Q1 Find the mean number of heads in three tosses of a fair coin.

(A) 1.5 (B) 4.5

(C) 2.5 (D) 3.5

Q2 If A and B are two events such that $P(A)=\frac{1}{2}$, $P(B)=\frac{1}{2}$ and $P(A|B)=\frac{1}{4}$, is then $P\left(A'\cap B'\right)$

(A) 1/8 (B) 3/16

(C) 1/12 (D) 3/4

Q3 A pandemic has been spreading all over the world. The probabilities are 0.7 that there will be a lockdown, 0.8 that the pandemic is controlled in one month if there is a lockdown and 0.3 that it is controlled in one month if there is no lockdown. The probability that the pandemic will be controlled in one month is

(A) 0.65 (B) 1.65

(C) 1.46 (D) 0.46

Q4 If A and B are two independent events such that

$P\left(\overline{A}\right)=0.75,\ P(A\cup B)=0.65 \text{ and } P(B)$
$=x,$

then find the value of x.

(A) 5/14 (B) 8/15

(C) 9/14 (D) 7/15

Q5 Suppose that the number of elements in set A is p, the number of elements in set B is q and the number of elements A x B is 7, then p^2 + q^2 =

(A) 50 (B) 51

(C) 42 (D) 49

Q6 The domain of the function $f(x)=\frac{1}{\log_{10}(1-x)}+\sqrt{x+2}$ is

(A) $[-2,0)\cap(0,1)$

(B) [-2, 1)

(C) [-2, 0)

(D) [-2, 0) U (0, 1)

Q7 The trigonometric function y = tan x in the II quadrant

(A) decreases from 0 to ∞

(B) decreases from -∞ to 0

(C) increases from 0 to ∞

(D) increases from -∞ to 0

Q8 The degree measure of p/32 is equal to

(A) $5^{\circ}30'20''$ (B) $5^{\circ}37'20''$

(C) $5^{\circ}37'30''$ (D) $4^{\circ}30'30''$

Q9 The value of $\sin\frac{5\pi}{12}\sin\frac{\pi}{12}$ is

(A) 0 (B) 1

(C) 1/2 (D) 1/4

Q10 $\sqrt{2+\sqrt{2+\sqrt{2+2\cos 8\theta}}}=$

(A) sin 2q (B) 2 cos q

(C) 2 sin q (D) 2 cos q/2

Q11 If A = {1, 2, 3,.....,10}, then number of subsets of A containing only odd numbers is

(A) 31 (B) 27

(C) 32 (D) 30

Q12 If all permutations of the letters of the word MASK are arranged in the order as in dictionary with or without meaning, which one of the following is 19th word?

(A) KAMS (B) SAKM

(C) AKMS (D) AMSK

Q13 If $a_1, a_2, a_3, \ldots\ldots, a_{10}$ is a geometric progression and $\frac{a_3}{a_1}=25$, then $\frac{a_9}{a_5}$ equals

(A) $3(5^2)$ (B) 5^4

(C) 5^3 (D) $2(5^2)$

Q14 If the straight line 2x - 3y + 17 = 0 is perpendicular to the line passing through the points (7, 17) and (15, b), then b equals$(7,17)$

(A) -5 (B) 5

(C) 29 (D) -29

Q15 The octant in which the point (2, -4, -7) lies is

(A) Eighth (B) Third

(C) Fourth (D) Fifth

Q16 If $f(x)=\begin{cases} x^2-1, & 0<x<2 \\ 2x+3, & 2\le x<3 \end{cases}$ the quadratic equation whose roots are $\lim_{x\to 2^-} f(x)$ and $\lim_{x\to 2^+} f(x)$ is

(A) $x^2 - 14x + 49 = 0$

(B) $x^2 - 10x + 21 = 0$

(C) $x^2 - 6x + 9 = 0$

(D) $x^2 - 7x + 8 = 0$

Q17 If 3x + i (4x - y) = 6 - i where x and y are real numbers, then the values of x and y are respectively,

(A) 3, 9 (B) 2, 4

(C) 2, 9 (D) 3, 4

Q18 If the standard deviation of the numbers -1, 0, 1, k is $\sqrt{5}$ where k > 0, then k is equal to

(A) $4\sqrt{\frac{5}{3}}$ (B) $\sqrt{6}$

(C) $2\sqrt{\frac{10}{3}}$ (D) $2\sqrt{6}$

Q19 If the set x contains 7 elements adn set y contains 8 elements, then the number of bijections from x to y is

(A) 0 (B) $8P_7$

(C) 7! (D) 8!

Q20 If $f: R \to R$ be defined by $f(x)=\begin{cases} 2x: & x>3 \\ x^2: & 1<x\le 3 \\ 3x: & x\le 1 \end{cases}$ then f(-1) + f(2) + f(4) is

(A) 5 (B) 10

(C) 9 (D) 14

Q21 Let the relation R is defined in N by aRb, if 3a + 2b = 27 then R is

(A) {(1, 12), (3, 9), (5, 6), (7, 3)}

(B) {(0, 27/2), (1, 12), (3, 9), (5, 6), (7, 3)}

(C) {(1, 12), (3, 9), (5, 6), (7, 3), (9, 0)}

(D) {(2, 1), (9, 3), (6, 5), (3, 7)}

Q22 $\lim_{y\to 0} \frac{\sqrt{3+y^3}-\sqrt{3}}{y^3} =$

(A) $\frac{1}{2\sqrt{3}}$ (B) $\frac{1}{3\sqrt{2}}$

(C) $2\sqrt{3}$ (D) $3\sqrt{2}$

Q23 If A is a matrix of order 3 x 3, then $\left(A^2\right)^{-1}$ is equal to

(A) $(-A^2)^2$ (B) $(A^1)^2$

(C) A^2 (D) $(-A)^{-2}$

Q24 If $A=\begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}$, then the inverse of the matrix A^3 is

(A) A (B) -1

(C) 1 (D) -A

Q25 If A is a skew symmetric matrix, then A^{2021} is

(A) Row matrix

(B) Column matrix

(C) Symmetric matrix

(D) Skew symmetric matrix

Q26 If $A=\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, then $(aI + bA)^n$ is (where I is the identify matrix of order 2)

(A) $a^2 I + a^{n-1} b \cdot A$

(B) $a^n I + n \cdot a^{n-1} b \cdot A$

(C) $a^n I + na^n bA$

(D) $a^n I + b^n A$

Q27 If A is a 3 x 3 matrix such that |5 . adj A| = 5, then |A| is equal to

(A) ± 1 (B) $\pm 1/25$

(C) $\pm 1/5$ (D) ± 5

Q28 If there are two values of 'a' which makes determinant

$$\Delta = \begin{vmatrix} 1 & -2 & 5 \\ 2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix} = 86$$

Then, the sum of these number is

(A) -4 (B) 9

(C) 4 (D) 5

Q29 If the vertices of a triangle are (-2, 6), (3, -6) and (1, 5) then the area of the triangle is

(A) 40 sq. units (B) 15.5 sq. units

(C) 30 sq. units (D) 35 sq. units

Q30 Domain $\cos^{-1}[x]$ is, where [x] denotes a greatest integer function

(A) (-1, 2] (B) (-1, 2)

(C) [-1, 2] (D) [-1, 2)

Q31 If $y=\left(1+x^2\right)\tan^{-1} x-x,$ then $\frac{dy}{dx}$ is

(A) $2x\ \tan^{-1} x$

(B) $\frac{\tan^{-1} x}{x}$

(C) $x^2 \tan^{-1} x$

(D) $x \tan^{-1} x$

Q32 If $x=e^{\theta} \sin \theta,\ y=e^{\theta} \cos \theta$ where θ is a parameter, then dy/dx at (1, 1) is equal to

(A) 0 (B) 1/2

(C) -1/2 (D) -1/4

Q33 If $y=e\sqrt{x\sqrt{x}\sqrt{x}......x<1},$ then $\frac{d^2 y}{dx^2} at x=\log_e 3$ is

(A) 3 (B) 5

(C) 0 (D) 1

Q34 If $f(1)=1, f'(l)=3$ then the derivation of $f(f(f)(f))+(f(x))^2$ at x = 1 is

(A) 10 (B) 33

(C) 35 (D) 12

Q35 If $y=x^{\sin x}+\left(\sin x\right)^{x}$, then $\frac{dy}{dx}$ at $x=\frac{\pi}{2}$ is

(A) $\frac{4}{\pi}$

(B) $\pi \log \frac{\pi}{2}$

(C) 1

(D) $\frac{\pi^2}{2}$

Q36 If

$A_n=\begin{bmatrix}1-n & n \\ n & 1-n\end{bmatrix}$, then $|A_1|+|A_2|+ \ldots .|A_{2021}|=$

