KCET PYQ K (2024)

Kannada

MATHEMATICS

Instructions:

1. The question paper has five parts namely *A, B, C, D* and *E*. Answer all the parts.
2. Part-*A* has **10** multiple choice questions, **5** fill in the blanks and **5** very short answer questions of **1 marks** each.
3. Use the graph sheet for question on linear programming problem in Part *E*.

Part-A

I. Answer all the multiple choice questions: **15×1= 15**

1. The relation R in the set $\{1, 2, 3\}$ given $R = \{(1, 1), (2, 2), (1, 2), (2, 3), (3, 3)\}$ is

(A) Reflexive
(B) Reflexive and Symmetric
(C) Reflexive and Transitive
(D) Symmetric and Transitive

2. If $f : Z \to Z$, where Z is the set of integers is defined as $f(x) = 3x$ then

(A) f is both one-one and onto
(B) f is many one and onto
(C) f is one-one but not onto
(D) f is neither one-one nor onto

3. The principal value of branch of $\sin^{-1} x$ is

(A) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ (B) $(0, \pi)$
(C) $[0, \pi]$ (D) $(0, 2\pi)$

4. If $A=\left[a_{ij}\right]$ is a 2×2 matrix whose elements are given by $a_{ij} = \frac{i}{j}$ then A is

(A) $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & \frac{1}{2} \\ 2 & 1 \end{bmatrix}$
(C) $\begin{bmatrix} 0 & 2 \\ \frac{1}{2} & 0 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 2 \\ \frac{1}{2} & 1 \end{bmatrix}$

5. If A is an invertible matrix of order 2×2 $\det(A) = 5$ then $\det(A^{-1})$ is

(A) 5 (B) $\frac{1}{25}$
(C) $\frac{1}{5}$ (D) 25

6. The function $f:R\to R$ defined as $f(x)=[x]$ m denotes the greatest integer less than or equal to x. For what values of x in the interval $2 < x < 5$ given below $f(x)$ is not differentiable?

(A) 2 and 5
(B) 3 and 5
(C) 4 and 5
(D) 3 and 4

7. If $y=\sin\left(x^2+5\right)$ then $\frac{dy}{dx}$ is

(A) $\cos\left(x^2+5\right)$
(B) $-2x\cos\left(x^2+5\right)$
(C) $\cos\left(x^2+5\right)(2x+5)$
(D) $2\times\cos\left(x^2+5\right)$

8. The maximum value of the function $f(x)=x,\ x\in(1, 2)$ is

(A) 1
(B) do not have maximum value
(C) 3
(D) 2

9. $\int \sec x(\sec x+\tan x)dx$ is

(A) $\sec^2 x+\tan x+c$
(B) $\sec x+\tan x+c$
(C) $\sec x-\tan x+c$
(D) $-\tan x-\sec x+c$

10 $\int e^x(\sin x+\cos x)dx$ is equal to

(A) $e^x\cos x+c$ (B) $e^x\tan x+c$

(C) $e^x\sin x+c$ (D) $-e^x\cos x+c$

11. The projection of the vector $\vec{a}=\hat{i}+3\hat{j}+7\hat{k}$ along x-axis is

(A) 1 (B) 3

(C) 7 (D) 0

12 The unit vector in the direction of $\vec{a}=\hat{i}+\hat{j}+2\hat{k}$ is

(A) $\frac{\hat{i}-\hat{j}-2\hat{k}}{6}$ (B) $\frac{\hat{i}+\hat{j}+2\hat{k}}{\sqrt{6}}$

(C) $\frac{\hat{i}-\hat{j}+2\hat{k}}{6}$ (D) $\frac{\hat{i}+\hat{j}-2\hat{k}}{\sqrt{6}}$

13. If a line makes 90°, 135°, 45° with the x, y and z axis respectively then direction cosines are

(A) $0, \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$ (B) $0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

(C) $1, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$ (D) $1, \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}$

