# KCET PYQ K (2020)

# Kannada

## MATHEMATICS

### Part-A

Answer **all** the **ten** questions: 10 × 1 = 10

**1.** Let * be the binary operation on $N$ given by $a*b$ = L.C.M of $a$ and $b$. Find 5*7.

**2.** Write the range of the function $y = \sec^{-1} x$.

**3.** If a matrix has 5 elements, what are the possible orders it can have?

**4.** Find the values of $x$ for which $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$

**5.** If $y = \tan\left(\sqrt{x}\right)$, find $\frac{dy}{dx}$.

**6.** Find $\int \left(2x^2 + e^x\right) dx$.

**7.** Define Negative of a vector.

**8.** If a line makes angles 90°, 135° and 45° with the $X$, $Y$ and $Z$-axes respectively, find its direction cosines.

**9.** Define optimal solution in Linear programming problem.

**10.** If $P(A) = \frac{3}{5}$ and $P(B) = \frac{1}{5}$ find $P(A \cap B)$ if $A$ and $B$ are independent events.

### Part-B

Answer **any ten** questions: 10 × 2 = 20

**11.** If $f:R \to R$ and $g:R \to R$ are given by $f(x)= \cos x$ and $g(x) = 3x^2$. Find $g \circ f$ and $f \circ g$.

**12.** Prove that $\cot^{-1}(-x) = \pi - \cot^{-1}x, \forall x \in R$.

**13.** Find the value of $\sin^{-1}(\sin \frac{3\pi}{5})$.

**14.** Find the area of the triangle whose vertices are (–2, -3), (3, 2) and (–1, -8) using determinant method.

**15.** Find $\frac{dy}{dx}$, if $\sin^2 x + \cos^2 y = 1$.

**16.** If $y = x^x$, find $\frac{dy}{dx}$.

**17.** Find the interval in which the function f given by $f(x) = x^2 - 4x + 6$ is strictly decreasing.

**18.** Find $\int \cot x \log(\sin x)\, dx$.

**19.** Find $\int x \sec^2 x dx$.

**20.** Find the order and degree (if defined) of the differential equation

$$\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0.$$

**21.** Find the projection of the vector $\vec{a} = \hat{i} + 3\hat{j} + 7\hat{k}$ on the vector $\vec{b} = 7\hat{i} - \hat{j} + 8\hat{k}$.

**22.** Find the area of the parallelogram whose adjacent sides are determined by the vectors $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$.

**23.** Find the equation of the plane with intercepts 2, 3 and 4 on the *X*, *Y* and *Z*-axes respectively.

**24.** A random variable *X* has the following probability distribution. Find the value of *K*..

| $X$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $P(X)$ | 0.1 | $K$ | $2K$ | $2K$ | $K$ |

## PART – C

Answer any ten questions: 10 × 3 =30

**25.** Show that the relation $R$ defined in the set $A$ of all triangles as $R=\{(T_1, T_2): T_1 \text{ is similar to } T_2\}$ is an equivalence relation.

**26.** Prove that $2\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{7}\right)=\tan^{-1}\left(\frac{31}{17}\right)$.

**27.** If $F(x)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$ then show that $F(x)F(y)=F(x+y)$.

**28.** If $x = 2at^2$, $y = at^4$ then find $\frac{dy}{dx}$.

**29.** Verify Mean Value Theorem for the function $f(x) = x^2 - 4x - 3$, $x \in [1, 4]$.

**30.** Use differential to approximate $\sqrt{36.6}$.

**31.** Find $\int \frac{(x-3)e^x}{(x-1)^3}dx$.

**32.** Evaluate: $\int_0^{\pi/2}\cos^2 x\,dx$.

**33.** Find the area of the region bounded by $x^2 = 4y$, $y=2$, $y=4$ and the $y$-axes in the first quadrant.

**34.** Find the equation of a curve passing through the point (–2, 3), given that the slope of the tangent to the curve at any point $(x, y)$ is $\frac{2x}{y^2}$.

**35.** Find a unit vector perpendicular to each of the vector $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$ where $\vec{a}=3\hat{i}+2\hat{j}+2\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}-2\hat{k}$.

**36.** Find $x$ such that the four points $A(3, 2, 1)$, $B(4, x, 5)$, $C(4, 2, -2)$ and $D(6, 5, -1)$ are coplanar.

**37.** Find the equation of the plane through the intersection of the planes $3x-y+2z-4=0$ and $x + y + z - 2 = 0$ and the point (2, 2, 1).

**38.** A man is known to speak truth 3 out of 4 times. He throws a dice and reports that it is a six. Find the probability that it is actually a six.

## Part-D

Answer any six questions: 6 × 5 = 30

**39.** Show that the function $f:R\to R$ given by $f(x)=4x+3$ is invertible. Find the inverse of $f$.

**40.** If $A = \begin{bmatrix}1 & 2 & -3\\ 5 & 0 & 2\\ 1 & -1 & 1\end{bmatrix}$, $B = \begin{bmatrix}3 & -1 & 2\\ 4 & 2 & 5\\ 2 & 0 & 3\end{bmatrix}$ and $C = \begin{bmatrix}4 & 1 & 2\\ 0 & 3 & 2\\ 1 & -2 & 3\end{bmatrix}$ then compute $(A+B)$ and $(B-C)$. Also verify that $A+(B-C) = (A+B) - C$.

**41.** Solve the system of linear equations by matrix method $2x+3y+3z = 5$ $x - 2y + z = -4$ $3x-y-2z = 3$

**42.** If $y = (\tan^{-1} x)^2$, show that $(x^2+1)^2 y_2 + 2x(x^2+1)y_1 = 2$.

**43.** Sand is pouring from a pipe at the rate of 12 cm³/s. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4 cm?

**44.** Find the integral of $\frac{1}{x^2+a^2}$ w.r.t. $x$ and hence evaluate $\int \frac{1}{x^2+2x+2}dx$.

**45.** Using the method of integration, find the area of the smaller region bounded by the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ and the line $\frac{x}{3}+\frac{y}{2}=1$.

**46.** Find the general solution of the differential equation $x\frac{dy}{dx}+2y=x^2 \log x$.

**47.** Derive the equation of a line in space passing through a given point and parallel to a given vector in both Vector and Cartesian form.

**48.** A person buys a lottery ticket in 50 lotteries, in each of which his chance of winning a prize is $\frac{1}{100}$. What is the probability that he will win a prize

(a) exactly once

(b) atleast once?

