CBSE :
Class -

2023-24

Board:

Inverse Trigonometric Functions

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Principal Values and Domains of Inverse Trigonometric/circular Functions

	Function	Domain	Range
(i)	$y=\sin^{-1}x$	$-1\le x\le 1$	$-\frac{\pi}{2}\le y\le\frac{\pi}{2}$
(ii)	$y=\cos^{-1}x$	$-1\le x\le 1$	$y=\sin^{-1}x$
(iii)	$y=\tan^{-1}x$	$x\in R$	$-\frac{\pi}{2}<y<\frac{\pi}{2}$
(iv)	$y=\text{cosec}^{-1}x$	$x\le -1$ or $x\ge 1$	$-\frac{\pi}{2}\le y\le\frac{\pi}{2};y\ne 0$
(v)	$y=\sec^{-1}x$	$x\le -1$ or $x\ge 1$	$0\le y\le\pi;y\ne\frac{\pi}{2}$
(vi)	$y=\cot^{-1}x$	$x\in R$	$0<y<\pi$

Properties of Inverse circular Functions

P-1 :

(i) $y=\sin\left(\sin^{-1}x\right)=x, x\in\left[-1,1\right], y\in\left[-1,1\right], y$ is aperiodic.

(ii) $y=\cos\left(\cos^{-1}x\right)=x, x\in\left[-1,1\right], y\in\left[-1,1\right], y$ is aperiodic.

(iii) $y=\tan\left(\tan^{-1}x\right)=x, x\in R, y\in R, y$ is aperiodic.

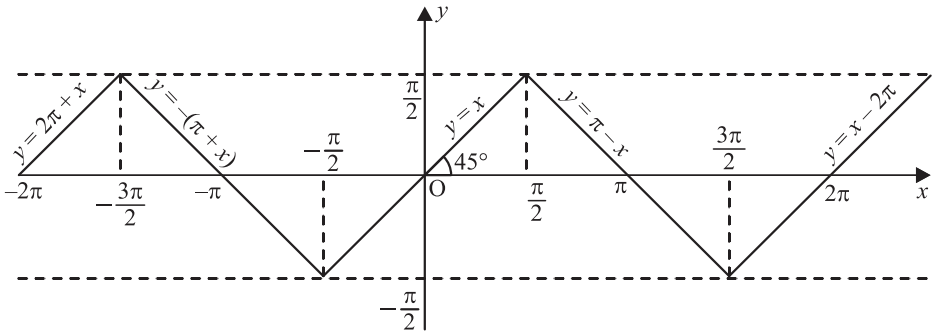
(iv) $y=\cot\left(\cot^{-1}x\right)=x, x\in R, y\in R, y$ is aperiodic.

(v) $y=\text{cosec}\left(\text{cosec}^{-1}x\right)=x, |x|\ge 1, |y|\ge 1, y$ is aperiodic.

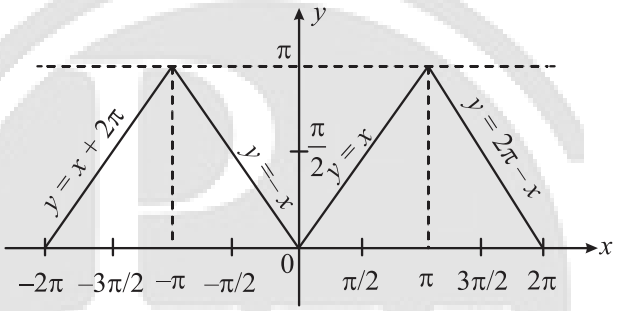
(vi) $y=\sec\left(\sec^{-1}x\right)=x, |x|\ge 1; |y|\ge 1, y$ is aperiodic.

