KCET PYQ K (2020)

Kannada

MATHEMATICS

Instructions:

1. The question paper has five parts namely *A, B, C, D* and *E*. Answer all the parts.
2. Part-*A* has **10** multiple choice questions, **5** fill in the blanks and **5** very short answer questions of **1 marks** each.
3. Use the graph sheet for question on linear programming problem in Part *E*.

Part-A

I. Answer all the multiple choice questions: **10×1= 10**

1. The relation R in the set {1, 2, 3} given by $R = \{(1, 2), (2, 1)\}$ is

(A) Reflexive
(B) Symmetric
(C) Transitive
(D) Equivalence relation

2. The principal value branch of $tan^{-1}x$ is

(A) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
(B) $[0, \pi]$
(C) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
(D) $(0, \pi)$

3. If a matrix has 18 elements, then the number of matrices having all possible orders is

(A) 4 (B) 6
(C) 2 (D) 8

4. If *A* is a square matrix of order *n*, then $|adjA|$ is

(A) $|A|^{n-1}$ (B) $|A|^n$
(C) $|A|^{n^2}$ (D) $n|A|$

5. The derivative of $\sin^{-1} x$ exists in the interval

(A) $[-1, 1]$ (B) $(-1, 1)$
(C) R (D) $(-\frac{\pi}{2}, \frac{\pi}{2})$

6. $\int e^x \sec x(1+\tan x)\,dx$ equals

(A) $e^x \cos x + c$
(B) $e^x \tan x + c$
(C) $e^x \sin x + c$
(D) $e^x \sec x + c$

7. The unit vector in the direction of the vector $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ is

(A) $\frac{\hat{i} + \hat{j} + 2\hat{k}}{\sqrt{6}}$
(B) $\frac{\hat{i} + \hat{j} + 2\hat{k}}{6}$
(C) $\frac{\hat{i} + \hat{j} + 2\hat{k}}{4}$
(D) $\frac{\hat{i} + \hat{j} + 2\hat{k}}{2}$

8. If a line makes angle 90°, 60° and 30° with the positive direction of *X, Y* and *Z* axis respectively, then its direction cosines are

(A) $1, \frac{1}{2}, \frac{\sqrt{3}}{2}$
(B) $0, \frac{\sqrt{3}}{2}, \frac{1}{2}$
(C) $1, \frac{\sqrt{3}}{2}, \frac{1}{2}$
(D) $0, \frac{1}{2}, \frac{\sqrt{3}}{2}$

9. The corner points of the feasible region determined by the system of linear constraints are (0, 0), (0, 50), (30, 0), (20, 30). The maximum value of the objective function $z = 4x + y$ is

(A) 210 (B) 150
(C) 110 (D) 120

10. Let E and F be events of a sample space S of an experiment, then $P(E/F)+P(E'/F)$ is

(A) $P(E)$
(B) $P(F)$
(C) $P(S)$
(D) $P(E/F)$

II. Fill in the blanks by choosing the appropriate answer from those given in the bracket: **5 × 1 = 5**

(0, 2, 11, 3, 4)

11. The value of x in which $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$ is ___________

(A) 0 (B) 2
(C) 11 (D) 3
(E) 4

12. The slope of the tangent to the curve $y = x^3 - x$ at $x = 2$ is ___________

(A) 0 (B) 2
(C) 11 (D) 3
(E) 4

13. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} sin^7 xdx$ is ___________.

(A) 0 (B) 2
(C) 11 (D) 3
(E) 4

14. The order of the differential equation $\frac{d^4y}{dx^4} + sin(y''') = 0$ is ___________

(A) 0 (B) 2
(C) 11 (D) 3
(E) 4

15. The distance of the point (2, 3, –5) from the plane $x + 2y - 2z = 9$ is ___________

(A) 0 (B) 2
(C) 11 (D) 3
(E) 4

III. Answer all the following questions:

16. Let * be the binary operation on N of natural numbers given by $a * b$ = LCM of a and b. Find 5 * 7.

17. Find the derivative of the function $\sin(x^2 + 5)$ with respect to x.

18. Define coinitial vectors.

19. Define feasible region in a linear programming problem.

20. If A and B are two events such that $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{8}$, find P(not A and not B).