**Part-E**

Answer any one of the following questions $1 \times 10 = 10$

**49.** **(**a) Prove that

$$\int_{-a}^{a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx, & \text{iff }(x)\text{is an even function} \\ 0, & \text{iff }(x)\text{is an odd function} \end{cases}$$

and hence evaluate $\int_{-1}^{1}\sin^5 x\cos^4 x dx$. (6)

(b) Show that

$$\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)(4-x)^2. \text{ (4)}$$

**50.** **(**a) Maximise $z$ = 4x + $y$ Subject to constraints: $x + y \le 50$ $3x + y \le 90$ $x \ge 0$ $y \ge 0$ by graphical method. (6)

**(**b) Find the value of $K$, if $f(x)=\begin{cases} Kx+1, & \text{if} x\le \pi \\ \cos x, & \text{if} x>\pi \end{cases}$ is continuous at $x = \pi$. (4)

# CETKCET PYQ K (2020)

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# Mathematics

## Hint & Solution

**1.** Given operation:

$a*b=$ L.C.M of $a$ and $b$

For $5*7$:

$5*7=$ L.C.M of 5 and 7

Since 5 and 7 are prime numbers, their L.C.M is their product:

$5\times 7=35$

**2.** The principal value of branch (range) of their Inverse secant function is : $[0,\pi]-\left\{\frac{\pi}{2}\right\}$

**3.** The order for matrix with $n$ elements is given by $m\times n$, where $m\times n=5$. Since 5 is a prime number, its only positive integral factors are 1 and 5.

Therefore, the possible orders are:

* $1\times 5$
* $5\times 1$

**4.** Evaluate the determinants on both sides:

$(x\cdot x)-(2\cdot 18)=(6\cdot 6)-(2\cdot 18)$

$x^2-36=36-36$

$x^2-36=0$

$x^2=36$

$x=\pm\sqrt{36}=\pm 6$

$x=\pm 6$

**5.** Using the chain rule of differentiation:

$$\frac{dy}{dx}=\frac{d}{dx}[\tan(\sqrt{x})]$$

$$\frac{dy}{dx}=\sec^2(\sqrt{x})\cdot\frac{d}{dx}(\sqrt{x})$$

$$\frac{dy}{dx}=\sec^2(\sqrt{x})\cdot\frac{1}{2\sqrt{x}}$$

$$\frac{dy}{dx}=\frac{\sec^2(\sqrt{x})}{2\sqrt{x}}$$

**6.** Integrate term by $m$: $\int 2x^2dx+\int e^x dx$ $

$$=2\left(\frac{x^3}{3}\right)+e^x+C$$

$=\frac{2}{3}x^3+e^x+C$ Where C is constant of integration.

**7.** A vector having the same magnitude as that of given vector(say, $\vec{a}$ ), but direction opposite to it, is called the negative of the given vector and is denoted as $-\vec{a}$ .

**8.** Let $\alpha=90^\circ,\beta=135^\circ$, and $\gamma=45^\circ$. The direction cosines $(l,m,n)$ are given by:

- $l=\cos 90^\circ=0$
- $m=\cos 135^\circ=\cos(180^\circ-45^\circ)=-\cos 45^\circ=-\frac{1}{\sqrt{2}}$
- $n=\cos 45^\circ=\frac{1}{\sqrt{2}}$

The direction cosines are $\left(0,-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$.

**9.** Any feasible solution in a Linear Programming Problem that optimizes (maximizes or minimizes) the objective function is called an optimal solution.

**10.** For two independent events $A$ and $B$:

$P(A \cap B) = P(A) \cdot P(B)$

Substitute the given values:

$P(A \cap B) = \frac{3}{5} \times \frac{1}{5} = \frac{3}{25}$ (or 0.12)

**11.** 1. Finding $g^\circ f$:

$(g^\circ f)(x) = g(f(x)) = g(\cos x)$

Since $g(x) = 3x^2$, substitute $x = \cos x$:

$(g^\circ f)(x) = 3(\cos x)^2 = 3\cos^2 x$

2. Finding $f^\circ g : (f^\circ g)(x) = f(g(x)) = f(3x^2)$

Since $f(x) = \cos x$, substitute $x = 3x^2$:

$(f^\circ g)(x) = \cos(3x^2)$

**12.** Let $\cot^{-1}(-x) = \theta$

Taking cot on both sides:

$-x = \cot\theta \Rightarrow x = -\cot\theta$

Using the trigonometric identity $\cot(\pi - \theta) = -\cot\theta$:

$x = \cot(\pi - \theta)$

Taking $\cot^{-1}$ on both sides:

$\cot^{-1} x = \pi - \theta$

Rearranging the terms:

$\theta = \pi - \cot^{-1} x$

Substitute $\theta = \cot^{-1}(-x)$ back:

$\cot^{-1}(-x) = \pi - \cot^{-1} x$ (Hence Proved)

**13.** The principal value branch of $\sin^{-1} x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Since $\frac{3\pi}{5} \notin \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, we simplify it:

$\frac{3\pi}{5} = \pi - \frac{2\pi}{5}$

Using the identity $\sin(\pi - \theta) = \sin\theta$:

$\sin\left(\frac{3\pi}{5}\right) = \sin\left(\pi - \frac{2\pi}{5}\right) = \sin\left(\frac{2\pi}{5}\right)$

Now, evaluate the inverse:

$\sin^{-1}\left(\sin\frac{3\pi}{5}\right) = \sin^{-1}\left(\sin\frac{2\pi}{5}\right) = \frac{2\pi}{5}$

(Since $\frac{2\pi}{5} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$)

Answer: $\frac{2\pi}{5}$

**14.** The formula for the area of a triangle using determinants is:

$$\text{Area} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Substituting the vertices $(-2,-3),(3,2),(-1,-8)$:

$$\text{Area} = \frac{1}{2}\begin{vmatrix} -2 & -3 & 1 \\ 3 & 2 & 1 \\ -1 & -8 & 1 \end{vmatrix}$$

Expanding along the first row $(R_1)$:

Area =

$\frac{1}{2}[-2(2-(-8)) - (-3)(3-(-1)) + 1(-24-(-2))]$

$= \frac{1}{2}[-2(10) + 3(4) + 1(-22)]$

$= \frac{1}{2}[-20 + 12 - 22]$

$= \frac{1}{2}|-30| = 15$

**15.** Differentiating both sides with respect to $x$:

$\frac{d}{dx}(\sin^2 x) + \frac{d}{dx}(\cos^2 y) = \frac{d}{dx}(1)$

Using the chain rule:

$2\sin x \cdot \frac{d}{dx}(\sin x) + 2\cos y \cdot \frac{d}{dx}(\cos y) = 0$

$2\sin x \cos x + 2\cos y\left(-\sin y \frac{dy}{dx}\right) = 0$

Using double angle identities $2\sin\theta\cos\theta = \sin 2\theta$

$\sin 2x - \sin 2y \frac{dy}{dx} = 0$

$$\sin 2y \frac{dy}{dx} = \sin 2x$$

$$\frac{dy}{dx} = \frac{\sin 2x}{\sin 2y}$$

**16.** Taking natural logarithm ( log ) on both sides:

$$\log y = \log(x^x)$$

$$\log y = x \log x$$

Differentiating both sides with respect to $x$ using the product rule on the right-hand side:

$$\frac{1}{y}\frac{dy}{dx} = x \cdot \frac{d}{dx}(\log x) + \log x \cdot \frac{d}{dx}(x)$$

$$\frac{1}{y}\frac{dy}{dx} = x\left(\frac{1}{x}\right) + \log x(1)$$

$$\frac{1}{y}\frac{dy}{dx} = 1 + \log x$$

$$\frac{dy}{dx} = y(1 + \log x)$$

Substitute $y = x^x$ back:

$$\frac{dy}{dx} = x^x(1 + \log x)$$

**17.** First, find the derivative $f'(x)$:

$$f'(x) = \frac{d}{dx}(x^2 - 4x + 6) = 2x - 4$$

For a function to be strictly decreasing, $f'(x) < 0$:

$$2x - 4 < 0$$

$$2x < 4 \Rightarrow x < 2$$

Thus, $x \in (-\infty, 2)$.

The function is strictly decreasing in the interval $(-\infty, 2)$.

**18.** Let $u = \log(\sin x)$

Differentiating with respect to $x$:

$$du = \frac{1}{\sin x} \cdot \cos x dx = \cot x dx$$

Substitute $u$ and $du$ into the integral:

$$\int \cot x \log(\sin x) dx = \int u du$$

$$= \frac{u^2}{2} + C$$

Substitute $u = \log(\sin x)$ back:

$$= \frac{[\log(\sin x)]^2}{2} + C$$

**19.** Using integration by parts formula:

$$\int uv dx = u\int v dx - \int (u' \int v dx) dx$$

Following ILATE rule, let $u = x$ (Algebraic) and $v = \sec^2 x$ (Trigonometric).

* $u' = 1$
* $\int v dx = \int \sec^2 x dx = \tan x$

Applying the formula:

$$\int x \sec^2 x dx = x(\tan x) - \int (1 \cdot \tan x) dx$$

$$= x \tan x - \log|\sec x| + C$$

or $\quad x \tan x + \log|\cos x| + C$

**20.**
1. Order : The highest or aeroderivative present in the equations $\frac{d^2y}{dx^2}$, so the order = 2.
2. Degree: The equation can not be written as polynomial in its $\sin\left(\frac{dy}{dx}\right)$ term. Therefore, the Degree is not defined.

**21.** The formula for the projection of $\vec{a}$ on $\vec{b}$ is:

$$\text{Projection} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$$

1. Calculate dot product $\vec{a}.\vec{b}$

$$\vec{a} \cdot \vec{b} = (1)(7) + (3)(-1) + (7)(8)$$

$$= 7 - 3 + 56 = 60$$

2. Calculate magnitude $|\vec{b}|$:

$$|\vec{b}| = \sqrt{7^2 + (-1)^2 + 8^2}$$

$$= \sqrt{49 + 1 + 64} = \sqrt{114}$$

3. Compute the final projection:

$$\text{Projection} = \frac{60}{\sqrt{114}}$$

**22.** The area of a parallelogram is given by $|\vec{a}\times\vec{b}|$.

1. Find cross product $\vec{a}\times\vec{b}$ :

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & 3\\ 2 & -7 & 1\end{vmatrix}$$

$$=\hat{i}((-1)(1)-(3)(-7))-\hat{j}((1)(1)-(3)(2))$$
$$+\hat{k}((1)(-7)-(-1)(2))$$
$$=\hat{i}(-1+21)-\hat{j}(1-6)+\hat{k}(-7+2)$$
$$=20\hat{i}+5\hat{j}-5\hat{k}$$

2. Calculate magnitude :

$$|\vec{a}\times\vec{b}|=\sqrt{20^2+5^2+(-5)^2}$$
$$=\sqrt{400+25+25}=\sqrt{450}$$
$$=\sqrt{225\times 2}=15\sqrt{2}\ \text{sq unit.}$$

**23.** The intercept form of a plane equation is:

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

Where $a=2, b=3$, and $c=4$ are the intercepts (p. 3).

Substitute the values:

$$\frac{x}{2}+\frac{y}{3}+\frac{z}{4}=1$$

Taking the LCM of denominators (LCM of 2, 3, 4 is 12):

$$\frac{6x+4y+3z}{12}=1$$
$$6x+4y+3z=12$$

**24.** The sum of all probabilities in a probability distribution must equal 1:

$$\sum P(X)=1$$
$$0.1+K+2K+2K+K=1$$
$$0.1+6K=1$$
$$6K=1-0.1$$
$$6K=0.9$$
$$K=\frac{0.9}{6}=\frac{9}{60}=0.15\ \left(\text{or } \frac{3}{20}\right)$$

**25.** To prove $R$ is an equivalence relation, we must show it is Reflexive, Symmetric, and Transitive.

1. Reflexive:

Every triangle $T_1$ is similar to itself $(T_1\sim T_1)$

$\Rightarrow (T_1,T_1)\in R,\quad \forall T_1\in A$.

Therefore, $R$ is reflexive.

2. Symmetric:

Let $(T_1,T_2)\in R$.

$\Rightarrow T_1$ is similar to $T_2$.

$\Rightarrow T_2$ is similar to $T_1$.

$\Rightarrow (T_2,T_1)\in R$.

Therefore, $R$ is symmetric.

3. Transitive:

Let $(T_1,T_2)\in R$ and $(T_2,T_3)\in R$.

$\Rightarrow T_1$ is similar to $T_2$, and $T_2$ is similar to $T_3$

$\Rightarrow T_1$ is similar to $T_3$.

$\Rightarrow (T_1,T_3)\in R$.

Therefore, $R$ is transitive.

Since $R$ is reflexive, symmetric, and transitive, it is an equivalence relation.

