



THE ASSOCIATION OF MATHEMATICS TEACHERS OF INDIA

Screening Test – Bhaskara Contest

NMTC at JUNIOR LEVEL – IX & X Standards

2026 – 2027

Test Paper With Solution

1. If $x \neq y \neq z$ with $x + y + z = 0$ and $x^3 + x + 1 = 0, y^3 + y + 1 = 0, z^3 + z + 1 = 0$, then the numerical value of $x^9 + y^9 + z^9$ is
 (A) 7 (B) -2 (C) -6 (D) 6

Sol. (D)

Since x, y, z are distinct roots satisfying $t^3 + t + 1 = 0$, we find the sums of their powers using Vieta's formulas:

$$S_0 = x^0 + y^0 + z^0 = 3$$

$$S_1 = x + y + z = 0$$

$$S_2 = (x + y + z)^2 - 2(xy + yz + zx) = 0 - 2(1) = -2$$

From $t^3 = -t - 1$, the power sum relation is $S_{n+3} = -S_{n+1} - S_n$:

$$S_3 = -S_1 - S_0 = 0 - 3 = -3$$

$$S_4 = -S_2 - S_1 = 2 - 0 = 2$$

$$S_5 = -S_3 - S_2 = 3 - (-2) = 5$$

$$S_6 = -S_4 - S_3 = -2 - (-3) = 1$$

$$S_7 = -S_5 - S_4 = -5 - 2 = -7$$

$$S_8 = -S_6 - S_5 = -1 - 5 = -6$$

$$S_9 = -S_7 - S_6 = 7 - 1 = 6$$

2. The number of natural numbers n satisfying the equation $\frac{8^n - 2^n}{6^n - 3^n} = 2$, is

(A) only one (B) two (C) four (D) infinitely many

Sol. (A)

$$\text{Numerator: } 8^n - 2^n = (2^3)^n - 2^n = 2^n(4^n - 1) = 2^n(2^n - 1)(2^n + 1)$$

$$\text{Denominator: } 6^n - 3^n = (2 \times 3)^n - 3^n = 3^n(2^n - 1)$$

Substitute these back into the original equation:

$$\frac{2^n(2^n - 1)(2^n + 1)}{3^n(2^n - 1)} = 2$$

Since n is a natural number ($n \geq 1$), $2^n - 1 \neq 0$. We can safely cancel it out:

$$\frac{2^n(2^n + 1)}{3^n} = 2$$

Multiply both sides by 3^n :



$$2^n (2^n + 1) = 2 \cdot 3^n$$

$$4^n + 2^n = 2 \cdot 3^n$$

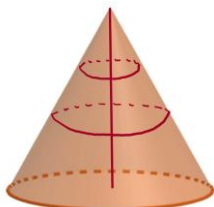
For $n = 1$: $4^1 + 2^1 = 6$ and $2 \cdot 3^1 = 6$.

For $n = 2$: $4^2 + 2^2 = 20$ and $2 \cdot 3^2 = 18$. ($20 \neq 18$)

For $n \geq 2$: The left-hand side (4^n) grows much faster than the right-hand side ($2 \cdot 3^n$), so no other natural number solutions exist.

Thus, there is only one solution.

3. A right circular cone is cut into three parts of volume v_1, v_2, v_3 from the vertex respectively, by giving two cuts parallel to the base, which trisect the altitude of the cone. Then $v_1 : v_2 : v_3 =$



- (A) 1:2:3 (B) 1:4:9 (C) 1:8:27 (D) 1:7:19

Sol. (D)

Let the height of the top section be h .

Small top cone: Height = h

Medium cone (top + middle): Height = $2h$

Full large cone: Height = $3h$

the total volumes of these three nested cones:

The volume of similar geometric shapes scales with the cube of their linear dimensions (height):

$$V_{\text{small}} \propto h^3 = 1$$

$$V_{\text{medium}} \propto (2h)^3 = 8$$

$$V_{\text{large}} \propto (3h)^3 = 27$$

Top part (v_1): This is just the small cone.

$$v_1 = 1$$

Middle part (v_2): Subtract the small cone from the medium cone.