14. Which of the following is a non-negative constraints in a Linear Programming Problem?

(A) $x\geq 0, y\leq 0$ (B) $x\leq 0, y\leq 0$

(C) $x\geq 0, y\geq 0$ (D) $x\leq 0, y\geq 0$

15. If two cards are drawn without replacement from a pack of 52 playing cards then the probability that both the cards are black is

(A) $\frac{1}{26}$ (B) $\frac{1}{4}$

(C) $\frac{25}{104}$ (D) $\frac{25}{102}$

II. Fill in the blanks by choosing the appropriate answer from those given in the bracket: 5 × 1 = 5

$(0, 1, 2, 3, \frac{1}{2}, 6)$

16. The value of $\sin\left(\operatorname{cosec}^{-1}2\right)$ is ……..

17. If A is a square matrix of order 2 × 2 and $|A|=8$ then $\left|\frac{1}{2}A\right|$ is …….

18. The order of the differential equation $\frac{d^3y}{dx^3}+y^2+e^{\frac{dy}{dx}}=0$ is ……….

19. Two lines with direction ratios 1, 3, 5 and 2, K, 10 are parallel then the value of K is ……

20. If F is an event of the sample space S and $P(F)\neq 0$ then $P(S/F)$ is ………..

Part-B

Answer any nine questions: **6 × 2 = 12**

21. Show that $\sin^{-1}\left(2x\sqrt{1-x^2}\right)=2\sin^{-1}x$, $-\frac{1}{\sqrt{2}}\leq x\leq\frac{1}{\sqrt{2}}$.

22. Find the equation of the line joining the points (3, 1) and (9, 3) using determinants.

23. If $2x+3y=\sin y$ then find $\frac{dy}{dx}$.

24. The radius of a circle is increasing at the rate of 0.7 cm/s. What is the rate of increase of its circumference?

25. Find the interval in which the function f given by $f(x)=2x^2-3x$ is decreasing.

26. Find $\int\frac{x}{(x+1)(x+2)}dx$.

27. Evaluate $\int_1^{\sqrt{3}}\frac{dx}{1+x^2}$.

28. Consider two points P and Q with position vector $\overrightarrow{OP} = 3\vec{a} - 2\vec{b}$ and $\overrightarrow{OQ} = \vec{a} + \vec{b}$. Find the position vector of a point R which divides the line joining P and Q internally in the ratio 2:1.

29. Find the angle between the lines $\frac{x}{2} = \frac{y}{2} = \frac{z}{2}$ and $\frac{x-5}{4} = \frac{y-2}{1} = \frac{z-3}{8}$.

30. Two coins are tossed once, where the events E and F are defined as
E : Tail appears on one coin.
F : One coin shows Head
Find $P(E/F)$.

31. Let A and B be 2 independent events such that $P(A) = 0.3$ and $P(B) = 0.6$. Find
(a) $P(A$ and not $B)$
(b) $P($neither A nor $B)$

Part-C

Answer any six questions: **6 x 3 = 18**

32. Show that the relation R in the set of real numbers R defined as $R = \{(a,b) : a \le b\}$ is Reflexive and Transitive but not symmetric.

33. Write $\tan^{-1}\left(\frac{x}{\sqrt{a^2 - x^2}}\right), |x| < a$ in the simplest form.

34. **E**xpress $A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix.

35. Differentiable $(\log x)^{\cos x}$, $x > 0$ with respect to x.

36. Find $\frac{dy}{dx}$ if $x = a(\theta + \sin\theta)$, $y = a(1 = \cos\theta)$.

37. Find two positive numbers x and y such that $x + y = 60$ and xy^3 is maximum.

38. Find $\int x \tan^{-1} x \, dx$,

39. Find the equation of a curve passing through the point (0, 1) and whose differential equation is given by $\frac{dy}{dx} = y \tan x \left(y \ne 0 \text{ and } 0 \le x < \frac{\pi}{2} \right)$.

40. If $\vec{a}, \vec{b}$ and $\vec{c}$ be three vector such that $|\vec{a}| = 3, |\vec{b}| = 4, |\vec{c}| = 5$ and $\vec{a}$ is perpendicular to $(\vec{b} + \vec{c})$, $\vec{b}$ is perpendicular to $(\vec{c} + \vec{a})$ and $\vec{c}$ is perpendicular to $(\vec{a} + \vec{b})$ then find $|\vec{a} + \vec{b} + \vec{c}|$.