P-2:

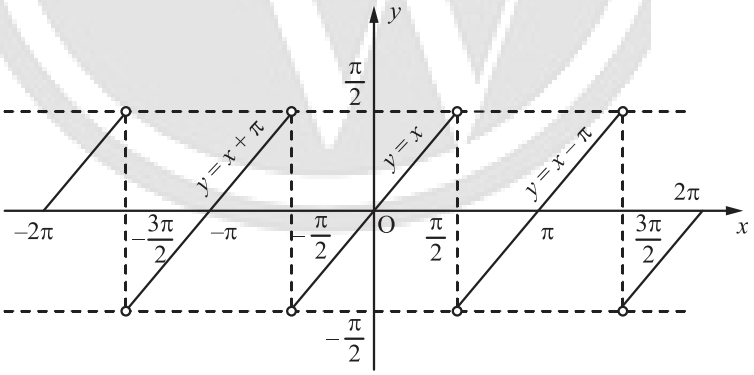
(i) $y=\sin^{-1}(\sin x), x\in R, y\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$. Periodic with period 2π.



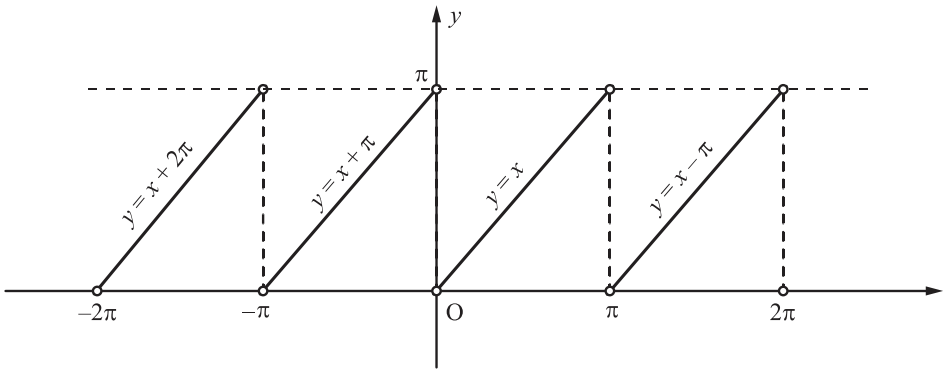
(ii) $y=\cos^{-1}(\cos x), x\in R, y\in[0,\pi]$, periodic with period 2π.



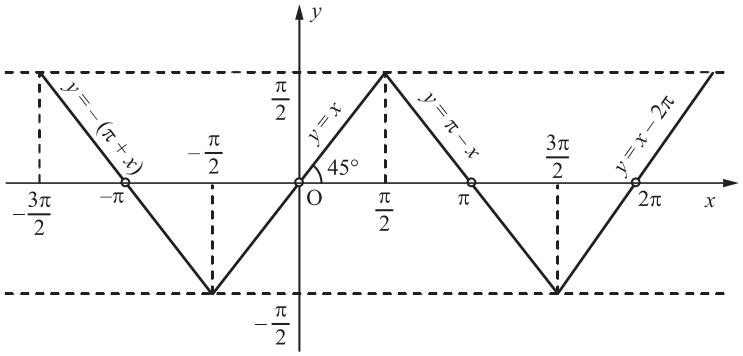
(iii) $y=\tan^{-1}(\tan x), x\in R-\left\{\frac{n\pi}{2}\right\}, n\in I$



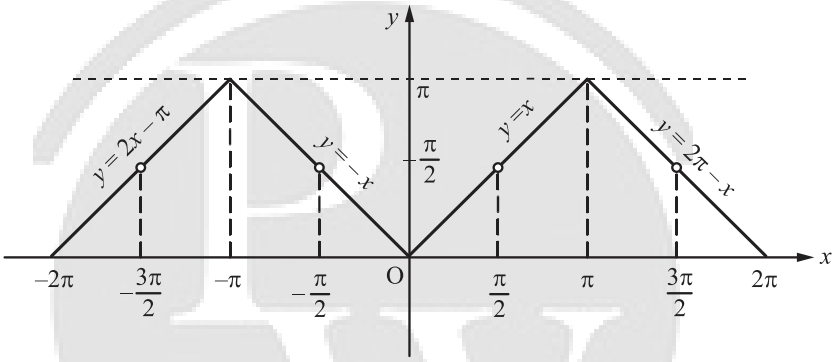
(iv) $y=\cot^{-1}(\cot x), x\in R-\{n\pi\}, n\in I, y\in(0,\pi)$, periodic with period π.