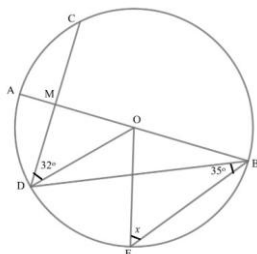
$$v_2 = V_{\text{medium}} - V_{\text{small}} = 8 - 1 = 7$$

Bottom part (v_3): Subtract the medium cone from the full large cone.

$$v_3 = V_{\text{large}} - V_{\text{medium}} = 27 - 8 = 19$$

$$v_1 : v_2 : v_3 = 1 : 7 : 19$$

4. In the adjoining figure, AOB is a diameter of the circle with center O. Point M is the mid-point of chord CD. $\angle CDO = 32^\circ$, $\angle DBE = 35^\circ$ and $\angle BEO = x^\circ$, then $x =$



- (A) 74° (B) 54° (C) 64° (D) 66°

Sol. (C)

$\angle MOD$: Since $OM \perp CD$, $\angle MOD = 90^\circ - 32^\circ = 58^\circ$.

$\angle DOB$: Linear pair with $\angle MOD \Rightarrow 180^\circ - 58^\circ = 122^\circ$.

$\angle OBD$: $\triangle ODB$ is isosceles ($OD = OB$) $\Rightarrow \frac{180^\circ - 122^\circ}{2} = 29^\circ$.

total $\angle OBE$: $\angle OBD + \angle DBE = 29^\circ + 35^\circ = 64^\circ$.

x : $\triangle OBE$ is isosceles ($OB = OE$) $\Rightarrow x = \angle OBE = 64^\circ$.

5. During a wartime, a group of n soldiers had enough food to last for 30 days. After 10 days, 50 more soldiers joined. If the food now can only last for 16 days, the value of n is
- (A) 180 (B) 200 (C) 220 (D) 310

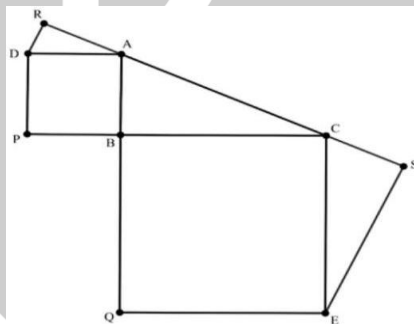
Sol. (B)

Remaining days for initial group: $30 - 10 = 20$ days

Equation: $20 \times n = 16 \times (n + 50)$

for n : $4n = 800 \Rightarrow n = 200$

6. Let ABC be a right-angled triangle, $\angle ABC = 90^\circ$. $AB = 5\text{cm}$, $BC = 12\text{cm}$. Squares $ABPD$ and $BCEQ$ are drawn in the exterior of the $\triangle ABC$. DR is drawn perpendicular to line AC at R and ES is perpendicular to the line AC at S . Let $DR = h_1$, and $ES = h_2$, then $\frac{h_1 + h_2}{AC} =$



- (A) $\frac{5}{13}$ (B) $\frac{12}{13}$ (C) 2 (D) 1

Sol. (D)

$\triangle ABC$:

By the Pythagorean theorem,

$$AC = \sqrt{5^2 + 12^2} = 13 \text{ cm}.$$

Let $\angle BAC = \theta$, so $\cos\theta = \frac{5}{13}$ and $\sin\theta = \frac{12}{13}$.

$h_1 = \angle DAC = 180^\circ - 90^\circ - \theta = 90^\circ - \theta$, in right-angled $\triangle DRA$:

$$h_1 = AD \cdot \sin(90^\circ - \theta) = 5 \cos\theta = 5 \left(\frac{5}{13} \right) = \frac{25}{13}$$

$h_2 =$

Since $\angle ECS = \theta$, in right-angled $\triangle ESC$:

$$h_2 = CE \cdot \sin\theta = 12 \sin\theta = 12 \left(\frac{12}{13} \right) = \frac{144}{13}$$

$$h_1 + h_2 = \frac{25 + 144}{13} = \frac{169}{13} = 13$$

$$\frac{h_1 + h_2}{AC} = \frac{13}{13} = 1$$

7. If $(\sqrt{2})^{\sqrt{x}} - (\sqrt{2})^{\sqrt{y}} = 48$, where x and y are natural numbers, then the value of $x + y$ is

(A) 208 (B) 112 (C) 132 (D) 96

Sol. (A)

$$\frac{\sqrt{x}}{2^2} - \frac{\sqrt{y}}{2^2} = 48$$

$$\frac{\sqrt{y}}{2^2} \left(2^{\frac{\sqrt{x}-\sqrt{y}}{2}} - 1 \right) = 16 \times 3 = 2^4 \times (2^2 - 1)$$

$$\frac{\sqrt{y}}{2} = 4 \Rightarrow \sqrt{y} = 8 \Rightarrow y = 64$$

$$\frac{\sqrt{x} - \sqrt{y}}{2} = 2 \Rightarrow \sqrt{x} - 8 = 4 \Rightarrow \sqrt{x} = 12 \Rightarrow x = 144$$

$$\text{So } x + y = 144 + 64 = 208$$

8. x and y are real numbers which satisfy $x^2 + y^2 + 2x + 6y + 10 = 0$. Then the numerical value of $\frac{x+y}{x-y}$ is