41. Find the area of a triangle having the points $A(1, 1, 2)$, $B(2, 3, 5)$ and $C(1, 5, 5)$ as its vertices.

42. In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{4}$ be the probability that he knows the answer and $\frac{1}{4}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{4}$. What is the probability that the student knows the answer given that he answered it correctly?

Part-D,

Answer any four questions 4x5=20

43. Let $f : N \to Y$ be a function defined as $f(x) = 4x + 3$, where $Y = \{y \in N : y = 4x + 3$ for some x, $N)$ Show that f is invertible. Find the inverse of f.

44. If $A = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix}$, $C = \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$ then compute $(A + B)$ and $(B - C)$. Also verify that $A+(B - C)$. Also verify that $A + (B - C) = (A + B) - C$

45. Solve the following system of Linear equations by matrix method:
$x+y+z=6$
$y+3z=11$
$x-2y+z=0$.

46. If $y=Ae^{mx}+Be^{nx}$ then show that $\frac{d^2y}{dx^2}-(m+n)\frac{dy}{dx}+mny=0$.

47. Find the integral of $\frac{1}{x^2+a^2}$ with respect to x and hence find $\int\frac{dx}{x^2+2x+2}$.

48. Find the area enclosed by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ by the method of integration.

49. Find the particular solution of the differential equation $(1+x^2)\frac{dy}{dx}+2xy=\frac{1}{1+x^2}: y=0$ when $x=1$.

50. Derive the equation of a line in space which passes through a given point and parallel to a given vector both in vector and Cartesian form.

Part-E

Answer the following questions:

51. (a) Prove that $\int_0^a f(x)dx=\int_0^a f(a-x)dx$ and hence evaluate $\int_0^{\frac{\pi}{2}}\frac{\sin x-\cos x}{1+\sin x\cos x}dx$.

(6)

OR

(b) Solve the following Linear Programming Problem graphically:
Maximise $Z = 4x + y$ ……….(1)
Subject to the constraints:
$x+y\leq 50$ ……….(2)
$3x+y\leq 90$ ………(3)
$x\geq 0,\ y\geq 0$ ………(4)

52. (a) Show that the matrix $A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$ satisfies the equation $A^2-5A+7I=O$, where I is 2 × 2 identify matrix and O is 2 × 2 zero matrix. Using this equation find A^{-1}.

OR

(b) Find the value of K so that the function f defined as $f(x)=\begin{cases}Kx+1, & if \quad x\leq\pi\\ \cos x, & if \quad x>\pi\end{cases}$ is continuous at $x=\pi$.

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Mathematics

Hint and Solution

1. (a) or Reflexive

2. (c) or writing f is one-one but not onto

3. (a) $[-\frac{\pi}{2}, \frac{\pi}{2}]$

4. (b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

5. (c) $\frac{1}{5}$

6. (d) 3 and 4

7. (d) $2x\cos(x^2+5)$

8. (b) do not have maximum value

9. (b) $\sec x + \tan x + C$

10. (c) $e^x \sin x + C$

11. (a) 1

12. (b) $\frac{\vec{i}+\vec{j}+2\vec{k}}{\sqrt{6}}$

13. (a) $0, \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

14. (c) $x \geq 0, y \geq 0$

15. (d) $\frac{25}{102}$

16. $\frac{1}{2}$

17. 2

18. 3

19. 6

20. 1

21. $x = \sin\theta$ and $\theta = \sin^{-1} x$

$\sin^{-1}(2x\sqrt{1-x^2}) = \sin^{-1}(\sin 2\theta) = 2\theta = 2\sin^{-1} x$

22. $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0 \; OR \; \begin{vmatrix} x & y & 1 \\ 3 & 1 & 1 \\ 9 & 3 & 1 \end{vmatrix} = 0$