(v) $y=\text{cosec}^{-1}(\text{cosec}x), x\in R-\{n\pi\}, n\in I, y\in \left[-\frac{\pi}{2},0\right)\cup\left(0,\frac{\pi}{2}\right]_y$, is periodic with period 2π.



(vi) $y=\sec^{-1}(\sec x), y$ is periodic with period 2π $x\in R-\left\{(2n-1)\frac{\pi}{2}\right\}, n\in I, y\in\left[0,\frac{\pi}{2}\right)\cup\left(\frac{\pi}{2},\pi\right]$



P-3:

(i) $\text{cosec}^{-1}x=\sin^{-1}\frac{1}{x};\ x\le -1, x\ge 1$

(ii) $\sec^{-1}x=\cos^{-1}\frac{1}{x}; x\le -1, x\ge 1$

(iii) $\cot^{-1}x=\begin{cases}\tan^{-1}\frac{1}{x}; & x>0\\ \pi+\tan^{-1}\frac{1}{x}; & x<0\end{cases}$

P-4:

(i) $\sin^{-1}(-x)=-\sin^{-1}x, -1\le x\le 1$

(ii) $\tan^{-1}(-x)=-\tan^{-1}x, x\in R$

(iii) $\cos^{-1}(-x)=\pi-\cos^{-1}x, -1\le x\le 1$

(iv) $\cot^{-1}(-x)=\pi-\cot^{-1}x, x\in R$

(v) $\sec^{-1}(-x)=\pi-\sec^{-1}x, x\le -1$ or $x\ge 1$

(vi) $\text{cosec}^{-1}(-x)=-\text{cosec}^{-1}x, x\le -1$ or $x\ge 1$

P-5:

(i) $\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2};\ -1\le x\le 1$

(ii) $\tan^{-1}x+\cot^{-1}x=\frac{\pi}{2};\ x\in R$

(iii) $\operatorname{cosec}^{-1}x+\sec^{-1}x=\frac{\pi}{2};\ |x|\ge 1$

P-6:

(i) $\tan^{-1}x+\tan^{-1}y=\begin{cases}\tan^{-1}\frac{x+y}{1-xy} \\ \text{where } x>0, y>0 \text{ and } xy<1 \\ \pi+\tan^{-1}\frac{x+y}{1-xy} \\ \text{where } x>0, y>0 \text{ and } xy>1 \\ \frac{\pi}{2}, \text{ where } x>0, y>0 \text{ and } xy=1\end{cases}$

(ii) $\tan^{-1}x-\tan^{-1}y=\begin{cases}\tan^{-1}\frac{x-y}{1+xy}, xy>-1 \\ \pi+\tan^{-1}\left(\frac{x-y}{1+xy}\right), \\ \text{where } x>0, y>0 \text{ and } xy<-1 \\ -\pi+\tan^{-1}\left(\frac{x-y}{1+xy}\right), \\ \text{where } x<0, y<0 \text{ and } xy>1\end{cases}$

(iii) $\sin^{-1}x+\sin^{-1}y=\sin^{-1}\left(x\sqrt{1-y^2}+y\sqrt{1-x^2}\right)$, where $x\ge 0, y\ge 0 \,\&\, \left(x^2+y^2\right)<1$

Note that: $x^2+y^2<1\Rightarrow 0<\sin^{-1}x+\sin^{-1}y<\frac{\pi}{2}$

(iv) $\sin^{-1}x+\sin^{-1}y=\pi-\sin^{-1}\left(x\sqrt{1-y^2}+y\sqrt{1-x^2}\right)$, where $x>0, y>0$ and $x^2+y^2>1$.

Note that: $x^2+y^2>1\Rightarrow \frac{\pi}{2}<\sin^{-1}x+\sin^{-1}y<\pi$.

(v) $\sin^{-1}x-\sin^{-1}y=\sin^{-1}\left[x\sqrt{1-y^2}-y\sqrt{1-x^2}\right]$ where $x>0, y>0$.