(A) –2021 (B) $-(2021)^2$

(C) $(2021)^2$ (D) 4042

Q37 The Function $f(x)=\log(1+x)-\frac{2x}{2+x}$ is increasing on

(A) $(-\infty, \infty)$

(B) $(\infty,-1)$

(C) $(-1, \infty)$

(D) $(-\infty, 0)$

Q38 The coordinates of the point on the $\sqrt{x}+\sqrt{y}=6$ at which the tangent is equally inclined to the axes is

(A) (4,4) (B) (1,1)

(C) (9,9) (D) (6,6)

Q39 The function $f(x)=4\sin^3 x-6\sin^2 x+12\sin x+100$ is strictly

(A) decreasing in $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

(B) Decreased in $\left[0, \frac{\pi}{2}\right]$

(C) Increasing in $\left(\pi, \frac{3\pi}{2}\right)$

(D) Decreasing in $\left(\frac{\pi}{2}, \pi\right)$

Q40 If $[x]$ is the greatest integer function not greater than x, then $\int_0^8 [x]dx$ is equal to

(A) 28 (B) 30

(C) 29 (D) 20

Q41 $\int_0^{\pi/2} \sqrt{\sin\theta}\cos^3\theta d\theta$ is equal to

(A) $\frac{8}{23}$ (B) $\frac{7}{23}$

(C) $\frac{8}{21}$ (D) $\frac{7}{21}$

Q42 If $e^y+xy=e$ the ordered pair $\left(\frac{dy}{dx}, \frac{d^2 y}{dx^2}\right)$ at x=0 is equal to

(A) $\left(\frac{1}{e}, \frac{1}{e^2}\right)$

(B) $\left(\frac{-1}{e}, \frac{-1}{e^2}\right)$

(C) $\left(\frac{1}{e}, \frac{-1}{e^2}\right)$

(D) $\left(\frac{-1}{e}, \frac{1}{e^2}\right)$

Q43 $\int \frac{\cos 2x-\cos 2\alpha}{\cos x-\cos \alpha} dx$ is equal to

(A) $2(\sin x-x\cos\alpha)+c$

(B) $2(\sin x+x\cos\alpha)+c$

(C) $2(\sin x-2x\cos\alpha)+c$

(D) $2(\sin x+2x\cos\alpha)+c$

Q44 $\int_0^1 \frac{xe^x}{(2+x)^3} dx$ is equal to

(A) $\frac{1}{27}.e - \frac{1}{8}$ (B) $\frac{1}{27}.e + \frac{1}{8}$

(C) $\frac{1}{9}.e + \frac{1}{4}$ (D) $\frac{1}{9}.e - \frac{1}{4}$

Q45 if $\int \frac{dx}{(x+2)(x^2+1)} = a\log\left|1+x^2\right| + b\tan^{-1}x$

$+\frac{1}{5}\log\left|x+2\right| + c$

then

(A) $a = \frac{-1}{10}, b = \frac{2}{5}$

(B) $a = \frac{1}{10}, b = \frac{2}{5}$

(C) $a = \frac{-1}{10}, b\frac{-2}{5}$

(D) $a = \frac{1}{10}, b\frac{-2}{5}$

Q46 Area of the region bounded by the curve $y = \tan x$, the X-axis and line $x = \frac{\pi}{3}$ is

(A) $\log\frac{1}{2}$ (B) log 2

(C) 0 (D) – log 2

Q47 Evaluate $\int_2^3 x^2 dx$ as the limit of a sum

(A) 72/6 (B) 53/9

(C) 25/7 (D) 19/3

Q48 $\int_0^{\pi/2} \frac{\cos x \sin x}{1+\sin x} dx$ is equal to

(A) log 2 – 1

(B) log 2

(C) – log 2

(D) 1 – log 2

Q49 If $\frac{dy}{dx} + \frac{y}{x} = x^2$, then $2y\left(2\right) - y\left(1\right) =$

(A) $\frac{11}{4}$ (B) $\frac{15}{4}$

(C) $\frac{9}{4}$ (D) $\frac{13}{4}$

Q50 The solution of the differential equation $\frac{dy}{dx} = (x+y)^2 is$

(A) $\tan^{-1}(x+y) = x + C$

(B) $\tan^{-1}(x+y) = 0$

(C) $\cos^{-1}(x+y) = C$

(D) $\cot^{-1}(x+y) = x + C$

Q51 If $y(x)$ is the solution of differential equation $x\log x\frac{dy}{dx} + y = 2x\log x, y\left(e\right)$ is equal to

(A) e (B) 0

(C) 2 (D) 2e

Q52 If $\left|a\right| = 2$ *and* $\left|b\right| = 3$ and the angle between a and b is 120°, then the length of the vector $\left|\frac{a}{2} - \frac{b}{3}\right|$ is

(A) 2 (B) 3

(C) 1/6 (D) 1

Q53 If $\left|a \times b\right|^2 + \left|a.b\right|^2 = 36\ and\ \left|a\right|$ equal to $= 3\ then\ \left|b\right|\ is$

(A) 9 (B) 36

(C) 4 (D) 2

Q54 if $\alpha = \hat{i} - 3\hat{j}, \beta = \hat{i} + 2\hat{j} - \hat{k}$ then express β i the form $\beta = \beta_1 + \beta_2$ Where β_1 is parallel to $\alpha\ and\ \beta_2$ is perpendicular to α, then β_2 is given by

(A) $\frac{-1}{2}\hat{i} + \frac{3}{2}\hat{j}$

(B) $\frac{5}{8}\left(\hat{i} + 3\hat{j}\right)$

(C) $\hat{i} - 3\hat{j}$

(D) $\hat{i} + 3\hat{j}$

Q55 The sum of the degree and order of the differential equation $\left(l + y_1^2\right)^{2/3} = y_2$ is

(A) 4 (B) 6

(C) 5 (D) 7

Q56 The coordinates of foot of the perpendicular drawn from the origin to the plane $2x - 3y + 4z = 29$ are

(A) $(2, 3, 4)$

(B) $(2, -3, -4)$

(C) $(2, -3, 4)$

(D) $(-2, -3, 4)$

Q57 The angle between the pair of lines $\frac{x+3}{3}=\frac{y-1}{5}=\frac{z+3}{4}$ and $\frac{x+1}{1}=\frac{y-4}{4}=\frac{z-5}{2}$ is

(A) $\theta=\cos^{-1}\left[\frac{31}{5\sqrt{42}}\right]$

(B) $\theta=\cos^{-1}\left[\frac{8\sqrt{3}}{15}\right]$

(C) $\theta=\cos^{-1}\left[\frac{19}{21}\right]$

(D) $\theta=\cos^{-1}\left[\frac{5\sqrt{3}}{16}\right]$

Q58 The corner points of the feasible region of an LPP are $(0,2),(3,0),(6,0),(6,8)$ *and* $(0,5)$, then the minimum value of $z=4x+6y$ occurs at

(A) Finite number of points

(B) Infinite number of points

(C) Only one point

(D) Only two points

Q59 A dietician has to develop a special diet using two foods X and Y. Each packet (containing 30 g) of food. X contains 12 units of calcium, 4 units of iron, 6 units of cholesterol and 6 units of vitamin A. Each packet of the same quantity of food Y contains 3 units of calcium, 20 units of iron, 4 units of cholesterol and 3 units of vitamin A. The diet requires at least 240 units of calcium, atleast 460 units of iron and atmost 300 units of cholesterol. The corner points of the feasible region are

(A) (2,72), (40, 15), (15, 20)

(B) (2,72), (15, 20), (0, 23)

(C) (0, 23), (40, 15), (2, 72)

(D) (2,72), (40, 15), (115,0

Q60 The distance of the point whose position vector is $\left(2\hat{i}+\hat{j}-\hat{k}\right)$ from the plane $r.\left(\hat{i}-2\hat{j}+4\hat{k}\right)=4$ is

(A) $\frac{8}{\sqrt{21}}$

(B) $8\sqrt{21}$

(C) $-\frac{8}{\sqrt{21}}$

(D) $-\frac{8}{21}$

Answer Key

Q1	A
Q2	A
Q3	A
Q4	B
Q5	A
Q6	D
Q7	D
Q8	C
Q9	D
Q10	B
Q11	C
Q12	B
Q13	B
Q14	B
Q15	A
Q16	B
Q17	C
Q18	D
Q19	A
Q20	C
Q21	A
Q22	A
Q23	D
Q24	A
Q25	D
Q26	B
Q27	C
Q28	A
Q29	B
Q30	D
Q31	A
Q32	A
Q33	A
Q34	B
Q35	C
Q36	B
Q37	C
Q38	C
Q39	D
Q40	A
Q41	C
Q42	D
Q43	B
Q44	D
Q45	A
Q46	B
Q47	D
Q48	D
Q49	B
Q50	A
Q51	C
Q52	B
Q53	D
Q54	A
Q55	C
Q56	C
Q57	A
Q58	D
Q59	A
Q60	A

Vernacular

Hints & Solutions

Note: scan the QR code to watch video solution

Q1 Text Solution:

Given three coins are tossed.