(A) 1 (B) 2 (C) -2 (D) $-\frac{1}{2}$

Sol. (C)

Rearrange the equation into perfect squares:

$$(x^2 + 2x + 1) + (y^2 + 6y + 9) = 0$$

$$(x+1)^2 + (y+3)^2 = 0$$

Since squares of real numbers are non-negative, each term must be zero:

$$x + 1 = 0 \Rightarrow x = -1$$

$$y + 3 = 0 \Rightarrow y = -3$$

Substitute $x = -1$ and $y = -3$:

$$\frac{x+y}{x-y} = \frac{-1+(-3)}{-1-(-3)} = \frac{-4}{2} = -2$$

9. If $x = \frac{\sqrt{101}+1}{2}$, then the find the value of $(x^3 - 26x - 23)^5$ is

(A) 1 (B) $\frac{1}{32}$ (C) 32 (D) 81

Sol. (C)

Form a quadratic equation:

$$2x - 1 = \sqrt{101} \Rightarrow (2x - 1)^2 = 101$$

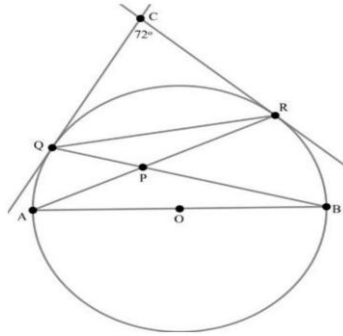
$$4x^2 - 4x + 1 = 101 \Rightarrow x^2 - x - 25 = 0 \Rightarrow x^2 = x + 25$$

$$x^3 = x(x^2) = x(x + 25) = x^2 + 25x = (x + 25) + 25x = 26x + 25$$

$$x^3 - 26x - 23 = (26x + 25) - 26x - 23 = 2$$

$$(2)^5 = 32$$

10. In the adjoining figure, AOB is a diameter of the circle with center O . QR is a chord and the tangents to the circle at points Q and R include an angle of measure 72° . Chords BQ and AR intersect at P , then the measure of $\angle RPB$ is



- (A) 40° (B) 34° (C) 38° (D) 36°

Sol. (D)

minor arc QR : The angle between tangents is 72° , so the intercepted minor arc $QR = 180^\circ - 72^\circ = 108^\circ$.

Since AOB is a diameter, arc $AB = 180^\circ$.

angle $\angle APB$: The angle between two intersecting chords inside a circle is half the sum of their intercepted arcs:

$$\angle APB = \frac{\text{arc}AB + \text{arc}QR}{2} = \frac{180^\circ + 108^\circ}{2} = 144^\circ$$

$\angle RPB$: Since $\angle APB$ and $\angle RPB$ form a linear pair on the straight line AR :

$$\angle RPB = 180^\circ - 144^\circ = 36^\circ$$

11. If the arithmetic mean of two numbers is 25 and their geometric mean is 24, then an equation with the given numbers as its roots, is

- (A) $x^2 + 50x + 576 = 0$ (B) $x^2 - 50x - 576 = 0$ (C) $x^2 + 50x - 576 = 0$ (D) $x^2 - 50x + 576 = 0$

Sol. (D)

Sum of roots: $2 \times 25 = 50$

Product of roots: $24^2 = 576$

Formula $x^2 - (\text{Sum})x + \text{Product} = 0$

$$x^2 - 50x + 576 = 0$$

12. The number of real values of x satisfying the equation $\frac{3x^2 - 21x}{x^2 - 7x} = x - 4$, is

- (A) Zero (B) One (C) Two (D) Three

Sol. (A)

$$\frac{3x(x-7)}{x(x-7)} = x-4$$

The denominator cannot be zero, so $x \neq 0$ and $x \neq 7$.

$$3 = x - 4 \Rightarrow x = 7$$

Since $x = 7$ is excluded by the domain restriction, there are no valid real solutions.