Part-B

Answer any nine questions: **9 × 2 = 18**

21. Find $g \circ f$ and $g \circ f$, if $f: R \to R$ and $g: R \to R$ are given by $f(x) = \cos x$ and $g(x) = 3x^2$.

22. Prove that $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2},\ x \in [-1, 1]$.

23. Find the value of $\sin\left(\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right)$.

24. Find the area of the triangle whose vertices are (1, 0), (6, 0) and (4, 3) using determinants.

25. Find $\frac{dy}{dx}$, if $y + \sin y = \cos x$.

26. Find $\frac{dy}{dx}$, if $y = x^x$, $x > 0$.

27. Using differential, find the approximate value of $\sqrt{25.3}$.

28. Evaluate $\int \frac{2 - 3\sin x}{\cos^2 x} dx$.

29. Evaluate $\int_{1}^{\sqrt{3}} \frac{dx}{1+x^2}$.

30. Form the differential equation representing the family of parabolas having vertex at origin and axis along positive direction of x-axis.

31. Find the projection of the vector $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ on the vector $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$.

32. Find the area of the parallelogram whose adjacent sides are determined by the vectors $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$.

33. Find the angle between the pair of lines given by $\vec{r} = \left(2\hat{i} - 5\hat{j} + \hat{k}\right) + \lambda\left(3\hat{i} + 2\hat{j} + 6\hat{k}\right)$ and $\vec{r} = \left(7\hat{i} - 6\hat{k}\right) + \mu\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$

34. Find the probability distribution of number of tails in the simultaneous tosses of three coins.

Part-C

Answer any nine questions: **9 × 3 = 27**

35. Let T be the set of all triangles in a plane with 'R' a relation in T given by $R=\{(T_1, T_2): T_1$ is congruent to $T_2\}$. Show that R is an equivalence relation.

36. Prove that $2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{31}{17}$

37. Express $\begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}$ as the sum of a symmetric and skew symmetric matrix.

38. Find $\frac{dy}{dx}$, if $x = a(\theta - \sin\theta)$ and $y = a(1+\cos\theta)$.

39. Verify Rolle's theorem for the function $y = x^2 + 2$, $a = -2$ and $b = 2$.

40. Find the intervals in which the function f given by $f(x) = 2x^2 - 3x$ is

(a) Increasing

(b) Decreasing

41. Find $\int \frac{dx}{(x+1)(x+2)}$

42. Evaluate $\int x \sin 3x\, dx$.

43. Find the area of the region bounded by $x^2 = 4y$, $y = 2$, $y = 4$ and the y-axis in the first quadrant.

44. Find the general solution of the differential equation $\frac{dy}{dx} = \left(1+x^2\right)\left(1+y^2\right)$.

45. Show that the position vector of the point P, which divides the line joining the points A and B having position vectors $\vec{a}$ and $\vec{b}$ internally in the ratio $m : n$ is $\frac{m\vec{b} + n\vec{a}}{m+n}$

46. Prove that $\left[\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}\right] = 2\left[\vec{a}, \vec{b}, \vec{c}\right]$

47. Find the distance between the lines given by $\vec{r} = \left(\hat{i} + 2\hat{j} - 4\hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)$ and $\vec{r} = \left(3\hat{i} + 3\hat{j} - 5\hat{k}\right) + \mu\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)$.

48. Of the students in a college, it is known that 60% reside in hostel and 40% are day scholars (not residing in hostel). Previous year results report that 30% of all students who reside in hostel attain A grade and 20% of day scholars attain A grade in their annual examination. At the end of the year, one student is chosen at random from the college and he has an A grade, what is the probability that the student is a hostlier?

Part-D

Answer any five questions: **5 × 5 = 25**

49. Let $f: N \to Y$ be a function defined as $f(x)=4x+3$, where $Y = \{y \in N: y = 4x + 3$, for some $x \in N\}$. Show that f is invertible. Find the inverse of f.

50. If $A = \begin{bmatrix} 0 & 6 & 7 \\ -6 & 0 & 8 \\ 7 & -8 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$

Calculate AC, BC and $(A+B)C$. Verify that $(A+B)C = AC + BC$.