Therefore, sample space,

S = {TTT, TTH, THT, HTT, HHT, HTH, THH, HHH}

\ n(S) = 8

Let X represents 'number of heads'

∵ X = 0, 1, 2 or 3.

Probability distribution of X is

X	0	1	2	
$P(X)$	1/8	3/8	3/8	1/8

Required mean $= \Sigma X_i P_i$

$= 0\left(\frac{1}{8}\right)+1\left(\frac{3}{8}\right)+2\left(\frac{3}{8}\right)+3\left(\frac{1}{8}\right)$

$= 0 + \frac{3}{8} + \frac{6}{8} + \frac{3}{8} = \frac{12}{8} = 1.5$

Q2 Text Solution:

Given, $P(A) = \frac{1}{2}, P(B)$

$= \frac{1}{2}$ and $P(A \mid B) = \frac{1}{4}$

$P\left(A \mid B\right) = \frac{P(A\cap B)}{P(B)} \Rightarrow \frac{1}{4} = \frac{P(A\cap B)}{1/2}$

$\Rightarrow P(A \cap B) = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$

Now, $P(A' \cap B') = P(A \cup B)' = 1$

$- P(A \cup B)$

$= 1 - [P(A) + P(B) - P(A \cap B)] = 1$

$-[\frac{1}{2} + \frac{1}{2} - \frac{1}{8}]$

$= 1 - 1 + \frac{1}{8} = \frac{1}{8}$

Q3 Text Solution:

Let event E_1 = There is a lockdown,

Event E_2 = There is no lockdown, and A = Pandemic is controlled in one month

Now, $P(E_1) = 0.7, P(E_2) = 0.3$

$P(A \mid E_1) = 0.8$ and $P(A \mid E_2) = 0.3$

$P(A) = 0.7(0.8) + 0.3(0.3) = 0.56 + 0.09$
$= 0.65$

Q4 Text Solution:

Given, $P(\overline{A}) = 0.75, P(A \cup B)$
$= 0.65$ and $P(B) = x$

$P(A) = 1 - P(\overline{A}) = 1 - 0.75 = 0.25$

Also, A and B are independent.

$\Rightarrow P(A \cap B) = P(A) \cdot P(B) = 0.25x$

Using $P(A \cup B) = P(A) + P(B)$
$- P(A \cap B)$

$\Rightarrow 0.65 = 0.25 + x - 0.25x \Rightarrow 0.40$
$= 0.75x$

$\Rightarrow x = \frac{40}{75} = \frac{8}{15}$

Q5 Text Solution:

Given, $n(A) = p, n(B) = q$ and $(A \times B) = 7$

Since, $n(A \times B) = n(A) \times n(B)$

$\Rightarrow 7 = p \times q \Rightarrow pq = 7$

So, possible values of p and q are 7, 1 respectively.

$\Rightarrow p^2 + q^2 = 7^2 + 1^2 = 49 + 1 = 50$

Q6 Text Solution:

Given, function $f(x) = \frac{1}{\log_{10}(1-x)}$
$+ \sqrt{x+2}$

$f(x)$ will be defined if,

$1 - x > 0, \neq 1$ and $x + 2 \geq 0$

or $1 > x, 1 - x \neq 1$ and $x \geq -2$

or $x < 1,\ x \neq 0$ and $x \geq -2$

$\Rightarrow x \in [-2, 0) \cup (0, 1)$

Q7 Text Solution:

Given, function y = tan x

We know that, tan x is increasing in all the quadrants and for II quadrant the value of x is considered as (-∞, 0)

Hence, y = tan x is increase from -∞ to 0.

Q8 Text Solution:

Given, angle in radian $= \frac{\pi}{32}$

Degree measure $= \frac{180}{\pi} \times$ radian measure

$= \frac{180}{\pi} \times \frac{\pi}{32} = \left(\frac{45}{8}\right)^{\circ} = \left(5\frac{5}{8}\right)^{\circ}$

$= 5^{\circ}\left(\frac{5}{8} \times 60\right)' = 5^{\circ}\left(\frac{75}{2}\right)'$

$= 5^{\circ}37'\left(\frac{1}{2} \times 60\right)'' = 5^{\circ}37'30''$

Q9 Text Solution:

Given, expression is $\sin\frac{5\pi}{12}\sin\frac{\pi}{12}$

We know that,

2 sin A sin B = cos (A - B) - cos (A + B)

Therefore, $\sin\frac{5\pi}{12}\sin\frac{\pi}{12}$

$= \frac{1}{2}\left[\cos\left(\frac{5\pi}{12} - \frac{\pi}{12}\right) - \cos\left(\frac{5\pi}{12} + \frac{\pi}{12}\right)\right]$

$= \frac{1}{2}\left[\cos\left(\frac{4\pi}{12}\right) - \cos\left(\frac{6\pi}{12}\right)\right]$

$= \frac{1}{2}\left[\cos\left(\frac{\pi}{3}\right) - \cos\left(\frac{\pi}{2}\right)\right] = \frac{1}{2}\left[\frac{1}{2} - 0\right] = \frac{1}{4}$

Q10 Text Solution:

$$y = \sqrt{2 + \sqrt{2 + \sqrt{2 + 2\cos 8\theta}}}$$

We know, $1 + \cos 2A = 2\cos^2 A$

Therefore, y

$$= \sqrt{2 + \sqrt{2 + \sqrt{2 \cdot (1 + \cos 8\theta)}}}$$

$$= \sqrt{2 + \sqrt{2 + \sqrt{2 \cdot 2\cos^2 4\theta}}}$$

$$= \sqrt{2 + \sqrt{2 + 2\cos 4\theta}}$$

$$= \sqrt{2 + \sqrt{2 \cdot 2\cos^2 2\theta}}$$

$$= \sqrt{2 + 2\cos 2\theta} = \sqrt{2 \cdot 2\cos^2\theta} = 2\cos\theta$$

Q11 Text Solution:

Given, A = {1, 2, 3, ..., 10}

Set A containing only odd number = {1, 3, 5, 7, 9}

Number of subsets = 2^5

The required number of subsets of A containing only odd number = 2^5

= 32

Q12 Text Solution:

To find the 19th word in the dictionary order of permutations of the letters in the word "MASK," it is helpful first to determine the total number of permutations for the word. Since "MASK" consists of 4 unique letters, the number of different permutations can be calculated using the factorial of 4:

4! = 4 x 3 x 2 x 1 = 24

So, there are 24 possible permutations. Arranging these permutations in dictionary order means starting with the permutations beginning with 'A', followed by those starting with 'K', 'M', and 'S'. Now let's list them out to find the 19th word:

We will list the permutations in blocks based on their starting letter. With each letter in the first position, the remaining three letters can be arranged in 3! or 6 different ways.

Starting with 'A':

- 1. A-K-M-S
- 2. A-K-S-M
- 3. A-M-K-S
- 4. A-M-S-K
- 5. A-S-K-M
- 6. A-S-M-K

Starting with 'K': (Since the dictionary order will have all permutations starting with 'A' first, we continue from the count of 6.)