**26.** Using the identity: $2\tan^{-1}x=\tan^{-1}\left(\frac{2x}{1-x^2}\right)$

1. Simplify the first term:

$$2\tan^{-1}\left(\frac{1}{2}\right)=\tan^{-1}\left(\frac{2\cdot\frac{1}{2}}{1-(\frac{1}{2})^2}\right)$$

$$=\tan^{-1}\left(\frac{1}{1-\frac{1}{4}}\right)$$

$$=\tan^{-1}\left(\frac{1}{\frac{3}{4}}\right)=\tan^{-1}\left(\frac{4}{3}\right)$$

2. Substitute back into LHS:

$$\text{LHS}=\tan^{-1}\left(\frac{4}{3}\right)+\tan^{-1}\left(\frac{1}{7}\right)$$

3. Using identity:

$$\tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right)$$

$$\text{LHS}=\tan^{-1}\left(\frac{\frac{4}{3}+\frac{1}{7}}{1-\frac{4}{3}\cdot\frac{1}{7}}\right)=\tan^{-1}\left(\frac{\frac{28+3}{21}}{1-\frac{4}{21}}\right)$$

$$=\tan^{-1}\left(\frac{\frac{31}{21}}{\frac{17}{21}}\right)=\tan^{-1}\left(\frac{31}{17}\right)=\text{RHS}$$

(Hence proved)

**27.** Given $F(x)$, then $F(y)=\begin{bmatrix}\cos y & -\sin y & 0\\ \sin y & \cos y & 0\\ 0 & 0 & 1\end{bmatrix}$.

Now, perform matrix multiplication for $F(x)F(y)$:

$$F(x)F(y)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos y & -\sin y & 0\\ \sin y & \cos y & 0\\ 0 & 0 & 1\end{bmatrix}$$

Multiplying row 1 by column 1, 2, 3:

$R_1C_1=\cos x\cos y-\sin x\sin y+0=\cos(x+y)$

$R_1C_2=-\cos x\sin y-\sin x\cos y+0$

$=-(\sin x\cos y+\cos x\sin y)=-\sin(x+y)$

$R_1C_3 = 0 + 0 + 0 = 0$

Multiplying row 2 by column 1, 2, 3:

$R_2C_1=\sin x\cos y+\cos x\sin y+0=\sin(x+y)$

$R_2C_2=-\sin x\sin y+\cos x\cos y+0$

$=\cos x\cos y-\sin x\sin y=\cos(x+y)$

$R_2C_3 = 0 + 0 + 0 = 0$

Multiplying row 3 by column 1, 2, 3:

$R_3C_1 = 0 + 0 + 0 = 0$

$R_3C_2 = 0 + 0 + 0 = 0$

$R_3C_3 = 0 + 0 + 1 = 1$

Putting it back together:

$$F(x)F(y)=\begin{bmatrix}\cos(x+y) & -\sin(x+y) & 0\\ \sin(x+y) & \cos(x+y) & 0\\ 0 & 0 & 1\end{bmatrix}$$

$=F(x+y)$

(Hence Proved)

**28.** This is a parametric equation problem. Differentiate both $x$ and $y$ with respect to the parameter $t$.

1. Differentiate $x$:

   $$\frac{dx}{dt}=\frac{d}{dt}(2at^2)=2a(2t)=4at$$

2. Differentiate $y$:

   $$\frac{dy}{dt}=\frac{d}{dt}(at^4)=a(4t^3)=4at^3$$

3. Find $\frac{dy}{dx}$

   $$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{4at^3}{4at}=t^2$$ Answer: $\frac{dy}{dx}=t^2$

**29.** 1. Check Conditions:

   1. $f(x)$ is a polynomial function, so it is continuous on [1, 4].
   2. $f'(x)=2x-4$, which exists for all values in $(1,4)$, so it is differentiable on $(1,4)$.

2. Apply Mean Value Theorem Formula:
   There must exist at least one $c\in(1,4)$ such that: $f'(c)=\frac{f(b)-f(a)}{b-a}$

3. Calculate values for $a$ = 1 and $b$ = 4.

   1. $f(1)=(1)^2-4(1)-3=1-4-3=-6$
   2. $f(4)=(4)^2-4(4)-3$
      $=16-16-3=-3$

4. Solve for $c$:

   $$2c-4=\frac{-3-(-6)}{4-1}$$

   $$2c-4=\frac{3}{3}$$

   $2c-4=1\Rightarrow 2c=5\Rightarrow c=2.5$

   Since $c=2.5\in(1,4)$, Mean Value Theorem is verified.

**30.** Let $y=\sqrt{x}$. Take $x=36$ and $\Delta x=0.6$.

1. Calculate $y$:

$$y=\sqrt{36}=6$$

2. Find the derivative $\frac{dy}{dx}$:

$$\frac{dy}{dx}=\frac{1}{2\sqrt{x}}$$

3. Find by :

$$dy\approx\left(\frac{dy}{dx}\right)\Delta x=\frac{1}{2\sqrt{36}}\cdot(0.6)$$

$$=\frac{1}{2\cdot 6}\cdot 0.6=\frac{0.6}{12}=0.05$$

4. Approximate the value:

$$\sqrt{36.6}=y+dy=6+0.05=6.05$$

**31.** Rewrite the numerator to match the denominator structure:

$$\int\frac{(x-1-2)e^x}{(x-1)^3}dx=\int e^x\left[\frac{x-1}{(x-1)^3}-\frac{2}{(x-1)^3}\right]dx$$

$$=\int e^x\left[\frac{1}{(x-1)^2}+\left(\frac{-2}{(x-1)^3}\right)\right]dx$$

This is in the standard form $\int e^x[f(x)+f'(x)]dx=e^x f(x)+C$.

Let $f(x)=\frac{1}{(x-1)^2}\Rightarrow f'(x)=\frac{-2}{(x-1)^3}$.

Therefore, the value of the integral is:

$$=\frac{e^x}{(x-1)^2}+C$$

**32.** Let I = $=\int_0^{\pi/2}\cos^2 x dx$ --- (Equation 1)

Using the property $\int_0^a f(x)dx=\int_0^a f(a-x)dx$:

$$I=\int_0^{\pi/2}\cos^2\left(\frac{\pi}{2}-x\right)dx$$

$$I=\int_0^{\pi/2}\sin^2 x dx \quad \text{--- (Equation 2)}$$

Add Equation 1 and Equation 2:

$$2I=\int_0^{\pi/2}(\cos^2 x+\sin^2 x)dx$$

Since $\cos^2 x+\sin^2 x=1$:

$$2I=\int_0^{\pi/2}1dx$$

$$2I=[x]_0^{\pi/2}=\frac{\pi}{2}-0$$

$$I=\frac{\pi}{4}$$

**33.** The equation $x^2=4y$ represents an upward-opening parabola. We need to find the area bounded along the $y$-axis from $y=2$ to $y=4$ in the first quadrant ( $x\ge 0$ ).

