KCET PYQ K (2025)

Kannada

MATHEMATICS

Instructions:

1. The question paper has five Parts namely A, B, C, D and E. Answer all parts.
2. PART-A has 15 M.C.Q.'s. 5 fill in the blanks of 1 mark each.
3. For PART-A questions, only the first written Answer will be considered for awarding marks.
4. Use graph sheet for question on linear Programming in PART-E.
5. For questions having figure/graph, alternate questions are given at the end of question paper in separate PART-F for visually challenged students.

PART-A

I. Answer all the multiple choice questions:

1. A relation R in a set A is called Reflexive relation if

(a) $(a, a) \in R$ for all $a \in A$

(b) $(a, a) \in R$ for atleast one $a \in A$

(c) $(a, b) \in R$ implies $(b, a) \in R$

(d) $(a, b) \in R$ and $(b, c) \in R$ implies $(a, c) \in R$

2. The principal value of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

3. Match List -I with List-II:

	List-I		List-II
(A)	Domain of $\sin^{-1}x$	(i)	$\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$
(B)	Range of $\tan^{-1} x$	(ii)	$[0, \pi]$
(C)	Range of $\cos^{-1}x$	(iii)	$[-1, 1]$

Choose the correct answer from the options given below:

(a) A-i, B-ii, C-iii (b) A-iii, B-ii, C-i

(c) A-ii, B-i, C-iii (d) A-iii, B-i, C-ii

4. For a 2 × 2 matrix $A = [a_{ij}]$ whose elements are given by $a_{ij} = 2i - j$ then A is equal to

(a) $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

5. Let A be a nonsingular matrix of order 3 × 3 then $|adj\ A|$ is equal to

(a) $|A|$ (b) $3\ |A|$

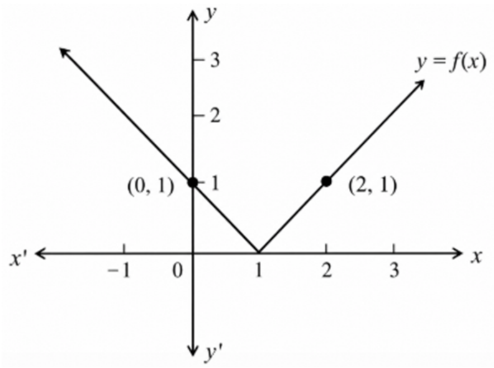
(c) $|A|^3$ (d) $|A|^2$

6. If $f(x) = \cos 2x$, then $f'\left(\frac{x}{4}\right)$ is

(a) 2 (b) –2

(c) $\sqrt{2}$ (d) $-\sqrt{2}$

7. For the given figure consider the following statements 1 and 2:



Statement 1: Left hand derivative of $y = f(x)$ at $x = 1$ is –1.

Statement 2: The function $y = f(x)$ is differentiable at $x = 1$.

Then which of the following are true?

(a) Statement-I is true, Statement-2 is false.

(b) Statement-1 is false, Statement-2 is true

(c) Both Statement 1 and 2 are true

(d) Both Statement-1 and 2 are false.

8. The absolute maximum value of the function f given by $f(x)=x^3,\ x\,[-2,2]$ is

(a) 2 (b) 0

(c) –2 (d) 8

9. $\int e^x(\sin x-\cos x)dx$ is

(a) $-e^x\cos x$ (b) $e^x\cos x$

(c) $e^x\sin x$ (d) $e^x\sin^2 x$

10. The degree of differential equation $\frac{d^3y}{dx^3}+\frac{d^2y}{dx^2}+e^{\frac{dy}{dx}}=0$ is

(a) 1 (b) 3

(c) 2 (d) not defined

11. The direction cosines of the vector $\vec{a}=\hat{i}-\hat{j}-2\hat{k}$ are

(a) $\frac{1}{\sqrt{5}},\frac{-1}{\sqrt{5}},\frac{2}{\sqrt{5}}$ (b) $\frac{1}{\sqrt{6}},\frac{-1}{\sqrt{6}},\frac{2}{6}$

(c) $\frac{1}{6},\frac{-1}{6},\frac{2}{6}$ (d) $\frac{-1}{\sqrt{6}},\frac{1}{\sqrt{6}},\frac{2}{\sqrt{6}}$

12. The angle between two vectors $\vec{a}$ and $\vec{b}$ with $|\vec{a}|=\sqrt{3},|\vec{b}|=2$ and $\vec{a}.\vec{b}=\sqrt{6}$ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

13. The equation of y-axis in space is

(a) $x=0,y=0$ (b) $x=0,z=0$

(c) $y=0,\ z=0$ (d) $y=0$

14. If $P(A)=\frac{1}{2},\ P(B|A)=\frac{2}{3}$ then $P(A\cap B)$ is

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$

(c) 1 (d) $\frac{3}{5}$

15. **Assertion [A]** : For two events E and F if $P(E)=\frac{1}{5},\ P(F)=\frac{1}{2}$ and $P(E/F)=\frac{1}{5}$ them E and F are independent events.

Reason [R]: If E and F are two independent events then $P(E/F)=P(F)$.

The which of the following are true?

(a) [A] is true but [R] is false

(b) Both [A] and [R] are false

(c) Both [A] and [R] are true

(d) [A] is false but [R] is true

II. Fill in the blanks by choosing the appropriate answer from those given in the bracket:

$[0, 2, 1, \frac{5}{9}, -1, 6]$

16. The value of $\cos\left(\sec^{-1}(2)-\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)$ is ……..

17. If $y=\sin^{-1}(\cos x)$ then $\frac{dx}{dy}=$.......

18. The value of $\int_7^{13} 1\,dx=$........... .

19. The projection of vector $\hat{i}+\hat{j}$ along the vector $\hat{i}-\hat{j}$ is ……

20. If $P(A\cap B)=\frac{4}{13}$ and $P(B)=\frac{9}{13}$ then $P(A'|B)=$...........

Part-B

III. Answer any six of the following questions:

21. Find the equation of the line through the points (1, 2) and (3, 6) using determinants.

22. If $\sqrt{x}+\sqrt{y}=\sqrt{10}$ then show that $\frac{dy}{dx}+\sqrt{\frac{y}{x}}=0$.

23. A balloon which is always remains spherical has a variable radius. Find the rat eat which its volume is increasing with radius when the radius is 10 cms.