(vi) $\cos^{-1}x+\cos^{-1}y=\cos^{-1}\left(xy-\sqrt{1-x^2}\sqrt{1-y^2}\right); x, y\ge 0$

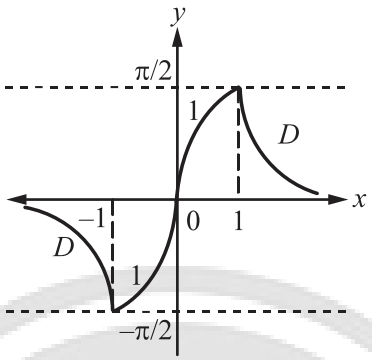
(vii) $\cos^{-1}x-\cos^{-1}y=\begin{cases}\cos^{-1}\left(xy+\sqrt{1-x^2}\sqrt{1-y^2}\right), x>0, y>0 \text{ and } x<y \\ -\cos^{-1}\left(xy+\sqrt{1-x^2}\sqrt{1-y^2}\right), x>0, y>0 \text{ and } x>y\end{cases}$

(viii) $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right]$ if $x>0, y>0, z>0 \,\&\, xy+yz+zx<1$.

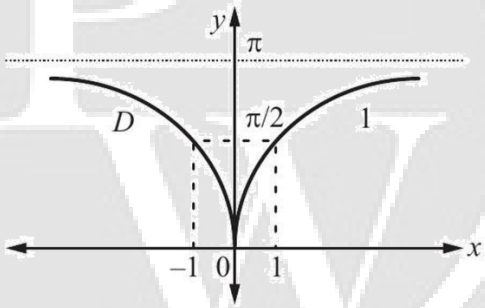
Note that: In the above results $x \,\&\, y$ are taken positive. In case if these are given as negative, we first apply $P-4$ and then use above results.

Simplified Inverse Trigonometric Functions

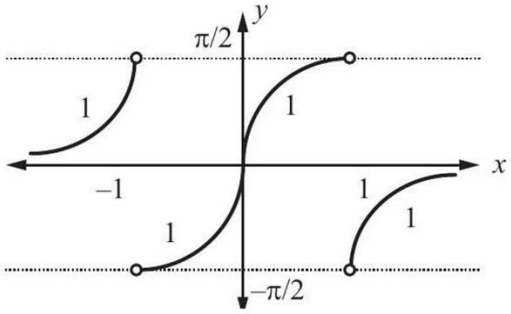
(a) $y = f(x) = \sin^{-1}\left(\dfrac{2x}{1+x^2}\right) = \begin{cases} 2\tan^{-1}x & \text{if} \quad |x| \le 1 \\ \pi - 2\tan^{-1}x & \text{if} \quad x > 1 \\ -\left(\pi + 2\tan^{-1}x\right) & \text{if} \quad x < -1 \end{cases}$



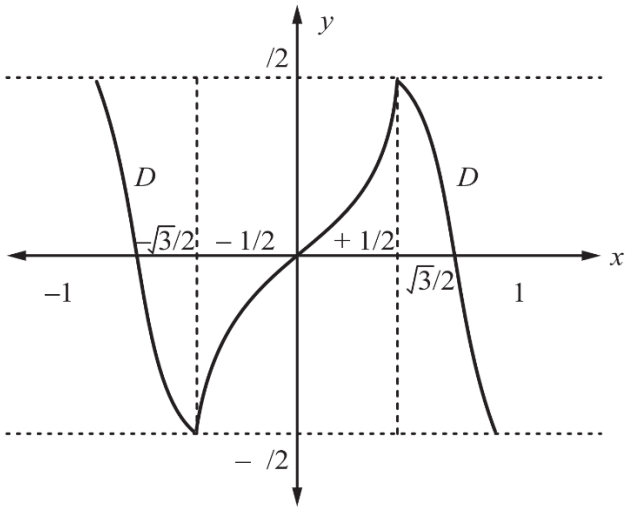
(b) $y = f(x) = \cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right) = \begin{cases} 2\tan^{-1}x \text{ if } x \ge 0 \\ -2\tan^{-1}x \text{ if } x < 0 \end{cases}$



