Subject : Mathematics
Class : XI

Kannada

Limit and Derivatives

Limit

- The expected value of the function as dictated by the points to the left of a point defines the left hand limit of the function at that point. Similarly the right hand limit.
- Limit of a function at a point is the common value of the left and right hand limits, if they coincide.
- For a function f and a real number a, $\lim\limits_{x\to a} f(x)$ and $f(a)$ may not be same (In fact, one may be defined and not the other one).

Fundamental Theorems on Limits

- For functions f and g if $\lim\limits_{x\to a} f(x)$ and $\lim\limits_{x\to a} g(x)$ exists, then the following holds :

$$\lim_{x\to a}\left[f(x)\pm g(x)\right]=\lim_{x\to a} f(x)\pm\lim_{x\to a} g(x)$$

$$\lim_{x\to a}\left[f(x)\cdot g(x)\right]=\lim_{x\to a} f(x)\cdot\lim_{x\to a} g(x)$$

$$\lim_{x\to a}\left[\frac{f(x)}{g(x)}\right]=\frac{\lim\limits_{x\to a} f(x)}{\lim\limits_{x\to a} g(x)}\; g(x)\neq 0 \;\&\; \lim_{x\to a} g(x)\neq 0$$

Theorem

Following are some of the standard limits

$$\lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1},\ \text{for any } +ve \text{ integer}$$

$$\lim_{x\to a}\frac{\log(1+x)}{x}=1$$

$$\lim_{x\to 0}\frac{\sin x}{x}=\lim_{x\to 0}\frac{\tan x}{x}=1$$

$$\lim_{x\to 0}\frac{1-\cos x}{x}=0$$

$$\lim_{x\to 0}\left(1+\frac{1}{x}\right)^x=e$$

$$\lim_{x\to 0}\frac{e^x-1}{x}=1$$

$$\lim_{x\to 0}\frac{a^x-1}{x}=\log_e a$$

L'hopital's Rule

If $\lim_{x\to 0}\frac{f(x)}{g(x)}$ is of form $\frac{0}{0}$

Then $\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}$

Derivatives

Derivative of a function f at any point x is defined by

$$f'(x)=\frac{df(x)}{dx}=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

For functions u and v the following holds :

$(u\pm v)'=u'\pm v'$

$(uv)'=u'v+uv'$

$\left(\frac{u}{v}\right)'=\frac{u'v-uv'}{v^2}$ provided all are defined.

Following are some of the standard derivatives.

$\frac{d}{dx}(x^n)=nx^{n-1}$

$\frac{d}{dx}(\sin x)=\cos x$

$\frac{d}{dx}(\cos x)=-\sin x$

$\frac{d}{dx}(e^x)=e^x$

$\frac{d}{dx}(a^x)=a^x\log a$

$\frac{d}{dx}(\ln x)=\frac{1}{x}$

$\frac{d}{dx}(\tan x)=\sec^2 x$

$\frac{d}{dx}(\operatorname{cosec} x)=-\operatorname{cosec} x\cot x$

$\frac{d}{d}(\sec x)=\sec x\tan x$



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