- 7. K-A-M-S
- 8. K-A-S-M
- 9. K-M-A-S
- 10. K-M-S-A
- 11. K-S-A-M
- 12. K-S-M-A

Starting with 'M':

- 13. M-A-K-S
- 14. M-A-S-K
- 15. M-K-A-S
- 16. M-K-S-A
- 17. M-S-A-K
- 18. M-S-K-A

At this point, we've reached the 18th permutation, which implies the next one will be the 19th. Since we are looking for the 19th word and we have finished with all permutations starting with 'M', we know the 19th word will begin with 'S', as it is the next letter in alphabetical order. The 19th permutation will be:

1. S – A – K – M

Hence, the 19th word formed from the permutation of the letters in "MASK" in dictionary order is "SAKM".
Therefore, the correct answer is:
Option B: SAKM

Q13 Text Solution:

Given, $a_1, a_2, a_3, \ldots, a_{10}$

and $\frac{a_3}{a_1} = 25$

$\Rightarrow \frac{ar^2}{a} = 25 \Rightarrow r^2 = 25$

$\frac{a_9}{a_5} = \frac{ar^8}{ar^4} = r^4 = \left(r^2\right)^2 = (25)^2 = 5^4$

Q14 Text Solution:

Equation of line passing through (7, 17) and (15, b) is

$\left(y - 17\right) = \left(\frac{\beta-17}{15-7}\right)\left(x - 7\right)$

$\Rightarrow y - 17 = \frac{(\beta-17)}{8}\left(x - 7\right)$

$\Rightarrow y = \left(\frac{\beta-17}{8}\right)x - \frac{7(\beta-17)}{8} + 17$

On comparing to y = mx + c, we get slope, $m_1 = \frac{\beta-17}{8}$

It is given that above line is perpendicular to

$2x - 3y + 17 = 0$

$\Rightarrow y = \frac{2}{3}x + \frac{17}{3}$

$\therefore$ Slope, $m_2 = \frac{2}{3}$

Condition for perpendicular lines, $m_1 \cdot m_2 = -1$

$\left(\frac{\beta-17}{8}\right) \times \frac{2}{3} = -1$

$\Rightarrow \beta - 17 = -12 \Rightarrow \beta = 5$

Q15 Text Solution:

The following table shows the signs of the coordinates in eight octants.

Coordinates ↓	I	II	III	IV	V	VI	VII	VIII
x	+	−	−	+	+	−	−	+
y	+	+	−	−	+	+	−	−
z	+	+	+	+	−	−	−	-

Given, point is (2, -4, -7).
It lies in VIII quadrant.

Q16 Text Solution:

$f(x) = \begin{cases} x^2 - 1, & 0 < x < 2 \\ 2x + 3, & 2 \le x < 3 \end{cases}$

Now, $\lim\limits_{x \to 2^-} f(x) = \lim\limits_{x \to 2^-} x^2 - 1$

$= \lim\limits_{h \to 0}\left[(2 - h)^2 - 1\right] = \lim\limits_{h \to 0}\left[4 + h^2 - 4h - 1\right]$

$= 4 + 0 - 0 - 1 = 3$

and $\lim\limits_{x \to 2^+} f(x) = \lim\limits_{x \to 2^+} (2x + 3)$

$= \lim\limits_{h \to 0} 2(2 + h) + 3$

$= \lim\limits_{h \to 0} 4 + 2h + 3 = 4 + 0 + 3 = 7$

Therefore, the quadratic equation whose roots are 3 and 7 is given by $x^3 - (3 + 7)x + (3 \times 7) = 0$ which is $x^2 - 10x + 21 = 0$

Q17 Text Solution:

3x + i (4x - y) = 6 - i

On comparing real and imaginary parts of LHS and RHS, we get

$3x = 6$ and $4x - y = -1$

$\Rightarrow 3x = 6 \Rightarrow x = 2$

Put $x = 2$ into $4x - y = -1$, we get

$4 \times 2 - y = -1$

$\Rightarrow 8 + 1 = y \Rightarrow y = 9$

Q18 Text Solution:

Given, numbers are -1, 0, 1, k

Standard deviation, $\sigma = \sqrt{5}$

$\sigma^2 = \frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2$

$\Rightarrow 5 = \frac{1+0+1+k^2}{4} - \left(\frac{-1+0+1+k}{4}\right)^2$

$\Rightarrow 5 = \frac{2+k^2}{4} - \frac{k^2}{16} \Rightarrow 80 = 8 + 4k^2 - k^2$

$\Rightarrow 72 = 3k^2 \Rightarrow k^2 = 24 \Rightarrow k = 2\sqrt{6}$

Q19 Text Solution:

Number of bijections from x to y will be zero because only sets with the same cardinality have bijections between them and it is given that number of elements in x and y are not same.

Q20 Text Solution:

$$f(x)=\begin{cases} 2x: & x>3 \\ x^2: & 1<x\leq 3 \\ 3x: & x\leq 1 \end{cases}$$

$$f(-1)+f(2)+f(4)=3(-1)(2)^2+2(4)$$

$$=-3+4+8=9$$

Q21 Text Solution:

Given relation is 3a + 2b = 27, where $a,\ b\in N$

$$3a+2b=27$$

$$\Rightarrow b=\frac{27-3a}{2}$$

$$a=1\Rightarrow b=\frac{27-3}{2}=\frac{24}{2}=12$$

$$a=3\Rightarrow b=\frac{27-9}{2}=\frac{18}{2}=9$$

$$a=5\Rightarrow b=\frac{27-15}{2}=\frac{12}{2}=6$$

$$a=7\Rightarrow b=\frac{27-21}{2}=\frac{6}{2}=3$$

Hence, we can see that the elements of option (a) satisfies the given relation. Hence, option (a) is correct.

Q22 Text Solution:

$$\text{Let } L=\lim_{y\to 0}\frac{\sqrt{3+y^3}-\sqrt{3}}{y^3}$$

$$L=\lim_{y\to 0}\frac{\left(\sqrt{3+y^3}-\sqrt{3}\right)}{y^3}\times\frac{\left(\sqrt{3+y^3}+\sqrt{3}\right)}{\left(\sqrt{3+y^3}+\sqrt{3}\right)}$$

$$=\lim_{y\to 0}\frac{3+y^3-3}{y^3\left(\sqrt{3+y^3}+\sqrt{3}\right)}=\lim_{y\to 0}\frac{y^3}{y^3\left(\sqrt{3+y^3}+\sqrt{3}\right)}$$

$$=\lim_{y\to 0}\frac{1}{\sqrt{3+y^3}+\sqrt{3}}=\frac{1}{\sqrt{3}+\sqrt{3}}=\frac{1}{2\sqrt{3}}$$

Q23 Text Solution:

$$\left(A^2\right)^{-1}=(A)^{-2}=\left(A^{-1}\right)^2=\left(-A^{-1}\right)^2$$

$$=(-A)^{-2}$$

$$\left(A^2\right)^{-1}\neq A^2,$$

$$\left(A^2\right)^{-1}=(-A)^{-2}$$

Q24 Text Solution:

$$A=\begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}$$

$$|A|=-4+3=-1$$

$$\text{adj}\left(A\right)=\begin{bmatrix} -2 & -3 \\ -(-1) & 2 \end{bmatrix}^T$$

$$=\begin{bmatrix} -2 & -3 \\ 1 & 2 \end{bmatrix}^T=\begin{bmatrix} -2 & 1 \\ -3 & 2 \end{bmatrix}$$

$$A^{-1}=\frac{\text{adj}\ (A)}{|A|}=\frac{\begin{bmatrix} -2 & 1 \\ -3 & 2 \end{bmatrix}}{(-1)}=\begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}$$

$$=A$$

$$\Rightarrow A^{-1}=A\Rightarrow A\cdot A^{-1}=A\cdot A$$

$$\Rightarrow I=A^2\Rightarrow A\cdot I=A\cdot A^2$$

$$\Rightarrow A=A^3\Rightarrow (A)^{-1}=\left(A^3\right)^{-1}$$

$$\Rightarrow A=\left(A^3\right)^{-1}\Rightarrow \left(A^3\right)^{-1}=A$$

Q25 Text Solution:

Given, $A^T=-A$

Let $P=A^{2021}$

$$P^T=\left[A^{2021}\right]^T=\left[A^T\right]^{2021}$$

$$=[-A]^{2021}=-[-A]^{2021}=-P$$

Hence, A^{2021} is also a skew symmetric matrix.

Q26 Text Solution:

$$\text{Given, } A=\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

Let $P\left(n\right):(aI+bA)^n=a^nI+na^{n-1}bA$

For $n=1, p\left(1\right):(aI+bA)^1=a^1I+1$

$\cdot\, a^{1-1}bA$

aI + bA = aI + bA

It is ture for n = 1.

Let P(n) is true for n = K.

So, $P\left(K\right):(AI+bA)^K=a^KI+Ka^{K-1}bA$

Now, for n = K + 1

$P\left(K+1\right):(aI+bA)^{K+1}=a^{K+1}I$

$+\left(K+1\right)a^{K+1-1}bA \quad ...(ii)$

$LHS = (aI + bA)^{K+1} = (aI + bA)^K (aI + bA)$

$= (a^K I + Ka^{K-1} bA)(aI + bA)$ [from Eq. (i)]

$= a^{K+1} I + Ka^K IbA + a^K IbA + Ka^{K-1} b^2 A^2$

$= a^{K+1} I + Ka^K b(IA) + a^K b(IA) + Ka^{K-1} b^2 (A^2)$

$= a^{K+1} I + Ka^K bA + a^K bA + Ka^{K-1} b^2 \times 0$

$[\because\ IA = A$ and

$A^2 = A \cdot A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$= 0]$

$= a^{K+1} I + (K+1) a^K bA = RHS$

Hence, P(n) is true for all $n \in N$.

