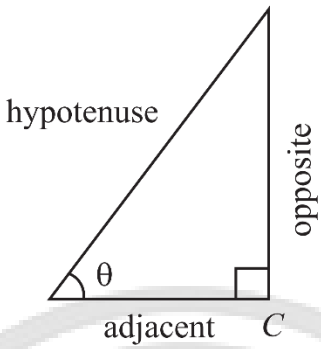
Subject : Mathematics
Class : XI

Kannada

Trigonometric Ratios and Identities

1. Trigonometric Ratio - Basic Terminology



The six trigonometric ratios of θ are defined as follows:

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}, \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}, \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\cot\theta = \frac{\text{adjacent}}{\text{opposite}}, \ \text{cosec}\theta = \frac{\text{hypotenuse}}{\text{opposite}}, \sec\theta = \frac{\text{hypotenus}}{\text{adjacent}}$$

2. Trigonometric identities:

(i) $\sin^2\theta + \cos^2\theta = 1$

(ii) $\sec^2\theta - \tan^2\theta = 1$

(iii) $\text{cosec}^2\theta - \cot^2\theta = 1$

3. Allied angles:

Two angles are said to be allied when their sum or difference is either zero or a multiple of $\frac{\pi}{2}$, two angles x, y are allied angles iff $|x \pm y| = 0$ or $\frac{n\pi}{2}, n \in N$.

$q \to$	$\frac{\pi}{2} - \theta$	$\frac{\pi}{2} + \theta$	$\pi - \theta$	$\pi + \theta$	$\frac{3\pi}{2} - \theta$	$\frac{3\pi}{2} + \theta$	$2\pi - \theta$	$2\pi + \theta$	$-\theta$
sin	$\cos\theta$	$\cos\theta$	$\sin\theta$	$-\sin\theta$	$\sin\theta$	$-\cos\theta$	$-\sin\theta$	$\sin\theta$	$-\sin\theta$
cos	$\sin\theta$	$-\sin\theta$	$-\cos\theta$	$-\cos\theta$	$-\sin\theta$	$-\sin\theta$	$\cos\theta$	$\cos\theta$	$\cos\theta$
tan	$\cot\theta$	$-\cot\theta$	$-\tan\theta$	$\tan\theta$	$\cot\theta$	$-\cot\theta$	$-\tan\theta$	$\tan\theta$	$-\tan\theta$
cot	$\tan\theta$	$-\tan\theta$	$-\cot\theta$	$\cot\theta$	$\tan\theta$	$-\tan\theta$	$-\cot\theta$	$\cot\theta$	$-\cot\theta$
sec	$\text{cosec}\,\theta$	$-\text{cosec}\,\theta$	$-\sec\theta$	$-\sec\theta$	$-\text{cosec}\,\theta$	$\text{cosec}\,\theta$	$\sec\theta$	$\sec\theta$	$\sec\theta$
cosec	$\sec\theta$	$\sec\theta$	$\text{cosec}\,\theta$	$-\text{cosec}\,\theta$	$-\sec\theta$	$-\sec\theta$	$-\text{cosec}\,\theta$	$\text{cosec}\,\theta$	$-\text{cosec}\,\theta$

4. Sum & Difference Formula:

(i) $\sin(A+B)=\sin A\cos B+\cos A\sin B$ (ii) $\sin(A-B)=\sin A\cos B-\cos A\sin B$

(iii) $\cos(A+B)=\cos A\cos B-\sin A\sin B$ (iv) $\cos(A-B)=\cos A\cos B+\sin A\sin B$

(v) $\tan(A+B)=\dfrac{\tan A+\tan B}{1-\tan A\tan B}$ (vi) $\tan(A-B)=\dfrac{\tan A-\tan B}{1+\tan A\tan B}$

(vii) $\cot(A+B)=\dfrac{\cot B\cot A-1}{\cot B+\cot A}$ (viii) $\cot(A-B)=\dfrac{\cot B\cot A+1}{\cot B-\cot A}$