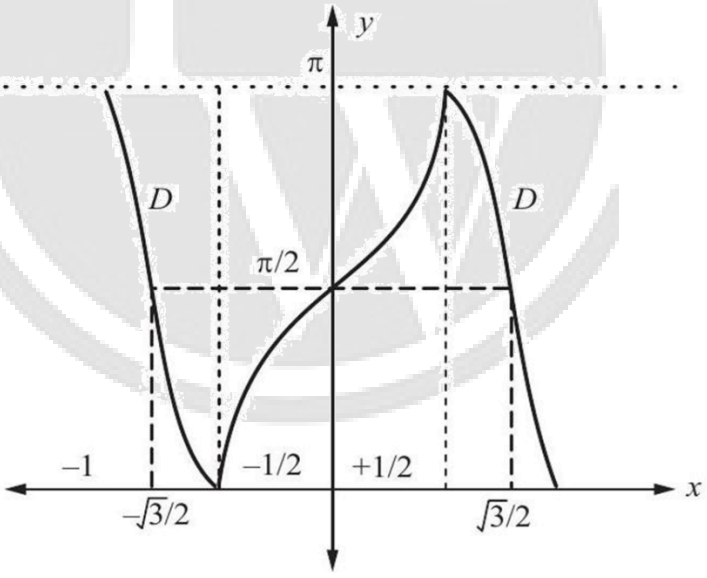
(c) $y = f(x) = \tan^{-1}\dfrac{2x}{1-x^2} = \begin{cases} 2\tan^{-1}x & \text{if} \quad |x| < 1 \\ \pi + 2\tan^{-1}x & \text{if} \quad x < -1 \\ -\left(\pi - 2\tan^{-1}x\right) & \text{if} \quad x > 1 \end{cases}$



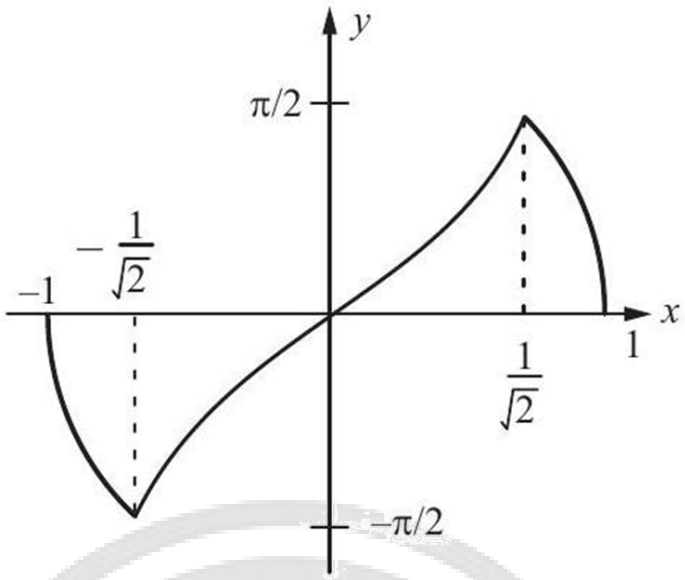
(d) $y = f(x) = \sin^{-1}\left(3x - 4x^3\right) = \begin{cases} -\left(\pi + 3\sin^{-1}x\right) & \text{if} \quad -1 \le x \le -\dfrac{1}{2} \\ 3\sin^{-1}x & \text{if} \quad -\dfrac{1}{2} \le x \le \dfrac{1}{2} \\ \pi - 3\sin^{-1}x & \text{if} \quad \dfrac{1}{2} \le x \le 1 \end{cases}$



(e) $y = f(x) = \cos^{-1}(4x^3 - 3x) = \begin{cases} 3\cos^{-1}x - 2\pi & \text{if} \quad -1 \le x \le -\dfrac{1}{2} \\ 2\pi - 3\cos^{-1}x & \text{if} \quad -\dfrac{1}{2} \le x \le \dfrac{1}{2} \\ 3\cos^{-1}x & \text{if} \quad \dfrac{1}{2} \le x \le 1 \end{cases}$



(f) $\sin^{-1}\left(2x\sqrt{1-x^2}\right) = \begin{cases} -\left(\pi + 2\sin^{-1}x\right) & -1 \le x \le -\dfrac{1}{\sqrt{2}} \\ 2\sin^{-1}x & -\dfrac{1}{\sqrt{2}} \le x \le \dfrac{1}{\sqrt{2}} \\ \pi - 2\sin^{-1}x & \dfrac{1}{\sqrt{2}} \le x \le 1 \end{cases}$





PW Web/App - https://smart.link/7wwosivoicgd4

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