Subject : Mathematics
Class : XI

Kannada

Permutation & Combination

Fundamental Principle of Counting (Counting without actually counting)

If an event can occur in ' m ' different ways, following which another event can occur in ' n ' different ways, then the total number of different ways of

(a) Simultaneous occurrence of both events in a definite order is $m \times n$. This can be extended to any number of events (known as multiplication principle).

(b) Happening of exactly one of the events is $m + n$ (known as addition principle)

Factorial

A Useful Notation : $n! = n(n-1)(n-2).....3 \cdot 2 \cdot 1$;

$n! = n.(n-1)$! where $n \in W$

0! = 1! = 1

$(2n)! = 2^n \cdot n!\left[1 \cdot 3 \cdot 5 \cdot 7 \ldots (2n-1)\right]$

Permutation

(a) ${}^nP_r$ denotes the number of permutations of n different things, taken r at a time $(n \in N, r \in W, n \geq r)$

$${}^nP_r = n(n-1)(n-2)\ldots(n-r+1) = \frac{n!}{(n-r)!}$$

(b) The number of permutations of n things taken all at a time when p of them are similar of one type, q of them are similar of second type, r of them are similar of third type and the remaining $n-(p+q+r)$ are all different is : $\frac{n!}{p!q!r!}$.

(c) The number of permutation of n different objects taken r at a time, when a particular object is always to be included is $r! \cdot {}^{n-1}C_{r-1}$.

(d) The number of permutation of n different object taken r at a time, when repetition be allowed any number of times is $n \times n \times n \ldots r$ times $= n^r$.

(e) (i) The number of circular permutations of n different things taken all at a time is ; $(n-1)! = \frac{{}^nP_n}{n}$.

If clockwise & anti-clockwise circular permutations are considered to be same, then it is $\frac{(n-1)!}{2}$.

(ii) The number of circular permutation of n different things taking r at a time distinguishing clockwise & anticlockwise arrangement is $\frac{{}^nP_r}{r}$.

Combination

(a) ${}^nC_r$ denotes the number of combinations of n different things taken r at a time, and ${}^nC_r = \frac{n!}{r!(n-r)!} = \frac{{}^nP_r}{r!}$

where $r \le n; n \in N$ and $r \in W \cdot {}^nC_r$ is also denoted by $\binom{n}{r}$ or A_r^n

or $C(n, r)$.

(b) The number of combination of n different things taking r at a time.

(i) When p particular things are always to be included $= {}^{n-p}C_{r-p}$.

(ii) When p particular things are always to be excluded $= {}^{n-p}C_r$.

(iii) When p particular things are always to be included and q particular things are to be excluded $= {}^{n-p-q}C_{r-p}$.

(c) Given n different objects, the number of ways of selecting atleast one of them is,
${}^nC_1 + {}^nC_2 + {}^nC_3 + \ldots + {}^nC_n = 2^n - 1$. This can also be stated as the total number of combinations of n distinct things.

(d) (i) Total number of ways in which it is possible to make a selection by taking some or all out of $p + q + r$ ... things, where p are alike of one kind, q a like of a second kind, r alike of third kind and so on is given by : $(p + 1)(q + 1)(r + 1)\ldots - 1$.

(ii) The total number of ways of selecting one or more things from p identical of one kind, q identical things of second kind, r identical things of third kind and n different things is $(p+1)(q+1)(r+1)2^n - 1$.

Divisors

Let $N = p^a \cdot q^b \cdot r^c \ldots$ where $p, q, r \ldots$ are distinct primes and $a, b, c \ldots$ are natural numbers then :

(a) The total numbers of divisors of N including $1 \& N$ is $= (a+1)(b+1)(c+1)\ldots$.

(b) The sum of these divisors is $= \left(p^0 + p^1 + p^2 + \ldots + p^a\right)\left(q^0 + q^1 + q^2 + \ldots + q^b\right)\left(r^0 + r^1 + r^2 + \ldots + r^c\right)\ldots$

(c) Number of ways in which N can be resolved as a product of two factor is $= \frac{1}{2}(a+1)(b+1)(c+1)\ldots$ if N is not a perfect square $\frac{1}{2}\left[(a+1)(b+1)(c+1)\ldots+1\right]$ if N is a perfect square

(d) Number of ways in which a composite number N can be resolved into two factors which are relatively prime (or coprime) to each other is equal to 2^{n-1} where n is the number of different prime factors in N.

Division and Distribution

(a) (i) The number of ways in which $(m + n)$ different things can be divided into two groups containing m and n things respectively is :

$\frac{(m+n)!}{m!n!}(m \ne n)$.

(ii) If $m = n$, it means the groups are equal & in this case the number of subdivision is $\frac{(2n)!}{n!n!2!}$; for in any one way it is possible to inter change the two groups without obtaining a new distribution.

(iii) If $2n$ things are to be divided equally between two persons then the number of ways $= \frac{(2n)!}{n!n!(2!)} \times 2!$.

(b) (i) Number of ways in which $(m+n+p)$ different things can be divided into three groups containing m, n and p things respectively is $\frac{(m+n+p)!}{m!n!p!}, m \neq n \neq p$.

(ii) If $m = n = p$ then the number of groups $= \frac{(3n)!}{n!n!n!3!}$.

(iii) If $3n$ things are to be divided equally among three people then the number of ways in which it can be done is $\frac{(3n)!}{(n!)^3}$. (c) In general, the number of ways of dividing n distinct object into l groups containing p objects each, m groups containing q objects each is equal to $\frac{n!(l+m)!}{(p!)^l (q!)^m l!m!}$.

Here $lp + mq = n$.

(d) Number of ways in which n distinct things can be distributed to p persons if there is no restriction to the number of things received by them = p^n.

(e) Number of ways in which n identical things may be distributed among p persons if each person may receive one, one or more things is; ${}^{n+p-1}C_n$.

Dearrangement

Number of ways in which n letters can be placed in n directed envelopes so that no letter goes into its own envelope is $= n!\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\ldots+(-1)^n\frac{1}{n!}\right]$

Important Result

(a) Number of rectangle of any size in a square of size $n \times n$ is $\sum_{r=1}^{n} r^3$ and number of square of any size is $\sum_{r=1}^{n} r^2$.

(b) Number of rectangle of any size in a rectangle of size $n \times \ p(n<p)$ is $\frac{np}{4}(n+1)(p+1)$ and number of squares of any size is $\sum_{r=1}^{n}(n+1-r)(p+1-r)$.

(c) If there are n points in a plane of which $m(< n)$ are collinear:

(i) Total number of lines obtained by joining these points is ${}^nC_2 - {}^mC_2 + 1$.

(ii) Total number of different triangle ${}^nC_3 - {}^mC_3$.

(d) Maximum number of point of intersection of n circles is ${}^nP_2$ and n lines is ${}^nC_2$.



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