$-2x + 6y = 0$ or $-x + 3y = 0$ or $x - 3y = 0$

23. $2 + 3\frac{dy}{dx} = \cos y \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{2}{3-\cos y}$ or $\frac{dy}{dx} = \frac{2}{\cos y - 3}$

24. $\frac{dr}{dt} = 0.7$ or $\frac{dc}{dt} = 2\pi\frac{dr}{dt}$

$\frac{dc}{dt} = 1.4\pi$ cm/s

25. $\frac{dy}{dx}$ or $f'(x) = 4x - 3$

$(-\infty, \frac{3}{4})$

26. $\frac{x}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$

$A = -1$ and $B = 2$

$-\log|x+1| + 2\log|x+2| + C$

27. $\int_1^{\sqrt{3}} \frac{1}{1+x^2} dx = [\tan^{-1} x]_1^{\sqrt{3}}$

$\frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$

28. $\overrightarrow{OR} = \frac{2(\vec{a}+\vec{b}) + 1(3\vec{a} - 2\vec{b})}{2+1}$

$\overrightarrow{OR} = \frac{5\vec{a}}{3}$

29. $\vec{b}_1 = 2\vec{i} + 2\vec{j} + \vec{k},\ \vec{b}_2 = 4\vec{i} + \vec{j} + 8\vec{k}$

$\vec{b}_1 \cdot \vec{b}_2 = 18$ and $|\vec{b}_1| = 3$ and $|\vec{b}_2| = 9$

$\theta = \cos^{-1}(\frac{18}{27})$ or $\theta = \cos^{-1}(\frac{2}{3})$

30. $n(E) = 2, n(F) = 2$ and $n(E \cap F) = 2$

OR

$P(E) = \frac{1}{2},\ P(F) = \frac{1}{2}$ and $P(E \cap F) = \frac{1}{2}$

$P(E|F) = 1$

31. $P(\text{A and not } B) = P(A)P(B') = (0.3)(0.4) = 0.12$

$P(\text{neither } A \text{ nor } B) = P(A')P(B') = (0.7)(0.4) = 0.28$

32. Reflexive: $a \le a \forall a \in R \Rightarrow (a,a) \in R \therefore R$ is reflexive.

By giving any counter example as (1, 2) in $\in R$ but $(2,1) \notin R \Rightarrow R$ is not symmetric.

$(a,b) \in R$ and $(b,c) \in R \Rightarrow a \le b$ and $b \le c \Rightarrow a \le c \Rightarrow (a,c) \in R$ is transitive.

33. $x = a\sin\theta$ OR $\theta = \sin^{-1}\frac{x}{a}$

$\tan^{-1}\left(\frac{x}{\sqrt{a^2 - x^2}}\right) = \tan^{-1}\left(\frac{a\sin\theta}{\sqrt{a^2 - a^2\sin^2\theta}}\right)$

OR

$\tan^{-1}\left(\frac{a\sin\theta}{a\cos\theta}\right)$

$\tan^{-1}(\tan\theta) = \theta = \sin^{-1}\frac{x}{a}$

34. $(A + A') = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix}$ OR $\frac{1}{2}(A + A') = \begin{bmatrix} 1 & \frac{11}{2} \\ \frac{11}{2} & 7 \end{bmatrix}$

$(A - A') = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ OR $\frac{1}{2}(A - A') = \begin{bmatrix} 0 & -\frac{1}{2} \\ \frac{1}{2} & 0 \end{bmatrix}$

$\frac{1}{2}(A + A') + \frac{1}{2}(A - A') = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} = A$

35. $\log y = \cos x \log(\log x)$

$\frac{1}{y}\frac{dy}{dx} = \cos x\left(\frac{1}{\log x}\right)\frac{1}{x} + \log(\log x)(-\sin x)$

$\frac{dy}{dx} = y\left[\frac{\cos x}{x\log x} - \sin x \log(\log x)\right]$

OR

$\frac{dy}{dx} = (\log x)^{\cos x}\left[\frac{\cos x}{x\log x} - \sin x\log(\log x)\right]$

