Q1 If $2^x + 2^y = 2^{x+y}$, then $\frac{dy}{dx}$ is **[KCET 2020]**

(A) 2^{y-x}

(B) -2^{y-x}

(C) 2^{x-y}

(D) $\frac{2^y-1}{2^x-1}$

Q2 If $f\left(x\right) = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$, then $f'\left(\sqrt{3}\right)$ is **[KCET 2020]**

(A) $-\frac{1}{2}$ (B) $\frac{1}{2}$

(C) $\frac{1}{\sqrt{3}}$ (D) $-\frac{1}{\sqrt{3}}$

Q3 The right hand and left hand limit of the function are respectively.

$$f\left(x\right) = \begin{cases} \frac{e^{1/x}-1}{e^{1/x}+1}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$ **[KCET 2020]**

(A) 1 and 1

(B) 1 and –1

(C) –1 and –1

(D) –1 and 1

Q4 If $y = 2x^{n+1} + \frac{3}{x^n}$, then $x^2\frac{d^2y}{dx^2}$ is **[KCET 2020]**

(A) $6n(n+1)y$ (B) $n(n+1)y$

(C) $x\frac{dy}{dx} + y$ (D) y

Q5 If the curves 2x = y^2 and 2xy = K intersect perpendicularly, then the value of K^2 is **[KCET 2020]**

(A) 4 (B) $2\sqrt{2}$

(C) 2 (D) 8

Q6 If $(xe)^y = e^y$, then $\frac{dy}{dx}$ is **[KCET 2020]**

(A) $\frac{\log x}{(1+\log x)^2}$ (B) $\frac{1}{(1+\log x)^2}$

(C) $\frac{\log x}{(1+\log x)}$ (D) $\frac{e^x}{x(y-1)}$

Q7 If the side of a cube is increased by 5%, then the surface area of a cube is increased by **[KCET 2020]**

(A) 10% (B) 60%

(C) 6% (D) 20%

Q8 The value of $\int \frac{1+x^4}{1+x^6}dx$ is [KCET 2020]

(A) $\tan^{-1}x + \tan^{-1}x^3 + C$

(B) $\tan^{-1}x + \frac{1}{3}\tan^{-1}x^3 + C$

(C) $\tan^{-1}x - \frac{1}{3}\tan^{-1}x^3 + C$

(D) $\tan^{-1}x + \frac{1}{3}\tan^{-1}x^2 + C$

Q9 The maximum value of $\frac{\log_e x}{x}$, if $x > 0$ is **[KCET 2020]**

(A) e (B) 1

(C) $\frac{1}{e}$ (D) $-\frac{1}{e}$

Q10 The value of $\int e^{\sin x}\sin 2x\,dx$ is **[KCET 2020]**

(A) $2e^{\sin x}\left(\sin x - 1\right) + C$

(B) $2e^{\sin x}\left(\sin x + 1\right) + C$

(C) $2e^{\sin x}\left(\cos x + 1\right) + C$

(D) $2e^{\sin x}\left(\cos x - 1\right) + C$

Q11 The value of $\int_{-1/2}^{1/2}\cos^{-1}x\,dx$ is **[KCET 2020]**

(A) π (B) $\frac{\pi}{2}$

(C) 1 (D) $\frac{\pi^2}{2}$

Q12 If $\int \frac{3x+1}{(x-1)(x-2)(x-3)}dx A\log\left|x-1\right| B\log\left|x-2\right| + C\log\left|x-3\right| + C,$

then the values of A, B and C are respectively **[KCET 2020]**

(A) $5, -7, -5$

(B) $2, -7, -5$

(C) $5, -7, 5$

(D) $2, -7, 5$

Q13 Find the value of $\int_0^1 \frac{\log(1+x)}{1+x^2}dx$ is **[KCET 2020]**

(A) $\frac{\pi}{2}\log 2$ (B) $\frac{\pi}{4}\log 2$

(C) $\frac{1}{2}$ (D) $\frac{\pi}{8}\log 2$

Q14 The area of the region bounded by the curve y^2 = 8x and the line y = 2x is **[KCET 2020]**

(A) $\frac{16}{3}$ sq. units

(B) $\frac{4}{3}$ sq. units

(C) $\frac{3}{4}$ sq. units

(D) $\frac{8}{3}$ sq. units

Q15 The value of $\int_{-\pi/2}^{\pi/2} \frac{\cos x}{1+e^x} dx$ is **[KCET 2020]**

(A) 2 (B) 0

(C) 1 (D) –2

Q16 The order of the differential equation obtained by eliminating arbitrary constants in the family of curves $c_1 y = (c_2 + c_3)e^{x+c_4}$ is **[KCET 2020]**

(A) 1 (B) 2

(C) 3 (D) 4

Q17 The general solution of the differential equation $x^2 dy - 2xy dx = x^4 \cos x dx$ is **[KCET 2020]**

(A) $y = x^2 \sin x + cx^2$ (B) $y = x^2 \sin x + c$

(C) $y = \sin x + cx^2$ (D) $y = \cos x + cx^2$

Q18 The area of the region bounded by the line y = 2x + 1, X-axis and the ordinates x = -1 and x = 1 is **[KCET 2020]**

(A) $\frac{9}{4}$ (B) 2

(C) $\frac{5}{2}$ (D) 5

Q19 The two vector $\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} + 3\hat{j} + 5\hat{k}$ represent the two sides $\overline{AB}$ and $\overline{AC}$ respectively of a ABC. The length of the median through A is **[KCET 2020]**

(A) $\frac{\sqrt{14}}{2}$ (B) 14

(C) 7 (D) $\sqrt{14}$

Q20 If a and b are unit vectors and θ is the angle between a and b, then sin $\frac{\theta}{2}$ is equal to **[KCET 2020]**

(A) |a + b| (B) $\frac{|a+b|}{2}$

(C) $\frac{|a-b|}{2}$ (D) |a – b|

Q21 The curve passing through the point (1, 2) given that the slope of the tangent at any point (x, y) is $\frac{3x}{y}$ represents **[KCET 2020]**

(A) Circle (B) Parabola

(C) Ellipse (D) Hyperbola

Q22 2 If $|a + b|^2 + |a . b|^2 = 144|a| = 6$, then |b| is equal to **[KCET 2020]**

(A) 6 (B) 3

(C) 2 (D) 4

Q23 The point (1, -3, 4) lies in the octant **[KCET 2020]**

(A) Second (B) Third

(C) Fourth (D) Eighth

Q24 If the vectors $2\hat{i} + 3\hat{j} + 4\hat{k}, 2\hat{i} + \hat{j} - \hat{k}$ and $\lambda\hat{i} - \hat{j} + 2\hat{k}$ are coplanar, then the value of λ is **[KCET 2020]**

(A) 6 (B) –5

(C) –6 (D) 5

Q25 The distance of the point (1, 2, –4) from the line $\frac{x-3}{2} = \frac{y-3}{3} = \frac{z+5}{6}$ is **[KCET 2020]**

(A) $\frac{293}{7}$ (B) $\frac{\sqrt{293}}{7}$

(C) $\frac{293}{49}$ (D) $\frac{\sqrt{293}}{49}$

Q26 The sine of the angle between the straight line $\frac{x-2}{3} = \frac{3-y}{-4} = \frac{z-4}{5}$ and the plane $2x - 2y + z = 5$ is **[KCET 2020]**