5. Product to sum:

(i) $2\sin A\cos B=\sin(A+B)+\sin(A-B)$

(ii) $2\cos A\sin B=\sin(A+B)-\sin(A-B)$

(iii) $2\cos A\cos B=\cos(A+B)+\cos(A-B)$

(iv) $2\sin A\sin B=\cos(A-B)-\cos(A+B)$

6. Trigonometric transformations:

(i) $\sin C+\sin D=2\sin\left(\dfrac{C+D}{2}\right)\cos\left(\dfrac{C-D}{2}\right)$

(ii) $\sin C-\sin D=2\cos\left(\dfrac{C+D}{2}\right)\sin\left(\dfrac{C-D}{2}\right)$

(iii) $\cos C+\cos D=2\cos\left(\dfrac{C+D}{2}\right)\cos\left(\dfrac{C-D}{2}\right)$

(iv) $\cos C-\cos D=-2\sin\left(\dfrac{C+D}{2}\right)\sin\left(\dfrac{C-D}{2}\right)$

$=2\sin\left(\dfrac{C+D}{2}\right)\sin\left(\dfrac{D-C}{2}\right)$

7. Some standard trigonometric values

Some useful Formulae:

(i) $\sin(A+B)\sin(A-B)=\sin^2A-\sin^2B=\cos^2B-\cos^2A$

(ii) $\cos(A+B)\cos(A-B)=\cos^2A-\sin^2B=\cos^2B-\sin^2A$

(iii) $\tan(A+B)\tan(A-B)=\dfrac{\tan^2A-\tan^2B}{1-\tan^2A\tan^2B}$

(iv) $\cot(A+B)\cot(A-B)=\dfrac{\cot^2B\cot^2A-1}{\cot^2B-\cot^2A}$

angle $\rightarrow$ / Trigonometric Function $\rightarrow$	$15°$	$18°$	$22\frac{1}{2}°$	$36°$
sin	$\frac{\sqrt{3}-1}{2\sqrt{2}}$	$\frac{\sqrt{5}-1}{4}$	$\frac{1}{2}\sqrt{2-\sqrt{2}}$	$\frac{\sqrt{10-2\sqrt{5}}}{4}$
cos	$\frac{\sqrt{3}+1}{2\sqrt{2}}$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\frac{1}{2}\sqrt{2+\sqrt{2}}$	$\frac{\sqrt{5}+1}{4}$
tan	$2-\sqrt{3}$	$\frac{\sqrt{25-10\sqrt{5}}}{5}$	$\sqrt{2}-1$	$\sqrt{5-2\sqrt{5}}$

8. Double Angle / Triple Angle

(i) $\sin 2A = 2\sin A\cos A$

(ii) $\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$

Suppose that A is not an odd multiple of $\frac{\pi}{2}$. Then

(iii) $\sin 2A = \frac{2\tan A}{1+\tan^2 A}$

(iv) $\cos 2A = \frac{1-\tan^2 A}{1+\tan^2 A}$

(v) $\tan 2 = \frac{2\tan}{1-\tan^2}$ (Here $2A$ is also not an odd multiple of $\frac{\pi}{2}$)

(vi) $\sin 3A = 3\sin A - 4\sin^3 A, \forall A \in R$

(vii) $\cos 3A = 4\cos^3 A - 3\cos A, \forall A \in R$

(viii) $\tan 3A = \frac{3\tan A - \tan^3 A}{1-3\tan^2 A}$ (3A, A are not odd multiples of $\frac{\pi}{2}$)

(ix) $\cot 3A \frac{3\cot A\cot^3 A}{13\cot^2 A}\left(3A, A \text{ are not multiples of } \frac{\pi}{2}\right)$

9. Half Angle

(i) $\sin A = 2\sin\frac{A}{2}\cos\frac{A}{2}$

(ii) $\cos A = \cos^2\frac{A}{2} - \sin^2\frac{A}{2} = 2\cos^2\frac{A}{2} - 1 = 1 - 2\sin^2\frac{A}{2}$

(iii) $\tan A = \frac{2\tan\frac{A}{2}}{1-\tan^2\frac{A}{2}}\left(\text{ where } \frac{A}{2} \neq (2n+1)\frac{\pi}{2}, n \in Z\right)$

10. Conditional Identities

In a Triangle If $A+B+C=\pi$, then

(i) $\sin 2A+\sin 2B+\sin 2C=4\sin A\sin B\sin C$

(ii) $\sin A+\sin B+\sin C=4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