51. Solve the following system of Linear equations by matrix method $3x–2y+3z = 8$ $2x + y – z = 14x – 3y + 2z = 4$

52. If $y = 3e^{2x} + 2e^{3x}$, prove that $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$

53. The length x of a rectangle is decreasing at the rate of 5 cm/min and the width y is increasing at the rate of 4 cm/min. When $x = 8$ cm and $y = 6$ cm, find the rates of change of

(a) the perimeter, and

(b) the area of the rectangle.

54. Find the integral of $\frac{1}{\sqrt{x^2 + a^2}}$ with respect to x and hence evaluate $\int \frac{dx}{\sqrt{x^2 + 2x + 2}}$

55. Find the area enclosed by the circle $x^2 + y^2 = a^2$ by the method of integration.

56. Find the general solution of the differential equation $x\frac{dy}{dx} + 2y = x^2 \, (x \neq 0)$.

57. Derive the equation of a plane perpendicular to a given vector and passing through a given point both in vector and Cartesian form.

58. A die is thrown 6 times. If 'getting an odd number' is a success, what is the probability of

(a) 5 successes?

(b) at least 5 successes?

(c) at most 5 successes?

Part-E

Answer the following questions:

59. (a) Minimize $Z = -3x + 4y$ Subject to constraints $x+2y \leq 8$ $3x + 2y \leq 12$ $x \geq 0$ and $y \geq 0$ by graphical method.

OR

(b) Prove that $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$ and hence evaluate $\int_0^{\frac{\pi}{2}} \frac{\cos^5 x}{\sin^5 x + \cos^5 x}\,dx$.

60. (a) Find the value of K so that the function $f(x)$ given by f(x) = $\begin{cases} Kx^2, & \text{if} x \leq 2 \\ 3, & \text{if} x > 2 \end{cases}$ is continuous at $x = 2$.

OR

(b) Prove that

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$$

CETKCET PYQ K (2023)

Kannada

Mathematics

Hint and Solution

1. (b) Or Symmetric

2. (c) Or $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

3. (b) Or 6

4. (a) Or $|A|^{n-1}$

5. (b) Or (-1, 1)

6. (d) Or $e^x \sec x + c$

7. (a) Or $\frac{i+j+2k}{\sqrt{6}}$

8. (d) Or 0, $\frac{1}{2}$, $\frac{\sqrt{3}}{2}$

9. (d) Or 120

10. (c) Or $P(S)$

11. 2

12. 11

13. 0

14. 4

15. 3

16. 5*7=35

17. $\cos(x^2+5)2x$

18. Two or more vectors having the same initial point

19. The common region determined by all the constraints including non-negative constraints of a LPP is called feasible region

20. $\frac{3}{8}$

21. $(f\circ g)(x)=f(g(x))=f(3x^2)=\cos(3x^2)$
$(g\circ f)(x)=g(f(x))=g(\cos x)$
$=3(\cos x)^2=3\cos^2 x$

22. $\sin^{-1}x=\theta$ and $x=\sin\theta=\cos\left(\frac{\pi}{2}-\theta\right)$
$\cos^{-1}x+\theta=\frac{\pi}{2}$ or $\cos^{-1}x+\sin^{-1}x=\frac{\pi}{2}$

23. $\sin\left(\frac{\pi}{3}+\sin^{-1}\left(\frac{1}{2}\right)\right)$
$\sin\frac{\pi}{2}=1$

24. Area= $\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1\\ x_2 & y_2 & 1\\ x_3 & y_3 & 1\end{vmatrix}$ OR Area= $\frac{1}{2}\begin{vmatrix} 1 & 0 & 1\\ 6 & 0 & 1\\ 4 & 3 & 1\end{vmatrix}$
Area= $-\frac{15}{2}$

25. $\frac{dy}{dx}+\cos y\frac{dy}{dx}=-\sin x$
$\frac{dy}{dx}=\frac{-\sin x}{1+\cos y}$

