Q1 Two finite sets have m and n elements respectively. The total number of subsets of the first set is 56 more than the total number of subsets of the second set. The values of m and n, respectively are [KCET 2024]

(A) 7, 6 (B) 5, 1

(C) 6, 3 (D) 8, 7

Q2 If $[x]^2 - 5[x] + 6 = 0$, where [x] denotes the greatest integer function, then [KCET 2024]

(A) $x \in [3,4]$

(B) $x \in [2,4)$

(C) $x \in [2,3]$

(D) $x \in (2,3]$

Q3 If in two circles, arcs of the same length subtend angles 30^o and 78^o at the centre, then the ratio of their radii is

(A) 5/13 (B) 13/5

(C) 13/4 (D) 4/13

Q4 If DABC is right angled at C, then the value of tan A + tan B is

(A) a + b (B) a^2/bc

(C) c^2/ab (D) b^2/ac

Q5 The real value of 'a' for which $\frac{1-i\sin\alpha}{1+2i\sin\alpha}$ is purely real is

(A) $(n+1)\frac{\pi}{2},\ n \in Z$

(B) $(2n+1)\frac{\pi}{2},\ n \in Z$

(C) $n\pi,\ n \in N$

(D) $(2n-1)\frac{\pi}{2},\ n \in Z$

Q6 The length of a rectangle is five times the breadth. If the minimum perimeter of the rectangle is 180 cm , then

(A) Breadth $\leq$ 15 cm

(B) Breadth $\geq$ 15 cm

(C) Length $\leq$ 15 cm

(D) Length = 15 cm

Q7 The value of ${}^{49}C_3 + {}^{48}C_2 + {}^{47}C_3 + {}^{46}C_3 + {}^{45}C_3$ is $+ {}^{45}C_4$

(A) ${}^{50}C_4$ (B) ${}^{50}C_3$

(C) ${}^{50}C_2$ (D) ${}^{50}C_1$

Q8 In the expansion of $(1+x)^n \frac{C_1}{C_0} + 2\frac{C_2}{C_1} + 3\frac{C_3}{2} + \ldots + n\frac{C_n}{C_{n-1}}$ is equal to

(A) $\frac{n(n+1)}{2}$ (B) n/2

(C) $\frac{n+1}{2}$ (D) 3n(n + 1)

Q9 If S_n stands for sum of n-terms of a GP with a as the first term and r as the common ratio, then $S_n : S_{2n}$ is

(A) $r^n + 1$ (B) $\frac{1}{r^n+1}$

(C) $r^n - 1$ (D) $\frac{1}{r^n-1}$

Q10 If AM and GM of roots of a quadratic equation are 5 and 4, respectively, then the quadratic equation is

(A) $x^2 - 10x - 16 = 0$

(B) $x^2 + 10x + 16 = 0$

(C) $x^2 + 10x - 16 = 0$

(D) $x^2 - 10x + 16 = 0$

Q11 The angle between the line x + y = 3 and the line joining the points (1, 1) and (-3, 4) is

(A) $\tan^{-1}(7)$ (B) $\tan^{-1}(-1/7)$

(C) $\tan^{-1}(1/7)$ (D) $\tan^{-1}(2/7)$

Q12 The equation of parabola whose focus is (6, 0) and directrix is x = -6 is

(A) $y^2 = 24x$ (B) $y^2 = -24x$

(C) $x^2 = 24y$ (D) $x^2 = -24y$

Q13 $\lim_{x \to \frac{\pi}{4}} \frac{\sqrt{2}\cos x - 1}{\cos x - 1}$ is equal to

(A) 2 (B) $\sqrt{2}$

(C) 1/2 (D) $1/\sqrt{2}$

Q14 The negation of the statement "For every real number x; $x^2 + 5$ is positive" is

(A) For every real number x; $x^2 + 5$ is not positive

(B) For every real number x; $x^2 + 5$ is negative.

(C) There exists atleast one real number x, such that $x^2 + 5$ is not positive.

(D) There exists atleast one real number x, such that $x^2 + 5$ is positive.

Q15 Let a, b, c, d and e be the observations with mean m and standard deviation S. The standard deviation of the observations a + k, b + k, c + k, d + k and e + k is

(A) kS (B) S + k

(C) S/k (D) S

Q16 Let $f : R \to R$ be given f(x) = tan x. Then $f^{-1}(1)$ is

(A) $\frac{\pi}{4}$

(B) $\left\{n\pi + \frac{\pi}{4} : n \in Z\right\}$

(C) $\frac{\pi}{3}$

(D) $\left\{n\pi + \frac{\pi}{3} : n \in Z\right\}$

Q17 Let f : R → R be defined by $f(x) = x^2 + 1$. Then, the pre images of 17 and -3, respectively are

(A) $\phi, \ \{4, \ -4\}$

(B) $\{3, \ -3\}, \ \phi$

(C) $\{4, \ -4\}, \ \phi$

(D) {4, -4}, {2, -2}

Q18 Let $(g \text{ o } f)(x) = \sin\ x$ and $f \text{ o } g(x)$ Then, $= \left(\sin \sqrt{x}\right)^2$.

(A) $f(x) = \sin^2 x, \ g(x) = x$

(B) $f(x) = \sin \sqrt{x}, \ g(x) = \sqrt{x}$

(C) $f(x) = \sin^2 x, \ g(x) = \sqrt{x}$

(D) $f(x) = \sin \sqrt{x}, \ g(x) = x^2$

Q19 Let A = {2, 3, 4, 5,.....,16, 17, 18}. Let R be the relation on the set A of ordered pairs of positive integers defined by (a, b) R(c, d) if and only if ad = bc for all (a, b), (c, d) in A x A. Then the number of ordered pairs of the equivalence class of (3, 2) is

(A) 4 (B) 5

(C) 6 (D) 7

Q20 If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = 3\pi$, equals then $x(y+z)+y(z+x)+z(x+y)$ os

(A) 0 (B) 1

(C) 6 (D) 2

Q21 If $2\sin^{-1} x - 3\cos^{-1} x = 4, \ x \in [-1, \ 1]$, then $2\sin^{-1} x + 3\cos^{-1} x$ is equal to

(A) $\frac{4-6\pi}{5}$

(B) $\frac{6\pi-4}{5}$

(C) $\frac{3\pi}{2}$

(D) 0

Q22 If A is a square matrix, such that $A^2 = A$, then $(I + A)^3$ is equal to

(A) A - I (B) 7A

(C) 7A + I (D) I - 7A

Q23 If $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$, then A^{10} is equal to

(A) 2^8A (B) 2^9A

(C) 2^{10}A (D) 2^{11}A

Q24 If

$$f(x) = \begin{vmatrix} x-3 & 2x^2-18 & 2x^3-81 \\ x-5 & 2x^2-50 & 4x^3-500 \\ 1 & 2 & 3 \end{vmatrix},$$

then f(1) . f(3) + f(3) . f(5) + f(5) . f(1) is

(A) -2183328 (B) -2183330

(C) 2183328 (D) 2183330

Q25 If $P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ is the adjoint of a 3 x 3 matrix A and |A| = 4, then a is equal to

(A) 4 (B) 5

(C) 11 (D) 0

Q26 If $A = \begin{vmatrix} x & 1 \\ 1 & x \end{vmatrix}$ and $B = \begin{vmatrix} x & 1 & 1 \\ 1 & x & 1 \\ 1 & 1 & x \end{vmatrix}$, then $\frac{dB}{dx}$ is

(A) 3A (B) -3B

(C) 3B + 1 (D) 1 - 3A

Q27 Let $f(x)=\begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x & 2x \\ \sin x & x & x \end{vmatrix}$ is

. Then $\lim_{x \to 0} \frac{f(x)}{x^2}$

(A) -1 (B) 0
(C) 3 (D) 2

Q28 Which one of the following observations is correct for the features of logarithm function to any base b > 1?