24. Find the interval which the function given by $f(x) = 4x^3 - 6x^2 - 72x + 3$ is decreasing.

25. Find $\int \cos x.\log(\sin x)dx$.

26. Verify that the function $y = a\sin x + b\cos x$ is a solution of differential equation $\frac{d^2y}{dx^2} + y = 0$.

27. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$ then find the unit vector parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c}$.

28. If the lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{3}$ and $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-5}$ perpendicular to each other, then find the value of k.

29. An urn contains 10 block and 5 white balls. Two balls are drawn from the urn one after the other without replacement. What is the probability that both drawn balls black?

PART-C

IV. Answer **any six** of the following questions:

30. Check whether the relation R in R defined by $R = \{(a, b): a \leq b^3\}$ is reflexive, symmetric and transitive.

31. Prove that $\tan^{-1}\left(\frac{63}{16}\right) = \sin^{-1}\left(\frac{5}{13}\right) + \cos^{-1}\left(\frac{3}{5}\right)$.

32. Express $\begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}$ as the sum of a symmetric and the skew-symmetric matrix.

33. Find $\frac{dy}{dx}$ if $x = a\left(\cos t + \log\left(\tan\frac{t}{2}\right)\right)$ and $y = a\sin t$.

34. Find the two positive numbers x and y such that $x + y = 60$ and xy^3 is maximum.

35. Evaluate $\int \frac{2x}{x^2 + 3x + 2} dx$

36. Find the area of triangle ABC where position vectors of A, B, C are $\hat{i} - \hat{j} + 2\hat{k}$, $2\hat{j} + \hat{k}$, $\hat{j} + 3\hat{k}$ respectively/

37. Derive the equation of a line in space through a given point and parallel to a given vector $\vec{b}$ in the vector form.

38. In two identical boxes, box 1 contains 2 gold coins, while box II contains one gold and one silver coin. A person chooses a box at random and takes out a coin. If the coin is of gold, what is the probability that the other coin in the box is also a gold?

PART-D

V. Answer **any four** of the following questions: **4×5=20**

39. If $A = \mathbb{R} - \{3\}$ and $B = \mathbb{R} - [1]$ and $f: A \to B$ is a function defined by $f(x) = \left(\frac{x-2}{x-3}\right)$. Is f one-one and onto? Justify your answer.

40. If $A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}$, verify that $(AB)' = B'A'$.

41. Solve the following system of linear equations by matrix method $4x + 3y + 2z = 60$, $2x + 4y + 6z = 90$, $6x + 2y + 3z = 70$.

42. If $y = (\tan^{-1} x)^2$ then show that $(x^2 + 1)^2 y_2 + 2x(x^2 + 1)y_1 = 2$.]

43. Find the integral of $\frac{1}{x^2+a^2}$ with respect to 'x' and hence find $\int \frac{1}{x^2-6x+13}dx$.

44. Find the area of $x^2+y^2=a^2$ circle by method of integration.

45. **S**olve the differential equation $\cos^2 \times \frac{dy}{dx}\ y = \tan x \left(0 \le x < \frac{\pi}{2}\right)$.

PART-E

VI. Answer the following questions:

46. Prove that $\int_0^a f(x)dx = \int_0^a (a-x)dx$ and hence evaluate $\int_0^{\frac{\pi}{4}} \log(1+\tan x)dx$. (6)

OR,

Solve the following linear Programming Problem graphically: Minimise and Maximise $Z = 5x + 10y$.
Subject to:
$x+2y \le 120,$
$x+y \ge 60,$
$x-2y \ge 0,$
$x \ge 0, y \ge 0.$

47. If $A=\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2-5A+7I=O$ and hence find A^{-1}. (4)

OR

Determine the value of k if $f(x)=\begin{cases} \frac{k\cos x}{\pi-2x'} & x \ne \frac{\pi}{2} \\ 3, & x=\frac{\pi}{2} \end{cases}$ is continuous at $x=\frac{\pi}{2}$.

PART-F

VII. For visually challenged students only:

7. If **Statement 1:** Left hand derivative of $f(x)=|x|$ at $x = 0$ is –1.

Statement 2: The derivative of $f(x)=|x|$ exists at $x = 0$.

Then which of the following is true?

(a) Statement 1 is true, Statement 2 is false

(b) Statement 1 is false, Statement 2 is true.

(c) Statement 1 and 2 both are true.

(d) Statement 1 and 2 both are false.

CETKCET PYQ K (2020)

Kannada

Mathematics

Hint & Solution

1. By definition, a relation R on a set A is reflexive if every element of A is related to itself.
$\therefore (a,a)\in R, \quad \forall a\in A$
Correct Option: (a)

2. Let $y=\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)\Rightarrow \sin y=\frac{1}{\sqrt{2}}$.

Since the principal value branch of $\sin^{-1}x$ is $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$:

$\sin\left(\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}\Rightarrow y=\frac{\pi}{4}$
Correct Option: (c)

3. • Domain of $\sin^{-1}x$ is $[-1,1]\Rightarrow$ A-iii

• Range of $\tan^{-1}x$ is $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)\Rightarrow$ B-i

• Range of $\cos^{-1}x$ is $[0,\pi]\Rightarrow$ C-ii
Correct Option: (d)

4. Evaluate the elements for $i,j\in\{1,2\}$:
- $a_{11}=2(1)-1=1$
- $a_{12}=2(1)-2=0$
- $a_{21}=2(2)-1=3$
- $a_{22}=2(2)-2=2$

$A=\begin{bmatrix}1 & 0\\ 3 & 2\end{bmatrix}$
Correct Option: (b)

5. For any non-singular square matrix of or der n |adj |$adj\ A$| = |A|$^{n-1}$
Given order $n=3$:
$|adjA|=|A|^{3-1}=|A|^2$
Correct Option: (d)

6. Differentiating w.r.t x:
$f'(x)=-2\sin 2x$

Substitute $x=\frac{\pi}{4}$:

$f'\left(\frac{\pi}{4}\right)=-2\sin\left(2\cdot\frac{\pi}{4}\right)=-2\sin\left(\frac{\pi}{2}\right)=-2(1)=-2$
Correct Option: (b)

7. • From the graph, the line to the left of $x=1$ passes through $(0,1)$ and $(1,0)$. Its slope is $\frac{0-1}{1-0}=-1$.
Hence, the Left-Hand Derivative is -1 (Statement 1 is true)
• At $x=1$, the graph exhibits a sharp turn (corner point), meaning the left-hand slope (-1) does not equal the right-hand slope (1). Thus, it is not differentiable at $x=1$ (Statement 2 is false)
Correct Option: (a)

8. Evaluate the critical and endpoint values:
- $f'(x)=3x^2=0\Rightarrow x=0$
- $f(-2)=(-2)^3=-8$
- $f(0)=0^3=0$
- $f(2)=(2)^3=8$

The absolute maximum value is 8
Correct Option: (d)

9. Using the standard identity
$\int e^x[f(x)+f'(x)]dx=e^x f(x)$:
Let $f(x)=-\cos x\Rightarrow f'(x)=\sin x$.
$\int e^x[\sin x+(-\cos x)]dx=e^x(-\cos x)=-e^x\cos x$
Correct Option: (a)

10. Because the differential equation contains an exponential term involving a derivative $\left(e^{\frac{dy}{dx}}\right)$, it cannot be expressed as a polynomial equation in terms of its derivatives. Consequently, its degree is not defined.
Correct Option: (d)

11. Find the magnitude $|\vec{a}|$
$|\vec{a}| = \sqrt{1^2 + (-1)^2 + 2^2} = \sqrt{1+1+4} = \sqrt{6}$
The direction cosines are given by $\left(\frac{x}{|\vec{a}|}, \frac{y}{|\vec{a}|}, \frac{z}{|\vec{a}|}\right)$
$\left(\frac{1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{2}{\sqrt{6}}\right)$
Correct Option: (b)

12. Using the dot product formula:
$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \frac{\sqrt{6}}{\sqrt{3}\cdot 2} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$
$\theta = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$
Correct Option: (c)

13. At any point on the y-axis in a three-dimensional space, both the x-coordinate and the z-coordinate are identically zero.
$\therefore x = 0, \quad z = 0$
Correct Option: (b)

14. By the multiplication rule of probability:
$P(A \cap B) = P(A)\cdot P(B|A) = \frac{1}{2}\cdot\frac{2}{3} = \frac{1}{3}$
Correct Option: (a)

15. Assertion (A): Two events are independent if $P(E|F) = P(E)$ (p. 14). Here, $P(E|F) = \frac{1}{5}$ and $P(E) = \frac{1}{5}$. Since they are equal, the events are independent. Thus, (A) is true.
Reason (R): By definition, if E and F are independent, the occurrence of E does not affect the probability of F, so $P(F|E) = P(F)$. Thus, (R) is true.
Correct Option: (c)

16. 1. Find the individual inverse values:
1. $\sec^{-1}(2) = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$
2. $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$

2. Substitute these back into the expression:
$\cos\left(\frac{\pi}{3} - \frac{\pi}{3}\right) = \cos(0) = 1$

17. Using trigonometric identities, rewrite $\cos x$ as a sine function:
$y = \sin^{-1}\left(\sin\left(\frac{\pi}{2} - x\right)\right)$
Within the principal value branch:
$y = \frac{\pi}{2} - x$
Differentiating with respect to x:
$\frac{dy}{dx} = 0 - 1 = -1$
Answer: -1

18. Integrate the constant:
$\int_7^{13} 1 dx = [x]_7^{13}$
Substitute the upper and lower limits:
$= 13 - 7 = 6$

19. Let $\vec{a} = \hat{i} + \hat{j}$ and $\vec{b} = \hat{i} - \hat{j}$.
The projection of vector $\vec{a}$ along vector $\vec{b}$ is given by the formula:
$\text{Projection} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$
1. Calculate the dot product $\vec{a}\cdot\vec{b}$:
$(1)(1) + (1)(-1) = 1 - 1 = 0$

2. Compute the final projection value:

$$\text{Projection} = \frac{0}{\sqrt{1^2+(-1)^2}} = 0$$

Answer: 0

20. Using the complement rule for conditional probability:

$P(A'|B) = 1 - P(A|B)$

Substitute the conditional probability formula $P(A|B) = \dfrac{P(A\cap B)}{P(B)}$:

$$P(A'|B) = 1 - \frac{\frac{4}{13}}{\frac{9}{13}} = 1 - \frac{4}{9} = \frac{9-4}{9} = \frac{5}{9}$$

Answer: $\dfrac{5}{9}$

21. Let $P(x, y)$ be any point on the line joining $A(1,2)$ and $B(3,6)$. Since these points are collinear, the area of the triangle formed by them must be zero:

$$\frac{1}{2}\begin{vmatrix} x & y & 1 \\ 1 & 2 & 1 \\ 3 & 6 & 1 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} x & y & 1 \\ 1 & 2 & 1 \\ 3 & 6 & 1 \end{vmatrix} = 0$$

Expanding along the first row (R_1):

$x(2-6) - y(1-3) + 1(6-6) = 0$

$\Rightarrow x(-4) - y(-2) + 1(0) = 0$

$-4x + 2y = 0 \Rightarrow 2y = 4x$

$\Rightarrow y = 2x \quad \text{or} \quad 2x - y = 0$

22. Given equation: $\sqrt{x} + \sqrt{y} = \sqrt{10}$

Differentiating both sides with respect to x:

$$\frac{d}{dx}(x^{1/2}) + \frac{d}{dx}(y^{1/2}) = \frac{d}{dx}(\sqrt{10})$$

$$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}\frac{dy}{dx} = 0$$

Multiply the entire equation by 2:

$$\frac{1}{\sqrt{x}} + \frac{1}{\sqrt{y}}\frac{dy}{dx} = 0 \Rightarrow \frac{1}{\sqrt{y}}\frac{dy}{dx} = -\frac{1}{\sqrt{x}}$$

$$\frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}} \Rightarrow \frac{dy}{dx} = -\sqrt{\frac{y}{x}} \Rightarrow \frac{dy}{dx} + \sqrt{\frac{y}{x}} = 0$$

