Subject : Mathematics
Class : XI

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Binomial Theorem

Important terms in the binomial expansion are

(a) General term: The general term or the $(r+1)^{th}$ term in the expansion of $(x+y)^n$ is given by

$$T_{r+1} = {}^nC_r x^{n-r} \cdot y^r$$

(b) Middle term: The middle term (s) is the expansion of $(x+y)^n$ is (are):

(i) If n is even, there is only one middle term which is given by $T_{(n+2)/2} = {}^nC_{n/2} \cdot x^{n/2} \cdot y^{n/2}$

(ii) If n is odd, there are two middle terms which are $T_{(n+1)/2}$ and $T_{[(n+1)/2]+1}$

(c) Term independent of **x** : Term independent of x contains no x ; Hence find the value of r for which the exponent of x is zero.

If $(\sqrt{A}+B)^n = I + f$, where I & n are positive integers and $0 \le$ f < 1, then.

(a) $(I+f)\cdot f = K^n$ if n is odd & $A - B^2 = K > 0$

(b) $(I+f)(1-f) = k^n$ if n is even $\& \sqrt{A} - B < 1$

Some results on binomial coefficients

(a) ${}^nC_x = {}^nC_y \Rightarrow x = y$ or $x + y = n$

(b) ${}^nC_{r-1} + {}^nC_r = {}^{n+1}C_r$

(c) $C_0 + \frac{C_1}{2}, \frac{C_2}{3} \ldots \frac{C_n}{n+1} \quad \frac{2^{n+1}-1}{n+1}$

(d) $C_0 - \frac{C_1}{2} + \frac{C_2}{3} - \frac{C_3}{4} \ldots + \frac{(-1)^n C_n}{n+1} = \frac{1}{n+1}$

(e) $C_0 + C_1 + C_2 + \ldots = C_n = 2^n$

(f) $C_0 + C_2 + C_4 + \ldots = C_1 + C_3 + C_5 + \ldots = 2^{n-1}$

(g) $C_0{}^2 + C_1{}^2 + C_2{}^2 + \ldots + C_n{}^2 = {}^{2n}C_n = \frac{(2n)!}{n!n!}$

(h) $C_0 \cdot C_r + C_1 \cdot C_{r+1} + C_2 \cdot C_{r+2} + \ldots + C_{n-r} C_n = \frac{(2n)!}{(n+r)!(n-r)}$

Greatest coefficient and Greatest Term in Expansion of $(x + a)^n$

(a) If n is even greatest coefficient is ${}^nC_{n/2}$. If n is odd greatest coefficient is ${}^nC_{\left(\frac{n-1}{2}\right)}$ or ${}^nC_{\left(\frac{n+1}{2}\right)}$

(b) For greatest term: Greatest term $=\begin{cases} T_P \text{ and } T_{p+1} & \text{if } \dfrac{n+1}{\left|\dfrac{x}{a}\right|+1} \text{ is an integer} \\ T_{q+1} & \text{if } \dfrac{n+1}{\left|\dfrac{x}{a}\right|+1} \text{ is non integer and } \in (q, q+1), q \in I \end{cases}$

Multinomial Theorem

For any $n \in N$,

(i) $\left(x_1 + x_2 + \ldots + x_k\right)^n = \sum_{r_1+r_2+\ldots+r_k=n} \dfrac{n!}{r_1!r_2!\ldots r_k!} x_1^{r_1} x_2^{r_2} \ldots x_k^{r_k}$

(ii) The general term in the above expansion is $\dfrac{n!}{r_1!r_2!\ldots r_k!} x_1^{r_1} x_2^{r_2} \ldots x_k^{r_k}$

Here total number of terms in the expansion $= {}^{n+k-1}C_{k-1}$.

Binomial Theorem for Negative or Fractional Indices

If $n \in Q$, then $(1+x)^n = 1 + nx + \dfrac{n(n-1)}{2!}x^2 + \dfrac{n(n-1)(n-2)}{3!}x^3 + \ldots\infty$ provided $|x| < 1$.

Notes

(i) $(1-x)^{-1} = 1 + x + x^2 + x^3 + \ldots\infty$

(ii) $(1+x)^{-1} = 1 - x + x^2 - x^3 + \ldots\infty$

(iii) $(1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \ldots\infty$

(iv) $(1+x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + \ldots\infty$

Exponential series

(a) $e^x = 1 + \dfrac{x}{1!} + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \ldots\infty$; where x may be any real or complex number and $e = \lim_{n\to\infty}\left(1 + \dfrac{1}{n}\right)^n$.

(b) $a^x = 1 + \dfrac{x}{1!}\ln a + \dfrac{x^2}{2!}\ln^2 a + \dfrac{x^3}{3!}\ln^3 a + \ldots\infty$, where $a > 0$.

Logarithmic Series

(a) $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots\infty$, where $-1 < x \leq 1$.

(b) $\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \ldots\infty$, where $-1 \leq x < 1$.

(c) $\ln\frac{(1+x)}{(1-x)} = 2\left(x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots\infty\right), |x| < 1$.



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