Q27 Text Solution:

Given A is a 3 x 3 matrix and |5 . adj (A)| = 5

$|5 \cdot adj(A)| = 5 \Rightarrow 5^3 |adj(A)| = 5$

$\Rightarrow |\text{adj}(A)| = \frac{1}{5^2} \Rightarrow |A|^{3-1} = \frac{1}{5^2}$

$\Rightarrow |A^2| = \left(\frac{1}{5}\right)^2 \Rightarrow |A| = \pm\frac{1}{5}$

Q28 Text Solution:

Given, $\Delta = \begin{vmatrix} 1 & -2 & 5 \\ 2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix} = 86$

$\Rightarrow \begin{vmatrix} 1 & -2 & 5 \\ 2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix} = 86$

$\Rightarrow 1[2a^2 + 4] + 2[4a + 0] + 5[8 - 0] = 86$

$\Rightarrow 2a^2 + 4 + 8a + 40 = 86$

$\Rightarrow 2a^2 + 8a - 42 = 0 \Rightarrow a^2 + 4a - 21 = 0$

$\Rightarrow a^2 + 7a - 3a - 21 = 0$

$\Rightarrow (a+7)(a-3) = 0$

$a = -7,\ 3$

$\therefore$ Required sum $= -7 + 3 = -4$

Q29 Text Solution:

Given vertices of triangle are (-2, 6), (3, -6) and (1, 5).

Area of triangle $= \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$= \frac{1}{2}\begin{vmatrix} -2 & 6 & 1 \\ 3 & -6 & 1 \\ 1 & 5 & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} -5 & 12 & 0 \\ 2 & -11 & 0 \\ 1 & 5 & 1 \end{vmatrix}$

$[R_1 \leftrightarrow R_1 - R_2, R_2 \leftrightarrow R_2 - R_3]$

$= \frac{1}{2}[1(55 - 24)] = \frac{31}{2} = 15.5$ sq. units

Q30 Text Solution:

$y = \cos^{-1}[x]$

We know, for $\cos^{-1} x$

$\Rightarrow\ -1 \le x \le 1$

Therefore, for $\cos^{-1}[x]$

$\Rightarrow -1 \le [x] \le 1 \Rightarrow -1 \le x < 2 \Rightarrow x \in [-1, 2)$

Q31 Text Solution:

Given, $y = (1 + x^2)\tan^{-1} x - x$

Differentiating the given function w.r.t. x

$\frac{dy}{dx} = (1 + x^2)\frac{d}{dx}(\tan^{-1} x) + \tan^{-1} x \frac{d}{dx}(1 + x^2) - \frac{d}{dx}(x)$

$= (1 + x^2)\frac{1}{(1+x^2)} + \tan^{-1} x(2x) - 1$

$= 1 + 2x\tan^{-1} x - 1 = 2x\tan^{-1} x$

Q32 Text Solution:

Given, $x = e^\theta \sin\theta, y = e^\theta \cos\theta$

We have, $\frac{dx}{d\theta} = e^\theta \cos\theta + e^\theta \sin\theta$

$= e^\theta(\cos\theta + \sin\theta) \ldots$ (i)

and $\frac{dy}{d\theta} = e^\theta(-\sin\theta) + e^\theta \cos\theta$

$= e^\theta(\cos\theta - \sin\theta)$

On dividing Eq. (ii) by Eq. (i), we get

$\frac{\frac{dy}{d\theta}}{\frac{dz}{d\theta}} = \frac{e^\theta(\cos\theta - \sin\theta)}{e^\theta(\cos\theta + \sin\theta)}$

$\Rightarrow \frac{dy}{dx} = \frac{\cos\theta\left(1 - \frac{\sin\theta}{\cos\theta}\right)}{\cos\theta\left(1 + \frac{\sin\theta}{\cos\theta}\right)} = \frac{1 - \tan\theta}{1 + \tan\theta}$ ...(iii)

As we also have, $\frac{x}{y} = \frac{e^\theta \sin\theta}{e^\theta \cos\theta} \Rightarrow \frac{x}{y} = \tan\theta$

At $(x, y) = (1, 1) \Rightarrow \tan\theta = \frac{1}{1} = 1$

Putting tan q =1 into Eq. (iii), we get

$\frac{dy}{dx} = \frac{1-1}{1+1} = \frac{0}{2} = 0$

Q33 Text Solution:

Given,

$$y = e^{\sqrt{x\sqrt{x\sqrt{x......}}}}, x > 1$$

$$y = e^{\sqrt{x\sqrt{x\sqrt{x...}}}}$$

$$\log_e y = \left(\sqrt{x\sqrt{x\sqrt{x \ldots}}}\right) \log_e e$$

$$\Rightarrow \quad \log y = \sqrt{x\sqrt{x\sqrt{x \ldots \ldots}}} \Rightarrow \log y$$

$$= \sqrt{x \cdot \log y}$$

$$\Rightarrow \quad \left(\log y\right)^2 = x \log y \Rightarrow \frac{(\log y)^2}{\log y} = x$$

$$\Rightarrow \log y = x....(i)$$

Differentiating w.r,t x, we get

$$\frac{d^2 y}{dx^2} = \frac{dy}{dx} \Rightarrow \frac{d^2 y}{dx^2} = y[\text{from Eq. (ii)}].....$$

Differentiating w.r.t. x, we get

At x = loge 3, from Eq (i). we get

$\log y = \log 3 \Rightarrow y = 3$

Putting $y = 3$ into Eq. (iii), get

$$\frac{d^2 y}{dx^2} = 3$$

Q34 Text Solution:

$$y = f(f(f(x))) + (f(x))^2$$

Diffentiating w.r.t x, we get

$$\frac{dy}{dx} = f'\left(f\left(f\left(x\right)\right)\right) \cdot f'\left(f\left(x\right)\right) \cdot f'\left(x\right)$$

$$+ 2f\left(x\right) \cdot f'\left(x\right).$$

At $x = 1$

$$\frac{dy}{dx} = f'\left(f\left(f\left(1\right)\right)\right) \cdot f'\left(f\left(1\right)\right) \cdot f'\left(1\right)$$
$$+ 2f\left(1\right) \cdot f'\left(1\right)$$

$$\frac{dy}{dx} = f'\left(f\left(1\right)\right) \cdot f'\left(1\right) \cdot 3 + 2 \cdot 1 \cdot 3 = f'\left(1\right)$$
$$\cdot 3 \cdot 3 + 6$$

$$= 9f'\left(1\right) + 6 = 9 \cdot 3 + 6 = 27 + 6 = 33$$

Q35 Text Solution:

Given $y = x^{\sin x} + (\sin x)^x$

Let $\quad u = x^{\sin -1}$ and $v = (\sin x)^x$

Now, $u = x^{\sin x}, \log u = \sin x \log x$

Differentiating w.r.t. x, we get

$$\frac{1}{u} \cdot \frac{du}{dx} = \sin x \cdot \frac{1}{x} + \log x \times \cos x$$

$$\frac{du}{dx} = u\left(\frac{\sin x}{x} + \cos x \log x\right)$$

Now, $v = (\sin x)^x \Rightarrow \log v = x \log(\sin x)$

Differentiating w.r.t x, we get

$$\frac{1}{vdv} = x\frac{\cos x}{\sin x} + \log\left(\sin x\right)$$

$$\frac{dv}{dx} = v\left(x \cot x + \log \sin x\right)$$

adding Eqs. (i) and (ii), we get

$$\frac{du}{dx} + \frac{dv}{dx} = x^{\sin x}\left(\frac{\sin x}{x} + \cos x \log x\right)$$
$$+(\sin x)^x(x \cot x + \log \sin x)$$

At $x = \frac{\pi}{2}$

$$\frac{dy}{dx} = \left(\frac{\pi}{2}\right)\left(\frac{1}{\pi/2} + 0 \log \frac{\pi}{2}\right)$$
$$+ \left(1\right)^{\frac{\pi}{2}}\left(\frac{\pi}{2} \cdot 0 + \log\left(1\right)\right)$$

$$\frac{dy}{dx} = \left(\frac{\pi}{2}\right)\left(\frac{2}{\pi} + 0\right) + 1 \cdot \left(0 + 0\right) = 1$$