Express $x$ in terms of $y$:

$$x=\sqrt{4y}=2\sqrt{y}$$

The area $A$ is given by the integral:

$$A=\int_2^4 xdy=\int_2^4 2\sqrt{y}dy$$

$$A=2\left[\frac{y^{3/2}}{3/2}\right]_2^4=\frac{4}{3}[y^{3/2}]_2^4$$

Substitute the upper and lower limits:

$$A=\frac{4}{3}\left(4^{3/2}-2^{3/2}\right)$$

- $4^{3/2}=(\sqrt{4})^3=2^3=8$
- $2^{3/2}=(\sqrt{2})^3=2\sqrt{2}$

$$A=\frac{4}{3}\left(8-2\sqrt{2}\right)=\frac{8}{3}(4-\sqrt{2})\text{ sq. units}$$

**34.** The slope of the tangent to a curve at any point $(x,y)$ is given by $\frac{dy}{dx}$.

$$\frac{dy}{dx}=\frac{2x}{y^2}$$

Using slope of the tangent to a curve at any point (x, y) is given by $\frac{dy}{dx}$.

$$y^2dy=2xdx$$

Integrate both sides:

$$\int y^2 dy = \int 2x dx$$

$$\frac{y^3}{3} = \frac{2x^2}{2} + C \Rightarrow \frac{y^3}{3} = x^2 + C$$

Since the curve passes through the point $(-2,3)$, substitute $x=-2$ and $y=3$ to find the integration constant $C$:

$$\frac{3^3}{3} = (-2)^2 + C$$

$$9 = 4 + C \Rightarrow C = 5$$

Substitute $C=5$ back into the general equation:

$$\frac{y^3}{3} = x^2 + 5 \Rightarrow y^3 = 3x^2 + 15$$

**35.** 1. Find $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$:

$$\vec{a}+\vec{b} = (3+1)\hat{i} + (2+2)\hat{j} + (2-2)\hat{k}$$
$$= 4\hat{i} + 4\hat{j} + 0\hat{k}$$
$$\vec{a}-\vec{b} = (3-1)\hat{i} + (2-2)\hat{j} + (2-(-2))\hat{k}$$
$$= 2\hat{i} + 0\hat{j} + 4\hat{k}$$

2. Find the cross product: $\vec{c} = (\vec{a}+\vec{b}) \times (\vec{a}-\ \vec{b})$

$$\vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 4 & 0 \\ 2 & 0 & 4 \end{vmatrix}$$

$$\vec{c} = \hat{i}(16-0) - \hat{j}(16-0) + \hat{k}(0-8)$$
$$= 16\hat{i} - 16\hat{j} - 8\hat{k}$$

3. Find the magnitude $|\vec{c}|$:

$$|\vec{c}| = \sqrt{16^2 + (-16)^2 + (-8)^2}$$
$$= \sqrt{256+256+64} = \sqrt{576} = 24$$

4. Calculate the unit vector $\hat{n}$:

$$\hat{n} = \pm\frac{\vec{c}}{|\vec{c}|} = \pm\frac{16\hat{i} - 16\hat{j} - 8\hat{k}}{24}$$ $

Divide each term by 8:

$$\hat{n} = \pm\left(\frac{2}{3}\hat{i} - \frac{2}{3}\hat{j} - \frac{1}{3}\hat{k}\right)$$

**36.** Four points $A, B, C, D$ are coplanar if the vectors $\overrightarrow{AB}, \overrightarrow{AC}$, and $\overrightarrow{AD}$ are coplanar. This means their scalar triple product is zero: $[\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}] = 0$.

1. Find the components of the vectors:
   1. $\overrightarrow{AB} = (4-3)\hat{i} + (x-2)\hat{j} + (5-1)\hat{k}$
      $= 1\hat{i} + (x-2)\hat{j} + 4\hat{k}$
   2. $\overrightarrow{AC} = (4-3)\hat{i} + (2-2)\hat{j} + (-2-1)\hat{k}$
      $= 1\hat{i} + 0\hat{j} - 3\hat{k}$
   3. $\overrightarrow{AD} = (6-3)\hat{i} + (5-2)\hat{j} + (-1-1)\hat{k}$
      $= 3\hat{i} + 3\hat{j} - 2\hat{k}$
2. Set the determinant to zero:

$$\begin{vmatrix} 1 & x-2 & 4 \\ 1 & 0 & -3 \\ 3 & 3 & -2 \end{vmatrix} = 0$$

3. Expand along the second row ($R_2$) for easier calculation:

$$-1\cdot[(x-2)(-2)-4(3)] + 0 - (-3)\cdot$$
$$[1(3)-(x-2)(3)] = 0$$
$$-1\cdot[-2x+4-12] + 3\cdot[3-3x+6] = 0$$
$$(2x+8) + 3(9-3x) = 0$$
$$2x+8+27-9x = 0$$
$$-7x+35 = 0 \Rightarrow 7x = 35 \Rightarrow x = 5$$

**37.** The equation of a family of planes passing through the intersection of two given planes is:

$$(3x-y+2z-4) + \lambda(x+y+z-2) = 0$$

Since this plane passes through the point $(2,2,1)$, substitute $x=2, y=2,$ and $z=1$ into the equation to determine $\lambda$:

$$[3(2)-2+2(1)-4] + \lambda(2+2+1-2) = 0$$
$$[6-2+2-4] + \lambda(3) = 0$$
$$2 + 3\lambda = 0 \Rightarrow \lambda = -\frac{2}{3}$$

Substitute $\lambda = -\frac{2}{3}$ back into the equation:

$$(3x-y+2z-4) - \frac{2}{3}(x+y+z-2) = 0$$

Multiply the entire equation by 3 to remove the fraction:

$$3(3x-y+2z-4)-2(x+y+z-2)=0$$
$$9x-3y+6z-12-2x-2y-2z+4=0$$
$$7x-5y+4z-8=0$$

**38.** This is a problem based on Bayes' Theorem:
$E_1$ : Event that a six actually occurs.
$E_2$: Event that a six does not occur.
$A$ : Event that the man reports a six.

