1} x$	$[-1, 1]$	$[0, \pi]$
$\tan^{-1} x$	R	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$\cot^{-1} x$	R	$(0, \pi)$
$\sec^{-1} x$	$R - \{-1, 1\}$	$\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$
$\text{cosec}^{-1} x$	$R - \{-1, 1\}$	$\left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$
$\lvert x \rvert$	R	$[0, \infty)$
$\lvert x - a \rvert$	R	$[0, \infty)$
$x^2 + a$ $(a \in R)$	R	$[a, \infty)$
$ax + b$	R	R $(a \neq 0)$, $\{b\}$ $(a = 0)$
$\frac{ax + b}{cx + d}$ $(ad - bc \neq 0)$	$R - \left\{ -\frac{d}{c} \right\}$	$R - \left\{ \frac{a}{c} \right\}$
$\frac{1}{1 + x^2}$	R	$(0, 1]$
$\frac{e^x - 1}{e^x + 1}$	R	$(-1, 1)$
e^x	R	$(0, \infty)$
$\frac{1}{x}$	R_0	R_0
$[x]$ (Least Integer Function)	R	I (Integers)
$\{x\} = x - [x]$ (Fractional Part)	R	$[0, 1)$
$x!$ $(x \in I,\ x \geq 0)$	I, $x \geq 0$	I, $x \geq 1$
nCr $(n \geq r \geq 0,\ n, r \in I)$	$n \geq r \geq 0$	I, $x \geq 0$

Even and Odd function

(1) **Even function:** If we put (–x) in place of x in the given function and if $f(-x) = f(x)$, $\forall x \in$ domain then function $f(x)$ is called even function. *e.g.*

$f(x) = e^x + e^{-x}$, $f(x) = x^2$, $f(x) = x\sin x$, $f(x) = \cos x$, $f(x) = x^2\cos x$ all are even functions.

(2) **Odd function:** If we put (–x) in place of x in the given function and if

$f(-x) = -f(x), \forall x \in$domain then $f(x)$ is called odd function. *e.g.*,

$f(x) = e^x - e^{-x}$, $f(x) = x^3$, $f(x) = \sin x$, $f(x) = x\cos x$, $f(x) = x^2\sin x$ all are odd functions.

Properties of even and odd function

• The graph of even function is always symmetric with respect to y-axis. The graph of odd function is always symmetric with respect to origin.

• The product of two even/odd functions is an even function.

• The sum and difference of two even functions is an even function.

• The sum and difference of two odd functions is an odd function.

• The product of an even and an odd function is an odd function.

It is not essential that every function is even or odd. It is possible to have some functions which are neither even nor odd function. *e.g.* $f(x) = x^2 + x^3$, $f(x) = \log_e x$, $f(x) = e^x$.

• The sum of even and odd function is neither even nor odd function.

• Zero function $f(x) = 0$ is the only function which is even and odd both.

Algebra of real functions

(1) **Scalar multiplication of a function:**

$(cf)(x) = c.f(x)$ where c is a scalar. The new function $c.f(x)$ has the domain X_f

(2) **Addition/subtraction of functions:**

$(f \pm g)(x) = f(x) \pm g(x)$ The new function has the domain X.

(3) **Multiplication of functions:**

$(fg)(x) = (g.f)(x) = f(x).g(x)$ The product function has the domain X.

(4) **Division of functions:**

(i) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ The new function has the domain X, except for the values of x for which $g(x) = 0$

(ii) $\left(\frac{g}{f}\right)(x) = \frac{g(x)}{f(x)}$ The new function has the domain X, except for the values of x for which $f(x) = 0$

(5) **Equal functions:** Two function f and g are said to be equal functions, if and only if

(i) Domain of f = Domain of g (ii) Co-domain of f = Co-domain of g

(iii) $f(x) = g(x)\ \forall x \in$ their common domain

Relations:

1. **Let** $\left(\frac{x}{2}-1, \frac{y}{9}+1\right)=(2,1)$ **, then the values of** ***x*** **and** ***y*** **respectively re**

(1) 3, 5 (2) 6, 0 (3) 5, 3 (4) 0, 6

2. **If** $G=\{7,8\}$ and $H=\{5,4,2\}$ **then** $G\times H$ **is**

(1) $\{(7,5),(7,4),(7,2),(8,4),(8,2)\}$ (2) $\{(7,5),(7,2),(8,5),(8,4),(8,2)\}$

(3) $\{(7,5),(7,4),(7,2),(8,5),(8,4),(8,2)\}$ (4) $\{(5,7),(4,7),(2,7),(5,8),(4,8),(2,8)\}$

3. **If** $A\times B=\{(a,x),(a,y)(b,x),(b,y)\}$ **then** ***A*** **and** ***B*** **are**

(1) $\{a,b\}$ and $\{x\}$ (2) $\{a,b\}$ and $\{x,y\}$ (3) $\{(a,b)\}$ and $\{x,y\}$ (4) $\{a\}$ and $\{x,y\}$

4. **Which of the following is true**

(1) If $P=(m,n)$ and $Q=(n,m)$ then $P\times Q=\{(m,n)(n,m)\}$

(2) If *A* and *B* are nonempty sets, then $A\times B$ is a nonempty set of ordered pairs $(x\ y)$ such that $x\in B$ and $y\in A$

(3) If $A=\{1,2\}$ $B=\{3,4\}$ then $A\times(B\cap\phi)=\phi$

(4) If A and *B* are two sets then $A\times B=B\times A$

5. **Let** $A=\{1,2\}, B=\{\{1\},\{2\}\}, C=\{\{1\},\{1,2\}\}$ **. Then which of the following relation is true?**