26. $y = x^x$ and $\log y = x \log x$

$\frac{dy}{dx} = y(1+\log x)$ or $\frac{dy}{dx} = x^x(1+\log x)$

27. $f(x) = \sqrt{x}$ and $f'(x) = \frac{1}{2\sqrt{x}}$

$\sqrt{25.3} = 5.03$

28. $\int\left(2\sec^2 x - 3\sec x\tan x\right)dx$

$2\tan x - 3\sec x + c$

29. $\int_{1/\sqrt{3}}^{\sqrt{3}} \frac{1}{1+x^2}dx = \left[\tan^{-1} x\right]_{1/\sqrt{3}}^{\sqrt{3}}$

$\frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$

30. $y^2 = 4ax$ and $2y\frac{dy}{dx} = 4a$

$2y\frac{dy}{dx} = \frac{y^2}{x}$ OR $y^2 - 2xy\frac{dy}{dx} = 0$

31. $\vec{a}\cdot\vec{b} = 10$ and $\left|\vec{b}\right| = \sqrt{6}$ OR

Projection of $\vec{a}$ on $\vec{b} = \frac{\vec{a}\cdot\vec{b}}{\left|\vec{b}\right|}$

Projection of $\vec{a}$ on $\vec{b}$ is $\frac{10}{\sqrt{6}}$

32. $\vec{a}\times\vec{b} = 20\hat{i} + 5\hat{j} - 5\hat{k}$

Area of parallelogram= $\sqrt{450}$ sq units

OR $15\sqrt{2}$ sq units

33. $b_1 \cdot b_2 = 19$ and $|b_1| = 7, |b_2| = 3$ OR

$\cos\theta = \frac{b_1 \cdot b_2}{|b_1||b_2|}$

OR

$\theta = \cos^{-1}\left(\frac{19}{21}\right)$

34. S = {HHH,HHT,HTH,HTT,TTT,TTH,THT,THH}

X	0	1	2	3
$P(X)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

35. Reflexive: Every triangle is congruent to itself

OR

Showing R is symmetric Showing R is transitive

And Hence R is an equivalence relation

36. $2\tan^{-1}\frac{1}{2} = \tan^{-1}\frac{4}{3}$

$\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

OR $\tan^{-1}\frac{4}{3} + \tan^{-1}\frac{1}{7} = \tan^{-1}\left(\frac{\frac{4}{3}+\frac{1}{7}}{1-\frac{4}{3}\times\frac{1}{7}}\right)$

$\tan^{-1}\frac{31}{17}$

37. $A' = \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix}$

$\frac{1}{2}(A+A') = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix} = P$

$\frac{1}{2}(A-A') = \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix} = Q$ and $P+Q = \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}$

38. $\frac{dx}{d\theta} = a(1-\cos\theta)$

$\frac{dy}{d\theta} = -a\sin\theta$

$\frac{dy}{dx} = \frac{-a\sin\theta}{a(1-\cos\theta)}$ OR $\frac{dy}{dx} - \tan\frac{\theta}{2}$

39. f is continuous in [−2, 2] and f is differentiable in (–2, 2)

$f'(x) = 2x$

OR

$f(-2) = 6$ and $f(2) = 6$

$x = 0 \in (-2, 2)$

40. $f'(x) = 4x - 3$

OR $x = \frac{3}{4}$

f is increasing in $(\frac{3}{4}, \infty)$

f is decreasing in $(-\infty, \frac{3}{4})$

41. $\frac{1}{(x+1)(x+2)} = \frac{A}{(x+1)} + \frac{B}{(x+2)}$

$A = 1$ and $B = -1$

$I = \log(x+1) - \log(x+2) + c$

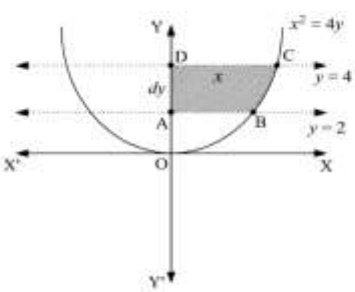
OR $I = \frac{\log(x+1)}{\log(x+2)} + c$

42. $I = x\int \sin 3x dx - \int\left(\frac{d(x)}{dx}\int \sin 3x dx\right)dx$

$I = -x\frac{\cos 3x}{3} + \int\frac{\cos 3x}{3}dx$

$I = -x\frac{\cos 3x}{3} + \frac{\sin 3x}{9} + c$

43.