(A) $\frac{3}{\sqrt{30}}$ (B) $\frac{3}{50}$

(C) $\frac{4}{5\sqrt{2}}$ (D) $\frac{\sqrt{2}}{10}$

Q27 If a line makes an angle of with each of X and Y-axis, then the acute angle made by Z-axis is **[KCET 2020]**

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{6}$

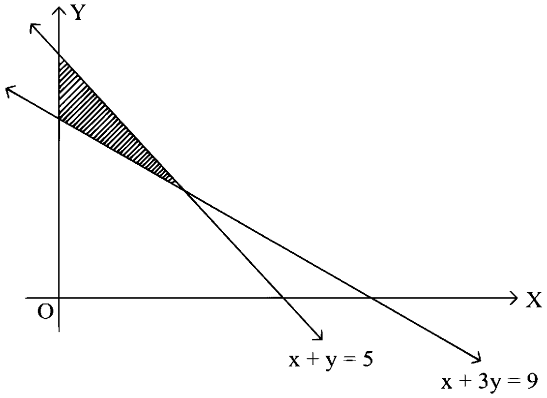
(C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

Q28 Corner points of the feasible region determined by the system of linear constraints are (0, 3), (1, 1) and (3, 0). Let z = px = qy, where, p, q > 0. Condition on p and q, so that the minimum of z occurs at (3, 0) and (1, 1) is **[KCET 2020]**

(A) p = 2q (B) p = $\frac{q}{2}$

(C) p = 3q (D) p = q

Q29 The feasible region of an LPP is shown in the figure. If z = 11x + 7y, then the maximum value of Z occurs at **[KCET 2020]**



(A) (0,5) (B) (3, 3)
(C) (5,0) (D) (3, 2)

Q30 A die is thrown 10 times, the probability that an odd number will come up at least one time is **[KCET 2020]**

(A) $\frac{1}{1024}$ (B) $\frac{1023}{1024}$
(C) $\frac{11}{1024}$ (D) $\frac{1013}{1024}$

Q31 If A and A are two events such that $P(A)=\frac{1}{3}, P(B)=\frac{1}{2}$ and $P(A \cap B)=\frac{1}{6}$, then $P(A'/B)$ is **[KCET 2020]**

(A) $\frac{2}{3}$ (B) $\frac{1}{3}$
(C) $\frac{1}{2}$ (D) $\frac{1}{12}$

Q32 Events E_1 and E_2 from a partition of the sample space S. A is any event such that $P(E_1)= P(\dot{E}_2)=\frac{1}{2}, P(E_2/A) = \frac{1}{2}$ and $P(A/E_2)=\frac{2}{3}$, then $P(E_1/A)$ is **[KCET 2020]**

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$
(C) 1 (D) $\frac{1}{4}$

Q33 The probability of solving a problem by three persons A, B and C independently is $\frac{1}{2}, \frac{1}{4}$ and $\frac{1}{3}$ respectively. Then the probability of the problem is solved by any two of them is **[KCET 2020]**

(A) $\frac{1}{12}$ (B) $\frac{1}{4}$
(C) $\frac{1}{24}$ (D) $\frac{1}{8}$

Q34 If n(A) = 2 and total number of possible relations from Set A to set B is 1024, then n(B) is **[KCET 2020]**

(A) 512 (B) 20
(C) 10 (D) 5

Q35 The value of $\sin^2 51° + \sin^2 39°$ is **[KCET 2020]**

(A) 1 (B) 0
(C) sin 12° (D) cos 12°

Q36 If tan A + cot A = 2, then the value of $\tan^4 A + \cot^4 A$ [KCET 2020]

(A) 2 (B) 1
(C) 4 (D) 5

Q37 If A = {1, 2, 3, 4, 5, 6}, then the number of subsets of A which contain at least two elements is **[KCET 2020]**

(A) 64 (B) 63
(C) 57 (D) 58

Q38 If z = x + iy, then the equation z + 1| = |2 - 1| represents **[KCET 2020]**

(A) a circle (B) a parabola
(C) X-axis (D) Y-axis

Q39 The value of ${}^{16}C_9 + {}^{16}C_{10} - {}^{16}C_6 - {}^{16}C_7$ is **[KCET 2020]**

(A) 0 (B) 1
(C) ${}^{n}C_0$ (D) ${}^{17}C_3$

Q40 The number of terms in the expansion of $(x + y + z)^{10}$ is **[KCET 2020]**

(A) 66 (B) 142
(C) 11 (D) 110

Q41 If P(n): $2^n < n!$ Then the smallest positive integer for which P(n) is true if **[KCET 2020]**

(A) 2 (B) 3
(C) 4 (D) 5

Q42 The two lines lx + my = n and l' * x + m'y = n' are perpendicular if **[KCET 2020]**

(A) ll' + mm' = 0 (B) lm' + ml'
(C) lm + l'm' = 0 (D) lm' + ml' = 0

Q43 If the parabola $x^2 = 4ay$ passes through the point (2, 1), then the length of the latus rectum is **[KCET 2020]**

(A) 1 (B) 4
(C) 2 (D) 3

Q44 If the sum of n terms of an AP is given by $S_n = n^2 + n$, then the common difference of the AP is **[KCET 2020]**

(A) 4 (B) 1
(C) 2 (D) 6

Q45 The negation of the statement "For all real numbers x and y, x + y = y + x" prime is

(A) For all real numbers x and y, $x + y \neq y + x$
(B) For some real numbers x and $y, x + y = y + x$
(C) For some real numbers x and $y, x + y \neq y + x$
(D) For some real numbers x and $y, x - y = y - x$

Q46 The standard deviation of the data 6, 7, 8, 9, 10 is **[KCET 2020]**

(A) $\sqrt{2}$ (B) $\sqrt{10}$
(C) 2 (D) 10

Q47 $\lim_{x \to 0}\left(\frac{\tan x}{\sqrt{2x+4}-2}\right)$ is equal to **[KCET 2020]**

(A) 2 (B) 3
(C) 4 (D) 6

Q48 If a relation R on the set {1, 2, 3} be defined by R = {(1, 1)}, then R is **[KCET 2020]**

(A) Reflexive and symmetric
(B) Reflexive and transitive
(C) Symmetric and transitive
(D) Only symmetric

Q49 Let f: [2,00) R be the function defined $f(x) = x^2 - 4x + 5$, then the ranges of f is **[KCET 2020]**

(A) $(-\infty, \infty)$
(B) $[1, \infty)$
(C) $(1, \infty)$
(D) $[5, \infty)$

Q50 If A, B, C are three mutually exclusive and exhaustive events of an experiment such that P(A) = 2P(B) = 3P(C) then P(B) is equal to **[KCET 2020]**

(A) $\frac{1}{11}$ (B) $\frac{2}{11}$
(C) $\frac{3}{11}$ (D) $\frac{4}{11}$

Q51 The domain of the function defined by $f(x) = \cos^{-1}\sqrt{x-1}$ is **[KCET 2020]**