(Hence proved)

23. The volume V of a sphere as a function of its radius r is given by:

$$V = \frac{4}{3}\pi r^3$$

Differentiating both sides with respect to the radius r:

$$\frac{dV}{dr} = \frac{4}{3}\pi\cdot(3r^2) = 4\pi r^2$$

Substitute the given radius $r = 10$ cm:

$$\left.\frac{dV}{dr}\right|_{r=10} = 4\pi(10)^2 = 4\pi(100) = 400\pi \text{ cm}^3/\text{cm}$$

24. First, find the derivative $f'(x)$:

$$f'(x) = \frac{d}{dx}(4x^3 - 6x^2 - 72x + 30)$$

$= 12x^2 - 12x - 72$

Factor out 12:

$f'(x) = 12(x^2 - x - 6) = 12(x-3)(x+2)$

For a function to be strictly decreasing, we must have $f'(x) < 0$:

$12(x-3)(x+2) < 0 \Rightarrow (x-3)(x+2) < 0$

By checking intervals along the critical points $x = -2$ and $x = 3$, the product is negative when $x \in (-2, 3)$.

The function is decreasing in the open interval (–2, 3)

25. Let $t = \log(\sin x)$.

Differentiating both sides with respect to x using the chain rule:

$$dt = \frac{1}{\sin x}\cdot\cos x dx = \cot x dx$$

Substitute t and dt back into the given integral:

$$\int \cot x\cdot\log(\sin x)dx = \int t dt = \frac{t^2}{2} + C$$

Substitute $t = \log(\sin x)$ back:

$$= \frac{[\log(\sin x)]^2}{2} + C$$

26. Given function: $y = a\sin x + b\cos x$

1. Differentiate once with respect to x:

$$\frac{dy}{dx} = a\cos x - b\sin x$$

2. Differentiate a second time to find $\frac{d^2y}{dx^2}$:

$$\frac{d^2y}{dx^2} = -a\sin x - b\cos x$$

3. Substitute into the LHS of the differential equation:

$$\text{LHS} = \frac{d^2y}{dx^2} + y$$

$$= (-a\sin x - b\cos x) + (a\sin x + b\cos x) = 0 = \text{RHS}$$

27. Let $\vec{v} = 2\vec{a} - \vec{b} + 3\vec{c}$.

1. Compute components of vector $\vec{v}$:

$$\vec{v} = 2(\hat{i} + \hat{j} + \hat{k}) - (2\hat{i} - \hat{j} + 3\hat{k}) + 3(\hat{i} - 2\hat{j} + \hat{k})$$

$$\vec{v} = (2 - 2 + 3)\hat{i} + (2 + 1 - 6)\hat{j} + (2 - 3 + 3)\hat{k}$$

$$\vec{v} = 3\hat{i} - 3\hat{j} + 2\hat{k}$$

2. Find the magnitude $|\vec{v}|$:

$$|\vec{v}| = \sqrt{3^2 + (-3)^2 + 2^2} = \sqrt{9 + 9 + 4} = \sqrt{22}$$

3. Compute the unit vector $\hat{v}$:

$$\hat{v} = \frac{\vec{v}}{|\vec{v}|} = \frac{3\hat{i} - 3\hat{j} + 2\hat{k}}{\sqrt{22}}$$

$$= \frac{3}{\sqrt{22}}\hat{i} - \frac{3}{\sqrt{22}}\hat{j} + \frac{2}{\sqrt{22}}\hat{k}$$

28. The direction ratios of the two lines are:

• Line 1: $(a_1, b_1, c_1) = (-3, 2k, 2)$

• Line 2: $(a_2, b_2, c_2) = (3k, 1, -5)$

Since the lines are mutually perpendicular, their dot product must equal zero $(a_1a_2 + b_1b_2 + c_1c_2 = 0)$:

$$(-3)(3k) + (2k)(1) + (2)(-5) = 0$$

$$-9k + 2k - 10 = 0 \Rightarrow -7k = 10 \Rightarrow k = -\frac{10}{7}$$

29. Total number of balls initially $= 10 + 5 = 15$.

• Probability that the first ball drawn is black:

$$P(B_1) = \frac{10}{15} = \frac{2}{3}$$

• After drawing one black ball without replacement, 9 black balls and 14 total balls remain. Probability that the second ball drawn is black:

$$P(B_2 \mid B_1) = \frac{9}{14}$$

• Total probability that both are black:

$$P(B_1 \cap B_2) = P(B_1) \cdot P(B_2 \mid B_1) = \frac{2}{3} \times \frac{9}{14} = \frac{18}{42} = \frac{3}{7}$$

30. 1. Reflexive:

Let $a = \frac{1}{2} \in \mathbb{R}$.

$$a^3 = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

Since $\frac{1}{2} \le \frac{1}{8}$ is false, $\left(\frac{1}{2}, \frac{1}{2}\right) \notin R$.

Therefore, R is not reflexive.

2. Symmetric:

Let $(1,2) \in R$ because $1 \le 2^3$ $(1 \le 8)$ is true.

However, $(2,1) \notin R$ because $2 \le 1^3$ $(2 \le 1)$ is false.

Therefore, R is not symmetric.

3. Transitive:

Let $a = 3$, $b = 1.5$, and $c = 1.2$.

1. $(3, 1.5) \in R$ because $3 \le (1.5)^3 \Rightarrow 3 \le 3.375$ (true).
2. $(1.5, 1.2) \in R$ because $1.5 \le (1.2)^3 \Rightarrow 1.5 \le 1.728$ (true).
3. But $3 \le (1.2)^3 \Rightarrow 3 \le 1.728$ is false, so $(3, 1.2) \notin R$.

Therefore, R is not transitive

31. Let $\sin^{-1}\left(\frac{5}{13}\right) = x \Rightarrow \sin x = \frac{5}{13} \Rightarrow \tan x = \frac{5}{12}$.

Let $\cos^{-1}\left(\frac{3}{5}\right) = y \Rightarrow \cos y = \frac{3}{5} \Rightarrow \tan y = \frac{4}{3}$.