13. The average marks obtained by Sunil in 6 subjects is 88. On subsequent verification, it was found that the marks obtained by him in a subject was wrongly copied was 86 instead of 68. The correct average marks obtained by him is $(ab)_{10}$, that is a two-digit number in base 10. Then $a^2 + b^2 =$

- (A) 89 (B) 88 (C) 100 (D) 113

Sol. (A)

$$\text{Net Change in Marks} = \text{Correct Mark} - \text{Wrong Mark} = 68 - 86 = -18$$

Change in Average: $\frac{-18}{6 \text{ subjects}} = -3$

Correct Average (ab): $88 - 3 = 85 \Rightarrow a = 8, b = 5$

so $a^2 + b^2 : 8^2 + 5^2 = 64 + 25 = 89$

14. The graph of $x^2 + y^2 = 25$ is plotted in a real xy -plane. The number of points (x, y) on this graph such that both x and y are integers, is

(A) 8 (B) 12 (C) 16 (D) 10

Sol. (B)

integer pairs (x, y) satisfying $x^2 + y^2 = 25$.

Case 1: One coordinate is zero

If one of the values is 0, the other must be ± 5 .

$(\pm 5, 0) \rightarrow 4$ points

$(0, \pm 5) \rightarrow 4$ points

Case 2: Neither coordinate is zero

The perfect squares that add up to 25 are 9 and 16 ($3^2 + 4^2 = 25$).

$(\pm 3, \pm 4) \rightarrow 4$ points

$(\pm 4, \pm 3) \rightarrow 4$ points

Total integer points: $4 + 4 + 4 + 4 = 12$ points.

15. If $f(n+1) = \frac{2 \times f(n) + 1}{2}$, for all natural numbers n and $f(1) = \frac{1}{2}$,

then $-f(1) + f(2) - f(3) + f(4) - f(5) + \dots - f(99) + f(100) =$

(A) 75 (B) 50 (C) -25 (D) 25

Sol. (D)

$$f(n+1) = \frac{2f(n) + 1}{2} = f(n) + \frac{1}{2}$$

This shows that the difference between any two consecutive terms is always $\frac{1}{2}$:

$$f(n+1) - f(n) = \frac{1}{2}$$

$$S = [f(2) - f(1)] + [f(4) - f(3)] + \dots + [f(100) - f(99)]$$

the total sum:

Each of the 50 pairs equals $\frac{1}{2}$:

$$S = 50 \times \frac{1}{2} = 25$$

16. In $\triangle ABC$, $\angle ACB = 30^\circ$, $AB = 5$ cm. Then the circumradius of $\triangle ABC$ is _____ cm.

Sol. (5)

For any triangle with side length c opposite to angle C , and circumradius R :

$$\frac{c}{\sin C} = 2R$$

Substitute the given values:

Here, side $AB = c = 5$ cm and angle $\angle ACB = C = 30^\circ$.

$$\frac{5}{\sin 30^\circ} = 2R$$

$$\text{Since } \sin 30^\circ = \frac{1}{2} :$$

$$\frac{5}{\frac{1}{2}} = 2R \Rightarrow 10 = 2R \Rightarrow R = 5 \text{ cm}$$

17. If $\frac{\sqrt{60} + \sqrt{95} + \sqrt{228} + 12}{\sqrt{5} + 2\sqrt{12} + \sqrt{19}} = \frac{\sqrt{a} + \sqrt{b}}{c}$

where a, b are square-free natural numbers and c is a natural number, then the value of $a + b + 7c =$ ____.

Sol. (38)

Notice the relationships between the terms:

$$\sqrt{60} = \sqrt{5} \times \sqrt{12}$$

$$\sqrt{95} = \sqrt{5} \times \sqrt{19}$$

$$\sqrt{228} = \sqrt{12} \times \sqrt{19}$$

$$12 = \sqrt{12} \times \sqrt{12}$$

Grouping the terms gives:

$$\text{Numerator} = (\sqrt{5} + \sqrt{12})(\sqrt{12} + \sqrt{19})$$

Let $x = \sqrt{5} + \sqrt{12}$ and $y = \sqrt{12} + \sqrt{19}$. The denominator can be split as $x + y$:

$$\text{Fraction} = \frac{x \cdot y}{x + y} = \frac{1}{\frac{1}{x} + \frac{1}{y}}$$

$$\frac{1}{x} = \frac{1}{\sqrt{12} + \sqrt{5}} = \frac{\sqrt{12} - \sqrt{5}}{7}$$

$$\frac{1}{y} = \frac{1}{\sqrt{19} + \sqrt{12}} = \frac{\sqrt{19} - \sqrt{12}}{7}$$

$$\frac{1}{x} + \frac{1}{y} = \frac{\sqrt{19} - \sqrt{5}}{7}$$

$$\text{Fraction} = \frac{7}{\sqrt{19} - \sqrt{5}} = \frac{7(\sqrt{19} + \sqrt{5})}{19 - 5} = \frac{\sqrt{5} + \sqrt{19}}{2}$$