(A) [1, 2] (B) [0, 2]
(C) [–1,1] (D) [0, 1]

Q52 The value of $\cos\left(\sin^{-1}\frac{\pi}{3} + \cos^{-1}\frac{\pi}{3}\right)$ is Does not exist **[KCET 2020]**

(A) 0 (B) 1
(C) –0 (D) Does not exist

Q53 If $A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ then A^4 is equal to **[KCET 2020]**

(A) A (B) 2A
(C) I (D) 4A

Q54 If A = {a, b, c} then the number of binary operations on A is **[KCET 2020]**

(A) 3 (B) 3^6
(C) 3^3 (D) 3^9

Q55 If $\begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ then the matrix a is **[KCET 2020]**

(A) $\begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}$
(B) $\begin{pmatrix} 2 & -1 \\ -3 & 2 \end{pmatrix}$
(C) $\begin{pmatrix} -2 & 1 \\ 3 & -2 \end{pmatrix}$
(D) $\begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix}$

Q56 If $f(x) = \begin{vmatrix} x^3 - x & a + x & b + x \\ x - a & x^2 - x & c + x \\ x - b & x - c & 0 \end{vmatrix}$, then **[KCET 2020]**

(A) $f(1) = 0$ (B) $f(2) = 0$
(C) $f(0) = 0$ (D) $f(-1) = 0$

Q57 If A and B are square matrices of same order and B is a skew symmetric matrix, then A'BA is **[KCET 2020]**

(A) Symmetric matrix
(B) Null matrix
(C) Diagonal matrix
(D) Skew symmetric matrix

Q58 If A is a square matrix of order 3 and |A| = 5, then | A adj. A | is **[KCET 2020]**

(A) 5 (B) 125

(C) 25 (D) 625

Q59 If $f(x) = \begin{cases} \frac{1-\cos Kx}{x\sin x}, & \text{if } x \neq 0 \\ \frac{1}{2}, & \text{if } x = 0 \end{cases}$ is continuous at $x = 0$, then the value of K is **[KCET 2020]**

(A) $\pm\frac{1}{2}$ (B) 0

(C) ± 2 (D) ± 1

Q60 If $a_1 a_2 a_3 \ldots a_9$ are in AP, then the value of $\begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$ is **[KCET 2020]**

(A) $\frac{9}{2}(a_1 + a_9)$ (B) $(a_1 + a_9)$

(C) $\log_e(\log_e e)$ (D) 1

Vernacular

Answer Key

Q1	B
Q2	B
Q3	B
Q4	B
Q5	D
Q6	A
Q7	A
Q8	B
Q9	C
Q10	A
Q11	B
Q12	D
Q13	D
Q14	B
Q15	C
Q16	A
Q17	A
Q18	C
Q19	D
Q20	C
Q21	D
Q22	C
Q23	C
Q24	A
Q25	B
Q26	D
Q27	A
Q28	B
Q29	D
Q30	B
Q31	A
Q32	A
Q33	B
Q34	D
Q35	A
Q36	A
Q37	C
Q38	D
Q39	A
Q40	A
Q41	C
Q42	A
Q43	B
Q44	C
Q45	C
Q46	A
Q47	A
Q48	C
Q49	B
Q50	C
Q51	A
Q52	D
Q53	C
Q54	D
Q55	B
Q56	C
Q57	D
Q58	B
Q59	D
Q60	C

Hints & Solutions

Note: scan the QR code to watch video solution

Q1 Text Solution:

We have,

$2^x + 2^y = 2^{x+y} \quad \ldots(i)$

On differentiating Eq. (i) w.r.t. x, we get

$2^x \log 2 + 2^y \log 2 \frac{dy}{dx}$

$= 2^{x+y} \log 2\left(1 + \frac{dy}{dx}\right)$

$\Rightarrow 2^x + 2^y \frac{dy}{dx} = 2^{x+y}\left(1 + \frac{dy}{dx}\right)$

$\Rightarrow 2^x - 2^{x+y} = \frac{dy}{dx}(2^{x+y} - 2^y)$

$\Rightarrow -2^y = \frac{dy}{dx}(2^x)$

$\Rightarrow \frac{dy}{dx} = -2^y/2^x = -2^{y-x}$

Q2 Text Solution:

We have, $f(x) = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

On putting

$x = \tan\theta$

$\theta = \tan^{-1} x$

Then we get, $f(x) = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$

$= \sin^{-1}(\sin 2\theta)$

$= 2\theta = 2\tan^{-1} x \quad \ldots(i)$

On differentiating Eq. (i) both sides w.r.t. x, we get

$f'(x) = \frac{2}{1+x^2}$

$\therefore f'\left(\sqrt{3}\right) = \frac{2}{1+\left(\sqrt{3}\right)^2} = \frac{2}{1+3} = \frac{2}{4} = \frac{1}{2}$

Q3 Text Solution:

We have, $f(x) = \begin{cases} \frac{e^{1/x}-1}{e^{1/x}+1}, & \text{if } x \neq 0 \\ 0, & x = 0 \end{cases}$

Right hand limit $= \lim_{x\to 0^+} f(x)$

$= \lim_{h\to 0} f(0+h)$

$= \lim_{h\to 0} f(h) = \lim_{h\to 0} \frac{e^{1/h}-1}{e^{1/h}+1}$

$= \lim_{h\to 0} \frac{1-e^{-1/h}}{1+e^{-1/h}}$

$= \frac{1-0}{1+0} = 1 \quad \left(\because \lim_{h\to 0} e^{-1/h} = 0\right)$

Left hand limit $= \lim_{x\to 0^-} f(x)$

$= \lim_{h\to 0} f(0-h)$

$= \lim_{h\to 0} f(-h)$

$= \lim_{h\to 0} \frac{e^{-1/h}-1}{e^{-1/h}+1}$

$= \lim_{h\to 0}\left(\frac{\frac{1}{e^{1/h}}-1}{\frac{1}{e^{1/h}}+1}\right) = \frac{0-1}{0+1} = -1$

Q4 Text Solution:

We have, $y = 2x^{n+1} + \frac{3}{x^n}$

$= 2x^{n+1} + 3x^{-n} \quad \ldots(i)$

On differentiating Eq. (i) both sides w.r.t. x, we get

$\frac{dy}{dx} = 2(n+1)x^n + 3(-n)x^{-n-1}$

Again differentiate w.r.t. to x, then we get

$\frac{d^2y}{dx^2} = 2(n+1)(n)x^{n-1}$

$+ 3(-n)(-n-1)x^{-n-2}$

$= n(n+1)(2x^{n-1} + 3x^{-n-2})$

$\Rightarrow \quad x^2\frac{d^2y}{dx^2} = n(n+1)(2x^{n+1} + \frac{3}{x^n})$

$\Rightarrow \quad x^2\frac{dy}{dx} = n(n+1)y \quad$ [using Eq. (i)]

Q5 Text Solution:

Given curves are

$2x = y^2 \quad \ldots(i)$

and 2xy = K ....(ii)

