Subject : Mathematics
Class : XI

Kannada

Complex Number – I

IOTA:

$i=\sqrt{-1}, i^2=-1, i^3=-i, i^4=1$

So $i^{4n+1}=i$, $i^{4n+2}=-1$, $i^{4n+3}=-i$, $i^{4n+4}=1$

In other words, $i^n=\begin{cases}(-1)^{n/2}, & \text{if } n \text{ is an even integer}\\ (-1)^{\frac{n-1}{2}}\cdot i, & \text{if } n \text{ is an odd integer}\end{cases}$.

The Complex Number System

$z=a+ib$, then $a-ib$ is called conjugate of z and is denoted by $\bar{z}$

Equality in Complex Number

$z_1=z_2 \Rightarrow \text{Re}(z_1)=\text{Re}(z_2)$ and $\text{Im}(z_1)=\text{Im}(z_2)$.

Conjugate Complex

If $z=a+ib$ then its conjugate complex is obtained by changing the sign of its imaginary part & is denoted by $\bar{z}$ i.e. $\bar{z}=a-ib$.

Note:

(i) $z+\bar{z}=2\,\text{Re}(z)$

(ii) $z-\bar{z}=2i\,\text{Im}(z)$

(iii) $z\bar{z}=a^2+b^2$ which is real

(iv) If z is purely real then $z-\bar{z}=0$

(v) If z is purely imaginary then $z+\bar{z}=0$

Important Properties of Conjugate

(a) $\overline{(\bar{z})}=z$

(b) $\overline{z_1+z_2}=\bar{z}_1+\bar{z}_2$

(c) $\overline{z_1-z_2}=\bar{z}_1-\bar{z}_2$

(d) $\overline{z_1z_2}=\bar{z}_1\cdot\bar{z}_2$

(e) $\overline{\left(\frac{z_1}{z_2}\right)}=\frac{\bar{z}_1}{\bar{z}_2};z_2\neq 0$

(f) If $f(\alpha+i\beta)=x+iy \Rightarrow f(\alpha-i\beta)=x-iy$

Important Properties of Modulus

(a) $|z| \geq 0$

(b) $|z| \geq \text{Re}(z)$

(c) $|z| \geq \text{Im}(z)$

(d) $|z| = |\bar{z}| = |-z| = |-\bar{z}|$

(e) $z\bar{z} = |z|^2$

(f) $|z_1 z_2| = |z_1| \cdot |z_2|$

(g) $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}, z_2 \neq 0$

(h) $|z^n| = |z|^n$

(i) $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2 + 2\text{Re}(z_1\bar{z}_2)$ or $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2 + 2|z_1||z_2|\cos(\theta_1 - \theta_2)$

(j) $|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2\left[|z_1|^2 + |z_2|^2\right]$

(k) $z_1| - |z_2 \leq |z_1 + z_2| \leq |z_1| + |z_2|$

[Triangle Inequality]

(l) $||z_1| - |z_2|| \leq |z_1 - z_2| \leq |z_1| + |z_2|$

[Triangle Inequality]

(m) If $\left|z + \frac{1}{z}\right| = a(a > 0)$, then $\max|z| = \frac{a + \sqrt{a^2 + 4}}{2}$ and $\min|z| = \frac{1}{2}\left(\sqrt{a^2 + 4} - a\right)$.

Important Properties of Amplitude

(a) $\text{amp}(z_1 \cdot z_2) = \text{amp}z_1 + \text{amp}z_2 + 2k\pi; k \in I$.

(b) $\text{amp}\left(\frac{z_1}{z_2}\right) = \text{amp}z_1 - \text{amp}z_2 + 2k\pi; k \in I$.

(c) $\text{amp}(z^n) = n\text{amp}(z) + 2k\pi$, where proper value of k must be chosen so that RHS lies in $(-\pi, \pi]$.

(d) $\log(z) = \log(re^{i\theta}) = \log r + i\theta = \log|z| + i\text{amp}(z)$.

Demoivre's Theorem

Case I: If n is any integer then

(i) $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

(ii) $(\cos\theta_1 + i\sin\theta_1)(\cos\theta_2 + i\sin\theta_2)(\cos\theta_3 + i\sin\theta_3)(\cos\theta_4 + i\sin\theta_4)\ldots(\cos\theta_n + i\sin\theta_n)$

$= \cos(\theta_1 + \theta_2 + \theta_3 + \ldots\theta_n) + i\sin(\theta_1 + \theta_2 + \theta_3 + \ldots\theta_n)$

Case II: If $p, q \in Z$ and $q \neq 0$ then $(\cos\theta + i\sin\theta)^{p/q}$

$= \cos\left(\frac{2k\pi + p\theta}{q}\right) + i\sin\left(\frac{2k\pi + p\theta}{q}\right)$

where $k = 0, 1, 2, 3\ldots q - 1$.

Cube Root of Unity

(i) The cube roots of unity are $1, \frac{-1+i\sqrt{3}}{2}, \frac{-1-i\sqrt{3}}{2}$.

(ii) If ω is one of the imaginary cube roots of unity then $1+\omega+\omega^2=0$. In general $1+\omega^t+\omega^{2t}=0$; where $t \in I$ but is not the multiple of 3.

(c) $a^2+b^2+c^2-ab-bc-ca=\left(a+b\omega+c\omega^2\right)\left(a+b\omega^2+c\omega\right)$ $a^3+b^3=(a+b)\left(a\omega+b\omega^2\right)\left(a\omega^2+b\omega\right)$

$a^3-b^3=(a-b)(a-\omega b)\left(a-\omega^2 b\right)$

$x^2+x+1=(x-\omega)\left(x-\omega^2\right)$

Square root of Complex Number

$$\sqrt{a+ib}=\pm\left\{\frac{\sqrt{|z|+a}}{2}+i\frac{\sqrt{|z|-a}}{2}\right\} \text{ for } b>0$$

and $\pm\left\{\frac{\sqrt{|z|+a}}{2}-i\frac{\sqrt{|z|-a}}{2}\right\}$ for $b<0$ where $|z|=\sqrt{a^2+b^2}$.



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