Comparing with $\frac{\sqrt{a} + \sqrt{b}}{c}$, we get $a = 5, b = 19$, and $c = 2$.

$$a + b + 7c = 5 + 19 + 7(2) = 38$$

18. Let $\frac{3x - 25}{x^2 - 5x + 6} = \frac{a}{x - 3} + \frac{b}{x - 2}$ be an identity on x . Then the numerical value of $b - a$ is ____.

Sol. (35)

$$x^2 - 5x + 6 = (x - 3)(x - 2)$$

$$3x - 25 = a(x - 2) + b(x - 3)$$

Put $x = 3$:

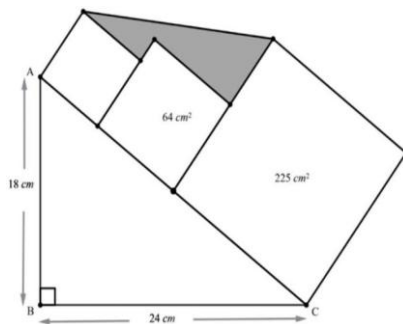
$$3(3) - 25 = a(3 - 2) \Rightarrow -16 = a$$

Put $x = 2$:

$$3(2) - 25 = b(2 - 3) \Rightarrow -19 = -b \Rightarrow b = 19$$

$$b - a = 19 - (-16) = 35$$

19. In the adjoining figure, ABC is a rightangled triangle. $\angle ABC = 90^\circ$, $AB = 18\text{cm}$, $BC = 24\text{cm}$. There are three squares drawn externally on the hypotenuse AC as shown, such that AC would be the sum of the side lengths of these squares. Area of two of these squares is 225cm^2 , and 64cm^2 respectively, as indicated in the figure. Then the area of the shaded region = _____ cm^2 .



Sol. (52)

Using the Pythagorean theorem in right-angled $\triangle ABC$:

$$AC = \sqrt{AB^2 + BC^2} = \sqrt{18^2 + 24^2} = \sqrt{324 + 576} = \sqrt{900} = 30 \text{ cm}$$

$$\text{Side length of the rightmost square} = \sqrt{225} = 15 \text{ cm}$$

$$\text{Side length of the middle square} = \sqrt{64} = 8 \text{ cm}$$

Since the sum of the side lengths of all three squares equals $AC = 30 \text{ cm}$, the side length of the leftmost square is:

$$30 - 15 - 8 = 7 \text{ cm}$$

Let the segment AC lie on the x -axis with A at the origin $(0, 0)$. The shaded region lies entirely above the first two squares (from $x = 0$ to $x = 15$):

The top-left vertex of the first square is at $(0, 7)$.

The top-left vertex of the third square is at $(15, 15)$.

The straight line bounding the top of the shaded region connects $(0, 7)$ and $(15, 15)$, forming a large trapezoid with the base from $x = 0$ to $x = 15$.

$$\text{Area of the large trapezoid} = \frac{\text{sum of parallel sides}}{2} \times \text{width} = \frac{7 + 15}{2} \times 15 = 11$$

$$\text{Area of the shaded region} = \text{Area of trapezoid} - (\text{Area of Square 1} + \text{Area of Square 2})$$

$$\text{Area of the shaded region} = 165 - (7^2 + 64) = 165 - (49 + 64) = 165 - 113 = 52$$

20. We say that a positive integer is arithmetically sequenced if its digits, in order, form an arithmetic sequence. Then the number of 4-digit positive integers which are arithmetically sequenced, is _____.