36. $\frac{dx}{d\theta} = a(1 + \cos\theta)$

$\frac{dy}{d\theta} = a\sin\theta$

$\frac{dy}{dx} = \frac{a\sin\theta}{a(1+\cos\theta)}$ OR $\frac{dy}{dx} = \tan\frac{\theta}{2}$

37. $P = xy^3$ and $P = (60 - y)y^3$

OR

$P = x(60 - x)^3$

$\frac{dP}{dy} = 60 \times 3y^2 - 4y^3$

OR

$\frac{dP}{dx} = -3x(60 - x)^2 + (60 - x)^3$

$x=15$ and $y=45$ and showing $\frac{d^2P}{dy^2}<0$

38. $I=\int x\tan^{-1}x\,dx$

$I=\tan^{-1}x\left(\frac{x^2}{2}\right)-\int\frac{x^2}{2}\left(\frac{1}{1+x^2}\right)dx$

$I=\frac{x^2}{2}\tan^{-1}x-\frac{1}{2}\int\frac{1+x^2-1}{1+x^2}dx$

$I=\frac{x^2}{2}\tan^{-1}x-\frac{1}{2}x+\frac{1}{2}\tan^{-1}x+C$

OR,

$I=\frac{1}{2}(x^2+1)\tan^{-1}x-\frac{1}{2}x+C$

39. $\frac{dy}{y}=\tan x\,dx$ OR $\int\frac{dy}{y}=\int\tan x dx$

$\log y=\log(\sec x)+C$

$C=0$, $\log y=\log(\sec x)$ OR $y=\sec x$

40. $|\vec{a}+\vec{b}+\vec{c}|^2=(\vec{a}+\vec{b}+\vec{c})\cdot(\vec{a}+\vec{b}+\vec{c})$

OR

$|\vec{a}+\vec{b}+\vec{c}|^2=\vec{a}\cdot\vec{a}+\vec{b}\cdot\vec{b}+\vec{c}\cdot\vec{c}+\vec{a}\cdot(\vec{b}+\vec{c})+\vec{b}\cdot(\vec{c}+\vec{a})+\vec{c}\cdot(\vec{a}+\vec{b})$

$|\vec{a}+\vec{b}+\vec{c}|^2=|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+0+0+0$

$|\vec{a}+\vec{b}+\vec{c}|^2=|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2=9+16+25=50$

$|\vec{a}+\vec{b}+\vec{c}|=\sqrt{50}=5\sqrt{2}$

41. $\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}=\vec{i}+2\vec{j}+3\vec{k}$

$\overrightarrow{AC}=\overrightarrow{OC}-\overrightarrow{OA}=0\vec{i}+4\vec{j}+3\vec{k}$

$$\overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix}\vec{i}&\vec{j}&\vec{k}\\1&2&3\\0&4&3\end{vmatrix}=-6\vec{i}-3\vec{j}+4\vec{k}$$

Area of $\Delta ABC=\frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AC}|=\frac{\sqrt{61}}{2}$

42. $P(E_1)=\frac{3}{4},P(E_2)=\frac{1}{4}$ and

$P(A|E_1)=1,P(A|E_2)=\frac{1}{4}$

$$P(E_1|A)=\frac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1)+P(E_2)P(A|E_2)}$$

$P(E_1|A)=\frac{12}{13}$

43. $f(x)=y=4x+3$

$g(y)=\frac{y-3}{4}$

Showing $fog(y)=y$, $gof(x)=x$

$f\circ g=I_Y$ and $g\circ f=I_X$ and concluding f is invertible or f^{-1} inverse exists.

$f^{-1}(x)=\frac{x-3}{4}$ or g is the inverse of f.

OR

$f(x_1)=f(x_2)\Rightarrow x_1=x_2$

Hence 'f' is one-one. Concluding that the function is one-one.

$x=\frac{y-3}{4}$ |1| Expressing x in terms of y as part of proving subjectivity.

$\forall y\in R$ there exist $x\in R$ such that $f(x)=y$ hence f is onto | 1 | Proving that the function is surjective (onto) by showing every value in the codomain has a pre-image in the domain.