Using the formula for $\tan(x+y)$:

$$\tan(x+y)=\frac{\tan x+\tan y}{1-\tan x\tan y}=\frac{\frac{5}{12}+\frac{4}{3}}{1-\left(\frac{5}{12}\times\frac{4}{3}\right)}$$

$$=\frac{\frac{21}{12}}{\frac{16}{36}}=\frac{21}{12}\times\frac{36}{16}=\frac{63}{16}$$

$$x+y=\tan^{-1}\left(\frac{63}{16}\right)\Rightarrow\sin^{-1}\left(\frac{5}{13}\right)+\cos^{-1}\left(\frac{3}{5}\right)$$

$$=\tan^{-1}\left(\frac{63}{16}\right)$$

(Hence Proved)

32. Let $A=\begin{bmatrix}1&5\\-1&2\end{bmatrix}\Rightarrow A'=\begin{bmatrix}1&-1\\5&2\end{bmatrix}$.

• Symmetric part (P):

$$P=\frac{1}{2}(A+A')=\frac{1}{2}\begin{bmatrix}1+1&5-1\\-1+5&2+2\end{bmatrix}=\begin{bmatrix}1&2\\2&2\end{bmatrix}$$

• Skew-Symmetric part (Q):

$$Q=\frac{1}{2}(A-A')=\frac{1}{2}\begin{bmatrix}1-1&5+1\\-1-5&2-2\end{bmatrix}=\begin{bmatrix}0&3\\-3&0\end{bmatrix}$$

$$A=\begin{bmatrix}1&2\\2&2\end{bmatrix}+\begin{bmatrix}0&3\\-3&0\end{bmatrix}$$

33. 1. Differentiate x w.r.t t:

$$\frac{dx}{dt}=a\left[-\sin t+\frac{1}{\tan(t/2)}\cdot\sec^2(t/2)\cdot\frac{1}{2}\right]$$

$$=a\left[-\sin t+\frac{1}{\sin t}\right]$$

2. Differentiate y w.r.t t:

$$\frac{dy}{dt}=a\cos t$$

3. Compute $\frac{dy}{dx}$:

$$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{a\cos t}{\frac{a\cos^2 t}{\sin t}}=\frac{\sin t}{\cos t}=\tan t$$

34. Substitute $x=60-y$ into product $P=xy^3$:

$$P=(60-y)y^3=60y^3-y^4\Rightarrow\frac{dP}{dy}=180y^2-4y^3$$

Set $\frac{dP}{dy}=0\Rightarrow 4y^2(45-y)=0\Rightarrow y=45$.

$$\frac{d^2P}{dy^2}=360y-12y^2\Rightarrow\left.\frac{d^2P}{dy^2}\right|_{y=45}$$

$$=360(45)-12(45)^2=-8100<0\text{ (Maxima)}$$

$$x=60-45=15$$

Answer The numbers are 15 and 45.

35. Using Partial Fractions:

$$\frac{2x}{(x+1)(x+2)}=\frac{-2}{x+1}+\frac{4}{x+2}$$

$$\int\left(\frac{-2}{x+1}+\frac{4}{x+2}\right)dx$$

$$=-2\log|x+1|+4\log|x+2|+C$$

$$=\log\left|\frac{(x+2)^4}{(x+1)^2}\right|+C$$

36. $\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}==-\hat{i}+3\hat{j}-\hat{k}$,

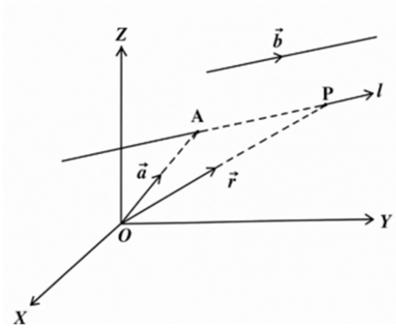
$\overrightarrow{AC}=\overrightarrow{OC}-\overrightarrow{OA}==-\hat{i}+2\hat{j}+\hat{k}$,

$$\overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\-1&3&-1\\-1&2&1\end{vmatrix}=5\hat{i}+2\hat{j}+\hat{k}$$

$$\Rightarrow|\overrightarrow{AB}\times\overrightarrow{AC}|=\sqrt{25+4+1}=\sqrt{30}$$

$$\text{Area}=\frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AC}|=\frac{\sqrt{30}}{2}\text{ sq. units.}$$

37. Let line pass through $A(\vec{a})$ and be parallel to $\vec{b}$. Let $P(\vec{r})$ be any point on the line.



Vector $\overrightarrow{AP}$ is parallel to $\vec{b} \Rightarrow \overrightarrow{AP} = \lambda\vec{b}$.

Since $\overrightarrow{AP} = \vec{r} - \vec{a}$:

$\vec{r} - \vec{a} = \lambda\vec{b} \Rightarrow \vec{r} = \vec{a} + \lambda\vec{b}$

38. By Bayes' Theorem: Let E_1 be Box I, E_2 be Box II, and G be drawing a gold coin.

$$P(E_1) = P(E_2) = \frac{1}{2}, \quad P(G \mid E_1) = \frac{2}{2} = 1,$$

$$P(G \mid E_2) = \frac{1}{2}$$

$$P(E_1 \mid G) = \frac{P(E_1)P(G \mid E_1)}{P(E_1)P(G \mid E_1) + P(E_2)P(G \mid E_2)}$$

$$= \frac{\frac{1}{2}\times 1}{\left(\frac{1}{2}\times 1\right) + \left(\frac{1}{2}\times\frac{1}{2}\right)} = \frac{\frac{1}{2}}{\frac{3}{4}} = \frac{2}{3}$$

39. 1. One-one (Injectivity):

Let $x_1, x_2 \in A$ such that $f(x_1) = f(x_2)$.

$$\frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3}$$

Cross-multiplying:

$(x_1 - 2)(x_2 - 3) = (x_2 - 2)(x_1 - 3)$

$x_1x_2 - 3x_1 - 2x_2 + 6 = x_1x_2 - 2x_1 - 3x_2 + 6$

Cancelling identical terms from both sides:

$-3x_1 - 2x_2 = -2x_1 - 3x_2$

$-3x_1 + 2x_1 = -3x_2 + 2x_2$

$-x_1 = -x_2 \Rightarrow x_1 = x_2$

Since $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$, the function f is one-one.