Sol. (30)

A 4-digit number $\overline{d_1 d_2 d_3 d_4}$ has digits forming an arithmetic sequence with a common difference c . Since it is a 4-digit number, $d_1 \in \{1, 2, \dots, 9\}$ and $d_4 \in \{0, 1, \dots, 9\}$.

the valid numbers based on the common difference c :

Constant Sequence ($c = 0$) :

$$d_1 = d_2 = d_3 = d_4 \Rightarrow 1111, 2222, \dots, 9999 \text{ (9 numbers)}$$

Increasing Sequences ($c > 0$) :

$$c = 1 \Rightarrow 1234, 2345, 3456, 4567, 5678, 6789 \text{ (6 numbers)}$$

$$c = 2 \Rightarrow 1357, 2468, 3579 \text{ (3 numbers)}$$

Decreasing Sequences ($c < 0$):

$c = -1 \Rightarrow 3210, 4321, \dots, 9876$ (7 numbers)

$c = -2 \Rightarrow 6420, 7531, 8642, 9753$ (4 numbers)

$c = -3 \Rightarrow 9630$ (1 number)

Total = $9 + 6 + 3 + 7 + 4 + 1 = 30$ numbers.

21. The smallest positive integer $(abc)_{10}$, in base 10, a, b, c are digits, with $500 > (abc)_{10} > 150$ such that the number formed by all possible arrangements (for example $(bac)_{10}, (cba)_{10}$) of all the digits of $(abc)_{10}$, is prime, is _____.

Sol. (199)

No digit can be even (2, 4, 6, 8) or 0 or 5, because any permutation ending in these digits would be divisible by 2 or 5, making it composite.

Therefore, the only allowed digits are $\{1, 3, 7, 9\}$.

Since $150 < (abc)_{10} < 500$, the first digit a must be either 1 or 3.

Case 1: If $a = 1$

Since the number must be greater than 150, the second digit b can only be 7 or 9 (as 1 and 3 would make the number less than 150).

Try digits $\{1, 7, c\}$:

If $c = 1 \Rightarrow 117$ is divisible by 9.

If $c = 3 \Rightarrow 371$ is divisible by 7 (7×53).

If $c = 7 \Rightarrow 177$ is divisible by 3.

If $c = 9 \Rightarrow 791$ is divisible by 7 (7×113).

Try digits $\{1, 9, c\}$:

If $c = 1 \Rightarrow 119$ is divisible by 7 (7×17).

If $c = 3 \Rightarrow 319$ is divisible by 11 (11×29).

If $c = 7 \Rightarrow 791$ is divisible by 7.

If $c = 9 \Rightarrow$ The digits are $\{1, 9, 9\}$. Let's check its permutations:

199 is a prime number.

919 is a prime number.

991 is a prime number.

Since all permutations are prime and $150 < 199 < 500$, 199 is a valid solution.

Any other valid set of digits starting with 3 (such as $\{3, 1, 1\} \Rightarrow 311$) will produce numbers greater than 199. Thus, 199 is the smallest possible integer.

22. Let circle-1 and circle-2 are orthogonal circles. The length of an arc measuring 60° of circle-1 is equal to the length of an arc measuring 45° of circle-2. Then the ratio of the area of square on the line-segment joining their centres to the area of square on the radius of circle-1, in its simplest form, is _____.

[Note: Let two distinct circles are intersecting in different points. If their tangents at the point of intersection are perpendicular to each other, then the circles are said to be Orthogonal circles]

Sol. (25/9)

Let r_1 be the radius of circle-1 and r_2 be the radius of circle-2.

$$\text{Arc length of circle-1} = \frac{60^\circ}{360^\circ} \times 2\pi r_1 = \frac{\pi r_1}{3}$$

$$\text{Arc length of circle-2} = \frac{45^\circ}{360^\circ} \times 2\pi r_2 = \frac{\pi r_2}{4}$$

Equating the two arc lengths:

$$\frac{\pi r_1}{3} = \frac{\pi r_2}{4} \Rightarrow \frac{r_2}{r_1} = \frac{4}{3}$$

If d is the distance between the centers of two orthogonal circles, then:

$$d^2 = r_1^2 + r_2^2$$

the ratio of the area of the square on the line connecting the centers (d^2) to the area of the square on the radius of circle-1 (r_1^2):

$$\text{Ratio} = \frac{d^2}{r_1^2} = \frac{r_1^2 + r_2^2}{r_1^2} = 1 + \left(\frac{r_2}{r_1}\right)^2$$

$$\text{Ratio} = 1 + \left(\frac{4}{3}\right)^2 = 1 + \frac{16}{9} = \frac{25}{9}$$

- 23.** If m and n are the roots of the equation $x^2 + 2mx + 5n = 0$; $m \neq 0, n \neq 0$, then the sum of roots of this equation is ____.