(1) A = B (2) $B\subseteq C$ (3) $A\in C$ (4) $A\subset C$

6. **If** ***A*** **is the null set and** ***B*** **is an infinite set, then** $A\times B$

(1) infinite set (2) ϕ (3) undefined (4) singleton set

7. **If** ***A*** **and** ***B*** **are two sets, then** $A\times B=B\times A$ **iff**

(1) $A\subset B$ (2) $B\subset A$ (3) $A=B$ (4) $A\cap B=\phi$

8. **Suppose that the number of elements in set A is p, the number of elements of in set B is q and the number of elements in A x B is 7 then** $p^2+q^2=$

(1) 50 (2) 42 (3) 51 (4) 49

9. **If two sets A and B have 99 elements in common, then the number of elements common to the sets A × B and B × A is**

(1) 2^{99} (2) 99^2 (3) 100 (4) 18

10. **If n(A) = 4, n(B) = 3, n(A × B × C) = 24, then n(C) is equal to**

(1) 288 (2) 12 (3) 2 (4) 17

11. **Let** $A=\{1,2,3\}$, $B=\{3,4\}$ **and** $C=\{4,5,6\}$, **then** $(A\times B)\cap(A\times C)=$

(1) $\{(1,4),(2,4),(3,4)\}$ (2) $\{(4,2),(3,4),(4,1)\}$ (3) $\{(2,4),(1,4)\}$ (4) $\{(1,4),(2,4)\}$

12. If $A=\{1,2,3\}$ and $B=\{3,8\}$ **then** $(A\cup B)\times(A\cap B)$ **is**

(1) $\{(3,1),(3,3),(3,8)\}$ (2) $\{(1,3),(2,3),(3,3),(8,3)\}$

(3) $\{(1,2),(2,2),(3,3),(8,8)\}$ (4) $\{(8,3),(8,2),(8,1),(8,8)\}$

13. If $A=\{x:x\in W, x<2\}$ $B=\{x:x\in N, 1<x<5\}$ $C=\{3,5\}$ **then** $A\times(B\cap C)$ **is**

(1) $\{(0,3),(1,3)\}$ (2) $\{(0,3)\}$ (3) $\{(1,3)\}$ (4) $\{(3,0),(3,1)\}$

14. The Cartesian product of $A\times A$ **has 9 elements, two of which are (–1, 0) and (0, 1), the remaining elements of** $A\times A$ **is given by**

(1) $\{(-1,1),(0,0),(-1,-1),(1,-1),(0,-1)\}$ (2) $\{(-1,1),(0,0),(1,-1),(1,0),(1,1),(0,1)\}$

(3) $\{(1,0),(0,-1),(0,0),(-1,-1),(1,-1),(1,1)\}$ (4) $\{(1,0),(0,-1)\}$

15. Let $A=\{1,2\}$ and $B=\{3,4\}$ **then number of subsets of** $A\times B$ **will have**

(1) 8 (2) 16 (3) 32 (4) 64

16. If number of elements in sets *A* and *B* are *p and q* respectively, then the number of relations from *A* to *B* is

(1) 2^{m+n} (2) 2^{mn} (3) $m+n$ (4) mn

17. Let $n(A)=m$ **and** $n(B)=n,$ **then the total number of nonempty relations that can be defined from A to *B* is**

(1) m^n (2) n^m-1 (3) $mn-1$ (4) $2^{mn}-1$

18. If n(A)=2 and total number of possible relations from set A to set B is 1024, then n(B) is

(1) 20 (2) 10 (3) 5 (4)512

19. The sets *A* and *B* are not singletons and $n(A\times B)=21.$ **If** $A\subset B, [n(A),n(B)]=$

(1) $(10,11)$ (2) $(11,10)$ (3) $(3,7)$ (4) $(7,5)$

20. Let $A=\{1,2,3,4,5,6,\}$ **define a relation *R* from *A* to *A* by** $R=\{(x,y);\ y=x+1\}$ **then *R* is**

(1) $R=\{(1,2),(2,3),(3,4),(4,5),(6,7)\}$ (2) $R=\{(1,2),(2,3),(3,4),(4,5)\}$

(3) $R=\{(1,2),(2,3),(3,4),(4,5),(5,6)\}$ (4) $R=\{(1,2),(2,3),(3,4)\}$

21. Let $A=\{1,2,3,4,6\}$**.Let*R* be the relation on *A* defined by** $\{(a,b):a,b\in A, b \text{ is exactly divisible by } a\}$ **then which of the following is false**

(1) $R=\{(1,1),(1,2),(1,3),(1,4),(1,6),(2,2),(2,4),(2,6),(3,3),(3,6),(4,4),(6,6)\}$

(2) Domain of R is $\{1,2,3,4,6\}$

(3) Range of R is $\{1,2,3,4\}$

(4) Range of R is $\{1,2,3,4,6\}$

22. The relation R defined on the set of natural numbers as {(a, b): a differs from b by 3} is given

(1) {(1, 4), (2, 5), (3, 6), ...} (2) {(5, 1), (6, 2), (7, 3), ...}

(3) {(1, 3), (2, 6), (3, 9), ...} (3) {(3, 1), (6, 2), (9, 3), ...}

23. The ordered pair (5,2) belongs to the relation

(1) $\{(x, y): y = x-5, x, y \in Z\}$ (2) $\{(x, y): y = x+5, x, y \in Z\}$

(3) $\{(x, y): y = x-3, x, y \in Z\}$ (4) $\{(x, y): x = y-3, x, y \in Z\}$

24. Let R be a relations in N defined by $R = \{(1+x, 1+x^2): x \le 5, x \in N\}$. Which of the following is false?

(1) R = {(2,2), (3,5), (4,10), (5,17), (6,25)} (2) Domain of R = {2,3,4,5,6}

(3) Range of R = {2,5,10,17,26} (4) R = {(2,2), (3,5), (4,10), (5,17), (6,26)}

25. If $R = \{(x, y) \mid y = 2x+7,$ where $x \in R$ and $-5 \le x \le 5\}$ is a relation, then which of the following is false?