On solving both Eqs. (i) and (ii), we get

$x = \frac{K^{2/3}}{2}$ and $= K^{1/3}$

$\therefore$ Intersecting point of both curves is

$$\left(\frac{K^{2/3}}{2}, K^{1/3}\right)$$

Now differentiate Eq. (i) w.r.t. x, we get

$2 = 2y\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{y}$

$\therefore$ Slope of tangent at $\left(\frac{K^{2/3}}{2}, K^{1/3}\right) = \frac{1}{K^{1/3}}$

And differentiate Eq. (ii) w.r.t. x, we get

$2\left(x\frac{dy}{dx} + y\right) = 0 \Rightarrow \frac{dy}{dx} = \frac{-y}{x}$

$\therefore$ Slope of tangent at $\left(\frac{K^{2/3}}{2}, K^{1/3}\right)$

$= \frac{-K^{1/3}}{K^{2/3}/2} = -2K^{-1/3}$

Since, both curves intersect perpendicularly

$\therefore \frac{1}{K^{1/3}} \times \left(-2K^{-1/3}\right) = -1$

$\Rightarrow -2K^{-2/3} = -1 \Rightarrow K^{2/3} = 2 \Rightarrow K^2 = 8$

Q6 Text Solution:

We have, $(xe)^y = e^x$

Taking log on both sides at base e, we get

$y\log(xe) = x\log e$

$\Rightarrow y(\log x + \log e) = x(\because \log_e e = 1)$

$\Rightarrow \quad y = \frac{x}{\log x + 1}$

On differentiating both sides w.r.t. x, we get

$\frac{dy}{dx} = \frac{(\log x+1) - x\left(\frac{1}{x}+0\right)}{(\log x+1)^2}$

$\Rightarrow \frac{dy}{dx} = \frac{\log x}{(\log x+1)^2}$

Q7 Text Solution:

We know that the surface area of cube is given by $= 6x^2$

$\therefore \frac{dS}{dx} = 12x^2$

$\Delta x = 5\%$ of $x = 0.05x$

The surface area of a cube is increased by $= \frac{\Delta S}{S} \times 100\%$

$= \frac{\left(\frac{dS}{dx}\right)\Delta x}{S} \times 100\%$

$= \frac{12x \times (0.05x)}{6x^2} \times 100\%$

$= 0.10 \times 100\% = 10\%$

Q8 Text Solution:

We have, $\int \frac{1+x^4}{1+x^6}dx$

$= \int \frac{1+x^4}{1+x^6} \times \frac{1+x^2}{1+x^2}dx$

$= \int \frac{1+x^2+x^4+x^6}{(1+x^6)}dx$

$= \int \frac{1+x^6}{(1+x^6)1+x^2}dx + \int \frac{x^2(1+x^2)}{(1+x^6)(1+x^2)}dx$

$= \int \frac{dx}{1+x^2} + \int \frac{x^2}{1+x^6}dx$

$= \tan^{-1} x + \int \frac{x^2}{1+(x^3)^2}dx$

$= \tan^{-1} x + \int \frac{dt/3}{1+t^2}$

Put $x^3 = t$

$\Rightarrow 3x^2dx = dt$

$\Rightarrow x^2dx = \frac{dt}{3}$

$= \tan^{-1} x + \frac{1}{3}\tan^{-1}(t) + C$

$= \tan^{-1} x + \frac{1}{3}\tan^{-1}(x^3) + C$

Q9 Text Solution:

Let $y = \frac{\log_e x}{x}$

$\Rightarrow \frac{dy}{dx} = \frac{x\cdot\frac{1}{x} - \log_e x\cdot 1}{x^2} = \frac{1-\log_e x}{x^2}$

Formaximum, put $\frac{dy}{dx} = 0$

$\Rightarrow 1 - \log_e x = 0$

$\Rightarrow \log_e x = 1$

$\Rightarrow x = e$

Now, $\frac{d^2y}{dx^2} = \frac{x^2\left(0-\frac{1}{x}\right) - (1-\log_e x)(2x)}{x^4}$

$= \frac{-x-2x(1-\log_e x)}{x^4}$

$\therefore \left(\frac{d^2y}{dx^2}\right)_{at\ x=e} < 0$

$\therefore y(e) = \frac{\log_e e}{e} = \frac{1}{e}$

Q10 Text Solution:

We have, $\int e^{\sin x}\sin 2x\,dx$

$= \int e^{\sin x}\cdot(2\sin x\cos x)\,dx$

$= 2\int e^{\sin x}\sin x\cos x\,dx$

let $\sin x = t$

$\Rightarrow \cos x dx \,\&= dt = 2\int e^{t}\cdot t\,dt$

$= 2\left[t\int e^{t}dt - \int\left(\frac{d}{dt}(t)\int e^{t}\,dt\right)dt\right]$

$= 2\left[t\cdot e^{t} - \int e^{t}\,dt\right]$

$= 2\left[t\cdot e^{t} - e^{t}\right]+C$

$= 2\left[e^{t}(t-1)\right]+C$

$= 2\left[e^{\sin x}(\sin x - 1)\right]+C$

$= 2e^{\sin x}(\sin x - 1) + C$

Q11 Text Solution:

We have, $\int_{-1/2}^{1/2}\cos^{-1}x\,dx$

$= \int_{-1/2}^{1/2}(\pi/2 - \sin^{-1}x)dx$

$= \int_{-1/2}^{1/2}\pi/2\,dx - \int_{-1/2}^{1/2}\sin^{-1}x\,dx$

$= \frac{\pi}{2}\int_{-1/2}^{1/2}dx$

$-0 \quad (\because \sin^{-1}x \text{ is odd function})$

$= \frac{\pi}{2}[x]_{-1/2}^{1/2} = \frac{\pi}{2}\left[\frac{1}{2} - (-1/2)\right] = \frac{\pi}{2}(1) = \frac{\pi}{2}$

Q12 Text Solution:

We have,

$\int \frac{3x+1}{(x-1)(x-2)(x-3)}dx$

Let $\frac{3x+1}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$

$\Rightarrow (3x+1) = A(x-2)(x-3)$

$+ B(x-1)(x-3) + C(x-1)(x-2)$

put $x - 1 = 0$

$\Rightarrow x = 1$

Then, $3\times 1 + 1 = A(-1)(-2)$

$\Rightarrow 4 = 2A$

$\Rightarrow A = 2$

Put $x - 2 = 0$

$\Rightarrow x = 2$

Then, $7 = B(2-1)(2-3)$

$\Rightarrow 7 = B(1)(-1)$

$\Rightarrow B = -7$

And put $x - 3 = 0$

$\Rightarrow x = 3$

Then, $10 = C(3-1)(3-2)$

$\Rightarrow 10 = C(2)(1)$

$\Rightarrow C = 5$

$\therefore \frac{3x+1}{(x-1)(x-2)(x-3)} = \frac{2}{x-1} - \frac{7}{x-2} + \frac{5}{x-3}$

$\therefore \int \frac{3x+1}{(x-1)(x-2)(x-3)}dx$

$= \int\frac{2}{x-1}dx - \int\frac{7}{x-2}dx + \int\frac{5}{x-3}dx$

$\Rightarrow \int \frac{3x+1}{(x-1)(x-2)(x-3)}dx$

$= 2\log|x-1| - 7\log|x-2| + 5\log|x-3| + C$

$= A\log|x-1| + B\log|x-2| + C\log|x-3| + C$ (Given)