(iii) $\cos 2A+\cos 2B+\cos 2C=-1-4\cos A\cos B\cos C$

(iv) $\sin^2 A+\sin^2 B+\sin^2 C=2\left(1+4\cos A\cos B\cos C\right)$

(v) $\cos^2 A+\cos^2 B+\cos^2 C=\left(1-2\cos A\cos B\cos C\right)$

11. Domains, Ranges and Periodicity of Trigonometric Functions:

T-Ratio	Domain	Range	Period
$\sin x$	R	$[-1,1]$	2π
$\cos x$	R	$[-1,1]$	2π
$\tan x$	$R-\left\{(2n+1)\frac{\pi}{2}\right\};n\in I$	R	π
$\cot x$	$R-\{n\pi:n\in I\}$	R	π
$\sec x$	$R-\left\{(2n+1)\frac{\pi}{2}\right\};n\in I$	$(-\infty,-1]\cup[1,\infty)$	2π
$\operatorname{cosec} x$	$R-\{n\pi:n\in I\}$	$(-\infty,-1]\cup[1,\infty)$	2π

12. Maxima-Minima

(i) $a\cos\theta+b\sin\theta$ will always lie in the interval $\left[-\sqrt{a^2+b^2},\sqrt{a^2+b^2}\right]$ i.e., the maximum and minimum values are $\sqrt{a^2+b^2},-\sqrt{a^2+b^2}$ respectively.

(ii) Minimum value of $a^2\tan^2\theta+b^2\cot^2\theta$ is $2ab$ where $a,b>0$.

(iii) $-\sqrt{a^2+b^2+2ab\cos(\alpha-\beta)}<a\cos(\alpha+\theta)+b\cos(\beta+\theta)\le\sqrt{a^2+b^2+2ab\cos(\alpha-\beta)}$ where α and β are known angles.

(iv) If $\alpha,\beta,\in\left(0,\frac{\pi}{2}\right)$ and $\alpha+\beta=\sigma$ (constant) then

(a) Maximum value of the expression $\cos\alpha\cos\beta,\cos\alpha+\cos\beta,\sin\alpha\sin\beta$ or $\sin\alpha+\sin\beta$ occurs when $\alpha=\beta=\frac{\sigma}{2}$

(b) Minimum value of $\sec\alpha+\sec\beta,\tan\alpha+\tan\beta,\text{cosec}\ \alpha+\text{cosec}\beta$ occurs when $\alpha=\beta=\frac{\sigma}{2}$

(v) If A,B,C are the angles of a triangle then maximum value of $\sin A+\sin B+\sin C$ and $\sin A\sin B\sin C$ occurs when $A=B=C=60^\circ$

(vi) In case a quadratic in $\sin\theta\ \&\ \cos\theta$ is given then the maximum or minimum values can be obtained by making perfect square.

13. Sum of Three or More Angles

(i) $\sin(A+B+C)=\sin A\cos B\cos C+\cos A\sin B\cos C+\cos A\cos B\sin C-\sin A\sin B\sin C=\sum\sin A\cos B\cos C-\prod\sin A$

(ii) $\cos(A+B+C)=\cos A\cos B\cos C-\cos A\sin B\sin C-\sin A\cos B\sin C-\sin A\sin B\sin C=\prod\cos A-\sum\cos A\sin B\sin C$

(iii) $\tan(A+B+C)=\frac{\tan A+\tan B+\tan C-\tan A\tan B\tan C}{1-\tan A\tan B-\tan B\tan C-\tan C\tan A}$

$=\frac{\sum\tan A-\Pi\tan A}{1-\sum\tan A\tan B}$

14. Summation of Series

$\sin\alpha+\sin(\alpha+\beta)+\sin(\alpha+2\beta)+\ldots+\sin(\alpha+\overline{n-1}\beta)$

$=\frac{\sin\left(\alpha+\frac{n-1}{2}\beta\right)\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}}$

$\cos\alpha+\cos(\alpha+\beta)+\cos(\alpha+2\beta)+\ldots+\cos(\alpha+\overline{n-1}\beta)$

$=\frac{\cos\left(\alpha+\frac{n-1}{2}\beta\right)\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}}$



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