(A) The domain of the logarithm function is R, the set of real numbers.
(B) The range of the logarithm function is R^+, the set of all positive real numbers
(C) The point (1,0) is always on the graph of the logarithm function
(D) The graph of the logarithm function is decreasing as we move from left to right.

Q29 The function f(x) = |cos x| is

(A) Everywhere continuous and differentiable
(B) Everywhere continuous but not differentiable at odd multiples of p/2
(C) Neither continuous nor differentiable at $(2n+1)\frac{\pi}{2},\ n \in Z$
(D) Not differentiable everywhere

Q30 If $y = 2x^{3x}$, then dy/dx at x = 1 is

(A) 2 (B) 6
(C) 3 (D) 1

Q31 Let the function satisfy the equation f(x + y) = f(x)f(y) for all x, y ∈ R, where $f(0) \neq 0$. If f(5) = 3 and f'(0) = 2, then f'(5) is

(A) 6 (B) 0
(C) 3 (D) -6

Q32 The value of C in (0, 2) satisfying the mean value theorem for the function $f(x)= x(x-1)^2,\ x \in [0,\ 2]$ is equal to

(A) 3/4 (B) 4/3
(C) 1/3 (D) 2/3

Q33 $\frac{d}{dx}\left[\cos^2\left(\cot^{-1}\sqrt{\frac{2+x}{2-x}}\right)\right]$ is

(A) -3/4 (B) -1/2
(C) 1/2 (D) 1/4

Q34 For the function $f(x)= x^3-6x^2+12x-3;\ x=2$ is

(A) A point of minimum
(B) A point of inflexion
(C) Not a critical point
(D) A point of maximum

Q35 The function $x^x;\ x>0$ is strictly increasing at

(A) $\forall x \in R$
(B) $x < \frac{1}{e}$
(C) $x > \frac{1}{e}$
(D) x < 0

Q36 The maximum volume of the right circular cone with slant height 6 units is

(A) $4\sqrt{3}\pi$ cu units
(B) $16\sqrt{3}\pi$ cu units
(C) $3\sqrt{3}\pi$ cu units
(D) $6\sqrt{3}\pi$ cu units

Q37 If $f(x)= xe^{x(1-x)}$, then $f(x)$ is

(A) Increasing in R
(B) Decreasing in R
(C) decreasing in [-1/2, 1]
(D) increasing in [-1/2, 1]

Q38 $\int \frac{\sin x}{3+4\cos^2 x}\,dx$

(A) $-\frac{1}{2\sqrt{3}}\tan^{-1}\left(\frac{2\cos x}{\sqrt{3}}\right)+C$
(B) $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{\cos x}{\sqrt{3}}\right)+C$
(C) $\frac{1}{2\sqrt{3}}\tan^{-1}\left(\frac{\cos x}{3}\right)+C$
(D) $-\frac{1}{2\sqrt{3}}\tan^{-1}\left(\frac{2\cos x}{3}\right)+C$

Q39 $\int_{-\pi}^{\pi}(1-x^2)\sin x \cdot \cos^2 x\, dx$ is

(A) $\pi - \frac{\pi^2}{3}$
(B) $2\pi - \pi^3$
(C) $\pi - \frac{\pi^3}{2}$
(D) 0

Q40 $\int \frac{1}{x[6(\log x)^2+7\log x+2]}dx$ is

(A) $\frac{1}{2}\log\left|\frac{2\ \log x+1}{3\log x+2}\right|+C$
(B) $\log\left|\frac{2\log x+1}{3\log x+2}\right|+C$
(C) $\log\left|\frac{3\log x+2}{2\log x+1}\right|+C$
(D) $\frac{1}{2}\log\left|\frac{3\log x+2}{2\log x+1}\right|+C$

Q41 $\int \frac{\sin\frac{3x}{2}}{\sin\frac{x}{2}}$dx is

(A) 2x + sin x + 2 sin 2x + C
(B) x + 2 sin x + 2 sin 2x + C
(C) x + 2 sin x + sin 2x + C
(D) 2x + sin x + sin 2x + C

Q42 $\int_{1}^{5}(|x-3|+|1-x|)dx =$

(A) 12 (B) 5/6
(C) 21 (D) 10

Q43 $\lim_{n\to\infty}\left(\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+\frac{n}{n^2+3^2}+\ldots+\frac{1}{5n}\right)=$

(A) p/4 (B) $\tan^{-1}3$
(C) $\tan^{-1}2$ (D) p/2

Q44 The area of the region bounded by the line y = 3x and the curve y = x^2 sq. units is

(A) 10 (B) 9/2
(C) 9 (D) 5

Q45 The area of the region bounded by the line y = x and the curve y = x^3 is

(A) 0.2 sq. unit (B) 0.3 sq. unit
(C) 0.4 sq. unit (D) 0.5 sq. unit

Q46 The solution of $e^{dy/dx} = x+1,\ y(0)= 3$ is

(A) y - 2 = x log x - x
(B) y - x - 3 = x log x
(C) y - x - 3 = (x + 1) log (x + 1)
(D) y + x - 3 = (x + 1) log (x + 1)

Q47 The family of curves whose x and y intercepts of a tangent at any point are respectively double the x and y coordinates of that point is

(A) xy = C (B) $x^2 + y^2$ = C
(C) $x^2 - y^2$ = C (D) y/x = C

Q48 The vectors $AB = 3\hat{i} + 4\hat{k}$ and $AC = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of a DABC, the length of the median through A is A

(A) $\sqrt{18}$ (B) $\sqrt{72}$
(C) $\sqrt{33}$ (D) $\sqrt{288}$

Q49 The volume of the parallelopiped whose co terminous edges are $\hat{j}+\hat{k},\ \hat{i}+\hat{k}$ and $\hat{i}+\hat{j}$ is

(A) 6 cu units (B) 2 cu units
(C) 4 cu units (D) 3 cu units

Q50 Let a and b be two unit vectors and q is the angle between them. Then, a + b is a unit vector. if

(A) $\theta = \frac{\pi}{4}$
(B) $\theta = \frac{\pi}{3}$
(C) $\theta = \frac{2\pi}{3}$
(D) $\theta = \frac{\pi}{2}$

Q51 If a, b and c are three non-coplanar vectors and p, q and r are vectors defined by $p = \frac{a\times c}{[abc]}, q = \frac{c\times a}{[abcb]}, r = \frac{a\times b}{[ab]},$ then $(a+b)\cdot p + (b+c)\cdot q + (c+a)\cdot r$ is

(A) 0 (B) 1
(C) 2 (D) 3

Q52 If lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-5}{1} = \frac{z-6}{-5}$ are mutually perpendicular, then k is equal to

(A) -10/7 (B) -7/10
(C) -10 (D) -7

Q53 The distance between the two planes 2x + 3y + 4z = 4 and 4x + 6y + 8z = 12 is

(A) 2 units (B) 8 units
(C) $\frac{2}{\sqrt{29}}$ unit (D) 4 units

Q54 The sine of the angle between the straight line $\frac{x-2}{3} = \frac{y-3}{4} = \frac{4-z}{-5}$ are the plane 2x - 2y + z = 5 is

(A) $\frac{1}{5\sqrt{2}}$ (B) $\frac{2}{5\sqrt{2}}$
(C) 3/50 (D) $\frac{3}{\sqrt{50}}$

Q55 The equation xy = 0 in three-dimensional space represents

(A) a pair of straight lines
(B) a plane
(C) a pair of planes at right angles
(D) a pair of parallel planes