Q36 Text Solution:

Given

$$A_n = \begin{bmatrix} 1 - n & n \\ n & 1 - n \end{bmatrix} \text{and}$$

$$|A_n| = \begin{bmatrix} 1 - n & n \\ n & 1 - n \end{bmatrix}$$

$$= (1 - n)^2 - n^2 = 1 + n^2 - 2n - n^2$$

$$|A_n| = 1 - 2n$$

$$\text{Now, } |A_1| + |A_2| + |A_3| + \ldots + |A_{2021}|$$
$$= (1 - 2) + (1 - 4) + (1 - 6) + \ldots$$
$$+ (1 - 4042)$$
$$= (1 + 1 + \ldots + 1) - (2 + 4 + 6 + \ldots$$
$$+ 4042)$$
$$= 2021 - \left[\frac{2021}{2}(2 + 4042)\right]$$
$$= 2021 - \frac{2021}{2} \times 4044 = 2021 - 2021$$
$$\times 2022$$
$$= 2021\left(1 - 2022\right) = -\left(2021\right)^2$$

Q37 Text Solution:

Given $x < -1 \Rightarrow$

$$f\left(x\right) = \log\left(1 + x\right) - \frac{2x}{2+x}$$

$f'(x) = \frac{1}{(1+x)}(0+1)$

$-2\left[\frac{(2+x)\times1-x(0+1)}{(2+x)^2}\right]$

$= \frac{1}{1+x} - 2\left[\frac{2+x-x}{(2+x)^2}\right] = \frac{1}{1+x} - \frac{4}{(2+x)^2}$

$= \frac{(2+x)^2-4(1+x)}{(1+x)(2+x)^2} = \frac{4+x^2+4x-4-4x}{(x+1)(x+2)^2}$

$= \frac{x^2}{(x+1)(x+2)^2} = \left(\frac{x}{x+2}\right)^2 \cdot \frac{1}{(x+1)}$

sign $f'(x)$ depends on the sign of $\frac{1}{(x+1)}$

$f'(x) > 0$ When $x > -1$ $\Rightarrow$increasing

$f'(x) < 0$ when $x < -1$ $\Rightarrow$decreasing

Hence $f(x)$ Is increasing on $(-1, \infty)$

Q38 Text Solution:

$\sqrt{x} + \sqrt{y} = 6$

$\sqrt{x} + \sqrt{y} = 6$

Differentiating w.r.t.x, we get

$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}}$

Since, the tangent is equally inclined to the axes.

$\frac{dy}{dx} = \pm1 \Rightarrow -\frac{\sqrt{y}}{\sqrt{x}} = \pm1$

$\Rightarrow \quad \sqrt{y} = \pm\sqrt{x} \Rightarrow y = x$

Therefore

$\sqrt{x} + \sqrt{y} = 6 \Rightarrow \sqrt{x} + \sqrt{x} = 6 \Rightarrow 2\sqrt{x} = 6$

$\Rightarrow \quad \sqrt{x} = 3 \Rightarrow x = 9$

$\therefore \quad x = 9 \text{and} y = 9$

Q39 Text Solution:

Given

$f(x) = 4\sin^3 x - 6\sin^2 x + 12\sin x + 100$

$f'(x) = 12\sin^2 x \cdot \cos x - 12\sin x\cos x + 12\cos x$

$= 12\cos x\left[\sin^2 x - \sin x + 1\right]$

$= 12\cos x\left[\sin^2 x + (1 - \sin x)\right]$

We know $1 - \sin x \geq 0$ and $\sin^2 x \geq 0$

$\therefore \sin^2 x + 1 - \sin x > 0$ and $f'(x) > 0$

Therfore $f'(x) > 0$ When $\cos x > 0 \Rightarrow x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

So, $f(x)$ is increasing when $x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ and

$f'(x) < 0$ When

$\cos x < 0 \Rightarrow x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

So, $f(x)$ is decreasing when $x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

Hence, $f(x)$ is decreaing in $\left(\frac{\pi}{2}, \pi\right)$

Q40 Text Solution:

Let $I = \int_0^8 [x]dx$

$I = \int_0^1 [x]dx + \int_1^2 [x]dx + \int_2^3 [x]dx + \int_3^4 [x]dx + \int_4^5 [x]dx + \int_5^6 [x]dx + \int_6^7 [x]dx + \int_7^8 [x]dx$

$= \int_0^1 0dx + \int_1^2 1dx + \int_2^8 2dx + \int_3^4 3dx + \int_4^5 4dx + \int_5^6 5dx + \int_6^7 6dx + \int_7^8 7dx$

$= 0 + 1[x]_1^2 + 2[x]_2^3 + 3[x]_3^4 + 4[x]_4^5 + 5[x]_5^6 + 6[x]_6^7 + 7[x]_7^8$

$= 0 + 1(2-1) + 2(3-2) + 3(4-3) + 4(5-4) + 5(6-5) + 6(7-6) + 7(8-7)$

$= 1 + 2 + 3 + 4 + 5 + 6 + 7$

$= \frac{7\cdot(7+1)}{2} = \frac{7\cdot8}{2} = 7\cdot4 = 28$

Q41 Text Solution:

Let $I = \int_0^{\pi/2} \sqrt{\sin\theta} \cdot \cos^3\theta d\theta$

Let $\sin\theta = t \Rightarrow \cos\theta d\theta = dt$

when $\quad \theta = 0 \Rightarrow t = 0$

When $\quad \theta = \frac{\pi}{2} \Rightarrow t = 1$

$I = \int_0^1 \sqrt{t}(1-t^2)\,dt = \int_0^1 (t^{1/2} - t^{5/2}) \cdot dt$

$= \left[\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^1 - \left[\frac{t^{\frac{7}{2}}}{\frac{7}{2}}\right]_0^1 = \frac{2}{3}\left[1-0\right] - \frac{2}{7}\left[1-0\right]$

$= \frac{2}{3} - \frac{2}{7} = \frac{14-6}{21} = \frac{8}{21}$

Q42 Text Solution:

Given $e^y + xy = e.....(i)$

Differentaing w.r.t x, we get

$$e^y \frac{dy}{dx} + x\frac{dy}{dx} + 1 \cdot y = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-y}{(e^y+x)} \qquad \dots (\text{iii})$$

Again, dirrentiantiang Eq. (ii) w.r.t x, we get

$$e^y \frac{d^2y}{dx^2} + e^y\left(\frac{dy}{dx}\right)^2 + x\frac{d^2y}{dx^2} + \frac{dy}{dx} + \frac{dy}{dx} = 0$$

$$\Rightarrow (e^y + x)\frac{d^2y}{dx^2} + e^y\left(\frac{dy}{dx}\right)^2 + 2\frac{dy}{dx} = 0$$

Now, on putting $x = 0$ in Eq, (i), we get

$e^y + 0.\, y = e \Rightarrow e^y = e^1 \Rightarrow y = 1$

On putting $x = 0, y = 1$ in Eq, (iii), we get

$\frac{dy}{dx} = \frac{-1}{e+0} = -\frac{1}{e}$

now, putting $x = 0, y = 1$and $\frac{dy}{dx} = \frac{-1}{e}$in Eq, (iv), we get

$$\left(e^1 + 0\right)\frac{d^2y}{dx^2} + e'\left(-\frac{1}{e}\right)^2 + 2\left(-\frac{1}{e}\right) = 0$$

$$\Rightarrow e\frac{d^2y}{dx^2} + \frac{1}{e} - \frac{2}{e} = 0 \Rightarrow \frac{d^2y}{dx^2} = \frac{1}{e^2}$$

Hence $\left(\frac{dy}{dx}, \frac{d^2y}{dx^2}\right)$at x = 0 is $\left(-\frac{1}{e}, \frac{1}{e^2}\right)$

Q43 Text Solution:

$$\text{Let} I = \int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha} dx$$

$$I = \int \frac{(2\cos^2 x - 1) - (2\cos^2 \alpha - 1)}{\cos x - \cos \alpha} dx$$

$$I = 2\int \frac{\cos^2 x - \cos^2 \alpha}{\cos x - \cos \alpha} dx$$

$$= 2\int \frac{(\cos x - \cos \alpha)(\cos x + \cos \alpha)}{(\cos x - \cos \alpha)} dx$$

$$= 2\int(\cos x + \cos \alpha)dx = 2(\sin x + x\cos \alpha) + c$$

Q44 Text Solution:

$$\text{Let} I = \int_0^1 \frac{xe^x}{(2+x)^3} dx = \int_0^1 e^x\left[\frac{(2+x)-2}{(2+x)^3}\right] dx$$

$$= \int_0^1 e^x\left[\frac{1}{(2+x)^2} - \frac{2}{(2+x)^3}\right] dx$$

Here $\frac{1}{(2+x)^2} = f\left(x\right)$ and $\frac{-2}{(2+x)^3} = f'\left(x\right)$