(1) Domain of R is $(-5,5)$ (2) Domain of R is $[-5,5]$

(3) Range of R is $[-3,17]$ (4) $R = \{(x, y) \mid y = 2x+7, x \in R \text{ and } -5 \le x \le 5\}$

26. Let *N* be the set of all natural numbers and let $R = \{(a,b): a, b \in N \text{ and } 2a+2b = 10\}$ then Range of *R* is

(1) $\{1,2,3,4\}$ (2) $\{2,4,6,8\}$ (3) $\{2,3,4\}$ (4) $\{2,6,8\}$

27. Let the relation R is defined in N by aRb, if 3a+2b=27 the R is

(1) $\{(1,12),(3,9),(5,6),(7,3)\}$ (2) $\{(1,12),(3,9),(5,6),(7,3),(9,0)\}$

(3) $\left\{\left(0,\frac{27}{2}\right),(1,12),(3,9),(5,6),(7,3)\right\}$ (4) $\{(2,1),(9,3),(6,5),(3,7)\}$

28. Which of the following is true

(1) The ordered pair $(5,2)$ belongs to the relation $R = \{(x, y): y = x-5,\ x,\ y \in z\}$

(2) If $(x-2,\ y+5) = \left(-2,\ \frac{1}{3}\right)$

(3) If $P = \{1,2\}$ then $P \times P \times P = \{(1,1,1),(2,2,2),(1,2,2),(2,1,1)\}$

(4) $A \times B = \{(a\ x),(a,\ y),(b,x),(b,y)\}$ then $A = \{a,b\}, B = \{x,y\}$

29. If the sets *A* and *B* are defined as

$A = \left\{(x,y) \mid y = \frac{1}{x}, x \ne 0, x \in R\right\}$ **and** $B = \{(x,y) \mid y = -x, x \in R\}$ **then number of elements in** $A \cap B$ **is**

(1) ϕ (2) 0 (3) 1 (4) 2

30. If $R = \{(x, y) \mid x \text{ and } y \text{ are integers and } x^2 + y^2 = 64\}$ is a relation, then find the value of R

(1) $R = \{(0,8),(8,0)\}$ (2) $R = \{(0,8),(-8,0),(8,0),(0,-8)\}$

(3) $R = \{(8,0)\}$ (4) $R = \{(-8,0),(8,0),(0,-8)\}$

RELATIONS									
1	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***	***10***
2	3	2	3	3	2	3	1	2	3
11	***12***	***13***	***14***	***15***	***16***	***17***	***18***	***19***	***20***
1	2	1	3	2	2	4	3	3	3
21	***22***	***23***	***24***	***25***	***26***	***27***	***28***	***29***	***30***
3	1	3	1	1	1	1	4	2	2

Functions:

1. **Which of the following relation is a function?**

(1) $\{(2,1),(5,1),(8,1),(11,1),(14,1),(17,1)\}$ (2) $\{(1,3),(1,5),(2,5)\}$

(3) $\{(2,2),(2,4),(3,3),(4,4)\}$ (4) $\{(4,6),(3,9),(-11,6),(3,11)\}$

2. **Which of the following relation is not a function**

(1) $\{(2,1),(4,2),(6,3),(8,4),(10,5),(12,6),(14,7)\}$

(2) $\{(2,1),(3,1),(4,2)\}$

(3) $\{(1,2),(2,3),(3,4),(4,5),(5,6),(6,7)\}$

(4) $\{(1,5),(1,6),(2,6)\}$

3. **If $f:A\to B$ is a function then which of the following is not true to define a function**

(1) No two elements of A have the same image is B

(2) A and B are finite non empty sets

(3) Elements in A has an image in B

(4) $f:A\to B$ is a relation from A to B

4. **Let A=[-1,1], B=[-1,1], $C=[0,\infty)$. Let $R_1=\{(x,y)\in A\times B : x^2+y^2=1\}$ and**

$R_2=\{(x,y)\in A\times C : x^2+y^2=1\}$

(1) R_1 defines a function from A to B (2) R_2 defines a function from A to C

(3) R_1 and R_2 are both functions (4) Both are not functions

5. **Is $g=\{(1,1),(2,3),(3,5),(4,7)\}$ a function? If this is described by the relation $g(x)=ax+b$ the what value should be assigned to a and b?**

(1) Yes, a=2, b=-1 (2) No, a=1,b=-1 (3) No, a=2, b=-1 (4) Yes, a=1, b=-1

6. **If $f(x)=ax+b$, where a and b are integers $f(-1)=-5$ *and* $f(3)=3$ then a and b are respectively**

(1) 0,2 (2) -3,-1 (3) 2,3 (4) 2,-3

7. **If** $f(x)=x^2$ **then** $\dfrac{f(1\cdot1)-f(1)}{1\cdot1-1}=$

(1) 2 (2) 2.1 (3) 2.5 (4) 0.21

8. **If** $f(x)=2x^2$ **then** $\dfrac{f(3\cdot8)-f(4)}{3.8-4}=$

(1) 156 (2) 0.156 (3) 1.56 (4) 15.6

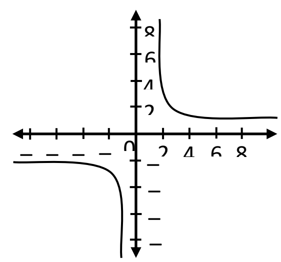
9. **If** $f(x)=x^3-\dfrac{1}{x^3}$, **then** $f(x)+f\left(\dfrac{1}{x}\right)$ **is equal to**