1. Find prior probabilities:
   1. $P(E_1)=\frac{1}{6}$ (Probability of getting a six)
   2. $P(E_2)=1-\frac{1}{6}=\frac{5}{6}$ (Probability of not getting a six)
2. Find conditional probabilities:
   1. $P(A|E_1)$ : Probability that he reports a six when it is actually a six (he speaks the truth) $=\frac{3}{4}$
   2. $P(A|E_2)$: Probability that he reports a six when it is not a six (he tells a lie) $=1-\frac{3}{4}=\frac{1}{4}$
3. Apply Bayes' Theorem to find $P(E_1|A)$:

$$P(E_1|A)=\frac{P(E_1)\cdot P(A|E_1)}{P(E_1)\cdot P(A|E_1)+P(E_2)\cdot P(A|E_2)}$$

$$P(E_1|A)=\frac{\frac{1}{6}\times\frac{3}{4}}{\left(\frac{1}{6}\times\frac{3}{4}\right)+\left(\frac{5}{6}\times\frac{1}{4}\right)}$$

$$P(E_1|A)=\frac{\frac{3}{24}}{\frac{3}{24}+\frac{5}{24}}=\frac{3}{3+5}=\frac{3}{8}$$

**39.** A function is invertible if it is both one-to-one (injective) and onto (surjective) (p. 13).

1. One-to-one (Injectivity):
   Let $x_1, x_2\in\mathbf{R}$ such that $f(x_1)=f(x_2)$
   $$4x_1+3=4x_2+3$$
   $$4x_1=4x_2\Rightarrow x_1=x_2$$
   Since $f(x_1)=f(x_2)\Rightarrow x_1=x_2$, the function $f$ is one-to-one.
2. Onto (Surjectivity):
   Let $y\in\mathbf{R}$ (co-domain). Set $y=f(x)$
   $$y=4x+3\Rightarrow 4x=y-3\Rightarrow x=\frac{y-3}{4}$$
   For every real number $y$, there exists a pre-image $x=\frac{y-3}{4}\in\mathbf{R}$.
   $$f(x)=f\left(\frac{y-3}{4}\right)=4\left(\frac{y-3}{4}\right)+3$$
   $$=y-3+3=y$$
   Since every element in the co-domain has a pre-image, the function f is onto.
   Since f is both one-to-one and onto, it is invertible.
3. Inverse Function:
   $$f^{-1}(y)=\frac{y-3}{4}\Rightarrow f^{-1}(x)=\frac{x-3}{4}$$

**40.**
1. Compute $(A+B)$:
   $$A+B=\begin{bmatrix}1+3 & 2+(-1) & -3+2\\ 5+4 & 0+2 & 2+5\\ 1+2 & -1+0 & 1+3\end{bmatrix}$$
   $$=\begin{bmatrix}4 & 1 & -1\\ 9 & 2 & 7\\ 3 & -1 & 4\end{bmatrix}$$
2. Compute (B –C) :
   $$B-C=\begin{bmatrix}3-4 & -1-1 & 2-2\\ 4-0 & 2-3 & 5-2\\ 2-1 & 0-(-2) & 3-3\end{bmatrix}$$
   $$=\begin{bmatrix}-1 & -2 & 0\\ 4 & -1 & 3\\ 1 & 2 & 0\end{bmatrix}$$

3. Verify LHS: $A+(B-C)$

$$\text{LHS}=\begin{bmatrix}1&2&-3\\5&0&2\\1&-1&1\end{bmatrix}+\begin{bmatrix}-1&-2&0\\4&-1&3\\1&2&0\end{bmatrix}$$

$$=\begin{bmatrix}0&0&-3\\9&-1&5\\2&1&1\end{bmatrix}$$

4. Verify RHS (A + B)–C:

$$\text{RHS}=\begin{bmatrix}4&1&-1\\9&2&7\\3&-1&4\end{bmatrix}-\begin{bmatrix}4&1&2\\0&3&2\\1&-2&3\end{bmatrix}$$

$$=\begin{bmatrix}0&0&-3\\9&-1&5\\2&1&1\end{bmatrix}$$

Since $\text{LHS}=\text{RHS}$, the relation is verified.

**41.** Write the system as $AX=B$:

$$A=\begin{bmatrix}2&3&3\\1&-2&1\\3&-1&-2\end{bmatrix},\quad X=\begin{bmatrix}x\\y\\z\end{bmatrix},\quad B=\begin{bmatrix}5\\-4\\3\end{bmatrix}$$

1. Find determinant $|A|$:

$$|A|=2(4-(-1))-3(-2-3)+3(-1-(-6))$$
$$|A|=2(5)-3(-5)+3(5)$$
$$=10+15+15=40$$

Since $|A|\neq 0$, a unique solution exists given by $X=A^{-1}B$.

2. Find the Matrix of Cofactors:

1. $A_{11}=5,\quad A_{12}=5,\quad A_{13}=5$
2. $A_{21}=3,\quad A_{22}=-13,\quad A_{23}=11$
3. $A_{31}=9,\quad A_{32}=1,\quad A_{33}=-7$

$$\text{Cofactor Matrix}=\begin{bmatrix}5&5&5\\3&-13&11\\9&1&-7\end{bmatrix}$$

3. Find adjoint matrix adj (A) (Transpose of cofactor matrix):

$$\text{adj}(A)=\begin{bmatrix}5&3&9\\5&-13&1\\5&1&-7\end{bmatrix}$$

4. Compute X $\frac{1}{|A|}\text{adj}(A)B$

$$\begin{bmatrix}x\\y\\z\end{bmatrix}=\frac{1}{40}\begin{bmatrix}5&3&9\\5&-13&1\\5&1&-7\end{bmatrix}\begin{bmatrix}5\\-4\\3\end{bmatrix}$$

$$=\frac{1}{40}\begin{bmatrix}25-12+27\\25+52+3\\25-4-21\end{bmatrix}=\frac{1}{40}\begin{bmatrix}40\\80\\0\end{bmatrix}=\begin{bmatrix}1\\2\\0\end{bmatrix}$$

Answer: x = 1, y = 2, z = 0

**42.** Given:

$y=(\tan^{-1}x)^2$

Differentiate with respect to $x$ using chain rule:

$$y_1=2(\tan^{-1}x)\cdot\frac{1}{x^2+1}$$

$$(x^2+1)y_1=2\tan^{-1}x$$

Differentiate both sides again with respect to $x$ using the Product Rule on the LHS:

$$(x^2+1)\cdot y_2+y_1\cdot(2x)=2\cdot\frac{1}{x^2+1}$$

Multiply the entire equation by $(x^2+1)$ to clear the denominator on the RHS:

$$(x^2+1)^2y_2+2x(x^2+1)y_1=2$$

(Hence proved)

**43.** Given $\frac{dV}{dt}=12\ \text{cm}^3/\text{s}$

Given $h=\frac{1}{6}r\Rightarrow r=6h$

The volume of a cone is:

$$V=\frac{1}{3}\pi r^2h$$

Substitute $r=6h$:

$$V=\frac{1}{3}\pi(6h)^2h=\frac{1}{3}\pi(36h^2)h=12\pi h^3$$

Differentiate both sides with respect to time $t$:

$$\frac{dV}{dt}=12\pi\cdot(3h^2)\frac{dh}{dt}=36\pi h^2\frac{dh}{dt}$$

Substitute $\frac{dV}{dt}=12$ and $h=4$ cm :

$$12=36\pi(4)^2\frac{dh}{dt}$$

$$12=576\pi\frac{dh}{dt}\Rightarrow\frac{dh}{dt}=\frac{12}{576\pi}=\frac{1}{48\pi}\ \text{cm/s}$$