Area, $A = \int_2^4 x dy = \int_2^4 2\sqrt{y}dy$

Area, $A = 2\left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_2^4$

Area $Area = \frac{4}{3}\left[4^{\frac{3}{2}} - 2^{\frac{3}{2}}\right]$ OR $\frac{4}{3}\left[8 - 2\sqrt{2}\right]$

44. $\frac{dy}{1+y^2} = \frac{dx}{1+x^2}$

$\int\frac{dy}{1+y^2} = \int\frac{dx}{1+x^2}$

$\tan^{-1} y = \tan^{-1} x + c$

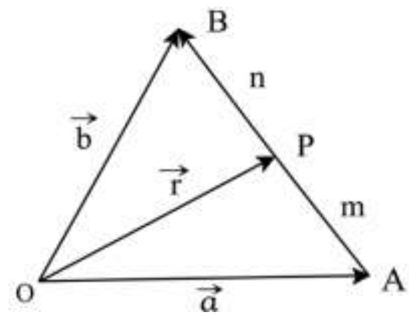
45. LHS = $(\vec{a}+\vec{b})\cdot((\vec{b}+\vec{c})\times(\vec{c}+\vec{a}))$

$(\vec{a}+\vec{b})\cdot[\vec{b}\times\vec{c}+\vec{b}\times\vec{a}+\vec{c}\times\vec{a}]$

RHS $= \vec{a}\cdot(\vec{b}\times\vec{c})+0+0+0+0+\vec{b}\cdot(\vec{c}\times\vec{a})$

$= 2[\vec{a}\vec{b}\vec{c}]$

46.



$\frac{m}{n} = \frac{\overrightarrow{AP}}{\overrightarrow{PB}}$

$m(\overrightarrow{OB}-\overrightarrow{OP}) = n(\overrightarrow{OP}-\overrightarrow{OA})$

$\overrightarrow{OP} = \frac{m\overrightarrow{OB}+n\overrightarrow{OA}}{m+n}$ OR $\overrightarrow{OP} = \frac{m\vec{b}+n\vec{a}}{m+n}$

47. $\vec{a_1} = \hat{i}+2\hat{j}-4\hat{k}$, $\vec{a_2} = 3\hat{i}+3\hat{j}-5\hat{k}$,

$\vec{b} = 2\hat{i}+3\hat{j}+6\hat{k}$ OR $(\vec{a_2}-\vec{a_1}) = 2\hat{i}+\hat{j}-\hat{k}$

OR $(\vec{a_2}-\vec{a_1})\times\vec{b} = 9\hat{i}-14\hat{j}+4\hat{k}$

OR $|(\vec{a_2}-\vec{a_1})\times\vec{b}| = \sqrt{293}$

OR $|\vec{b}| = 7$

$d = \frac{|(\vec{a_2}-\vec{a_1})\times\vec{b}|}{|\vec{b}|}$

$d = \frac{\sqrt{293}}{7}$

48. $P(E_1) = \frac{60}{100} = 0.6, P(E_2)$

$= \frac{40}{100} = 0.4, P(A|E_1) = \frac{30}{100} = 0.3$

$$P(A|E_2)=\frac{30}{100}=0.3$$

$$P(E_1|A)=\frac{P(A|E_1)P(E_1)}{P(A|E_1)P(E_1)+P(A|E_2)P(E_2)}$$

$$P(E_1|A)=\frac{9}{13}$$

49. $f(x)=y=4x+3$ and $g(y)=\dfrac{y-3}{4}$

Showing $fog(y)=y$

Showing $gof(x)=x$

$f\circ g=I_y$ and $g\circ f=I_N$ and concluding f is invertible or f^{-1} inverse exists.