(1) $2x^3$ (2) $2\dfrac{1}{x^3}$ (3) 0 (4) 1

10. **If *f* and *g* are real functions defined by** $f(x)=x^2+7$ **and** $g(x)=3x+5$ **then** $f(3)+g(-5)$ **and** $f(-2)+g(-1)$ **respectively be**

(1) 6,13 (2) 6, 12 (3) 12, 6 (4) 5, 12

11. **The graph of the following function represents the function**



(1) $f:R\to R$ defined by $f(x)=x^2\ x\in R$

(2) $f:R\to R$ defined by $f(x)=x^3$

(3) $f:R-\{0\}\to R$ defined by $f(x)=\dfrac{1}{x}, x\in R-\{0\}$

(4) $f:R\to R$ defined by $f(x)=\sqrt{x}$

12. **Which of the following is a identity function?**

(1) $f:R\to R$ defined by $f(x)=c, x\in R,$ the set of real numbers and C is constant

(2) $f:R\to R$ defined by $f(x)=x^2,\ x\in R,$ the set of real numbers

(3) $f:R\to R$ defined by $f(x)=x$ for each $x\in R$, the set of real numbers.

(4) $f:R\to R$ defined by $f(x)=|x|$ for each $x\in R,$ the set of real numbers

13. **Which of the following is not a constant function**

(1) $f:R\to R$ defined by $f(x)=5,\ x\in R$ (2) $f:R\to R$ defined by $f(x)=k,\ x\in R$ and k is constant

(3) $f:R\to R$ defined by $f(x)=2,\ x\in R$ (4) $f:R\to R$ defined by $f(x)=x,\ x\in R$

14. **Which of the following is not a polynomial function**

(1) $f:R\to R$ defined by $f(x)=x^3-x^2+2\ x\in R$

(2) $f:R\to R$ defined by $f(x)=x^{\frac{2}{3}}+2x\ x\in R$

(3) $f:R\to R$ defined by $f(x)=x^4+\sqrt{2}x\ x\in R$

(4) $f:R\to R$ defined by $f(x)=x^2\ x\in R$

15. **Which of the following is false for the function $f: R \to R$ defined by $f[x]=[x]$ where [*x*] is the greatest integer less than or equal to *x***

(1) $[x]=x$ is an integer

(2) $[x]=$ an inter immediately to the left of x if x is not an integer.

(3) $[3-0.9]=2$

(4) $[-3.29]=-3$

16. **For $f(x)=[x]$, where [x] is the greatest integer function, which of the following is true, for every $x \in R$**

(1) $[x]+1=x$ (2) $[x]+1>x$ (3) $[x]+1 \le x$ (4) $[x]+1<x$

17. **Which one of the following is correct in respect of graph of $y=\dfrac{1}{x-1}$**

(1) The domain is $\{x \in R \mid x \ne 1\}$ and the range is the set of reals

(2) The domain is $\{x \in R \mid x \ne 1\}$, the range is $\{y \in R \mid y \ne 0\}$ and the graph intersect y-axis at (0,-1)

(3) The domain is the set of reals and the range is the singleton set {0}

(4) The domain is $\{x \in R \mid x \ne 1\}$ and the range is the set of points on the y-axis

18. **Consider the following statements in respect of relations and functions:**

1. All relations are functions but all functions are not relations

2. A relation from A to B is a subset of Cartesian product A x B

3. A relation in A is a subset of Cartesian product A x A

Which of the above statement are correct?

(1) 1 and 2 only (2) 2 and 3 only (3) 1 and 3 only (4) 1,2 and 3

19. **If $f(x)=e^{-x}$, then $\dfrac{f(-a)}{f(b)}$ is equal to**

(1) $f(a+b)$ (2) $f(a-b)$ (3) $f(-a+b)$ (4) $f(-a-b)$

20. **If $f(x)=x^2-3x+1$ and $f(2\alpha)=2f(\alpha)$, then α is equal to**

(1) $\dfrac{1}{\sqrt{2}}$ (2) $-\dfrac{1}{\sqrt{3}}$ (3) $\dfrac{1}{\sqrt{2}}$ or $-\dfrac{1}{\sqrt{2}}$ (4) $\dfrac{1}{\sqrt{3}}$ or $-\dfrac{1}{\sqrt{3}}$

21. **If $f: R \to R$ be defined by $f(x)=\begin{cases} 2x & : \quad x>3 \\ x^2 & : 1<x \le 3 \\ 3x & : \quad x \le 1 \end{cases}$ then $f(-1)+f(2)+f(4)$ is**

(1) 5 (2) 9 (3) 10 (4) 14

22. Express the following as set of ordered pairs and determine their range.

$f: x \to R,\ f(x) = x^3 + 1,$ **where** $x = \{-1, 0, 3, 9, 7\}$

(1) $\{1, 28, 344\}$ (2) $\{0, 1, 28\}$ (3) $\{0, 1, 28, 344, 780\}$ (4) $\{344, 730\}$

23. If $f(x) = \sin[\pi^2]x - \sin[-\pi^2]x,$ **where [x] = greatest integer, then which of the following is not true?**

(1) $f\left(\frac{\pi}{2}\right) = 1$ (2) $f\left(\frac{\pi}{4}\right) = 1 + \frac{1}{\sqrt{2}}$ (3) $f(\pi) = -1$ (4) $f(0) = 0$