**44.** Part 1: Derivation

Let $I=\int\frac{1}{x^2+a^2}dx$

Put $x=a\tan\theta\Rightarrow dx=a\sec^2\theta d\theta$:

$$I=\int\frac{a\sec^2\theta}{a^2\tan^2\theta+a^2}d\theta=\int\frac{a\sec^2\theta}{a^2(\tan^2\theta+1)}d\theta$$

Since $\tan^2\theta+1=\sec^2\theta$:

$$I=\int\frac{a\sec^2\theta}{a^2\sec^2\theta}d\theta=\frac{1}{a}\int 1d\theta=\frac{1}{a}\theta+C$$

Since $\theta=\tan^{-1}\left(\frac{x}{a}\right)$:

$$\int\frac{1}{x^2+a^2}dx=\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+C$$

Part 2: Evaluation:

Given $\int\frac{1}{x^2+2x+2}dx$

Complete the square:

$x^2+2x+2=(x+1)^2+1^2$

Apply the derived standard formula

$(x\to x+1,\ a=1)$:

$$\int\frac{1}{(x+1)^2+1^2}dx=\tan^{-1}(x+1)+C$$

**45.** The boundary curves intersect the positive axes at $(3,0)$ and $(0,2)$

- Ellipse: $y_1=\frac{2}{3}\sqrt{9-x^2}$
- Line: $y_2=\frac{2}{3}(3-x)$

$$\text{Area}=\int_0^3(y_1-y_2)dx$$

$$=\frac{2}{3}\int_0^3\left(\sqrt{9-x^2}-(3-x)\right)dx$$

$$\text{Area}=\frac{2}{3}\left[\left(\frac{x}{2}\sqrt{9-x^2}+\frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right)\right)-\left(3x-\frac{x^2}{2}\right)\right]_0^3$$

Substituting limits:

$$\text{Area}=\frac{2}{3}\left[\left(0+\frac{9}{2}\sin^{-1}(1)\right)-\left(9-\frac{9}{2}\right)\right]-[0]$$

$$\text{Area}=\frac{2}{3}\left[\frac{9}{2}\left(\frac{\pi}{2}\right)-\frac{9}{2}\right]=3\left(\frac{\pi}{2}-1\right)=\left(\frac{3\pi}{2}-3\right)\ \text{sq. units}$$

**46.** Divide by $x$ to get the standard linear form $\frac{dy}{dx}+Py=Q$ :

$$\frac{dy}{dx}+\frac{2}{x}y=x\log x\Rightarrow P=\frac{2}{x},\quad Q=x\log x$$

1. Integrating Factor (I.F.):

$$\text{I.F.}=e^{\int Pdx}=e^{\int\frac{2}{x}dx}=e^{2\log x}=x^2$$

2. General Solution:

$$y\cdot(\text{I.F.})=\int Q\cdot(\text{I.F.})dx+C\ \$$$

$$y\cdot x^2=\int(x\log x)\cdot x^2dx+C$$

$$\Rightarrow x^2y=\int x^3\log xdx+C$$

Integrate $\int x^3\log xdx$ by parts

$(u=\log x, v=x^3)$:

$$\int x^3\log xdx=(\log x)\left(\frac{x^4}{4}\right)-\int\left(\frac{1}{x}\right)\left(\frac{x^4}{4}\right)dx$$

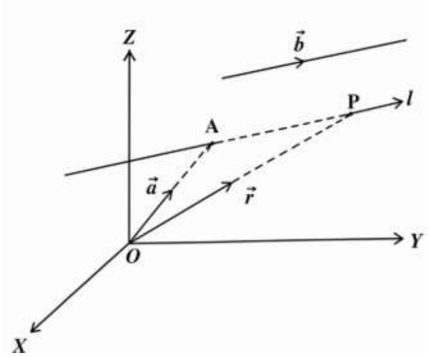
$$=\frac{x^4\log x}{4}-\frac{x^4}{16}$$

Substitute back and divide by $x^2$:

$$x^2y=\frac{x^4\log x}{4}-\frac{x^4}{16}+C$$

$$\Rightarrow y=\frac{x^2\log x}{4}-\frac{x^2}{16}+\frac{C}{x^2}$$

**47.** Let line pass through point A with position vector $\vec{a}$ and be parallel to vector $\vec{b}$




Let $P(\vec{r})$ be any point on the line.

1. Vector Form:

Vector $\overrightarrow{AP}$ is parallel to $\vec{b}$ :

$\overrightarrow{AP} = \lambda\vec{b}$ ($\lambda$ is a scalar parameter)

$\vec{r} - \vec{a} = \lambda\vec{b} \Rightarrow \vec{r} = \vec{a} + \lambda\vec{b}$

2. Cartesian Form:

Let A = (x_1, y_1, z_1), $\vec{b} = a\hat{i} + b\hat{j} + c\hat{k}$ and $P = (x, y, z)$ (p. 14).

$x\hat{i} + y\hat{j} + z\hat{k} = (x_1\hat{i} + y_1\hat{j} + z_1\hat{k})$
$+\lambda(a\hat{i} + b\hat{j} + c\hat{k})$

Equating components:

$$x = x_1 + \lambda a \Rightarrow \lambda = \frac{x - x_1}{a}$$

$$y = y_1 + \lambda b \Rightarrow \lambda = \frac{y - y_1}{b}$$

$$z = z_1 + \lambda c \Rightarrow \lambda = \frac{z - z_1}{c}$$

Equating values of $\lambda$ :

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

**48.** This follows a Binomial distribution with parameters n = 50 success probability $p = \frac{1}{100} = 0.01$, and failure probability $q$=0.90 ($p$.14)

Formula: $P(X = x) = \binom{n}{x} p^x q^{n-x}$

(a) Exactly once ($x = 1$):

$$P(X = 1) = \binom{50}{1}(0.01)^1(0.99)^{49}$$

$$= 50 \times 0.01 \times (0.99)^{49} = 0.5 \times (0.99)^{49}$$

(b) At least once ($x \geq 1$):

$$P(X \geq 1) = 1 - P(X = 0)$$

$$P(X = 0) = \binom{50}{0}(0.01)^0(0.99)^{50}$$

$$= (0.99)^{50}$$

$$P(X \geq 1) = 1 - (0.99)^{50}$$

**49.** (a) Proof:

Let the given integral be: $I = \int_{-a}^{a} f(x)dx$

Using the property of definite integrals, we can split the interval $[-a, a]$ at $0$ :

$$I = \int_{-a}^{0} f(x)dx + \int_{0}^{a} f(x)dx \quad \text{--- (Equation 1)}$$

Consider the first integral on the right hand side:

$I_1 = \int_{-a}^{0} f(x)dx.$

Substitute $x = -t \Rightarrow dx = -dt$ .