$\Rightarrow A = 2, B = -7, C = 5$

Q13 Text Solution:

Let $I = \int_0^1 \frac{\log(1+x)}{1+x^2} dx$

Put $x = \tan\theta$

$\Rightarrow \quad dx = \sec^2 \theta d\theta$

When $x = 0, \theta = 0$

and when $x = 1, \theta = \pi/4$

$\therefore \int_0^{\pi/4} \frac{\log(1+\tan\theta)}{1+\tan^2 \theta} \left(\sec^2 \theta\right) d\theta$

$I = \int_0^{\pi/4} \log\left(1 + \tan\theta\right) d\theta \ldots \left(i\right)$

$= \int_0^{\pi/4} \log\left[1 + \tan\left(\frac{\pi}{4} - \theta\right)\right] d\theta$

$= \int_0^{\pi/4} \log\left[1 + \frac{\tan\pi/4 - \tan\theta}{1+\tan\pi/4\tan\theta}\right] d\theta$

$= \int_0^{\pi/4} \log\left(1 + \frac{1-\tan\theta}{1+\tan\theta}\right) d\theta$

$= \int_0^{\pi/4} \log\left(\frac{2}{1+\tan\theta}\right) d\theta$

$\int_0^{\pi/4} \left[\log 2 - \log 1 + \tan\theta)\right] d\theta \ldots . \left(ii\right)$

On adding Eqs. (i) and (ii), we get

$2I = \int_0^{\pi/4} \log 2 d\theta$

$= \log 2 \int_0^{\pi/4} 1 d\theta = \log\left(\theta\right)_0^{\pi/4}$

$= \log(\pi/4 - 0) = \frac{\pi}{4} \log 2$

$\Rightarrow I = \frac{\pi}{8} \log 2$

Q14 Text Solution:

Given equation of curve, $y^2 = 8x$ and line $y = 2x$

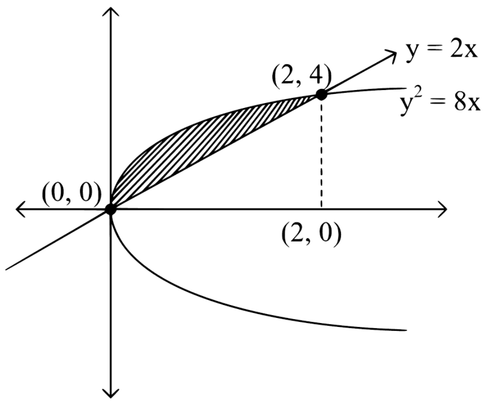
Now, intersecting points of given curve and line. $(2x)^2 = 8x$

$\Rightarrow 4x^2 = 8x$

$\Rightarrow x = 0, 2$

Putting values of x in $y = 2x$ then, we get $y = 0, 4$

$\therefore$Required area of bounded region.



$= \int_0^2 \left(\sqrt{8x} - 2x\right) dx$

$= \int_0^2 \left(2\sqrt{2} \cdot x^{1/2} - 2x\right) dx$

$= 2 \int_0^2 \left(\sqrt{2} \cdot x^{1/2} - x\right) dx$

$= 2\left[\sqrt{2} \int_0^2 x^{1/2} dx - \int_0^2 x \, dx\right]$

$= 2\left[\sqrt{2}\left(\frac{x^{3/2}}{3/2}\right)_0^2 - \left(\frac{x^2}{2}\right)_0^2\right]$

$= 2\left[\sqrt{2} \cdot \frac{2}{3}(2)^{3/2} - 2\right]$

$= 2\left[\frac{8}{3} - 2\right] = 2\left(\frac{2}{3}\right) = \frac{4}{3}$ sq. units

Q15 Text Solution:

We have, $\int_{-\pi/2}^{\pi/2} \frac{\cos x}{1+e^x} dx$

$= \int_0^{\pi/2} \left(\frac{\cos x}{1+e^x} + \frac{\cos(-x)}{(1+e^{-x})}\right) dx$

$\left[\because \int_{-a}^{a} f\left(x\right) dx = \int_0^a \left(f\left(x\right) + f\left(-x\right)\right) dx\right]$

$= \int_0^{\pi/2} \left(\frac{\cos x}{1+e^x} + \frac{e^x \cos x}{1+e^x}\right) dx$

$= \int_0^{\pi/2} \cos x \left(\frac{1+e^x}{1+e^x}\right) dx$

$= \int_0^{\pi/2} \cos x dx = (\sin x)_0^{\pi/2}$

$= \left(\sin \frac{\pi}{2} - \sin 0\right) = 1$

Q16 Text Solution:

We have, $c_1 y = (c_2 + c_3) e^{x+c_4}$

$\Rightarrow y = \left(\frac{c_2+c_3}{c_1}\right) e^{c_4} \cdot e^x$

$\Rightarrow y = ce^x$ where $c = \frac{c_2+c_3}{c_1} e^{c_4}$

Taking log on both sides, we get

$\log y = \log(ce^x)$

$\Rightarrow \log y = \log c + x \log e$

$\Rightarrow \log y = \log c + x$

On differentiating w.r.t. x, we get

$\frac{1}{y} \frac{dy}{dx} = 1 \Rightarrow \frac{dy}{dx} = y$

$\therefore$ order = 1

Q17 Text Solution:

We have, $x^2dy - 2xydx = x^4 \cos x dx$

$\Rightarrow \frac{dy}{dx} - \frac{2}{x}y = x^2 \cos x$

Comparing with $\frac{dy}{dx} + Py = Q$

Where $P = -\frac{2}{x}$ and $Q = x^2 \cos x$

$\therefore$ IF $= e^{\int \frac{-2}{x}dx} = e^{-2\log x} = e^{\log\left(\frac{1}{x^2}\right)} = \frac{1}{x^2}$

$\therefore$ The solution of the given differential 3

$y \cdot \text{IF} = \int (Q \times \text{IF})\, dx + c$

$\Rightarrow y \times \frac{1}{x^2} = \int (x^2 \cos x)\left(\frac{1}{x^2}\right)dx + c$

$\Rightarrow \frac{y}{x^2} = \int \cos x + c$

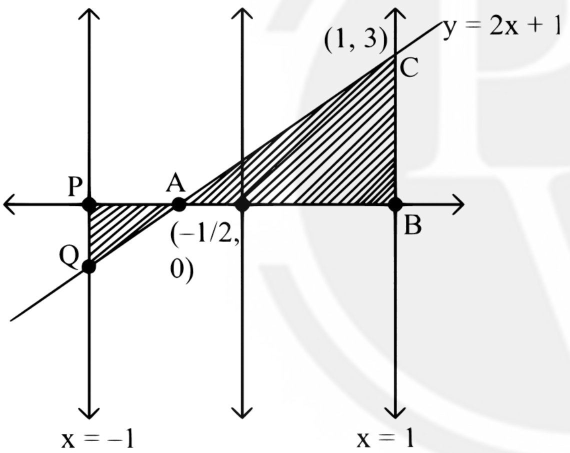
$\Rightarrow \frac{y}{x^2} = \sin x + c$

$\Rightarrow y = x^2 \sin x + cx^2$

Q18 Text Solution:

Given, equation of line $y = 2x + 1$ ....(i)

Eq. (i) passing through the points $\left(\frac{-1}{2}, 0\right)$ and $(0, 1)$.