24. The function $f(x) = |x-2| + |2+x|, -3 \le x < 3$ **is redefined as**

(1) $f(x) = \begin{cases} -2x & -3 \le x < -2 \\ 4 & -2 \le x < 2 \\ 2x & 2 \le x < 3 \end{cases}$

(2) $f(x) = \begin{cases} 2x & -3 \le x < -2 \\ 4 & -2 \le x < 2 \\ -2x & 2 \le x < 3 \end{cases}$

(3) $f(x) = \begin{cases} 4 & -3 \le x < -2 \\ 5 & -2 \le x < 2 \\ 2x & 2 \le x < 3 \end{cases}$

(4) $f(x) = \begin{cases} -2x & -3 \le x < -2 \\ 2x & -2 \le x < 3 \end{cases}$

25. Identify which of the following given relations are not function

1. $h = \{(4,6), (3,9), (-11,6), (3,11)\}$

2. $f = \{(x,x) \mid x\, is\, a\, real\, number\}$

3. $g = \left\{\left(x, \frac{1}{x}\right) \mid x\, is\, a\, postive\, \text{int}\, eger\right\}$

4. $s = \{(x, x^2) \mid x\, is\, a\, postive\, \text{int}\, eger\}$

5. $t = \{(x,3) \mid x\, is\, a\, postive\, \text{int}\, eger\}$

(1) only 1 (2) 1 and 3 (3) 2 and 5 (4) 1,2,3,4 and 5

FUNCTIONS									
1	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***	***10***
1	4	1	2	1	4	2	4	3	1
11	***12***	***13***	***14***	***15***	***16***	***17***	***18***	***19***	***20***
3	3	4	2	4	2	2	2	4	3
21	***22***	***23***	***24***	***25***					
2	3	3	1	1					

Domain & Range:

1. The domain of the function of defined by $f(x) = \frac{1}{\sqrt{x - |x|}}$ **is**

(1) R (2) R^+ (3) R^- (4) ϕ

2. The domain of $f(x) = \frac{1}{\sqrt{2x-1}} - \sqrt{1-x^2}$ **is**

(1) $\left(\frac{1}{2}, 1\right]$ (2) $[-1, \infty)$ (3) $[1, \infty)$ (4) $(-\infty, \infty)$

3. The domain of the function $f(x)=\log_e(x-[x])$ where $[x]$ is greatest integer, is

(1) R (2) $R-Z$ (3) $(0,\infty)$ (4) Z

4. The domain of the function $f(x)=\frac{x^2+2x+1}{x^2-8x+12}$

(1) $R-(2,6)$ (2) $R-\{-2,-6\}$ (3) $R-[2,6]$ (4) $R-\{2,6\}$

5. The domain of the function $f(x)=\frac{\log_3(x+7)}{x^2-5x+6}$ is

(1) $(-7,\infty)-\{2,3\}$ (2) $(-3,\infty)-\{3,2\}$ (3) $(-7,\infty)-\{3\}$ (4) $(-3,\infty)-\{3\}$

6. The domain of the function $f(x)=\log_2(\log_3(\log_4 x))$ is

(1) $(-\infty,4)$ (2) $(4,\infty)$ (3) $(0,4)$ (4) $(1,\infty)$

7. The domain of the function $f(x)=\sqrt{7-3x}+\log_e x$ is

(1) $0<x<\infty$ (2) $0<x\le\frac{7}{3}$ (3) $-\infty<x<0$ (4) $-\infty<x\le\frac{7}{3}$

8. The domain of the function $f(x)=\frac{1}{\log_{10}(1-x)}+\sqrt{x+2}$ is

(1) $[-2,0)\cap(0,1)$ (2) $[-2,1)$ (3) $[-2,0)$ (4) $[-2,0)\cup(0,1)$

9. The domain of the function $f(x)=\log_e[x-[x]]$, where $[x]$ is greatest integer is

(1) R (2) $R-Z$ (3) $(0,\infty)$ (4) Z

10. The domain of the function $\sqrt{\frac{x-7}{9-x}}$ is

(1) $(7,9)$ (2) $[7,9)$ (3) $[7,9]$ (4) $(7,9]$

11. The domain of the real valued function $f(x)=\sqrt{x^2-4}+\frac{1}{\sqrt{x^2-7x+6}}$ is

(1) $R-[-6,-2)$ (2) $R-[-2,6)$ (3) $R-(-2,6]$ (4) $R-(-2,6)$

12. The domain of the function $f(x)=\frac{1}{\sqrt{[x]^2-7[x]+6}}$ where $[x]$ denotes greatest integer is

(1) $(-\infty,1)\cup[7,\infty)$ (2) $(-\infty,1]\cup(7,\infty)$ (3) $(1,7)$ (4) $(-\infty,1)\cup(7,\infty)$

13. The domain of the function $f(x)=\frac{1}{\sqrt{(x-2)(x-5)}}$ is

(1) $(-\infty,2)\cup(5,\infty)$ (2) $(-\infty,3)\cup[5,\infty)$ (3) $(-\infty,3]\cup(5,\infty)$ (4) $(-\infty,2]\cup[5,\infty)$

14. $f(x)=\sqrt{\frac{(x+1)(x-3)}{x-2}}$ **is real valued function in the domain**

(1) $(-\infty,-1]\cup[3,\infty)$ (2) $(-\infty,-1]\cup(2,3]$ (3) $[-1,2)\cup(3,\infty)$ (4) $[-1,2)$

15. If $[x]$ **denotes the greatest integer function** $\leq x$ **then**

$\left[\frac{1}{2}+\frac{1}{1000}\right]+\left[\frac{1}{2}+\frac{2}{1000}\right]++...+\left[\frac{1}{2}+\frac{999}{1000}\right]=$