* When $x = -a, t = a$ .

* When $x = 0, t = 0$ .

Substitute these variables into $I_1$ :

$$I_1 = \int_{a}^{0} f(-t)(-dt) = -\int_{a}^{0} f(-t)dt$$

Using the property $\int_{a}^{b} f(t)dt = -\int_{b}^{a} f(t)dt$ :

$$I_1 = \int_{0}^{a} f(-t)dt$$

Since the variable of integration is a dummy variable, we can replace $t$ with $x$ :

$$I_1 = \int_{0}^{a} f(-x)dx$$

Substitute $I_1$ back into Equation 1:

$$I = \int_{0}^{a} f(-x)dx + \int_{0}^{a} f(-x)dx$$

$$= \int_{0}^{a} [f(x) + f(-x)]dx$$

Case 1: If $f(x)$ is an even function, then $f(-x) = f(x)$ .

$I=\int_0^a [f(x)+f(x)]dx = 2\int_0^a f(x)dx$

Case 2:} If $f(x)$ is an odd function, then $f(-x)=-f(x)$.

$I=\int_0^a [f(x)-f(x)]dx = 0$

(b) Let $f(x) = \sin^5 x\cos^4 x$.

Test if the function is even or odd by substituting $x\to -x$:

$f(-x)=\sin^5(-x)\cos^4(-x)=(-\sin x)^5(\cos x)^4$

$=-\sin^5 x\cos^4 x=-f(x)$

Since $f(-x)=-f(x)$, the function is odd.

Using the proven property:

$$\int_{-1}^{1}\sin^5 x\cos^4 x dx=0$$

Show that:

Let $\Delta=\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$

Apply the row operation $R_1\to R_1+R_2+R_3$:

$$\Delta=\begin{vmatrix} (x+4+2x+2x) & (2x+x+4+2x) & (2x+2x+x+4) \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$$

$$\Delta=\begin{vmatrix} 5x+4 & 5x+4 & 5x+4 \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$$

Take the common factor $(5x+4)$ out from the first row ($R_1$):

$$\Delta=(5x+4)\begin{vmatrix} 1 & 1 & 1 \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$$

Apply column operations $C_2\to C_2-C_1$ and $C_3\to C_3-C_1$:

$$\Delta=(5x+4)\begin{vmatrix} 1 & 0 & 0 \\ 2x & (x+4-2x) & 0 \\ 2x & 0 & (x+4-2x) \end{vmatrix}$$

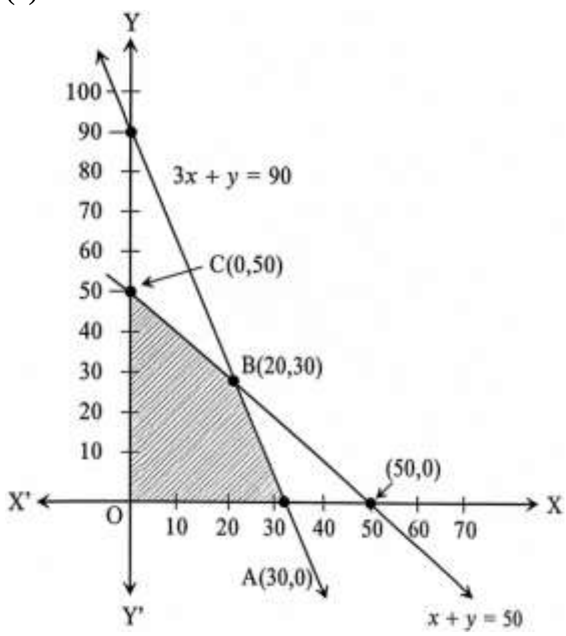
$$\Delta=(5x+4)\begin{vmatrix} 1 & 0 & 0 \\ 2x & 4-x & 0 \\ 2x & 0 & 4-x \end{vmatrix}$$

Expanding along the first row ($R_1$)

$\Delta=(5x+4)\cdot 1\cdot[(4-x)(4-x)-0]$

$=(5x+4)(4-x)^2$

**50.** (a)




1. Convert inequalities to equations to find boundary lines;
   1. x + y = 50 passes through points $(50,0)$ and $(0,50)$.
   2. Line 2: $3x+y=90$ passes through points $(30,0)$ and $(0,90)$.
2. Find the intersection point of the two lines:
   Subtract Line 1 from Line 2:
   $(3x+y)-(x+y)=90-50$
   $\Rightarrow 2x=40\Rightarrow x=20$
   Substitute $x=20$ into Line 1:
   $20+y=50\Rightarrow y=30$
   The lines intersect at (20, 30).

3. Identity the corner points of the bounded feasible region:
Since both constraints are '≤' and include non-negative restrictions (x, y ≥0), the feasible region lies in the first quadrant enclosed by the corner points:
1. $O(0, 0)$
2. $A(30,0)$
3. $B(20,30)$
4. $C(0,50)$
4. Evaluate the objective function $z=4x+y$ at each corner point:

| Corner Point | Value of $z=4x+y$ |
|---|---|
| $O(0,0)$ | $z=4(0)+0=0$ |
| $A(30,0)$ | $z=4(30)+0=120$ |
| $B(20,30)$ | $z=4(20)+30=110$ |
| $C(0,50)$ | $z=4(0)+50=50$ |

The objective function $z$ reaches its maximum value of 120 at the point $(\mathbf{30},\mathbf{0})$.

(b) For a function to be continuous at x = x, the Left-Hand Limit (LHL), Right-Hand Limit (RHL), and function value of must be equal:

$$\lim_{x\to\pi^-} f(x)=\lim_{x\to\pi^+} f(x)=f(\pi)$$

1. Calculate the Left-Hand Limit (LHL):
$$\text{LHL}=\lim_{x\to\pi^-}(Kx+1)=K\pi+1$$
2. Calculate the Right-Hand Limit (RHL):
$$\text{RHL}=\lim_{x\to\pi^+}(\cos x)=\cos\pi=-1$$
3. Equate LHL and RHL:
$$K\pi=-1-1$$
$$K\pi=-2\Rightarrow K=-\frac{2}{\pi}\text{ ft}$$