2. Onto (Surjectivity):

Let $y \in B$ (co-domain). Set $y = f(x)$:

$$y = \frac{x-2}{x-3} \Rightarrow y(x-3) = x - 2$$

$xy - 3y = x - 2 \Rightarrow xy - x = 3y - 2$

$$x(y-1) = 3y - 2 \Rightarrow x = \frac{3y-2}{y-1}$$

Since $y \in B \Rightarrow y \neq 1$ (p. 17), the denominator $(y-1) \neq 0$. Thus, x is a well-defined real number.

Let's check if $x = 3$:

$$3 = \frac{3y-2}{y-1} \Rightarrow 3y - 3 = 3y - 2 \Rightarrow -3 = -2$$

(which is impossible)

Therefore, $x \neq 3 \Rightarrow x \in A$.

Since every element $y \in B$ has a pre-image $x \in A$ such that $f(x) = y$, the function f is onto.

Conclusion: The function f is both one-one and onto.

40. 1. Find AB and then $(AB)'$:

$$AB = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}_{3\times 1} \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}_{1\times 3}$$

$$AB = \begin{bmatrix} 1(-1) & 1(2) & 1(1) \\ -4(-1) & -4(2) & -4(1) \\ 3(-1) & 3(2) & 3(1) \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 2 & 1 \\ 4 & -8 & -4 \\ -3 & 6 & 3 \end{bmatrix}$$

Taking the transpose:

$$\text{LHS} = (AB)' = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$$

2. Find B, A' and then $B'A'$:

$$B' = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \quad A' = \begin{bmatrix} 1 & -4 & 3 \end{bmatrix}$$

$$\text{RHS} = B'A' = \begin{bmatrix} -1(1) & -1(-4) & -1(3) \\ 2(1) & 2(-4) & 2(3) \\ 1(1) & 1(-4) & 1(3) \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$$

Conclusion: Since LHS = RHS, the property $(AB)' = B'A'$ is verified.

41. The system can be written in the form $AX = B$, where:

$$A = \begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix} \quad [0.1.17]$$

1. Find Determinant $|A|$:

$$|A| = 4(12-12) - 3(6-36) + 2(4-24)$$
$$|A| = 4(0) - 3(-30) + 2(-20)$$
$$= 0 + 90 - 40 = 50$$

Since $|A| = 50 \neq 0$, A^{-1} exists and the system has a unique solution given by $X = A^{-1}B$.

2. Find the Cofactors of matrix A:

1. $A_{11} = +(12-12) = 0$; $A_{12} = -(6-36) = 30$; $A_{13} = +(4-24) = -20$

2. $A_{21} = -(9-4) = -5$; $A_{22} = +(12-12) = 0$; $A_{23} = -(8-18) = 10$

3. $A_{31} = +(18-8) = 10$; $A_{32} = -(24-4) = -20$; $A_{33} = +(16-6) = 10$

$$\text{Cofactor Matrix} = \begin{bmatrix} 0 & 30 & -20 \\ -5 & 0 & 10 \\ 10 & -20 & 10 \end{bmatrix}$$

3. Find Adjoint Matrix $\text{adj}(A)$:

$$\text{adj}(A) = \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$$

4. Compute $X = A^{-1}B = \frac{1}{|A|}\text{adj}(A)B$:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{50}\begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}\begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{50}\begin{bmatrix} 0(60) - 5(90) + 10(70) \\ 30(60) + 0(90) - 20(70) \\ -20(60) + 10(90) + 10(70) \end{bmatrix}$$

$$= \frac{1}{50}\begin{bmatrix} -450 + 700 \\ 1800 - 1400 \\ -1200 + 900 + 700 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{50}\begin{bmatrix} 250 \\ 400 \\ 400 \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix}$$

Answer: $x = 5$, $y = 8$, $z = 8$

42. Given equation:

$y = (\tan^{-1} x)^2$

Differentiating both sides with respect to x:

$$y_1 = 2(\tan^{-1} x) \cdot \frac{d}{dx}(\tan^{-1} x)$$

$$y_1 = \frac{2\tan^{-1} x}{x^2 + 1}$$

Cross-multiplying to eliminate the fractional format:

$(x^2 + 1)y_1 = 2\tan^{-1} x$

Differentiating both sides again with respect to x using the Product Rule on the LHS:

$$(x^2 + 1) \cdot \frac{d}{dx}(y_1) + y_1 \cdot \frac{d}{dx}(x^2 + 1) = 2 \cdot \frac{d}{dx}(\tan^{-1} x)$$

$$(x^2 + 1)y_2 + y_1(2x) = \frac{2}{x^2 + 1}$$

Multiplying the entire equation by $(x^2 + 1)$ to isolate constants:

$(x^2 + 1)^2 y_2 + 2x(x^2 + 1)y_1 = 2$

(Hence Proved)

43. Part 1: Derivation of the standard formula

Let $I = \int \frac{1}{x^2 + a^2} dx$

Substitute $x = a\tan\theta \Rightarrow dx = a\sec^2\theta d\theta$:

$$I = \int \frac{a\sec^2\theta}{a^2\tan^2\theta + a^2} d\theta = \int \frac{a\sec^2\theta}{a^2(\tan^2\theta + 1)} d\theta$$

Using the identity $\tan^2\theta + 1 = \sec^2\theta$:

$$I = \int \frac{a\sec^2\theta}{a^2\sec^2\theta} d\theta = \int \frac{1}{a} d\theta = \frac{1}{a}\theta + C$$

Since $\tan\theta = \frac{x}{a} \Rightarrow \theta = \tan^{-1}\left(\frac{x}{a}\right)$:

$$\int \frac{1}{x^2 + a^2} dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$

Part 2: Evaluation of the given integral

Given: $\int \frac{1}{x^2 - 6x + 13} dx$ (p. 17)

Complete the square for the quadratic expressions in the denominator:

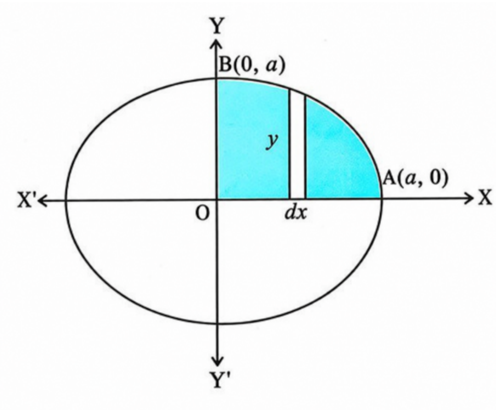
$$x^2 - 6x + 13 = (x^2 - 6x + 9) - 9 + 13$$

$$= (x-3)^2 + 4 = (x-3)^2 + 2^2$$

Now apply the derived standard formula with $(x \to x-3)$ and $a = 2$:

$$\int \frac{1}{(x-3)^2 + 2^2} dx = \frac{1}{2}\tan^{-1}\left(\frac{x-3}{2}\right) + C$$

44. The circle $x^2 + y^2 = a^2$ is perfectly symmetrical about both the X-axis and Y-axis.



Therefore, the total area A is equal to 4 times the area of the region bounded in the first quadrant (from $x = 0$ to $x = a$).

In the first quadrant, expressing y explicitly as a positive function of x:

$$y = \sqrt{a^2 - x^2}$$

The total area expression is setup using the definite integral profile:

$$A = 4\int_0^a y dx = 4\int_0^a \sqrt{a^2 - x^2} dx$$

Using the standard integration formula $\int \sqrt{a^2 - x^2} dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)$:

$$A = 4\left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]_0^a$$

Substitute the upper and lower limits:

• Upper limit ($x = a$):

$$4\left[0 + \frac{a^2}{2}\sin^{-1}(1)\right] = 4 \cdot \frac{a^2}{2} \cdot \frac{\pi}{2} = \pi a^2$$

• Lower limit ($x = 0$): $4[0 + 0] = 0$

Answer: The total area enclosed by the circle is πa^2 sq. units.

45. Divide the entire differential equation by $\cos^2 x$ to structure it into the canonical linear form $\frac{dy}{dx} + Py = Q$:

$$\frac{dy}{dx} + \frac{1}{\cos^2 x} y = \frac{\tan x}{\cos^2 x}$$

$$\frac{dy}{dx} + (\sec^2 x)y = \sec^2 x \tan x$$

Here, $P = \sec^2 x$ and $Q = \sec^2 x \tan x$.

1. Calculate the Integrating Factor (I.F.):

   $$\text{I.F.} = e^{\int P dx} = e^{\int \sec^2 x dx} = e^{\tan x}$$

2. Formulate the general solution expression:

   $$y \cdot (\text{I.F.}) = \int Q \cdot (\text{I.F.}) dx + C$$

   $$y \cdot e^{\tan x} = \int (\sec^2 x \tan x) \cdot e^{\tan x} dx + C$$

3. Integrate the RHS using method of substitution:

Let $t = \tan x \Rightarrow dt = \sec^2 x dx$.

$\int \sec^2 x \tan x e^{\tan x} dx = \int t e^t dt$

Applying integration by parts

$(\int uv' = uv - \int u'v)$:

$\int t e^t dt = t e^t - \int 1 \cdot e^t dt = t e^t - e^t = e^t (t-1)$

Replacing t back with $\tan x$:

$= e^{\tan x}(\tan x - 1)$

4. Combine the resolved blocks into final solution profile:

$y \cdot e^{\tan x} = e^{\tan x}(\tan x - 1) + C$

Dividing through by $e^{\tan x}$:

$y = \tan x - 1 + Ce^{-\tan x}$

46. Proof:

Let the given definite integral be:

$$I = \int_0^a f(x)dx$$

Substitute $t = a - x \Rightarrow dx = -dt$.

Now, change the integration limits accordingly:

• When $x = 0 \Rightarrow t = a - 0 = a$

• When $x = a \Rightarrow t = a - a = 0$

Substitute *x*, *dx*, and the new limits back into I:

$$I = \int_a^0 f(a-t)(-dt)$$

$$I = -\int_a^0 f(a-t)dt$$

Using the property of definite integrals, $\int_a^b f(t)dt = -\int_b^a f(t)dt$, we can reverse the limits to remove the negative sign:

$$I = \int_0^a f(a-t)dt$$

Since the variable of integration is a dummy variable, we can replace t with x:

$$\int_0^a f(x)dx = \int_0^a f(a-x)dx$$

(Hence Proved)

Evaluation:

Let $I = \int_0^{\pi/4} \log(1+\tan x)dx$ --- (Equation 1)

Applying the proven property $\left(x \to \frac{\pi}{4} - x\right)$:

$$I = \int_0^{\pi/4} \log\left(1 + \tan\left(\frac{\pi}{4} - x\right)\right)dx$$

Using the identity $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$:

$$\tan\left(\frac{\pi}{4} - x\right) = \frac{\tan(\pi/4) - \tan x}{1 + \tan(\pi/4)\tan x} = \frac{1 - \tan x}{1 + \tan x}$$

Substitute this back into the integral expression:

$$I = \int_0^{\pi/4} \log\left(1 + \frac{1 - \tan x}{1 + \tan x}\right)dx$$

$$I = \int_0^{\pi/4} \log\left(\frac{1 + \tan x + 1 - \tan x}{1 + \tan x}\right)dx$$

$$I = \int_0^{\pi/4} \log\left(\frac{2}{1 + \tan x}\right)dx$$

Using the logarithmic identity $\log\left(\frac{M}{N}\right) = \log M - \log N$:

$$I = \int_0^{\pi/4} \log 2 dx - \int_0^{\pi/4} \log(1 + \tan x)dx$$

From Equation 1, the second integral is equal to *I*:

$$I = \log 2 \int_0^{\pi/4} 1 dx - I$$

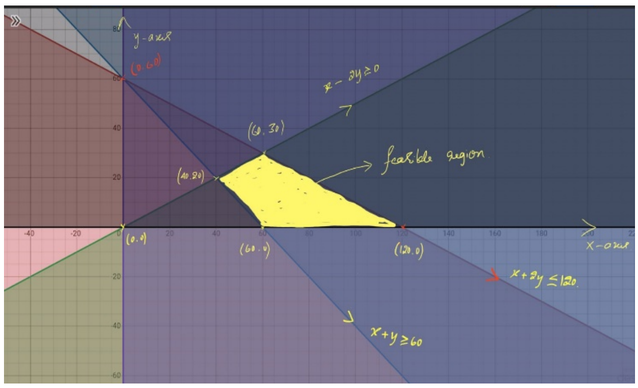
$$2I = \log 2 \cdot [x]_0^{\pi/4}$$

$$2I = \log 2 \cdot \left(\frac{\pi}{4} - 0\right)$$

$$2I = \frac{\pi}{4}\log 2 \Rightarrow I = \frac{\pi}{8}\log 2$$

Answer: $\frac{\pi}{8}\log 2$

OR



1. Convert inequalities to equations to find boundary lines:

1. Line 1: $x+2y=120$ passes through $(120,0)$ and $(0,60)$.
2. Line 2: $x+y=60$ passes through $(60,0)$ and $(0,60)$.
3. Line 3: $x-2y=0 \Rightarrow x=2y$ passes through $(0,0),(60,30)$, and $(120,60)$.

2 Find intersecting corner points of the region:

1. Intersecting Line 1 and Line 3 $(x=2y)$:

   $2y+2y=120 \Rightarrow 4y=120$

   $\Rightarrow y=30 \Rightarrow x=60$

   Point of intersection is ** $B(60,30)$ **.
2. Intersecting Line 2 and Line 3 $(x=2y)$:

   $2y+y=60 \Rightarrow 3y=60 \Rightarrow y=20 \Rightarrow x=40$

   Point of intersection is ** $A(40,20)$ **.

3. Identify corner points of the bounded feasible region:

   By testing regions with origin tests (e.g., $0+0\le 120$ is true, $0+0\ge 60$ is false), the shared enclosed feasible region has the following corner points:

   1. $A(40,20)$
   2. $B(60,30)$
   3. $C(120,0)$
   4. $D(60,0)$

4. Evaluate the objective function $Z=5x+10y$ at each corner point:

Corner Point	Value of $Z=5x+10y$	Result
$A(40,20)$	$Z=5(40)+10(20)$ $=200+200=400$	
$B(60,30)$	$Z=5(60)+10(30)$ $=300+300=600$	Max.
$C(120,0)$	$Z=5(120)+10(0)$ $=600+0=600$	Max.
$D(60,0)$	$Z=5(60)+10(0)$ $=300+0=300$	Min.

• Minimum Value: 300 at the point $(60,0)$.

• Maximum Value: 600 occurring at all points on the line segment connecting $(60,30)$ and $(120,0)$.

47. 1. Compute A^2:

$$A^2=A\cdot A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$$

$$A^2=\begin{bmatrix}3(3)+1(-1) & 3(1)+1(2)\\ -1(3)+2(-1) & -1(1)+2(2)\end{bmatrix}$$

$$=\begin{bmatrix}9-1 & 3+2\\ -3-2 & -1+4\end{bmatrix}=\begin{bmatrix}8 & 5\\ -5 & 3\end{bmatrix}$$

2. Verify $A^2-5A+7I=O_2$:

$$\text{LHS}=\begin{bmatrix}8 & 5\\ -5 & 3\end{bmatrix}-5\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}+7\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$\text{LHS}=\begin{bmatrix}8 & 5\\ -5 & 3\end{bmatrix}-\begin{bmatrix}15 & 5\\ -5 & 10\end{bmatrix}+\begin{bmatrix}7 & 0\\ 0 & 7\end{bmatrix}$$

$$\text{LHS}=\begin{bmatrix}8-15+7 & 5-5+0\\ -5-(-5)+0 & 3-10+7\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

$=O_2=\text{RHS}$

(Hence Proved)

3. Find A^{-1} using the equation:

$A^2-5A+7I=O_2$

Post-multiplying the entire matrix equation by A^{-1}:

$A^2\cdot A^{-1}-5A\cdot A^{-1}+7I\cdot A^{-1}=O_2$

$A-5I+7A^{-1}=O_2$

Isolate the inverse term:

$7A^{-1}=5I-A$

Substitute the respective matrix values:

$$7A^{-1}=5\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}-\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$$

$$=\begin{bmatrix}5 & 0\\ 0 & 5\end{bmatrix}-\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}=\begin{bmatrix}2 & -1\\ 1 & 3\end{bmatrix}$$

$$A^{-1}=\frac{1}{7}\begin{bmatrix}2 & -1\\ 1 & 3\end{bmatrix}$$

48. For a function to be continuous at $x=\frac{\pi}{2}$, the limit of the function as $x\to\frac{\pi}{2}$ must exist and be exactly equal to the value of the function at that point:

$$\lim_{x\to\frac{\pi}{2}} f(x) = f\left(\frac{\pi}{2}\right)$$

Given that $f\left(\frac{\pi}{2}\right)=3$, we evaluate the limit:

$$\lim_{x\to\frac{\pi}{2}} \frac{k\cos x}{\pi-2x}=3$$

Let $x=\frac{\pi}{2}+h$. As $x\to\frac{\pi}{2}$, the new variable $h\to 0$

$$\lim_{h\to 0}\frac{k\cos\left(\frac{\pi}{2}+h\right)}{\pi-2\left(\frac{\pi}{2}+h\right)}=3$$

Using the trigonometric identity

$\cos\left(\frac{\pi}{2}+h\right)=-\sin h$:

$$\lim_{h\to 0}\frac{-k\sin h}{\pi-\pi-2h}=3$$

$$\lim_{h\to 0}\frac{-k\sin h}{-2h}=3$$

$$\frac{k}{2}\left(\lim_{h\to 0}\frac{\sin h}{h}\right)=3$$

Since the standard limit identity states $\lim_{h\to 0}\frac{\sin h}{h}=1$

$$\frac{k}{2}\cdot(1)=3$$

$$k=3\times 2\Rightarrow k=6$$

Answer: $k=6$.