inverse $f^{-1}(x)=\dfrac{x-3}{4}$

or g is the inverse of f

OR $f(x_1)=f(x_2)\Rightarrow x_1=x_2$

Hence f is one-one

$x=\dfrac{y-3}{4}$ $\forall y\in R$ there exist $x\in R$

such that $f(x)=y$ hence f is onto

OR $f\left(\dfrac{y-3}{4}\right)=y$

f is invertible and getting $f^{-1}(x)=\dfrac{x-3}{4}$

50. $A+B=\begin{bmatrix}0&7&8\\-5&0&10\\7&-8&0\end{bmatrix}$

$AC=\begin{bmatrix}9\\12\\30\end{bmatrix}$

$BC=\begin{bmatrix}1\\8\\-2\end{bmatrix}$

$(A+B)C=\begin{bmatrix}10\\20\\28\end{bmatrix}$

$AC+BC=\begin{bmatrix}10\\20\\28\end{bmatrix}$ and hence $(A+B) = AC + BC$

51. $A=\begin{bmatrix}3&-2&3\\2&1&-1\\4&-3&2\end{bmatrix}$, $X=\begin{bmatrix}x\\y\\z\end{bmatrix}$ and $B=\begin{bmatrix}8\\1\\4\end{bmatrix}$

OR

$|A| = -17 \neq 0$

$\text{adj}(A)=\begin{bmatrix}-1&-5&-1\\-8&-6&9\\-10&1&7\end{bmatrix}$

$X = A^{-1}B = \dfrac{1}{|A|}(\text{adj}A)B,$

$X=-\dfrac{1}{17}\begin{bmatrix}-1&-5&-1\\-8&-6&9\\-10&1&7\end{bmatrix}\begin{bmatrix}8\\1\\4\end{bmatrix}$

$x = 1, y = 2, z = 3$

52. $\dfrac{dy}{dx}=6e^{2x}+6e^{3x}$

$$\frac{d^2y}{dx^2}=6e^{2x}(2)+6e^{3x}(3)$$

$$\frac{d^2y}{dx^2}=12e^{2x}+18e^{3x}$$

Substituting the values of $\dfrac{d^2y}{dx^2}$, $\dfrac{dy}{dx}$, and y in the LHS

$$\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=0$$

53. $\dfrac{dx}{dt}=-5$ and $\dfrac{dy}{dt}=4$

Perimeter of the rectangle, $P = 2(x + y)$

$$\frac{dP}{dt}=2\left(\frac{dx}{dt}+\frac{dy}{dt}\right)=2(-5+4)=-2$$

Area of the rectangle, $A = x.\,y$

$$\frac{dA}{dt}=x\frac{dy}{dt}+y\frac{dx}{dt}=8(4)+6(-5)=2$$

54. Taking: $x = a\tan\theta$

$$\therefore I = \int \frac{a\sec^2\theta d\theta}{\sqrt{a^2\tan^2\theta + a^2}} = \int \sec\theta d\theta$$

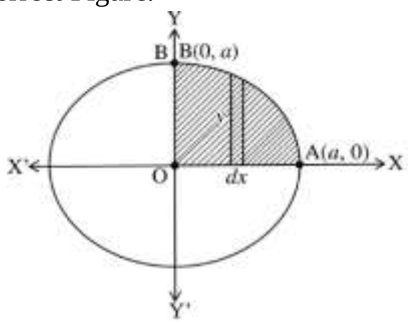
$I = \log|x + \sqrt{x^2 + a^2}| - \log|a| + C_1$

$= \log|x + \sqrt{x^2 + a^2}| + c,$

$$\int \frac{1}{\sqrt{x^2+2x+2}} dx = \int \frac{1}{\sqrt{(x+1)^2+1}} dx$$

$$I = \log\left|(x+1) + \sqrt{(x+1)^2+1}\right| + C$$

55. Correct Figure:



Area $A = 4\int_0^a y$ 1

Area $A = 4\int_0^a \sqrt{a^2 - x^2}\, dx$

Area $A = 4\left[\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a$

Area$= \pi a^2$ square unit. 1

56. $\frac{dy}{dx} + \frac{2}{x}y = x$ OR $P = \frac{2}{x}$ and $Q = x$

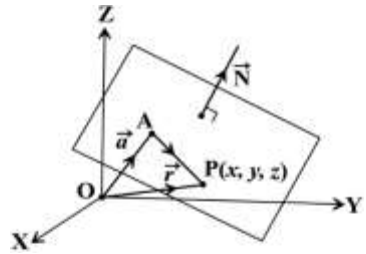
I.F. $= e^{\int P dx} = e^{\int \frac{2}{x} dx} = e^{2\log(x)} = x^2$