Q56 The plane containing the point (3, 2, 0) and the line $\frac{x-3}{1} = \frac{y-6}{5} = \frac{z-4}{4}$ is

(A) x - y + z = 1 (B) x + y + z = 5

(C) x + 2y - z = 1 (D) 2x - y + z = 5

Q57 Corner points of the feasible region for an LPP are (0, 2), (3, 0), (6, 0), (6, 8) and (0, 5). Let Z = 4x + 6y be the objective function. The minimum value of z occurs at

(A) Only (0, 2)

(B) Only (3, 0)

(C) The mid-point of the line segment joining the points (0, 2) and (3, 0)

(D) Any point on the line segment joining the points (0, 2) and (3, 0)

Q58 A die is thrown 10 times. The probability that an odd number will come up at least once is

(A) 11/1024 (B) 1013/1024

(C) 1023/1024 (D) 1/1024

Q59 A random variable X has the following probability distribution:

X	0	1	2
P(X)	25/36	k	1/36

If the mean of the random variable X is 1/3, then the variance is

(A) 1/18 (B) 5/18

(C) 7/18 (D) 11/18

Q60 If a random variable X follows the binomial distribution with parameters n = 5, p and P(X = 2) = 9P(X = 3), then p is equal to

(A) 10 (B) 1/10

(C) 5 (D) 1/5

Answer Key

Q1	C
Q2	B
Q3	B
Q4	C
Q5	C
Q6	B
Q7	A
Q8	A
Q9	B
Q10	D
Q11	C
Q12	A
Q13	C
Q14	C
Q15	D
Q16	A
Q17	C
Q18	C
Q19	C
Q20	C
Q21	B
Q22	C
Q23	B
Q24	A
Q25	C
Q26	A
Q27	B
Q28	C
Q29	B
Q30	B
Q31	A
Q32	B
Q33	D
Q34	B
Q35	C
Q36	B
Q37	D
Q38	A
Q39	D
Q40	B
Q41	C
Q42	A
Q43	C
Q44	B
Q45	D
Q46	D
Q47	A
Q48	C
Q49	B
Q50	C
Q51	D
Q52	A
Q53	C
Q54	A
Q55	C
Q56	A
Q57	D
Q58	C
Q59	B
Q60	B

Hints & Solutions

Note: scan the QR code to watch video solution

Q1 Text Solution:

Let set A have m elements and set B have n elements

Now, $2^m - 2^n = 56$

$\Rightarrow 2^n\left(2^{m-n} - 1\right) = 8 \times 7$

$\Rightarrow 2^n\left(2^{m-n} - 1\right) = 2^3 \times 7$

On comparing both sides, we get

$2^n = 2^3 \text{or } 2^{m-n} - 1 = 7$

Now, n = 3 ...(i)

So, $2^{m-n} - 1 = 7$

$\Rightarrow\ 2^{m-n} = 8$

$\Rightarrow\ m - n = 3 \quad ...(ii)$

From Eqs. (i) and (ii), we get

m = 6 and n = 3

Q2 Text Solution:

$[x]^2 - 5[x]+6 = 0$

$\Rightarrow [x]^2 - 3[x] - 2[x] + 6 = 0$

$\Rightarrow [x]([x] - 3) - 2([x] - 3) = 0$

$\Rightarrow ([x] - 3)([x] - 2) = 0$

$\Rightarrow [x] = 2 \text{ or } 3$

$[x] = 2 \text{ or } [x] = 3$

$x = [2, 3) \text{ or } x = [3, 4)$

Hence, $x \in [2, 4)$.

Q3 Text Solution:

Let the radii of the two circles be r_1 and r_2. Let an arc of length *l* subtend an angle of 30^o at the centre of the circle of radius r_1, while let an arc of length *l* subtend an angle of 78^o at the centre of the circle of radius r_2.

Now, $30^o = \frac{\pi}{6}$ radian and $78^o = \frac{13\pi}{30}$ radian

We know that in a circle of radius r unit, if an arc of length l unit subtend an angle q radian at the centre, then

$\theta = \frac{l}{r} \text{ or } l = r\theta$

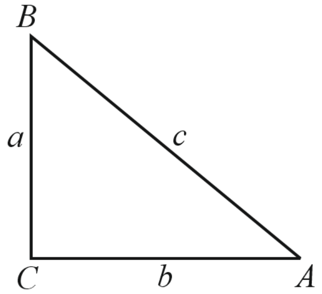
$\therefore l = \frac{r_1\pi}{6} \text{and} l = \frac{r_2 13\pi}{30}$

$\Rightarrow \frac{r_1\pi}{6} = \frac{r_2 13\pi}{30}$

$\Rightarrow r_1 = r_2 \frac{13}{5}$

$\Rightarrow \frac{r_1}{r_2} = \frac{13}{5}$

Q4 Text Solution:



$\because \tan A = \frac{a}{b}, \tan B =$

$\therefore \tan A + \tan B = \frac{a}{b} + \frac{b}{a}$

$= \frac{a^2+b^2}{ab}$

$= \frac{c^2}{ab} \quad \left[\because c^2 = a^2 + b^2\right]$

Q5 Text Solution:

Given, $\frac{1-i \sin \alpha}{1+2i \sin \alpha}$ is purely real

i.e. $\frac{1-i \sin \alpha}{1+2i \sin \alpha} \times \frac{(1-2i \sin \alpha)}{(1-2i \sin \alpha)}$

$= \frac{1-i \sin \alpha-2i \sin \alpha+2i^2 \sin^2 \alpha}{1-4i^2 \sin^2 \alpha}$

$= \frac{1-3i \sin \alpha-2 \sin^2 \alpha}{1+4 \sin^2 \alpha}$

$= \frac{1-2 \sin^2 \alpha}{1+4 \sin^2 \alpha} + i\left(\frac{-3 \sin \alpha}{1+4 \sin^2 \alpha}\right)$

which is given to purely real

$\Rightarrow \frac{-3 \sin \alpha}{1+4 \sin^2 \alpha} = 0$

$\Rightarrow -3 \sin \alpha = 0 \Rightarrow \sin \alpha = 0$

Hence, $\alpha = n\pi, n \in N$

Q6 Text Solution:

Since, perimeter ≥ 180

$\Rightarrow$ 2(length + breath ≥ 180)

$\Rightarrow$ 5 breath + breadth ≥ 90

$\left[\because \text{ length} = 5 \text{ breadth}\right]$

$\Rightarrow$ 6 breadth ≥ 90

Hence, breadth ≥ 15

Q7 Text Solution:

$\because {}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}$

Given, ${}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 + {}^{46}C_3 + {}^{45}C_3 + {}^{45}C_4$

$= {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 + {}^{46}C_3 + {}^{46}C_4$

$= {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 + {}^{47}C_4$

$= {}^{49}C_3 + {}^{48}C_3 + {}^{48}C_4$

$= {}^{49}C_3 + {}^{49}C_4 = {}^{50}C_4$

Q8 Text Solution:

$\because \frac{C_1}{C_0} + 2\cdot\frac{C_2}{C_1} + 3\cdot\frac{C_3}{C_2} + \ldots n\frac{C_n}{C_{n-1}}$

$= \frac{n}{1} + 2\cdot\frac{\frac{n(n-1)}{1\cdot 2}}{n} + 3\cdot\frac{\frac{n(n-1)(n-2)}{3\cdot 2\cdot 1}}{\frac{n(n-1)}{1\cdot 2}} + \ldots + n\cdot\frac{1}{n}$

$= n + \left(n-1\right) + \left(n-2\right)\ldots + 1 = \Sigma n$

$= \frac{n(n+1)}{2}$

Q9 Text Solution:

$S_n = \frac{a(1-r^n)}{1-r}$

$S_{2n} = \frac{a\left(1-r^{2n}\right)}{1-r}$

$\frac{S_n}{S_{2n}} = \frac{a(1-r^n)}{1-r} \times \frac{1-r}{a\left(1-r^{2n}\right)} = \frac{1-r^n}{(1+r^n)(1-r^n)}$

Hence, $S_n : S_{2n} = \frac{1}{1+r^n}$

Q10 Text Solution:

Let a and b be the roots the quadratic equation.