(1) 498 (2) 499 (3) 500 (4) 501

16. The range of the function $f(x)=\sin[x]-\frac{\pi}{4}<x<\frac{\pi}{4}$ **where** $[x]$ **denotes the greatest integer is**

(1) $\{0\}$ (2) $\{0,-\sin 1\}$ (3) $\{0\pm\sin 1\}$ (4) $\{0,\sin 1\}$

17. The range of the function $f(x)=\frac{1+x^2}{x^2}, x\in R-\{0\}$ **is**

(1) $[2,\infty)$ (2) $(1,\infty)$ (3) $[1,\infty)$ (4) $(-\infty,1]$

18. Let $f:R\to R$ **be a function defined by** $f(x)=x^2+9$**. The range of f is**

(1) $(-\infty,-9]\cup[9,\infty)$ (2) $[9,\infty)$ (3) $[3,\infty)$ (4) $(-\infty,\infty)$

19. The range of the function $f:R\to R$ **defined by** $f(x)=2, x\in R$

(1) $\{0,2\}$ (2) R (3) $\{2\}$ (4) $\{c\}$

20. The range of the function $f(x)=\frac{x^2-x+1}{x^2+x+1}x\in R$ **is**

(1) $(-\infty,3]$ (2) $(-\infty,\infty)$ (3) $[3,\infty)$ (4) $\left[\frac{1}{3},3\right]$

21. The range of the function $f(x)=\frac{1}{2-\cos 3x}$ **is**

(1) $(-2,\infty)$ (2) $\left[\frac{1}{3},1\right]$ (3) $\left(\frac{1}{2},1\right)$ (4) $[-2,3]$

22. Let $f=\left\{\left(x,\frac{x^2}{1+x^2}\right):x\in R\right\}$ **be a function from *R* to *R*. The range of *f* is**

(1) [0, 1] (2) [0, 1) (3) (0, 1) (4) (1, ∞)

23. Let $A=\{1,2,3\}$ *and* $B=\{2,4,6,8\}$**. Consider the function** $f:A\to B, f(x)=2x\,\forall x\in A$**.**

The domain, co-domain and range of f respectively are

(1) {1,2,3}, {2,4,6}, {2,4,6,8} (2) {1,2,3}, {2,4,6,8}, {2,4,6}

(3) {2,4,6,8}, {2,4,6,8}, {1,2,3} (4) {2,4,6}, {2,4,6,8}, {1,2,3}

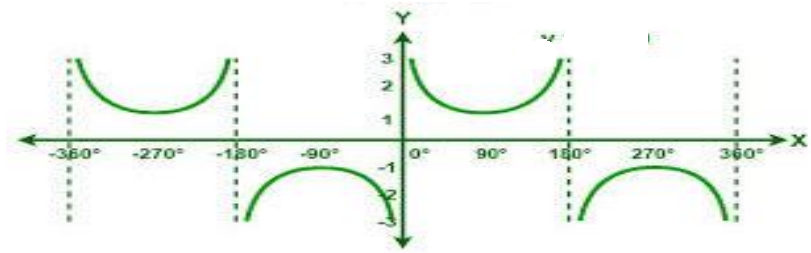
24. Domain and range of the function $f:R\to R$ **defined by** $f(x)=x^2$ **is**

(1) $R, [0,\infty)$ (2) $R,(-\infty,0)$ (3) R,R (4) $(-\infty,0), R$

25. **The domain and range of the function** $f : R \to R$ **defined by** $f(x)=\begin{cases} 1 & \text{if} \quad x>0 \\ 0 & \text{if} \quad x=0 \\ -1 & \text{if} \quad x<0 \end{cases}$ **is**

(1) R, R (2) $R,\{-1,1\}$ (3) $R,\{-1,0,1\}$ (4) $R,[0,\infty)$

26. **Find the domain and range of the function whose graph is shown below**



(1) Domain = $R-[-1,1]$, Range = [-1,1]

(2)Domain = $R-\{x : x = n\pi, n \in Z\}$, Range = $R-(-1,1)$

(3)Domain = $R-\{x : x=(2n+1)\pi, n \in Z\}$, Range = $R-[-1,1]$

(4) Domain = $R-\{x : x = n\pi, n \in Z\}$, Range = R

27. **The domain and range of the function** $f(x)=\sqrt{9-x^2}$ **is**

(1) $[-3,3]$ and $(0,3)$ (2) $[-3,3]$ and $[0, 3)$ (3) $[-3,3]$ and $[0,3]$ (4) $[-3, 3)$ and $(0,3)$

28. **The domain and range of real function *f* defined by** $f(x)=\frac{4-x}{x-4}$ **is given by**

(1) Domain = R, Range = $\{-1, 1\}$ (2) Domain $= R-\{1\}$, Range = R

(3) Domain = $R-\{4\}$, Range $=\{-1\}$ (4) Domain $= R-\{-4\}$, Range $=\{-1,1\}$

29. **The domain and range of the function** $f(x)=\frac{1}{\sqrt{x-2}}$ **is**

(1) Domain $=[2,\infty)$ Range $=R^+$ (2) Domain $=(2,\infty)$ Range $=(0,\infty)$

(3) Domain = $[2,\infty)$ Range $=(0,\infty)$ (4) Domain = $(-\infty,2)$ Range $=(0,\infty)$

30. **The domain and range of the function** $f(x)=\frac{1}{\sqrt{x-[x]}}$ **where** $[x]$ **is the greatest integer**

(1) $R-Z,(1,\infty)$ (2) $R,(1,\infty)$ (3) $R-Z$ $[1,\infty)$ (4) R, $[1,\infty)$