$y(I.F.) = \int Q(I.F.)dx + C$

$yx^2 = \int x.x^2 dx + C$ OR $yx^2 = \int x^3 dx + C$

General solution $yx^2 = \frac{x^4}{4} + C.$

57. Correct figure



$AP \cdot \vec{N} = 0$ 1

$(\overrightarrow{OP} - \overrightarrow{OA}) \cdot \vec{N} = 0$ and $(\vec{r} - \vec{a}) \cdot \vec{N} = 0$

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$,

$\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ and $\vec{N} = A\hat{i} + B\hat{j} + C\hat{k}$

Cartesian form

$A(x - x_1) + B(y - y_1) + C(z - z_1) = 0$

58. $n = 6, p = \frac{1}{2}$ and $q = \frac{1}{2}$

$P(X = x) = nC_x p^x q^{n-x}$

$P(X = 5) = 6C_5\left(\frac{1}{2}\right)^6 = \frac{3}{32}$

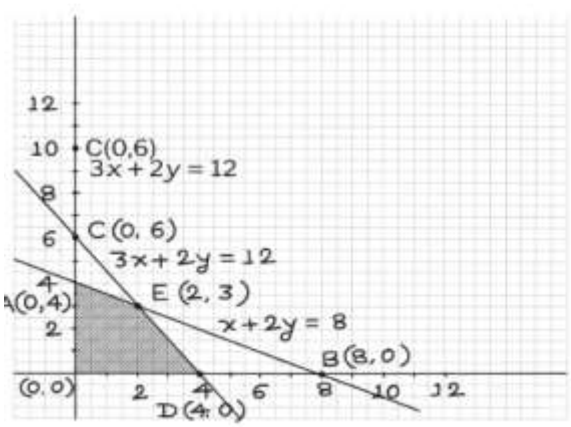
$P(X = 5) + P(X = 6) = 6C_5\left(\frac{1}{2}\right)^6 + 6C_6\left(\frac{1}{2}\right)^6 = \frac{7}{64}$

$P(X \le 5) = 1 - P(X = 6) = 1 - 6C_6\left(\frac{1}{2}\right)^6$

$= 1 - \frac{1}{64} = \frac{63}{64}$

P(none is a spade)

59.



Drawing the graph and shading the feasible region carries

Corner points $A(0,4), D(4,0), E(2,3)$ and $O(0,0)$

Corner points	$Z = -3x + 4y$
$A(0, 4)$	16
$D(0, 4)$	–12
$E(0, 4)$	6
$O(0, 0)$	0

Minimum of Z is –12 at (4, 0) OR showing it in the above second row.

$$I = \int_0^a f(x)dx$$

Putting $x = a - t$, then $dx = -t$ and $x = 0$, then $t = a$ and $x = a$, then $t = 0$.

$$I = \int_0^a f(a-t)dt$$

$$I = \int_0^a f(a-x)dx$$

$$I = \int_0^{\frac{\pi}{2}} \frac{\cos^5 x}{\sin^5 x + \cos^5 x} dx \text{ and}$$

$$I = \int_0^{\frac{\pi}{2}} \frac{\sin^5 x}{\sin^5 x + \cos^5 x} dx$$

$$2I = \int_0^{\frac{\pi}{2}} 1 dx$$

$$I = \frac{\pi}{4}$$

60. $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} = f(2)$

$$\lim_{x \to 2^-} f(x) = 4k$$

$$\lim_{x \to 2^+} f(x) = 3$$

$$k = \frac{3}{4}$$

Applying $R_1 \to R_1 + R_2 + R_3$ and

getting $\begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$

$$(a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

Applying $c_2 = c_2 - c_1$ and $c_3 \to c_3 - c_1$ and getting

$$(a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ 2b & -(a+b+c) & 0 \\ 2c & 0 & -(a+b+c) \end{vmatrix}$$

On expansion getting $(a + b + c)^3$