Then, the quadratic equation is

$x^2 - (a+b)x + ab = 0$ ...(i)

It is given that AM = 5 and GM = 4

i.e., $\frac{a+b}{2} = 5$ and $\sqrt{ab} = 4$

$\Rightarrow a + b = 10$ and $ab = 16$

On substituting these values in Eq. (i), we obtain.

Hence, $x^2 - 10x + 16 = 0$ is the required equation.

Q11 Text Solution:

$\because y = -x + 3;\ m_1 = -1 = \tan\theta_1$

and $m_2 = \frac{y-1}{x-1} = \frac{4-1}{-3-1}$

$\frac{y-1}{x-1} = \frac{3}{-4};\ m_2 = \frac{3}{-4} = \tan\theta_2$

Angle between two lines are

$= \tan|\theta_2 - \theta_1| = \frac{\tan\theta_2 - \tan\theta_1}{1+\tan\theta_1\cdot\tan\theta_2} = \frac{m_2-m_1}{1+m_1m_2}$

$= \frac{\frac{-3}{4}+1}{1+\frac{3}{4}} = \frac{1}{4}\times\frac{4}{7}$

Hence, $|\theta_2 - \theta_1| = \tan^{-1}\left(\frac{1}{7}\right)$

Q12 Text Solution:

∵ Focus has positive x-coordinate.

So, coordinate of focus = (a, 0) ≡ (6, 0)

$\therefore\ a = 6$ ...(i)

Hence, equation of parabola is

$y^2 = 24x\ \left[\because y^2 = 4ax \text{ and using Eq. (i)}\right]$

Q13 Text Solution:

$\because \lim_{x\to\frac{\pi}{4}} \frac{\sqrt{2}\cos x - 1}{\cot x - 1}$

$= \lim_{x\to\frac{\pi}{4}} \frac{-\sqrt{2}\sin x}{-cosec^2 x}$ [using L-Hospital rule]

$= \frac{-\sqrt{2}\times\frac{1}{\sqrt{2}}}{-2} = \frac{1}{2}$

Q14 Text Solution:

The negation of statement "For every real number x; $x^2 + 5$ is positive" is there exists at least one real number x such that $x^2 + 5$ is not positive.

Q15 Text Solution:

The given observation are a, b, c, d and e

Mean $= m = \frac{a+b+c+d+e}{5}$

$\Rightarrow \Sigma x_i = a + b + c + d + e = 5m$

$\therefore$ Standard deviation, $S = \sqrt{\frac{\Sigma x_i^2}{5} - m^2}$

Now, consider the observations a + k, b + k, c + k, d + k, e + k.

Now mean

Mean $= \frac{(a+k)+(b+k)+(c+k)+(d+k)+(e+k)}{5}$

$= \frac{a+b+c+d+e+5k}{5}$

$= \frac{5m+5k}{5} = m + k$

$\therefore$ New standard devaition

$= \sqrt{\frac{\sum(x_i+k)^2}{5} - (m+k)^2}$

$= \sqrt{\frac{\sum\left(x_i^2+k^2+2x_ik\right)}{5} - \left(m^2+k^2+2mk\right)}$

$= \sqrt{\frac{\sum x_i^2}{5} - m^2 + \frac{5k^2}{5} - k^2 + \frac{2k\Sigma x_i}{5} - 2mk}$

$= \sqrt{\frac{\sum x_i^2}{5} - m^2 + \frac{2k\times 5m}{5} - 2mk}$

$= \sqrt{\frac{\sum x_i^2}{5} - m^2} = S$

Q16 Text Solution:

$\because f(x) = \tan x$

Since, $f^{-1}$is inverse of f

$\Rightarrow f\left(f^{-1}(1)\right) = 1$

$\Rightarrow \tan\left(f^{-1}(1)\right) = 1$

But $\tan\left(\frac{\pi}{4}\right) = 1$

$\Rightarrow \tan\left(f^{-1}(1)\right) = \tan\left(\frac{\pi}{4}\right)$

$\therefore f^{-1}(l) = \frac{\pi}{4}$

Q17 Text Solution:

$\because$ For pre image of 17 , we have

$f(x) = x^2 + 1 = 17$

$\Rightarrow x^2 = 16 \Rightarrow x = \pm 4$

So, $x \in \{-4, 4\}$

For pre image of -3, we have

$f(x) = x^2 + 1 = -3$

$\Rightarrow x^2 = -4$ (not possible)

Hence, no possible pre image.

Q18 Text Solution:

$\because g(f(x)) = \sin x$ and $f(g(x)) = \left(\sin\sqrt{x}\right)^2$

(a) $f(x) = \sin^2 x$ and $g(x) = x$

Now, $f(g(x)) = f(x) = \sin^2 x$

and $g(f(x)) = g\left(\sin^2 x\right) = \sin^2 x$

(b) $f(x) = \sin\sqrt{x}$ and $g(x) = \sqrt{x}$

Now, $f(g(x)) = f\left(\sqrt{x}\right) = \sin\sqrt{\sqrt{x}}$

$= \sin(x)^{1/4}$

and $g(f(x)) = g\left(\sin\sqrt{x}\right) = \sqrt{\sin\sqrt{x}}$

(c) $f(x) = \sin^2 x$ and $g(x) = \sqrt{x}$

Now, $f(g(x)) = f\left(\sqrt{x}\right) = \sin^2\sqrt{x}$

and $g(g(x)) = g\left(\sin^2 x\right) = \sqrt{\sin^2 x} = |\sin x|$

(d) $f(x) = \sin\sqrt{x}$ and $g(x) = x^2$

Now, $f(g(x)) = f\left(x^2\right) = \sin|x|$

and $g(f(x)) = g\left(\sin\sqrt{x}\right) = \left(\sin\sqrt{x}\right)^2$

$= \sin^2\sqrt{x}$

Q19 Text Solution:

Let $(3, 2) R(x, y)$

$\Rightarrow 3y = 2x$

This is possible in the cases

x = 3, y = 2

x = 6, y = 4

x = 9, y = 6

x = 12, y = 8

x = 15, y = 10

x = 18, y = 12

Hence, total pairs are 6.

Q20 Text Solution:

Given, $\cos^{-1}(x) + \cos^{-1}(y) + \cos^{-1}(z) = 3\pi$

As, we know that $0 \leq \cos^{-1}(x) \leq \pi$

Thus, the maximum value of $\cos^{-1}(x) = \pi$ satisfies the given equation.