DOMAIN AND RANGE

1	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***	***10***
4	1	2	4	1	2	2	4	2	2
11	***12***	***13***	***14***	***15***	***16***	***17***	***18***	***19***	***20***
3	1	1	3	3	2	2	2	3	4
21	***22***	***23***	***24***	***25***	***26***	***27***	***28***	***29***	***30***
2	2	2	1	3	2	3	3	2	1

Algebra of real functions

1. **Let** $f(x)=\sqrt{1+x^2}$ **then**

(1) $f(xy)=f(x)f(y)$ (2) $f(xy)\geq f(x)f(y)$

(3) $f(xy)\leq f(x)f(y)$ (4) $f(xy)=\frac{f(x)}{f(y)}$

2. **Let *f* and *g* be real functions defined by** $f(x)=2x+1$ and $g(x)=4x-7$ **The value of *x* for which** $f(x)<g(x)$ **is**

(1) $x>2$ (2) $x<2$ (3) $x>4$ (4) $x<4$

3. **If *f* and *g* are two real valued functions defined as** $f(x)=2x+1$ **and** $g(x)=x^2+1$ **then consider the following algebra of functions given in column A and respective answers were given in column B**

Column A		Column B	
(a)	$f+g$	(i)	$-x^2+2x$
(b)	$f-g$	(ii)	$\frac{2x+1}{x^2+1}$
(c)	fg	(iii)	$2x^3+x^2+2x+1$
(d)	$\frac{f}{g}$	(iv)	x^2+2x+2

Which of the following is the correct match?

(1) (a) → (iii); (b) → (iv); (c) → (ii); (d) → (i) (2) (a) → (iv); (b) → (iii); (c) → (i); (d) → (ii)

(3) (a) → (iv); (b) → (i); (c) → (iii); (d) → (ii) (4) (a) → (i); (b) → (ii); (c) → (iv); (d) → (iii)

4. **Let *f* and *g* be two real functions given by**

$f=\{(2,4),(5,6),(8,-1),(10,-3)\}$ **and** $g=\{(2,5),(7,1),(8,4),(10,13),(11,5)\}$

Then consider the following columns

Column A		Column B	
(a)	$f-g$	(i)	$\left\{\left(2,\frac{4}{5}\right),\left(8,-\frac{1}{4}\right),\left(10,-\frac{3}{13}\right)\right\}$
(b)	$f+g$	(ii)	$\{(2,20),(8,-4),(10,-39)\}$
(c)	$f\cdot g$	(iii)	$\{(2,-1),(8,-5),(10,-16)\}$
(d)	$\frac{f}{g}$	(iv)	$\{(2,9),(8,3),(10,10)\}$

Which of the following is the correct match?

(1) (a) → (iii); (b) → (iv); (c) → (ii); (d) → (i) (2) (a) → (iv); (b) → (iii); (c) → (i); (d) → (ii)

(3) (a) → (i); (b) → (ii); (c) → (iv); (d) → (iii) (4) (a) → (ii); (b) → (i); (c) → (iii); (d) → (iv)

5. If $f(x)=\sqrt{x}$ and $g(x)=x$ be two functions defined in the domain $R^{+}\{0\}$, then consider the following algebra of functions given in column A and respective answers were given in column B

Column A		Column B	
(a)	$f+g$	(i)	$-x+\sqrt{x}$
(b)	$f-g$	(ii)	$x^{\frac{3}{2}}$
(c)	fg	(iii)	$x+\sqrt{x}$
(d)	$\frac{f}{g}$	(iv)	$\frac{1}{\sqrt{x}}$

Which of the following is the correct match?

(1) (a) $\rightarrow$ (iii); (b) $\rightarrow$ (i); (c) $\rightarrow$ (ii); (d) $\rightarrow$ (iv)(2) (a) $\rightarrow$ (iv); (b) $\rightarrow$ (iii); (c) $\rightarrow$ (i); (d) $\rightarrow$ (ii)

(3) (a) $\rightarrow$ (iv); (b) $\rightarrow$ (i); (c) $\rightarrow$ (iii); (d) $\rightarrow$ (ii)(4) (a) $\rightarrow$ (i); (b) $\rightarrow$ (ii); (c) $\rightarrow$ (iv); (d) $\rightarrow$ (iii)

6. The domain of which the function defined by $f(x)=3x^2-1$ *and* $g(x)=3+x$ are equal to

(1) $\left\{-1,\frac{4}{3}\right\}$ (2) $\left(-1,\frac{4}{3}\right)$ (3) $(-\infty,-1]\cup\left[\frac{4}{3},\infty\right)$ (4) $(-\infty,-1)\cup\left(\frac{4}{3},\infty\right)$

7. Let f and g be two real functions given by $f=\{(0,1),(2,0),(3,-4),(4,2),(5,1)\}$ and $g=\{(1,0),(2,2),(3,-1),(4,4),(5,3)\}$ then the domain of f.g is given by____

(1) $\{1,2,3,4,5\}$ (2) $\{-1,0,2,3,4\}$ (3) $\{2,3,4,5\}$ (4) $\{-4,0,1,2\}$

8. f (1) = 1, $n\geq 1\Rightarrow f(n+1)=2f(n)+1$ *then* $f(n)=$

(1) 2^{n+1} (2) 2^{n} (3) $2^{n}-1$ (4) $2^{n-1}-1$

9. If $f(x)=\frac{1}{2}-\tan\left(\frac{\pi x}{2}\right),-1<x<1$ *and* $g(x)=\sqrt{3+4x-4x^2}$ find the domain of $(f+g)$

(1) $\left[-\frac{1}{2},1\right)$ (2) $\left(-\frac{1}{2},1\right]$ (3) $\left[-\frac{1}{2},\frac{3}{2}\right]$ (4) $(-1,1)$

10. If $f(x)=4x^3+3x^2+3x+4$, *then* $x^3f\left(\frac{1}{x}\right)=$

(1) $f(-x)$ (2) $\frac{1}{f(x)}$ (3) $\left[f\left(\frac{1}{x}\right)\right]^2$ (4) $f(x)$

Algebra of real functions

1	2	3	4	5	6	7	8	9	10
3	4	3	1	1	1	3	3	1	4

RELATIONS & FUNCTIONS

CLASS 11

Build Strong Concepts. Solve with Confidence!