$\therefore$ Required area of shaded region

$= \int_{-1}^{-1/2} -(2x+1)dx +$ Area of ΔABC

$= -\left[2\left(\frac{x^2}{2}\right)+x\right]_{-1}^{-1/2} + \frac{1}{2} \times AB \times BC$

$= -[x^2 + x]_{-1}^{-1/2} + \frac{1}{2} \times \frac{3}{2} \times 3$

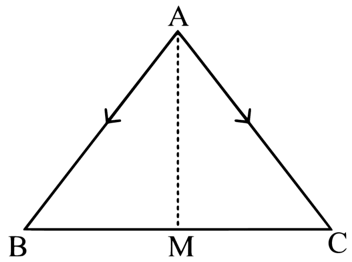
$= -\left[\left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right) - (-1)^2 - (-1)\right] + \frac{9}{4}$

$= -\left[\frac{1}{4} - \frac{1}{2}\right] + \frac{9}{4}$

$= \frac{1}{4} + \frac{9}{4} = \frac{10}{4} = \frac{5}{2}$ sq units

Q19 Text Solution:

We know that, the sum of three vectors of triangle is zero.



$\therefore AB + BC + CA = 0$

$\Rightarrow BC = AC - AB$

$\Rightarrow BM = \frac{AC-AB}{2}$ (since, M is a mid − point of BC)

And also,

$\Rightarrow AB + \frac{AC-AB}{2} = AM$

$\Rightarrow AM = \frac{AB+AC}{2}$

$\Rightarrow AM = \frac{(\hat{i}+\hat{j}+\hat{k})+(\hat{i}+3\hat{j}+5\hat{k})}{2}$

$\Rightarrow AM = \frac{2\hat{i}+4\hat{j}+6\hat{k}}{2}$

$\Rightarrow AM = \hat{i} + 2\hat{j} + 3\hat{k}$

$\therefore AM = \sqrt{(1)^2 + (2)^2 + (3)^2}$

$= \sqrt{1+4+9} = \sqrt{14}$

Q20 Text Solution:

Given a and b are unit vectors

$\therefore |a| = 1$ and $|b| = 1$

We know that, $|a-b|^2 = (a-b) \cdot (a-b)$

$= |a|^2 - 2a \cdot b + |b|^2$

$= |a|^2 - 2|a||b|\cos\theta + |b|^2$

$= 1 - 2 \cdot 1 \cdot 1 \cdot \cos\theta + 1$

$= 2 - 2\cos\theta$

$\Rightarrow |a-b|^2 = 2(1-\cos\theta)$

$= 2(2\sin^2 \theta/2)$

$= 4\sin^2 \theta/2$

$\Rightarrow \sin^2 \frac{\theta}{2} = \frac{1}{4}|a-b|$

$\Rightarrow \sin \frac{\theta}{2} = \frac{1}{2}|a-b|$

Q21 Text Solution:

We have, $\frac{dy}{dx} = \frac{3x}{y}$

$\Rightarrow \int y dy = \int 3 x dx$

$\Rightarrow \frac{y^2}{2} = 3 \cdot \frac{x^2}{2} + C \ldots (i)$

Since, Eq (i) passing through point $(1, 2)$

$\therefore \frac{4}{2} = \frac{3}{2}(1)^2 + c$

$c = 2 - \frac{3}{2} = \frac{1}{2}$

$\therefore$ Equation of curve, $\frac{y^2}{2} = \frac{3x^2}{2} + \frac{1}{2}$

$\Rightarrow y^2 = 3x^2 + 1 \Rightarrow y^2 - 3x^2 = 1$

$\Rightarrow \frac{y^2}{(1)^2} - \frac{x^2}{\left(1/\sqrt{3}\right)^2} = 1$

Which represents a Hyperbola.

Q22 Text Solution:

We have, $|a + b|^2 + |a \cdot b|^2 = 144$, and $|a| = 6$

We kniow that, $|a + b|^2 + |a \cdot b|^2 = (|a||b|)^2$

$\Rightarrow |a|^2|b|^2 = 144$

$\Rightarrow (6)^2|b|^2 = 144$

$\Rightarrow |b|^2 = 4$

$\Rightarrow |b| = 2$

Q23 Text Solution:

Clearly, point (1, -3, 4) lies in IV octant.

Q24 Text Solution:

Given vectors $2\hat{i} - 3\hat{j} + 4\hat{k}, 2\hat{i} + \hat{j} - \hat{k}$ and $\lambda\hat{i} - \hat{j} + 2\hat{k}$ are coplanar

$\therefore \begin{vmatrix} 2 & -3 & 4 \\ 2 & 1 & -1 \\ \lambda & -1 & 2 \end{vmatrix} = 0$

$\Rightarrow 2(2 - 1) + 3(4 + \lambda) + 4(-2 - \lambda) = 0$

$\Rightarrow 2 + 12 + 3\lambda - 8 + 4\lambda = 0$

$\Rightarrow 6 - \lambda = 0$

$\Rightarrow \lambda = 6$

Q25 Text Solution:

Given, point $(1, 2, -4)$.

and line $\frac{x-3}{2} = \frac{y-3}{3} = \frac{z+5}{6} = \lambda$ (let)

$\therefore x = 2\lambda + 3$

$\therefore x = 2\lambda + 3, y = 3\lambda + 3, z = 6\lambda - 5$

Let $P(2\lambda + 3, 3\lambda + 3, 6\lambda - 5)$ be the foot of perpendicular from $A(1, 2, 4)$

$\therefore$ Direction ratios of AP are

$2\lambda + 3 - 1, 3\lambda + 3 - 2, 6\lambda - 5 + 4$

i.e., $2\lambda + 2, 3\lambda + 1, 6\lambda - 1$

And direction ratios of given line are 2, 3, 6 : Since, $AP \perp$ given line

$\therefore 2(2\lambda + 2) + 3(3\lambda + 1) + 6(6\lambda - 1) = 0$

$\Rightarrow 4\lambda + 4 + 9\lambda + 3 + 36\lambda - 6 = 0$

$\Rightarrow 49\lambda + 1 = 0$

$\Rightarrow \lambda = -\frac{1}{49}$

$\therefore P(2\lambda + 3, 3\lambda + 3, 6\lambda - 5)$

$= 4\lambda^2 + 4 + 8\lambda + 9\lambda^2 + 1 + 6\lambda + 36\lambda^2 + 1 - 12\lambda$

$= 49\lambda^2 + 6 + 2\lambda$

$\therefore$ Distance of the point $(1, 2, -4)$ from the given line,

line,

$AP^2 = (2 + \lambda)^2 + (3\lambda + 1)^2 + (6\lambda - 1)^2$

$= 4\lambda^2 + 4 + 8\lambda + 9\lambda^2 + 1 + 6\lambda + 3$

$= 49\lambda^2 + 6 + 2\lambda$

$= 49\left(-\frac{1}{49}\right)^2 + 6 + 2\left(-\frac{1}{49}\right)$

$= \frac{1}{49} + 6 - \frac{2}{49}$

$= 6 - \frac{1}{49} = \frac{293}{49}$

$\Rightarrow AP = \frac{\sqrt{293}}{7}$

Q26 Text Solution:

Given straight line $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ and plane $2x - 2y + z = 5$

Let θ be the angle between line and plane then

$\sin\theta = \left|\frac{3\times2+(4)(-2)+5\times1}{\sqrt{(3)^2+(4)^2+(5)^2}\sqrt{(2)^2+(-2)^2+(1)^2}}\right|$

$= \left|\frac{6-8+5}{\sqrt{9+16+25}\sqrt{4+4+1}}\right|$

$= \left|\frac{3}{\sqrt{50}\sqrt{9}}\right| = \frac{1}{5\sqrt{2}} = \frac{\sqrt{2}}{10}$

Q27 Text Solution:

Given, $1 = m = \cos\frac{\pi}{3} = \frac{1}{2}$

$\therefore l^2 + m^2 + n^2 = 1$

$\Rightarrow \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + n^2 = 1$

$\Rightarrow \frac{1}{4} + \frac{1}{4} + n^2 = 1$

$\Rightarrow n^2 = \frac{1}{2} \Rightarrow n = \pm\frac{1}{\sqrt{2}}$

Then the acute angle made by Z

$-$ axis is $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$

Q28 Text Solution:

The minimum value of z is unique.

It is given that the minimum value of z occurs at two points (3, 0) and (1, 1)

$\Rightarrow p(3) + q(0) = p(1) + q(1)$

$\Rightarrow 3p = p + q$

$\Rightarrow 2p = q$

$\Rightarrow p = q/2$

Q29 Text Solution:

Given, maximize z = 11x + 7y

Intersecting point of lines x + y = 5 and x + 3y = 9 is (3, 2).

$\therefore$ Corner point is B(3, 2)

For corner points of the feasible region

We put,

x = 0 in x + 3y = 9

y = 3

$\Rightarrow$corner point is A(0, 3) and put x = 0 in x + y = 5 we get y = 5

$\Rightarrow$corner point is C(0,5) The values of z at these corner points are as follows

Corner points	x = 11x + 7y
A(0, 3)	11 × 0 + 7 × 3 = 21
B(3, 2)	11 × 3 + 7 × 2 = 47
C(0,5)	11 × 0 + 7 × 5 = 35

Therefore, maximum value of z is 47 at (3, 2).

Q30 Text Solution:

A die is thrown 10 times

Probability of success is an odd number

$\therefore n = 10, p = \frac{1}{2}, q = \frac{1}{2}$

Number of success is atleast one

Required probability

$P(x \geq 1) = 1 - P(x = 0)$

$= 1 - {}^{10}C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^{10}$

$= 1 - \frac{1}{1024} = \frac{1023}{1024}$

Q31 Text Solution:

Given $P(A)\& = \frac{1}{3}, P(B)$

$= \frac{1}{2}$ and $P(A \cap B) = \frac{1}{6}$

$P(A'/B)\& = \frac{P(A' \cap B)}{P(B)} = \frac{P(B) - P(A \cap B)}{P(B)}$

$= \frac{\frac{1}{2} - \frac{1}{6}}{1/2} = \frac{2}{3}$

Q32 Text Solution:

Given $P(E_1) = P(E_2) = \frac{1}{2}$

$P(E_2/A) = \frac{1}{2}, P(A/E_2) = \frac{2}{3}$

$P(E_1/A) = \frac{P(E_1) \times P(A/E_1)}{P(A)}$

$P(E_2/A) = \frac{P(E_2) \times P(A/E_2)}{P(A)}$

$P(E_1/A) + P(E_2/A) = 1$

$P(E_1/A) = 1 - P(E_2/A)$

$= 1 - \frac{1}{2} = \frac{1}{2}$

Q33 Text Solution:

Given, $P(A) = \frac{1}{2}, P(B) = \frac{1}{4}, P(C) = \frac{1}{3}$

$P(\overline{A}) = \frac{1}{2}, P(\overline{B}) = \frac{3}{4}, P(\overline{C}) = \frac{2}{3}$

Required probability $= P(AB\overline{C})$

$+ P(A\overline{B}C) + P(\overline{A}BC)$

$= \frac{1}{2} \times \frac{1}{4} \times \frac{2}{3} + \frac{1}{2} \times \frac{3}{4} \times \frac{1}{3} + \frac{1}{2} \times \frac{1}{4} \times \frac{1}{3}$

$= \frac{2+3+1}{24} = \frac{6}{24} = \frac{1}{4}$

Q34 Text Solution:

Given, $n(A) = 2$ let $n(B) = n$

Total number of relations from set A to set B is

$2^{2\pi} = 1024$

$2^{2n} = 2^{10} \Rightarrow 2n = 10 \Rightarrow n = 5$

Q35 Text Solution:

We have,

$\sin^2 51^\circ + \sin^2 39^\circ$

$\sin^2 51^\circ + \sin^2(90^\circ - 51^\circ)$

$\sin^2 51^\circ + \cos^2 51^\circ = 1$

Q36 Text Solution:

We have, $\tan A+\cot A=2$

$(\tan A+\cot A)^2=(2)^2$

$\tan^2 A+\cot^2+2=4$

$\tan^2 A+\cot^2 A=2$

$\left(\tan^2 A+\cot^2 A\right)^2=(2)^2$

$\tan^4 A+\cot^4 A+2=4$

$\tan^4 A+\cot^4 A=2$

Q37 Text Solution:

Given set A = {1, 2, 3, 4, 5, 6}

Number of subsets of A which contain at least two elements is

${}^6C_2+{}^6C_3+{}^6C_4+{}^6C_5+{}^6C_6$

$=2^6-{}^6C_0-{}^6C_1=64-1-6=57$

Q38 Text Solution:

Given,

$z=x+iy$

$|z+1|=|z-1||z+1|^2$

$=|z-1|^2|x+iy+1|^2$

$=|x+iy-1|^2(x+1)^2+y^2$

$=(x-1)^2+y^2(x+1)^2-(x-1)^2=0$

$(x+1-x+1)(x+1+x-1)=0$

$x=0$

$\therefore |z+1|=|z-1|$ represents $Y-$axis.

Q39 Text Solution:

We have,

${}^{16}C_9+{}^{16}C_{10}-{}^{16}C_6-{}^{16}C_7$

${}^{16}C_9+{}^{16}C_{10}-{}^{16}C_{10}$

$-{}^{16}C_9[\because {}^nC_r={}^nC_{n-r}]=0$

Q40 Text Solution:

We have,

$(x+y+z)^{10}$

Total number of terms in the expansion of

$(x+y+z)^{10}={}^{10+3-1}C_{3-1}={}^{12}C_2=66$

Q41 Text Solution:

We have,

$P(n)=2^n<n!$

$P(1)=2<1!$ False

$P(2)=2^2<2!$ False

$P(3)=2^3<3!$ False

$P(4)=2^4<4!$ True

$\therefore$ The smallest position integer for which P (n) is true for $n=4$

Q42 Text Solution:

Given lines $lx+my=n$ and $lx+m'y=n'$

Slope of line $lx+my-n=0$ is $-l/m$ and slope of line $lx+m'y-n'=0$ is $-l'/m'$ lines are perpendicular

$\therefore \left(\frac{-l}{m}\right)\left(\frac{-l'}{m'}\right)=-1$

$ll'+mm'=0$

Q43 Text Solution:

x^2 = 4ay

So the parameter is a itself.