$x = y = z = \cos\pi = -1$

Now, $x(y+z) + y(z+x) + z(x+y)$

$= xy + zx + yz + xy + zx + yz$

$= 2(xy + yz + zx)$

$= 2(1+1+1) = 6$

Q21 Text Solution:

$\because 2\sin^{-1}x - 3\cos^{-1}x = 4$

$\Rightarrow 2\sin^{-1}x - 3\left(\frac{\pi}{2} - \sin^{-1}x\right) = 4$

$\left[\because \cos^{-1}x = \frac{\pi}{2} - \sin^{-1}x\right]$

$\Rightarrow 5\sin^{-1}x = 4 + \frac{3\pi}{2}$

$\therefore \sin^{-1}x = \frac{4+\frac{3\pi}{2}}{5} = \frac{8+3\pi}{10}$

So, $2\sin^{-1}x + 3\cos^{-1}x$

$= 2\sin^{-1}x + 3\left(\frac{\pi}{2} - \sin^{-1}x\right)$

$= -\sin^{-1}x + \frac{3\pi}{2}$

$= \frac{-8-3\pi}{10} + \frac{3\pi}{2} = \frac{-8-3\pi+15\pi}{10}$

$= \frac{12\pi-8}{10} = \frac{6\pi-4}{5}$

Q22 Text Solution:

$(I+A)^3 = I^3 + A^3 + 3I^2A + 3A^2I$

$= I + A^3 + 3A + 3A^2$

$= I + A^2 \cdot A + 3A + 3A \quad \left[\because A^2 = A\right]$

$= I + A \cdot A + 6A$

$= I + A^2 + 6A$

$= I + A + 6A \quad \left[\because A^2 = A\right]$

$= I + 7A$

Q23 Text Solution:

$\because A^2 = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

$\therefore A^2 = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

$= \begin{pmatrix} 1+1 & 1+1 \\ 1+1 & 1+1 \end{pmatrix}$

$= \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix} = 2\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = 2A$

$\therefore A^3 = \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

$= \begin{pmatrix} 2+2 & 2+2 \\ 2+2 & 2+2 \end{pmatrix} = \begin{pmatrix} 4 & 4 \\ 4 & 4 \end{pmatrix}$

$= 4\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = 4A$

Hence, $A^{10} = 2^9 A \quad [\because A^n = 2^{n-1} A]$

Q24 Text Solution:

$f(x)$
$= 2(x$
$- 5)$
$\begin{bmatrix} x-1 & x^2-9 & 2x^3-81 \\ 1 & x+5 & 4(x^2+5x+25) \\ 1 & 1 & 1 \end{bmatrix}$

$\Rightarrow \ f(5) = 0$ and $f(1)$

$= 2(-4)\begin{bmatrix} -2 & -8 & -79 \\ 1 & 6 & 124 \\ 1 & 1 & 3 \end{bmatrix}$

$= -8[212 - 968 + 395] = 2888$

$f(3) = 2(-2)\begin{bmatrix} 0 & 0 & -27 \\ 1 & 8 & 196 \\ 1 & 1 & 3 \end{bmatrix}$

$= -4[0 + 0 - 27(1-8)] = -756$

So, $f(1)\cdot f(3) + f(3)\cdot f(5) + f(5)\cdot f(1)$

$= 2888 \times (-756) + 0 + 0 = -2183328$

Q25 Text Solution:

$P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$

For 3×3, matrix

$|\text{adj } A| = |A|^2$

$|\text{adj } A| = (4)^2 = 16$

Now, $P = \begin{vmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{vmatrix} = 16$

$\Rightarrow 1(12-12) - \alpha(4-6) + 3(4-6) = 16$

$\Rightarrow 0 + 2\alpha - 6 = 16$

$\Rightarrow 2\alpha = 22$

Hence, $\alpha = 11$

Q26 Text Solution:

$B = \begin{vmatrix} x & 1 & 1 \\ 1 & x & 1 \\ 1 & 1 & x \end{vmatrix}$

$= \begin{vmatrix} 1 & 1 & 1 \\ 0 & x & 1 \\ 0 & 1 & x \end{vmatrix} + \begin{vmatrix} x & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & x \end{vmatrix} + \begin{vmatrix} x & 1 & 0 \\ 1 & x & 0 \\ 1 & 1 & 1 \end{vmatrix}$

$= (x^2-1) + (x^2-1) + (x^2-1)$

$= 3(x^2-1)$

$= 3A \left[\because A = \begin{vmatrix} x & 1 \\ 1 & x \end{vmatrix} = x^2 - 1\right]$

Q27 Text Solution:

$\because f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x & 2x \\ \sin x & x & x \end{vmatrix}$

$= \cos x(x^2 - 2x^2) - x(2x\sin x - 2x\sin x)$

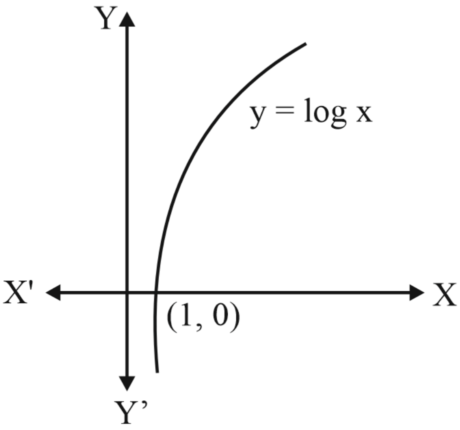
$= -x^2\cos x + x\sin x$

Now, $\lim_{x\to 0} \frac{f(x)}{x^2} = \lim_{x\to 0}\left(\frac{-x^2\cos x + x\sin x}{x^2}\right)$

$= \lim_{x\to 0}\left(-\cos x + \frac{\sin x}{x}\right)$

$= -1 + 1 \quad \left[\because \lim_{x\to 0}\frac{\sin x}{x} = 1\right] = 0$

Q28 Text Solution:

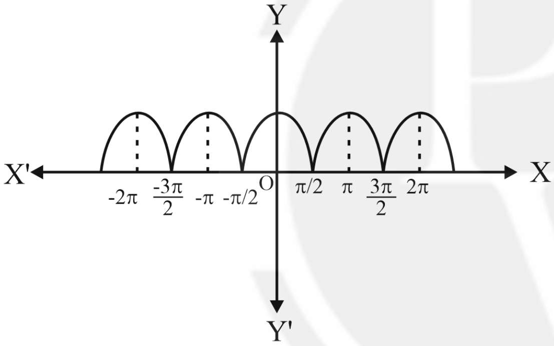


$\because$ Domain $\in R^+$ and range $\in R$ and graph of the logarithm function is increasing. So, point (1, 0) is always on the graph of logarithm function.

Q29 Text Solution:

f(x) = |cos x|

Graph of f(x) = |cos x|



From graph we can see that f(x) is not differentiable at points $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{-\pi}{2}$

Also, it is everywhere continuous.

Q30 Text Solution:

$y - 2x^{3x}$

$\frac{dy}{dx} = 2x^{3x}[3\log x + 3]$

So, $\left.\frac{dy}{dx}\right|_{x=1} = 2 \cdot (1)^{3(1)}[3 \times 0 + 3]$

$= 6 + 0 = 6$

Q31 Text Solution:

$f(x+y) = f(x) \cdot f(y)$

Put $x = 0, y = 5$, we get

$f(0+5) = f(0)f(5)$

$\Rightarrow f(5)[f(0) - 1] = 0$

$\Rightarrow f(0) = 1$

Consider $f'\left(5\right) = \lim_{h\to 0} \frac{f(5+h)-f(5)}{h}$

$= \lim_{h\to 0} \frac{f(5)f(h)-f(5)}{h}$

$= \lim_{h\to 0} \frac{f(5)[f(h)-1]}{h} = f\left(5\right) \lim_{h\to 0} \frac{f(h)-f(0)}{h}$

$= f\left(5\right) f'\left(0\right) = 2 \times 3 = 6$

Q32 Text Solution:

$f\left(x\right) = x(x-1)^2;\ x \in \left[0, 2\right]$

$f'(c) = \frac{f(b)-f(a)}{b-a};\ f\left(2\right) = 2, f\left(0\right) = 0$

$\therefore f'\left(x\right) = (x-1)^2 + 2x\left(x-1\right)$

$= x^2 + 1 - 2x + 2x^2 - 2x$

$= 3x^2 - 4x + 1$

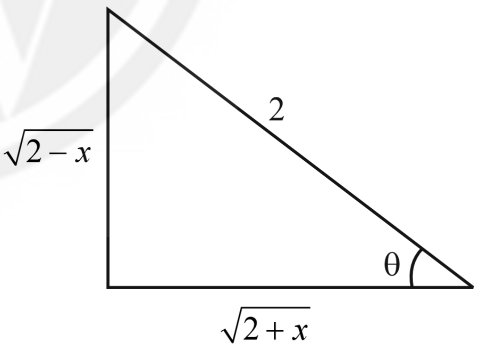
So, $f'(c) = 3c^2 - 4c + 1$

Thus, $3c^2 - 4c + 1 = \frac{f(2)-(0)}{2-0} = \frac{2-0}{2-0} = 1$

$\Rightarrow c(3c-4) = 0$

Hence, $c = \frac{4}{3}$

Q33 Text Solution:



$\because \frac{d}{dx}\left[\cos^2\left(\cot^{-1}\sqrt{\frac{2+x}{2-x}}\right)\right]$

$= \frac{d}{dx}\left(\cos\left(\cos^{-1}\left(\frac{\sqrt{2+x}}{2}\right)\right)\right)^2$

$= \frac{d}{dx}\left(\frac{2+x}{4}\right) = \frac{1}{4}$

$\left(\because \cot\theta = \sqrt{\frac{2+x}{2-x}}\ \therefore \cos\theta = \frac{\sqrt{2+x}}{2}\right)$

Q34 Text Solution:

$\because f(x) = x^3 - 6x^2 + 12x - 3$

$\Rightarrow f'(x) = 3x^2 - 12x + 12$

$\Rightarrow f''(x) = 6x - 12 = 6(x-2)$

$\Rightarrow f''(2) = 0$

Hence, $x = 2$is a point of inflexion.

Q35 Text Solution:

Let $y = x^x$

$\frac{dy}{dx} = x^x(1 + \log x)$

For increasing function, $\frac{dy}{dx} > 0$

$\Rightarrow x^x(1 + \log x) > 0$

$\Rightarrow 1 + \log x > 0$

$\Rightarrow \log_e x > \log_e \frac{1}{e}$

Hence, $x > \frac{1}{e}$

Q36 Text Solution:

$\because$ Slant height of the cone, L = 6 units

Let the radius be r and height be h.

and volume $e(V) = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi(L^2 - h^2)h \quad [\because L^2 = r^2 + h^2]$

$= \frac{1}{3}\pi(36 - h^2)h$

So, $\frac{dV}{dh} = \frac{1}{3}\pi(36 - 3h^2) = 0 \Rightarrow h$

$= 2\sqrt{3}$ units

Now, $\frac{d^2V}{dh^2} = -2\pi h < 0$ at $h = 2\sqrt{3}$ units

Hence, the maximum volume of the cone

$V = \frac{1}{3}\pi(36 - h^2)h$

$= \frac{1}{3}\pi\left(36 - \left(2\sqrt{3}\right)^2\right)2\sqrt{3}$

$= 16\sqrt{3}\pi$cu units

Q37 Text Solution:

Given $f(x) = xe^{x(1-x)}$

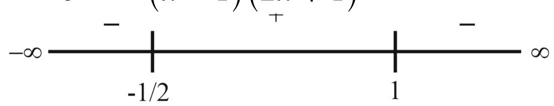
$f'(x) = e^{x(1-x)} + xe^{x(1-x)}(1-2x)$

$= e^{x(1-x)}(1 + x(1-2x))$

$= -e^{x(1-x)}(2x^2 - x - 1)$

$= -e^{x(1-x)}(2x^2 - 2x + x - 1)$

$= -e^{x(1-x)}(x-1)(2x+1)$



So, f(x) is increasing when $f'(x) \geq 0$ and decreasing when $f'(x) \leq 0$

Hence f is increasing in $\left[\frac{-1}{2}, 1\right]$

Q38 Text Solution:

$\because I = \int \frac{\sin x}{3+4\cos^2 x}dx$

$= \frac{\sin x}{3\left[1+\left(\frac{2\cos x}{\sqrt{3}}\right)^2\right]^2}dx$

put $t = \frac{2\cos x}{\sqrt{3}}$

$dt = \frac{-2\sin x}{\sqrt{3}}dx$

$I = \int -\frac{\sqrt{3}}{2(3t^2+3)}dt$

$= -\frac{1}{2\sqrt{3}}\tan^{-1}(t) + C$

$= \frac{-1}{2\sqrt{3}}\tan^{-1}\left(\frac{2\cos x}{\sqrt{3}}\right) + C$

Q39 Text Solution:

$\because$ Let $f(x) = (1 - x^2)\sin x \cdot \cos^2 x$

$f(-x) = (1 - (-x)^2)\sin(-x) \cdot \cos^2(-x)$

$= (1 - x^2)(-\sin x)(\cos^2 x)$

$= -(1 - x^2)(\sin x)(\cos^2 x)$

So, $f(x) = -f(-x)$

So, f(x) is an odd function

Hence, $\int_{-\pi}^{\pi}(1 - x^2)\sin x \cdot \cos^2 x \, dx = 0$

Q40 Text Solution:

$$I = \int \frac{1}{x[6(\log x)^2+7\log x+2]}dx$$

$$= \int \frac{dt}{6t^2+7t+2} \quad \left[\begin{array}{l} \because \text{put} t = \log x \\ dt = \frac{1}{x}dx \end{array}\right]$$

$$= \int \frac{dt}{(3t+2)(2t+1)}$$

Now, $\frac{1}{(3t+2)(2t+1)} = \frac{A}{(3t+2)} + \frac{B}{(2t+1)}$

$$= \frac{A(2t+1)+B(3t+2)}{(3t+2)(2t+1)}$$

$$\Rightarrow\ 1 = A(2t+1) + B(3t+2)$$

Now, on putting $t = \frac{-1}{2}$

$$\Rightarrow 1 = 0 + B\left[3\left(-\frac{1}{2}\right)+2\right]$$

$$\Rightarrow B = 2$$

and put $t = \frac{-2}{3}$

$$\Rightarrow 1 = A\left(2\times\left(-\frac{2}{3}\right)+1\right)+0 \Rightarrow 1 = A\left(-\frac{4}{3}+1\right)$$

$$\Rightarrow A = -3$$

$$\therefore I = \int\left(\frac{-3}{3t+2} + \frac{2}{2t+1}\right)dt$$

$$= \frac{-3\log|3t+2|}{3} + 2\frac{\log|2t+1|}{2} + C$$

$$= -\log|3t+2| + \log|2t+1| + C$$

$$= \log\left|\frac{2t+1}{3t+2}\right|+C$$

$$= \log\left|\frac{2\log x+1}{3\log x+2}\right|+C \quad [\because t = \log x]$$

Q41 Text Solution:

$$I = \int \frac{\sin\left(\frac{5x}{2}\right)}{\sin\left(\frac{x}{2}\right)}dx$$

$$= \int \frac{2\sin\left(\frac{5x}{2}\right)\cos\left(\frac{x}{2}\right)}{\sin x}dx = \int \frac{\sin 3x+\sin 2x}{\sin x}dx$$

$$= \int \frac{(3\sin x-4\sin^3 x)+2\sin x\cos x}{\sin x}dx$$

$$= \int(3 - 4\sin^2 x + 2\cos x)dx$$

$$= \int(3 - 2(1-\cos 2x) + 2\cos x)dx$$

$$= \int(3 - 2 + 2\cos 2x + 2\cos x)dx$$

$$= \int(1 + 2\cos 2x + 2\cos x)dx$$

$$= x + \sin 2x + 2\sin x + C$$

Q42 Text Solution:

$$\int_1^5 \left[|x-3|+|1-x|\right]dx$$

$$= \int_1^5 |x-3|dx + \int_1^5 |1-x|dx$$

$$= \int_1^3 |x-3|dx + \int_3^5 |x-3|dx + \int_1^5 |1 - x|dx$$

$$= \int_1^3 (3-x)dx + \int_3^5 (x-3)dx + \int_1^5 (x-1)dx$$

$$= \left[3x - \frac{x^2}{2}\right]_1^3 + \left[\frac{x^2}{2} - 3x\right]_3^5 + \left[\frac{x^2}{2} - x\right]_1^5$$

$$= \left(3\times 3 - \frac{9}{2}\right)^- - \left(3\times 1 - \frac{1}{2}\right)$$

$$+\left(\frac{5\times 5}{2} - 3\times 5\right) - \left(\frac{3\times 3}{2} - 3\times 3\right) + \left(\frac{5\times 5}{2} - 5\right) - \left(\frac{1}{2} - 1\right)$$

$$= \frac{9}{2} - \frac{5}{2} - \frac{5}{2} + \frac{9}{2} + \frac{15}{2} + \frac{1}{2}$$

$$= \frac{9-5-5+9+15+1}{2} = \frac{24}{2} = 12$$

Q43 Text Solution:

$$\lim_{n\to\infty}\left(\frac{n}{n^2+1^2} + \frac{n}{n^2+2^2} + \frac{n}{n^2+3^2} + \ldots + \frac{1}{5n}\right)$$

$$= \lim_{n\to\infty}\left(\frac{n}{n^2+1^2} + \frac{n}{n^2+2^2} + \frac{n}{n^2+3^2} + \ldots + \frac{n}{n^2+(2n)^2}\right)$$

$$= \lim_{n\to\infty} \, {}_{r=1}^{2n} \frac{n}{n^2+r^2}$$

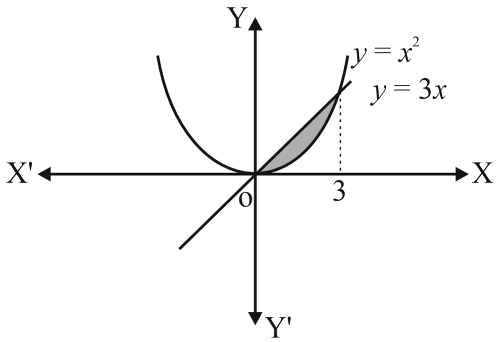
$$= \lim_{n\to\infty} \, {}_{r=1}^{2n} \frac{1}{n}\left(\frac{1}{1+\left(\frac{r}{n}\right)^2}\right)$$

$$= \int_0^2 \frac{dx}{1+x^2} = \tan^{-1}(2)$$

Q44 Text Solution:

We have, y = 3x and y = $x^2 \Rightarrow\ 3x = x^2$

x = 3 and x = 0 are the points of intersection.



$$\text{Now, area} = \int_0^3 (3x - x^2)dx$$

$$= \left.\frac{3x^2}{2}\right|_0^3 - \left.\frac{x^3}{3}\right|_0^3$$

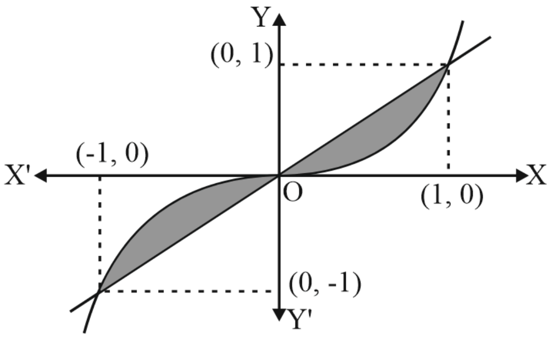
$$= 3\cdot\frac{9}{2} - \frac{27}{3}$$

$$= \frac{27}{2} - 9 = \frac{9}{2} \text{ sq. units}$$

Q45 Text Solution:

$\because y = x$ and $y = x^3$

The curve intersect at x = -1, y = -1; x = 0, y = 0 and x = 1, y = 1.



Therefore, area $= -\int_{-1}^{0}\left(x^3 - x\right)dx + \int_{0}^{1}\left(x - x^3\right)dx$

$= -\left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{-1}^{0} + \left[\frac{x^2}{2} - \frac{x^4}{4}\right]_{0}^{1}$

$= -\left(\frac{1}{4} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{4}\right)$

$= -\frac{1}{2} + 1 = \frac{1}{2} = 0.5$ sq. units

Q46 Text Solution:

$\because e^{\frac{dy}{dx}} = x + 1$

$\Rightarrow \frac{dy}{dx} = \log\left(x + 1\right)$

$\Rightarrow \int dy = \int \log\left(x + 1\right)dx$

$\Rightarrow y = x\log\left(x + 1\right) - \int \frac{x}{x+1}dx$

$\Rightarrow y = x\log\left(x + 1\right) - \int 1 \cdot dx + \int \frac{1}{1+x}dx$

$\Rightarrow y = x\log(x + 1) - x + \log(x + 1) + C$

$\Rightarrow y = (x + 1)\log(x + 1) - x + C$

Now, $y(0) = 3$

$3 = 0 + C \quad \Rightarrow C = 3$

Hence, $y = (x + 1)\log(x + 1) - x + 3$

$y + x - 3 = (x + 1)\log(x + 1)$

Q47 Text Solution:

Let the point on the curve be (h, k)

According to the question, the equation of the tangent will be

$\Rightarrow \frac{x}{2h} + \frac{y}{2k} = 1$

$\Rightarrow \frac{y}{2k} = 1 - \frac{x}{2h}$

$\Rightarrow y = -\left(\frac{2k}{2h}\right)x + 2k$

So, the slope of tangent will be

Now, $\frac{dy}{dx} = -\frac{h}{k} = -\frac{y}{x}$

$\Rightarrow \frac{dy}{y} = -\frac{dx}{x}$

On integrating both sides, we get

$\int \frac{dy}{dy} = -\int \frac{dx}{x}$

$\Rightarrow \ln y = -\ln x + \ln C$

$\Rightarrow \ln y = \ln \frac{C}{x}$

$\Rightarrow y = \frac{C}{x}$

$\therefore xy = C$

Q48 Text Solution:

Let A = origin

\ AB = Position vector of B.

AC = Position vector of C

\Position vector of mid-point of B and C

$= P = \frac{AB+AC}{2}$

$= \left(\frac{3+5}{2}\right)\hat{i} + \left(\frac{0-2}{2}\right)\hat{j} + \left(\frac{4+4}{2}\right)\hat{k}$

$= 4\hat{i} - \hat{j} + 4\hat{k}$

Median$= AP = 4\hat{i} - \hat{j} + 4\hat{k}$

Hence, length of median $= \left|AP\right|$

$= \sqrt{16 + 1 + 16} = \sqrt{33}$

Q49 Text Solution:

The volume of the parallelopiped whose coterminous edges are $\hat{j} + \hat{k},\ \hat{i} + \hat{k}$ and

$\hat{i} + \hat{j} = \begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix}$

$= 0 - 1(0 - 1) + 1(1 - 0)$

$= -1(-1) + 1$

$= 2$ cu units

Q50 Text Solution:

$\because a + b$ is a unit vector if$|a + b| = 1$

$\Rightarrow \left|a + b\right|^2 = 1$

$\Rightarrow \left(a + b\right) \cdot \left(a + b\right) = 1$

$\Rightarrow \left|a\right|^2 + \left|b\right|^2 + 2\left(a \cdot b\right) = 1$

$\Rightarrow 1 + 1 + 2\left(a \cdot b\right) = 1$

$\therefore 2(a \cdot b) = -1$

$\Rightarrow a \cdot b = -\frac{1}{2}$

$\Rightarrow \left|a\right|\left|b\right|\cos\theta = -\frac{1}{2}$

$\Rightarrow 1 \times 1 \times \cos\theta = -\frac{1}{2}$

$\Rightarrow \cos\theta = -\frac{1}{2}$

Hence, $\theta = \frac{2\pi}{3}$

Q51 Text Solution:

$p=\frac{b\times c}{[abc]}, q=\frac{c\times a}{[abc]}, r=\frac{a\times b}{[abc]}$

Now, $(a+b)\cdot p+(b+c)\cdot q+(c+a)\cdot r$

$=\frac{a\cdot(b\times a)}{[abc]}+\frac{b\cdot(b\times c)}{[abc]}+\frac{b\cdot(c\times a)}{[abc]}+\frac{c\cdot(c\times a)}{[abc]}$
$+\frac{c\cdot(a\times b)}{[abc]}+\frac{a\cdot(a\times b)}{[abc]}$

$=\frac{[abc]}{[abc]}+0+\frac{[bca]}{[abc]}+0+\frac{[cab]}{[abc]}+0$

[∵ scalar triple product is zero if two vectors are same]

$=\frac{[abc]}{[abc]}+\frac{[abc]}{[abc]}+\frac{[abc]}{[abc]}=3$

Q52 Text Solution:

$\frac{x-1}{-3}=\frac{y-2}{2k}=\frac{z-3}{2}$ and $\frac{x-1}{3k}=\frac{y-5}{1}=\frac{z-6}{-5}$

$\therefore b_1=-3\hat{i}+2k\hat{j}+2\hat{k}$ and $b_2=3\hat{i}+\hat{j}-5\hat{k}$

Now, $b_1\cdot b_2=0$

$\Rightarrow -9k+2k-10=0$

$\Rightarrow -7k=10$

$\therefore k=\frac{-10}{7}$

Hence, the value of k is -10/7.

Q53 Text Solution:

The equations of planes are

2x + 3y + 4z = 4 ...(i)

4x + 6y + 8z = 12

2x + 3y + 4z = 6 ...(ii)

It can be seen that the given planes are parallel. It is known that the distance between two parallel planes is $\left|\frac{d_2-d_1}{\sqrt{a^2+b^2+c^2}}\right|$

So, $D=\left|\frac{6-4}{\sqrt{(2)^2+(3)^2+(4)^2}}\right|=\frac{2}{\sqrt{29}}$ units

Q54 Text Solution:

We have, the equation of line as

$\frac{x-2}{3}=\frac{y-3}{4}=\frac{4-z}{-5}$

$\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$

This line is parallel to the vector $b=3\hat{i}+4\hat{j}+5\hat{k}$

Equation of plane is 2x - 2y + z = 5

Normal to the plane is $n=2\hat{i}-2\hat{j}+\hat{k}$

Then, $\sin\theta=\frac{b\cdot n}{|b||n|}$

$=\frac{\left|(3\hat{i}+4\hat{j}+5\hat{k})\cdot((\hat{i}-2\hat{j}+\hat{k})\right|}{\sqrt{3^2+4^2+5^2}\sqrt{4+4+1}}$

$=\frac{|6-8+5|}{\sqrt{50}\sqrt{9}}=\frac{3}{15\sqrt{2}}=\frac{1}{5\sqrt{2}}$

Q55 Text Solution:

In three-dimensional space, the equation xy = 0 represents a set of points where the product of the coordinates x and y is zero. This can be interpreted as follows:

The product xy = 0 is satisfied if either x = 0 or y = 0.

The condition x = 0 describes the yz-plane which is a plane that includes all points where the x-coordinate is zero.

Similarly, the condition y = 0describes the xz-plane, which is a plane that includes all points where the y-coordinate is zero.

Therefore, the equation xy = 0 in three-dimensional space actually represents two planes that intersect along the z-axis. These two planes:

The yz-plane (where x = 0)

The xz-plane (where y = 0)

These planes are perpendicular to each other because they intersect along the z-axis at right angles. Thus, the correct answer is:

Q56 Text Solution:

Given that point (3, 2, 0) lies on plane and line $\frac{x-3}{1}=\frac{y-6}{5}=\frac{z-4}{4}$ also lies on the plane.

∵ Plane contains points (3, 2, 0) and (3, 6, 4)

The DR's of line joining these two points is (0, 4, 4).

Therefore, DR's of normal to plane is

$(\hat{i}+5\hat{j}+4\hat{k})\times(4\hat{j}+4\hat{k})=4\hat{i}-4\hat{j}+4\hat{k}$

Therefore, the equation of plane is

4x - 4y + 4z = k

∵ Point (3, 2, 0) lies on planes. So we get

$\therefore k=4\times3-4\times2+4\times0=4$

Hence, the equation of plane is x - y + z = 1.

Q57 Text Solution:

S. N.	Corner Points	Value of Z = 4x + 6y
(i)	(0, 2)	Z = 4 x 0 + 6 x 2 = 12 (minimum)
(ii)	(3, 0)	Z = 4 x 3 + 6 x 0 = 12 (minimum)
(iii)	(6, 0)	Z = 4 x 6 + 6 x 0 = 24
(iv)	(6, 8)	Z = 4 x 6 + 6 x 8 = 72
(v)	(0, 5)	Z = 4 x 0 + 6 x 5 = 30

\ Minimum of Z occur at (0, 2) and (3, 0).

\ Minimum value of Z occur at any point on the line segment (0, 2) and (3, 0).

Q58 Text Solution:

Given, n = 10

Probability of odd number, p = 1/2

$\therefore q=\frac{1}{2}$

$\therefore$ Required probability $= p(X \geq 1)$

$= 1 - p(X = 0)$

$= 1 -{}^{10} C_0 \left(\frac{1}{2}\right)^{10-0}\left(\frac{1}{2}\right)^0$

$= 1 - \frac{1}{2^{10}}$

$= 1 - \frac{1}{1024} = \frac{1023}{1024}$

Q59 Text Solution:

X	0	1	2
P(X)	25/36	k	1/36

$\because$ Mean $= {}_{i=1}^{n} x_i P(x_i)$

$\Rightarrow \frac{1}{3} = 0 \times \frac{25}{36} + 1 \times k + 2 \times \frac{1}{36}$

$\Rightarrow \frac{1}{3} = k + \frac{1}{18}$

$\therefore k = \frac{1}{3} - \frac{1}{18} = \frac{6-1}{18} = \frac{5}{18}$

Now, variance $= \sum_{i=0}^{n} x_i^2 P_i(x) - (\text{mean})^2$

$= (0)^2 \times \frac{25}{36} + (1)^2 \times k + (2)^2 \times \frac{1}{36} - \frac{1}{9}$

$= 0 + k + \frac{1}{9} - \frac{1}{9} = k = \frac{5}{18}$

Hence, variance $= \frac{5}{18}$

Q60 Text Solution:

$\because p(X = 2) = 9 \times p(X = 3)$(where, n
$= 5$ and $q = 1 - p)$

$\Rightarrow {}^5C_2 p^2(1-p)^3 = 9 \cdot {}^5C_3 p^3(1-p)^2$

$\Rightarrow \frac{5!}{2!3!} p^2(1-p)^3 = 9 \cdot \frac{5!}{3!2!} p^3(1-p)^2$

$\Rightarrow \frac{p^2(1-p)^3}{p^3(1-p)^2} = 9 \Rightarrow \frac{1-p}{p} = 9$

$\Rightarrow 9p + p = 1 \Rightarrow 10p = 1$

$\therefore p = \frac{1}{10}$