RELATION

A relation from a set A to a set B is a subset of A × B.

If R is a relation from A to B, then $R \subseteq A \times B$

$\boxed{R \subseteq A \times B}$

FUNCTION

A relation f from A to B is a function if every element of A is related to exactly one element of B.

$\boxed{f : A \to B}$

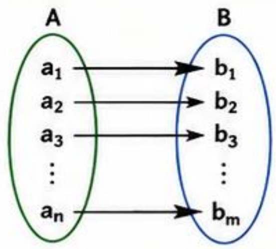
DOMAIN & RANGE

Let $f : A \to B$ be a function.

- **Domain (A)** : Set of all inputs (first elements).
- **Range (f)** : Set of all outputs actually obtained.

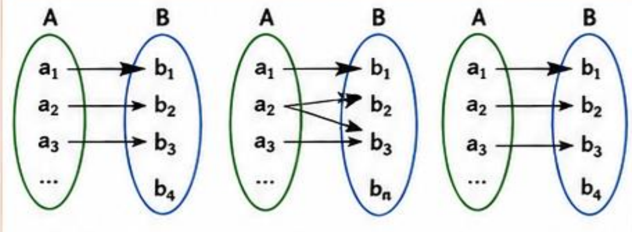
$$\text{Range}(f) = \{f(a) : a \in A\} \subseteq B$$

VISUAL REPRESENTATION



Arrows show mapping from A to B

TYPES OF MAPPINGS (BY DIAGRAM)



Valid Function (Every element of A has exactly one image)

Not a Function (One element of A has more than one image)

Not a Function (At least one element of A has no image)

ALGEBRA OF FUNCTIONS

OPERATIONS ON FUNCTIONS

Let $f, g : A \to \mathbb{R}$ be two functions.

1. **Sum**
   $(f + g)(x) = f(x) + g(x)$
2. **Difference**
   $(f - g)(x) = f(x) - g(x)$
3. **Product**
   $(f \cdot g)(x) = f(x)\, g(x)$
4. **Quotient**
   $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)}, \quad g(x) \neq 0$
5. **Scalar Multiple**
   $(kf)(x) = kf(x), \quad k \in \mathbb{R}$

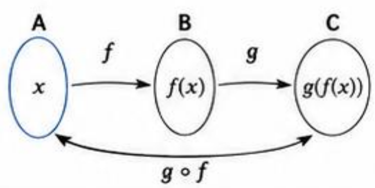
COMPOSITION OF FUNCTIONS

Let $f : A \to B$ and $g : B \to C$.

The composition of g and f, denoted by $g \circ f : A \to C$, is defined by

$$\boxed{(g \circ f)(x) = g(f(x))}$$

$(g \circ f)$ is read as "g of f" or "g after f".



IMPORTANT FORMULAS

- $(f + g)(x) = f(x) + g(x)$
- $(f - g)(x) = f(x) - g(x)$
- $(f \cdot g)(x) = f(x)\, g(x)$
- $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)}, \quad g(x) \neq 0$
- $(kf)(x) = kf(x)$
- $(g \circ f)(x) = g(f(x))$
- **Identity Function on A** : $I_A(x) = x$
- $(f \circ I_A)(x) = f(x) = (I_B \circ f)(x)$
- Domain$(g \circ f) = \{x \in A : f(x) \in Domain(g)\}$

EXAMPLES

Let $f(x) = 2x + 1$ and $g(x) = x^2 - 3$

- $(f + g)(x) = (2x + 1) + (x^2 - 3) = x^2 + 2x - 2$
- $(f - g)(x) = (2x + 1) - (x^2 - 3) = -x^2 + 2x + 4$
- $(f \cdot g)(x) = (2x + 1)(x^2 - 3) = 2x^3 + x^2 - 6x - 3$
- $\left(\dfrac{f}{g}\right)(x) = \dfrac{2x + 1}{x^2 - 3}, \quad x \neq \pm\sqrt{3}$
- $(g \circ f)(x) = g(f(x)) = g(2x + 1) = (2x + 1)^2 - 3$
  $= 4x^2 + 4x - 2$

KEY POINTS

- ✓ Relation is any subset of A × B.
- ✓ Function is a special relation where each input has exactly one output.
- ✓ Domain = set of all inputs.
- ✓ Range = set of actual outputs.
- ✓ Understand and use algebra of functions to solve problems easily.

QUICK CHECK

Q1. Is $R = \{(1, 2), (1, 3), (2, 4)\}$ a function from $\{1, 2\}$ to $\{2, 3, 4\}$?

Ans. No (1 has two images)

Q2. Domain of $f(x) = \sqrt{x - 1}$?

Ans. $[1, \infty)$

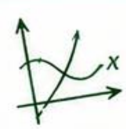
Q3. Find $(f \circ g)(x)$ if $f(x) = 3x - 1$, $g(x) = x^2 + 2$.

Ans. $(f \circ g)(x) = f(g(x)) = 3(x^2 + 2) - 1$
$= 3x^2 + 6 - 1 = 3x^2 + 5$

Practice! Understand! Excel!

MAPPING TODAY, MASTERY TOMORROW!

$f(x)$



$A \to B$