We know

$$\int e^x\left[f\left(x\right) + f'\left(x\right)\right] dx = e^x f\left(x\right) + c$$

$$I = \left[e^x \cdot \frac{1}{(2+x)^2}\right]_0^1 = \frac{e^x}{(2+1)^2} - \frac{e^0}{(2+0)^2} = \frac{e}{9} - \frac{1}{4}$$

Q45 Text Solution:

$$\int \frac{dx}{(x+2)(x^2+1)} = a\log\left|1 + x^2\right| + b\tan^{-1} x + \frac{1}{5}\log\left|x+2\right| + c \dots.$$

$$\text{Let} \frac{1}{(x+2)(x^2+1)} = \frac{A}{(x+2)} + \frac{Bx+C}{(x^2+1)}$$

$$\therefore \quad 1 = A\left(x^2+1\right) + \left(x+2\right)\left(Bx+C\right)$$

$$1 = Ax^2 + A + Bx^2 + Cx + 2Bx + 2C$$

$$1 = \left(A+B\right)x^2 + x\left(2B+C\right) + A + 2C$$

Here $A + B = 0$

$2B + C = 0 \text{and} A + 2C = 1$

On soving them, we get

$A = \frac{1}{5}, B = \frac{-1}{5}\text{and} C = \frac{2}{5}$

Therefore $\int \frac{dx}{(x+2)(x^2+1)}$

$$= \int\left(\frac{1}{5(x+2)} + \frac{\frac{-1}{5}x + \frac{2}{5}}{x^2+1}\right)$$

$$= \frac{1}{5}\int \frac{1}{(x+2)}dx - \frac{1}{5}\int \frac{x}{x^2+1}dx + \frac{2}{5}\int \frac{dx}{1+x^2}$$

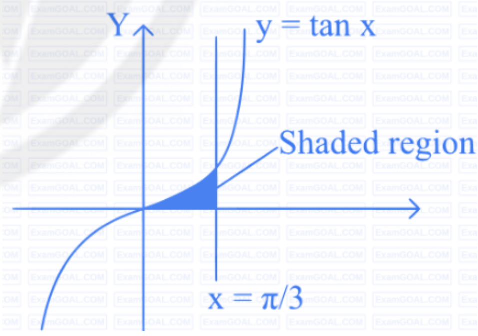
$$= \frac{1}{5}\log\left|x+2\right| - \frac{1}{10}\log\left|1+x^2\right| + \frac{2}{5}\tan^{-1} x + c$$

On comparing Eqs. (i) and (ii), we get

$a = \frac{1}{10}, and\, b\frac{2}{5}$

Q46 Text Solution:

Given, curve $y = \tan x$ and line $x = \frac{\pi}{3}$



$$\text{Required area} = \int_0^{\pi/3} \tan x dx = \left[\log\left|\sec x\right|\right]_0^{\pi/3}$$

$$= \log\left|\sec \frac{\pi}{3}\right| - \log\left|\sec 0\right|$$

$$= \log(2) - \log(1) = \log 2 \text{sq units}$$

Q47 Text Solution:

Let $I = \int_2^3 x^2 dx$

$I = \int_2^3 x^2 dx = \int_2^3 f(x) dx$, where $f(x) = x^2$

Also, $a = 2, b = 3$

$\lim_{h \to 0} h\left[f(2) + f(2+h) + f(2+2h) + \ldots + f(2+(n-1)h)\right]$

where, $nh = b - a = 3 - 2 = 1$

$= \lim_{h \to 0} h\left[2^2 + (2+h)^2 + (2+2h)^2 + \ldots + (2+(n-1)h)^2\right]$

$= \lim_{h \to 0} h\left[2^2 + (2^2 + h^2 + 4h) + (2^2 + 2^2h^2 + 8h) + \ldots + (2^2 + (n-1)^2h^2 + 4(n-1)h)\right]$

$= \lim_{h \to 0} h\left[2^2h + h^2\left(1^2 + 2^2 + \ldots \cdots + (n-1)^2\right) + 4h(1 + 2 + \ldots + (n-1))\right]$

$= \lim_{h \to 0} \left[4nh + \frac{h^3(n-1)n(2(n-1)+1)}{6} + 4h^2\frac{(n-1)h}{2}\right]$

$= \lim_{h \to 0} \left[4nh + \frac{(nh-h)(nh)(2nh-h)}{6} + 2(nh-h)(nh)\right]$

$= \lim_{h \to 0} \left[4 \times 1 + \frac{(1-h)\cdot 1(2-h)}{6} + 2(1-h)\cdot 1\right]$

$= 4 + \frac{2}{6} + 2 = 6 + \frac{1}{3} = \frac{19}{3}$

Q48 Text Solution:

Let $I = \int_0^{\pi/2} \frac{\cos x \sin x}{1+\sin x} dx$

$= \int_0^{\pi/2} \frac{\cos x(1+\sin x - 1)}{1+\sin x} dx$

$I = \int_0^{\pi/2} \left(\frac{\cos x(1+\sin x)}{1+\sin x} - \frac{\cos x}{1+\sin x}\right) dx$

$= \int_0^{\pi/2} \cos x dx - \int_0^{\pi/2} \frac{\cos x}{1+\sin x} dx$

$= \left[\sin x\right]_0^{\pi/2} - I_1 \ldots.\text{(i)}$

$I_1 = \int_0^{\pi/2} \frac{\cos x}{1+\sin x} dx$

Let $1 + \sin x = t$

$\Rightarrow \cos x dx = dt$

$I_1 = \int_0^{\pi/2} \frac{dt}{t} = [\log t]_0^{\pi/2} = [\log(1+\sin x)]_0^{\pi/2}$

$= \log(1+1) - \log(1+0) = \log 2$

Now, from Eq, (i) we have

$I = [1 - 0] - \log 2 = 1 - \log 2$

Q49 Text Solution:

$\frac{dy}{dx} + \frac{y}{x} = x^2$

On comparing to $\frac{dy}{dx} + P(x)y = Q(x)$

$P(x) = \frac{1}{x}, Q(x) = x^2$

$\text{IF} = e^{\int P dx} = e^{\int \frac{1}{x} dx} = e^{\ln x} = x$

$y \cdot (\text{IF}) = \int Q(x)(\text{IF}) dx + C$

$y \cdot x = \int x^2 x dx + C = \frac{x^4}{4} + C$

$y = \frac{x^3}{4} + \frac{C}{x}$

$y(2) = \frac{2^3}{4} + \frac{C}{2} = 2 + \frac{C}{2}$

$y(1) = \frac{1}{4} + \frac{C}{1} = \frac{1}{4} + C = x$

$2y(2) - y(1) = 2\left(2 + \frac{C}{2}\right) - \left(\frac{1}{4} + C\right)$

$= 4 + C - \frac{1}{4} - C = \frac{15}{4}$

Q50 Text Solution:

$\frac{dy}{dx} = (x+y)^2 \ldots.(i)$

$Let\ x + y = t \Rightarrow 1 + \frac{dy}{dx} = \frac{dt}{dx}$

Eq. (i) can be written as

$$\frac{dt}{dx} - 1 = t^2$$

$$\Rightarrow \quad \frac{dt}{dx} = t^2 + 1 \Rightarrow \frac{dt}{t^2+1} = dx$$

$$\Rightarrow \int \frac{dt}{t^2+1} = \int dx \Rightarrow \tan^{-1} t = x + C$$

$$\Rightarrow \quad \tan^{-1}(x+y) = x + C$$

Q51 Text Solution:

$x \log x \frac{dy}{dx} + y = 2x \log x \Rightarrow \frac{dy}{dx} + \frac{y}{x \log x} = 2$

$\text{IF} = e^{\int \frac{1}{x \log x} dx} = e^{\log(\log x)} = \log x$

$y(\log x) = \int (2)(\log x) \cdot dx + C$

$\Rightarrow \quad y \log x = 2\int \log x dx + C$

Putting $x = 1$ we get

$y \cdot 0 = 2[1 \times 0 - 1] + C \Rightarrow C = 2$

Putting $C = 2$ in Eq,(i), we get

$$y \log x = 2\left[x \log x - x\right] + 2$$

At $\quad x = e \Rightarrow y \cdot 1 = 2(e \times 1 - e) + 2$

$$y = 0 + 2 = 2$$

$$y(e) = 2$$

Q52 Text Solution:

We know that,

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| \cdot |\mathbf{b}| \cos\theta$$

$$\text{So,} \mathbf{a} \cdot \mathbf{b} = (2)(3)\cos 120^\circ \quad [\because \theta = 120^\circ]$$

$$= 6\cos(90^\circ + 30^\circ) = 6(-\sin 30^\circ) = -6 \times \tfrac{1}{2}$$

$$\mathbf{a} \cdot \mathbf{b} = -3$$

$$\text{Now,} \left|\tfrac{1}{2}\mathbf{a} - \tfrac{1}{3}\mathbf{b}\right|^2 = \left(\tfrac{1}{2}\mathbf{a} - \tfrac{1}{3}\mathbf{b}\right)\left(\tfrac{1}{2}\mathbf{a} - \tfrac{1}{3}\mathbf{b}\right)$$

$$= \tfrac{1}{4}|\mathbf{a}|^2 - \tfrac{1}{6}\mathbf{a} \cdot \mathbf{b} - \tfrac{1}{6}\mathbf{a} \cdot \mathbf{b} + \tfrac{1}{9}|\mathbf{b}|^2$$

$$= \tfrac{1}{4} \times 4 - \tfrac{1}{6} \times (-3) - \tfrac{1}{6}(-3) + \tfrac{1}{9} \times 9$$

$$= 1 + \tfrac{1}{2} + \tfrac{1}{2} + 1 = 2 + 1 = 3$$

Hence, Option (b) is correct

Q53 Text Solution:

$|\mathbf{a} \times \mathbf{b}| + |\mathbf{a} \cdot \mathbf{b}|^2 = 36$ and $|\mathbf{a}| = 3$

Correct one is $|\mathbf{a} \times \mathbf{b}|^2 + |\mathbf{a} \cdot \mathbf{b}|^2 = 36$ and $|\mathbf{a}| = 3$

We know $|\mathbf{a} \times \mathbf{b}|^2 + |\mathbf{a} \cdot \mathbf{b}|^2 = |\mathbf{a}|^2|\mathbf{b}|^2$

$$\therefore \quad |\mathbf{a}|^2|\mathbf{b}|^2 = 36 \Rightarrow (3)^2|\mathbf{b}|^2 = 36$$

$$\Rightarrow \quad |b|^2 = 4 \Rightarrow |b| = 2$$

Q54 Text Solution:

$$\alpha = \hat{\mathbf{i}} - 3\hat{\mathbf{j}}, \beta = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$$

$$\beta = \beta_1 + \beta_2$$

β_1 is parallelr to α

$$\Rightarrow \quad \beta_1 = \lambda\alpha \Rightarrow \beta_1 = \lambda\hat{\mathbf{i}} - 3\lambda\hat{\mathbf{j}}$$

$$\Rightarrow \text{Also,} \beta = \beta_1 + \beta_2$$

$$\beta_2 = \beta - \beta_1 = \left(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}\right) - \left(\lambda\hat{\mathbf{i}} - 3\lambda\hat{\mathbf{j}}\right)$$

$$\beta_2 = (1 - \lambda)\hat{\mathbf{i}} + (2 - 3\lambda)\hat{\mathbf{j}} - \hat{\mathbf{k}}$$

It is given β_2 is perpendicular to α.

$$\therefore (1 - \lambda)(1) + (2 + 3\lambda)(-3) + (-1)(0) = 0$$

$$\Rightarrow \quad 1 - \lambda - 6 - 9\lambda + 0 = 0$$

$$\Rightarrow \quad -5 - 10\lambda = 0$$

$$\Rightarrow \quad \lambda = -\tfrac{1}{2}$$

$$\beta_1 = \tfrac{-1}{2}\hat{\mathbf{i}} + \tfrac{3}{2}\hat{\mathbf{j}}$$

Q55 Text Solution:

$$\left(1 + y_1^2\right)^{2/3} = y_2$$

$$\Rightarrow \quad \left[\left(1 + y_1^2\right)^{2/3}\right]^3 = (y_2)^3 \Rightarrow \left(1 + y_1^2\right)^2 = y_2^3$$

Here, highest derivative is y_2.

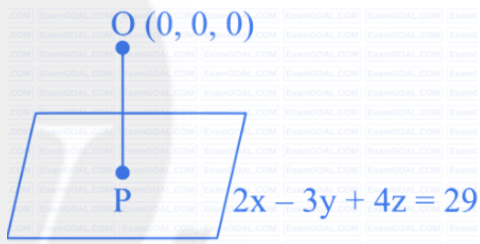
Therefore, order is 2 and power of y_2 is 3.

So, the degree is 3.

Hence, required sum = 2+3=5

Q56 Text Solution:

Given, equation of plane is $2x - 3y + 4z = 29$



Since, OP is perpendicular to the plane $2x - 3y + 4z = 29$

Therefore, Direction Ratios of OP will be $< 2, -3, 4 >$.

Equation of line OP will be

$\frac{x-0}{2} = \frac{y-0}{-3} = \frac{z-0}{4} = \lambda$ (say)

Coordinates of P will be $(2\lambda, -3\lambda, 4\lambda)$.

$\because P$ lies on plane, so

$$2(2\lambda) - 3(-3\lambda) + 4(4\lambda) = 29$$

$$\Rightarrow \quad 4\lambda + 9\lambda + 16\lambda = 29$$

$$\Rightarrow \quad \lambda = 1$$

$\therefore$ Coordinates of P will be $(2, -3, 4)$.

Q57 Text Solution:

Given, equation of lines are

$$\frac{x+3}{3} = \frac{y-1}{5} = \frac{z+3}{4} \quad \text{.... (i)}$$

$$\text{and} \quad \frac{x+1}{1} = \frac{y-4}{4} = \frac{z-5}{2} \quad \text{.... (ii)}$$

Direction ratios of line (i) :$< 3, 5, 4 >$

Direction ratios of line (ii) : $< 1, 4, 2 >$

We know angle between pair of lines is given by

$$\cos\theta = \left|\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right|$$

where, $< a_1, b_1, c_1 >$ and $< a_2, b_2, c_2 >$ are direction ratios of respectively lines.

$$\cos\theta = \left|\frac{3\times 1+5\times 4+4\times 2}{\sqrt{3^2+5^2+4^4}\sqrt{1^2+4^2+2^2}}\right|$$

$$= \left|\frac{3+20+8}{\sqrt{9+25+16}\sqrt{1+16+4}}\right| = \left|\frac{31}{5\sqrt{2}\sqrt{21}}\right| = \frac{31}{5\sqrt{42}}$$

$$\theta = \cos^{-1}\left(\frac{31}{5\sqrt{42}}\right)$$

Q58 Text Solution:

Given, the corner points of the feasible region of an LPP are (0, 2), (3, 0), (6, 0), (6, 8) and (0,5)

Minimum value of z occurs at (0, 2) and (3,0).

Hence, minimum value of z lies at any point on the line joining two points (0, 2) and (3,0).

Q59 Text Solution:

$x \geq 0,\ y \geq 0 \ \Rightarrow 12x + 3y \geq 240$

$\Rightarrow 4x + y \geq 80$

$4x + 20y \geq 460$

$\Rightarrow x + 5y \geq 115$

$6x + 4y \leq 300$

$\Rightarrow 3x + 2y \leq 150$

$x \geq 0,\ y \geq 0$

$4x + y \geq 80$

we can see that corner point of the feasible region are (2,72), (40, 15), (15, 20)

Q60 Text Solution:

Equation of plane $\mathbf{r}\cdot\left(\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 4\hat{\mathbf{k}}\right) = 4$

$\Rightarrow \quad \mathbf{r}\cdot\mathbf{n} = d$

Point $\mathbf{a} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$

$\mathbf{n} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 4\hat{\mathbf{k}}, \quad d = 4$

$$\text{Distance} = \frac{|\mathbf{a}\cdot\mathbf{n} - d|}{|\mathbf{n}|} = \left|\frac{(2\hat{\mathbf{i}}+\hat{\mathbf{j}}-\hat{\mathbf{k}})\cdot(\hat{\mathbf{i}}-2\hat{\mathbf{j}}+4\hat{\mathbf{k}})-4}{\sqrt{1^2+(-2)^2+4^2}}\right|$$

$$= \left|\frac{2-2-4-4}{\sqrt{1+4+16}}\right| = \left|\frac{-8}{\sqrt{21}}\right| = \frac{8}{\sqrt{21}}$$