Sol. **(-10)**

Product rule: $mn = 5n \Rightarrow m = 5$

Sum rule: $m + n = -2m$

Substitution: $m + n = -2(5) = -10$

- 24.** P is a point in the interior of an equilateral $\triangle ABC$. Line-segments PD, PE, PF are the perpendiculars drawn from P to the sides BC, CA, AB respectively with D lies on side BC, E lies on side CA and F lies on side AB . If $PD = 6\text{cm}, PE = 8\text{cm}, PF = 10\text{cm}$, then area of the triangle ABC is ____ cm^2 .

Sol. **($192\sqrt{3}$ or approx 332.55)**

$$h = PD + PE + PF = 6 + 8 + 10 = 24 \text{ cm}$$

Using the relation for an equilateral triangle's altitude $h = \frac{\sqrt{3}}{2}a$:

$$24 = \frac{\sqrt{3}}{2}a \Rightarrow a = \frac{48}{\sqrt{3}} = 16\sqrt{3} \text{ cm}$$

$$\text{Area} = \frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4}(16\sqrt{3})^2 = \frac{\sqrt{3}}{4}(768) = 192\sqrt{3} \text{ cm}^2$$

- 25.** Let the product $12 \times 15 \times 16 = 3146$ in base b , where b is a real number. Then the sum $s = 12 + 15 + 16$ in base b is ____.

Sol. **(44)**

$$(b+2)(b+5)(b+6) = 3b^3 + b^2 + 4b + 6$$

$$b^3 + 13b^2 + 52b + 60 = 3b^3 + b^2 + 4b + 6$$

$$2b^3 - 12b^2 - 48b - 54 = 0 \Rightarrow b^3 - 6b^2 - 24b - 27 = 0$$

By testing values, $b = 9$ is a root:

$$9^3 - 6(9^2) - 24(9) - 27 = 729 - 486 - 216 - 27 = 0$$

$$s = 12_9 + 15_9 + 16_9$$

Units place: $2 + 5 + 6 = 13$ in base 10 $\Rightarrow 14_9$ (Write 4, carry 1)

Tens place: $1 + 1 + 1 + \text{carry } 1 = 4$

Therefore, $s = 44_9$.

26. If $\frac{4^x}{2^{x+y}} = 8$, $\frac{9^{x+y}}{3^y} = 243$ then $x^2 + xy + 7y^2 = \underline{\hspace{2cm}}$.

Sol. (7)

$$\frac{2^{2x}}{2^{x+y}} = 2^3 \Rightarrow 2x - (x + y) = 3 \Rightarrow x - y = 3$$

$$\frac{3^{2(x+y)}}{3^y} = 3^5 \Rightarrow 2x + 2y - y = 5 \Rightarrow 2x + y = 5$$

Adding the equations: $3x = 8 \Rightarrow x = \frac{8}{3}$

Substituting back: $y = \frac{8}{3} - 3 = -\frac{1}{3}$

$$\begin{aligned} x^2 + xy + 7y^2 &= \left(\frac{8}{3}\right)^2 + \left(\frac{8}{3}\right)\left(-\frac{1}{3}\right) + 7\left(-\frac{1}{3}\right)^2 \\ &= \frac{64 - 8 + 7}{9} = \frac{63}{9} = 7 \end{aligned}$$

27. Let $a_1, a_2, a_3, \dots, a_n$ be all natural numbers and also $n \in \mathbb{N}$ such that $a_1 + a_2 + a_3 + \dots + a_n = 1000$. When $a_1 \times a_2 \times a_3 \times \dots \times a_n$ would be maximum, then $n = \underline{\hspace{2cm}}$.

Sol. (334 and 333) BONUS

The number of terms n is 334 (or 333, depending on whether the factor 4 is split into 2×2). Both configurations yield the identical maximum product.

To maximize the product of natural numbers $a_1 \times a_2 \times \dots \times a_n$ for a fixed sum $\sum a_i = 1000$, we apply the following mathematical rules:

No factor can be 1. Removing a factor of 1 and adding it to any other factor k always creates a larger product:

$$1 \times k < k + 1$$

No factor can be 5 or greater. Any integer $k \geq 5$ can be broken into 2 and $(k - 2)$, which yields a strictly greater product:

$$2 \times (k - 2) = 2k - 4 > k \quad (\text{for } k \geq 5)$$

Any three 2s can be replaced by two 3s because they have the same sum (6), but the 3s produce a higher product:

$$3 \times 3 = 9 > 2 \times 2 \times 2 = 8$$

Thus, an optimal configuration can contain at most two 2s.