Since the parabola passes through the point (2, 1), this point must satisfy the equation.

Substitute x = 2 and y = 1:

$(2)^2$ = 4a(1)

4 = 4a

a = 1

For the parabola x^2 = 4ay the length of the latus rectum is

4a

So,

4a = 4(1) = 4

The correct option is B, 4.

Q44 Text Solution:

Given sum of n terms of an AP.

$S_n=n^2+n$

$a_1=S_1=1+1=2$

$a_1+a_2=S_2=2^2+2=6$

$a_2=S_2-S_1=6-2=4$

$d=a_2-a_1=4-2=2$

Q45 Text Solution:

The negation of the statement "For all real x and y, x + y = y + x" is

For some real number

x and y, $x + y \neq y + x$.

Q46 Text Solution:

Given data 6, 7, 8, 9, 10

$\overline{x} = \frac{6+7+8+9+10}{5} = 8$

$SD = \sqrt{\frac{\Sigma x_i^2}{n} - (\overline{x})^2}$

$SD = \sqrt{\frac{6^2+7^2+8^2+9^2+10^2}{5} - (8)^2}$

$SD = \sqrt{\frac{36+49+64+81+100-320}{5}}$

$SD = \sqrt{\frac{330-320}{5}} = \sqrt{\frac{10}{5}} = \sqrt{2}$

Q47 Text Solution:

We have,

$\lim_{x\to0}\left(\frac{\tan x}{\sqrt{2x+4}-2}\right)$

$= \lim_{x\to0}\frac{(\tan x)\left(\sqrt{2x+4}+2\right)}{(2x+4)-4}$

$= \lim_{x\to0}\frac{\tan x\left(\sqrt{2x+4}+2\right)}{2x}$

$= \frac{1}{2} \times \left(\sqrt{4}+2\right) = \frac{1}{2}\left(2+2\right) = 2$

Q48 Text Solution:

Let

$A = \{1, 2, 3\}$

$R = \{1, 1\}$

$\{2, 2\} \notin R$,

R is not reflexive but R is symmetric and transitive.

Q49 Text Solution:

We have,

$f(x) = x^2 - 4x + 5$

$f(x) = x^2 - 4x + 4 + 1$

$f(x) = (x-2)^2 + 1$

$\therefore$ Range of $f(x)$ is $[1, \infty)$.

Q50 Text Solution:

A, B,

C are mutually exclusive and exhaustive events.

$\therefore P(A \cap B) = P(B \cap C)$

$= P(A \cap C) = 0 = P(A \cap B \cap C)$

and $P(A) + P(B) + P(C) = 1$

$2P(B) + P(B) + \frac{2}{3}P(B) = 1$

$[\because P(A) = 2P(B), 2P(B) = 3P(C)]$

$= \frac{11P(B)}{3} = 1$

$P(B) = \frac{3}{11}$

Q51 Text Solution:

We have, $f(x) = \cos^{-1}\sqrt{x-1}$

$f(x)$ is defined when $\sqrt{x-1} \in [0, 1]$

$\therefore 0 \le \sqrt{x-1} \le 1$

$0 \le x - 1 \le 1$

$1 \le x \le 2$

$\therefore$ Domain of $f(x)$ is $[1, 2]$.

Q52 Text Solution:

We have, $\cos\left(\sin^{-1}\frac{\pi}{3} + \cos^{-1}\frac{\pi}{3}\right)$

$\sin^{-1}x$ is defined $x \in [-1, 1]$ but $\frac{\pi}{3} \notin [-1, 1]$

$\therefore \cos\left(\sin^{-1}\left(\frac{\pi}{3}\right) + \cos^{-1}\left(\frac{\pi}{3}\right)\right)$ does not exists.

Q53 Text Solution:

We have,

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^4 = A^2 \cdot A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

Q54 Text Solution:

We have,

A = {a, b, c}

The number of binary operations on A is $3^{3^2} = 3^9$

Q55 Text Solution:

We have, $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Let $B = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$

$\therefore\ BA = I$

$A = B^{-1}I$

$A = B^{-1}$

$\therefore\ B^{-1} = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$

Hence, $A = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$

Q56 Text Solution:

We have,

$$f\left(x\right) = \begin{vmatrix} x^3 - x & a + x & b + x \\ x - a & x^2 - x & c + x \\ x - b & x - c & 0 \end{vmatrix}$$

$$f\left(0\right) = \begin{vmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{vmatrix}$$

$f(0) = 0$

$f\left(0\right)$ is skew symmetric matrix of order 3

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Q57 Text Solution:

Since B is skew-symmetric, we have

B' = –B

Now consider the matrix A'BA

To find its nature, take its transpose: (A'BA)' = A'B' A

because transpose of a product is taken in reverse order: (XYZ)' = Z'Y'X'

So, (A'BA)' = A'(-B)A (A'BA)' = -A'BA

This shows that the transpose of A'BA is the negative of the matrix itself.

So A'BA is a skew-symmetric matrix.

Hence, the correct option is

Option D: Skew symmetric matrix

Q58 Text Solution:

Given, $|A| = 5$

$|AadjA| = |A|^3$

$|AadjA| = (5)^3 = 125$

Q59 Text Solution:

We have,

$$f\left(x\right) = \begin{cases} \frac{1-\cos kx}{x\sin x} & , x \neq 0 \\ \frac{1}{2} & , x = 0 \end{cases}$$

$f\left(x\right)$ is continuous at $x = 0$

$\therefore \lim\limits_{x\to 0} \frac{1-\cos kx}{x\sin x} = \frac{1}{2}$

$\Rightarrow \lim\limits_{x\to 0} \frac{2\sin^2 \frac{k}{2}x}{x\sin x} = \frac{1}{2}$

$\Rightarrow \lim\limits_{x\to 0} 2\left(\frac{\sin \frac{kx}{2}}{\frac{kx}{2}}\right)^2 \times \lim\limits_{x\to 0} \frac{1}{\frac{\sin x}{x}} \times \frac{k^2}{4} = \frac{1}{2}$

$\Rightarrow 2 \times \frac{k^2}{4} = \frac{1}{2}$

$\Rightarrow k^2 = 1$

$k = \pm 1$

Q60 Text Solution:

$a_1, a_2, a_3, \ldots, a_9$ are in AP.

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$$

Apply $R_1 \to R_1 + R_2 - 2R_2$

$$\begin{vmatrix} a_1 + a_7 - 2a_4 & a_2 + a_8 - 2a_5 & a_3 + a_9 - 2a_6 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$$

$$\begin{vmatrix} 0 & 0 & 0 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$$

$[\because a_1 + a_7 - 2a_4 = a_1 + a_1 + 6d - 2a_1 - 6d = 0$

Similarly $a_2 + a_4 - 2a_5 = 0, a_3 + a_9 - 2a_6 = 0]$

$= 0$

$= \log_e 1 = \log_e\left(\log_e e\right)$