Optimizing for Sum = 1000

Dividing 1000 by 3 gives a remainder:

$$1000 = 3 \times 333 + 1$$

Since we cannot have a factor of 1, we combine one 3 with the 1 to form a 4. This leaves us with two equivalent choices for the maximum product:

Configuration	Factors	Number of Terms (n)	Product Value
Option A (Unsplit)	One 4 and 332 threes	$332 + 1 = 333$	$3^{332} \times 4$
Option B (Split)	Two 2s and 332 threes	$332 + 2 = 334$	$3^{332} \times 4$

Both configurations are mathematically correct as they achieve the absolute maximum possible product.

28. If $x = 2026$, and $y = \frac{1}{1013}$, then the value of $\left(\frac{x^2 + 1}{y}\right)\left(\frac{y^2 + 1}{x}\right) + \left(\frac{x^2 - 1}{y}\right)\left(\frac{y^2 - 1}{x}\right)$ is $\underline{\hspace{2cm}}$.

Sol. (5)



Given $x = 2026$ and $y = \frac{1}{1013}$:

$$xy = 2026 \times \frac{1}{1013} = 2$$

$$E = \frac{(x^2 + 1)(y^2 + 1) + (x^2 - 1)(y^2 - 1)}{xy}$$

$$(x^2 + 1)(y^2 + 1) = x^2y^2 + x^2 + y^2 + 1$$

$$(x^2 - 1)(y^2 - 1) = x^2y^2 - x^2 - y^2 + 1$$

Notice that x^2 and y^2 cancel out perfectly:

$$(x^2y^2 + x^2 + y^2 + 1) + (x^2y^2 - x^2 - y^2 + 1) = 2x^2y^2 + 2$$

$$E = \frac{2(xy)^2 + 2}{xy} = 2xy + \frac{2}{xy}$$

$$E = 2(2) + \frac{2}{2} = 4 + 1 = 5$$

29. If $\frac{1}{3^2+1} + \frac{1}{4^2+2} + \frac{1}{5^2+3} + \dots = \frac{a}{b}$
where a and b are coprime natural numbers, then $a + b = \underline{\hspace{2cm}}$.

Sol. (49)

Observe the pattern in the denominators:

$$\text{First term (n = 3): } \frac{1}{3^2+1} = \frac{1}{10}$$

$$\text{Second term (n = 4): } \frac{1}{4^2+2} = \frac{1}{18}$$

$$\text{Third term (n = 5): } \frac{1}{5^2+3} = \frac{1}{28}$$

The general term for $n \geq 3$ is:

$$T_n = \frac{1}{n^2 + (n-2)} = \frac{1}{n^2 + n - 2} = \frac{1}{(n-1)(n+2)}$$

$$T_n = \frac{1}{3} \left(\frac{1}{n-1} - \frac{1}{n+2} \right)$$

$$S = \frac{1}{3} \left[\left(\frac{1}{2} - \frac{1}{5} \right) + \left(\frac{1}{3} - \frac{1}{6} \right) + \left(\frac{1}{4} - \frac{1}{7} \right) + \left(\frac{1}{5} - \frac{1}{8} \right) + \left(\frac{1}{6} - \frac{1}{9} \right) + \dots \right]$$

Notice that the negative terms start canceling out with positive terms three steps ahead ($-\frac{1}{5}$ cancels with $+\frac{1}{5}$, $-\frac{1}{6}$

with $+\frac{1}{6}$, etc.). Only the first three positive fractions survive:

$$S = \frac{1}{3} \left(\frac{1}{2} + \frac{1}{3} + \frac{1}{4} \right) = \frac{1}{3} \left(\frac{6+4+3}{12} \right) = \frac{1}{3} \times \frac{13}{12} = \frac{13}{36}$$

Comparing $\frac{13}{36}$ with $\frac{a}{b}$, we have coprime integers $a = 13$ and $b = 36$:

$$a + b = 13 + 36 = 49$$

30. n is a natural number such that $(n+17)$ and $(n-40)$ both are perfect square numbers, then the square-root of greatest possible value of $(n-40)$ is ____ .

Sol. (28)

Define equations: Let $n+17 = a^2$ and $n-40 = b^2$.

Subtracting them gives $a^2 - b^2 = 57 \Rightarrow (a-b)(a+b) = 57$.

Maximize b : To find the greatest value of $b = \sqrt{n-40}$, use the factor pair with the smallest difference: $a-b=1$
 $a+b=57$

Solve for b : Subtracting the equations yields $2b = 56 \Rightarrow b = 28$.