OR

Proving $f\left(\frac{y-3}{4}\right)=y$ |1| An alternative method to prove surjectivity.

Writing f is invertible and getting $f^{-1}(x)=\frac{x-3}{4}$ | 1 | Concluding that since the function is both one-one and onto, it is invertible, and stating the final inverse function

44. $A+B=\begin{bmatrix}4&1&-1\\9&2&7\\3&-1&4\end{bmatrix}$ | 1 | Performing the matrix addition of A and B.

$B-C=\begin{bmatrix}-1&-2&0\\4&-1&3\\1&2&0\end{bmatrix}$ |1| Performing the matrix subtraction of C from B.

$A+(B-C) = \begin{bmatrix} 0 & 0 & -3 \\ 9 & -1 & 5 \\ 2 & 1 & 1 \end{bmatrix}$ | 1 | Adding the result of *B-C* to matrix *A*.

$(A+B)-C = \begin{bmatrix} 0 & 0 & -3 \\ 9 & -1 & 5 \\ 2 & 1 & 1 \end{bmatrix}$ | 1 | Subtracting matrix *C* from the result of *A+B*.

Showing *A+(B-C)=(A+B)-C* |1| Demonstrating that the associative property holds for matrix addition and subtraction

45. A = $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and B = $\begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$ | 1 |

Correctly identifying the coefficient matrix (*A*), the variable matrix (*X*), and the constant matrix (*B*) from the given system of linear equations.

OR

$|A| = 9 \neq 0$

|1| Calculating the determinant of matrix A and confirming it's not zero, which is a condition for the system to have a unique solution.

$adj(A) = \begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$ | 2 | Finding the adjoint matrix of A. The note below clarifies the marking criteria.

$X = A^{-1}B = \frac{1}{|A|}(adjA)B$

OR

$X = \frac{1}{9}\begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}\begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$ | 1 | Setting up the matrix equation to solve for X using the inverse matrix method.

x = 1, y =2, z =3 | 1 | Correctly solving for the variables x, y, and z.

46. $\frac{dy}{dx} = Ame^{mx} + Bne^{nx}$ | 1 | Correctly differentiating the given function $y = Ae^{mx} + Be^{nx}$ with respect to x.

$\frac{d^2y}{dx^2} = Am^2e^{mx} + Bn^2e^{nx}$ | 1 | Correctly finding the second derivative of the function.

Substituting the values of $\frac{d^2y}{dx^2}, \frac{dy}{dx}\,\frac{dy}{dx}$ and y in

$\frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny$ |1| Substituting the expressions for the derivatives and the original function into the left-hand side of the given differential equation.

LHS: $Am^2e^{mx} + Bn^2e^{nx} - (m+n)(Ame^{mx} + Bne^{nx})$

$+mn(Ae^{mx} + Be^{nx})$ |1| Expanding the substituted expression on the left-hand side.

LHS = 0 |1| Showing that the expanded expression simplifies to zero, confirming the solution to the differential equation.

47. $x = a\tan\theta \Rightarrow \tan^{-1}\left(\frac{x}{a}\right) = \theta$ and $dx = a\sec^2\theta d\theta$ |1|

Performing the correct trigonometric substitution to evaluate the integral.

$\int \frac{dx}{x^2+a^2} = \int \frac{a\sec^2\theta d\theta}{a^2\tan^2\theta + a^2} = \int \frac{a\sec^2\theta d\theta}{a^2(\tan^2\theta+1)} = \int \frac{a\sec^2\theta d\theta}{a^2\sec^2\theta} = \frac{1}{a}\int 1 d\theta$

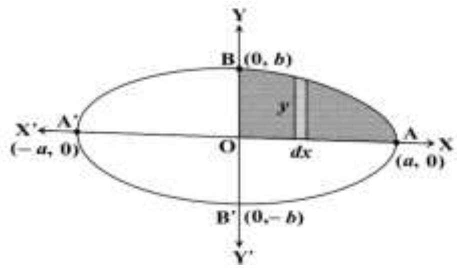
$= \frac{1}{a}(\theta) + c$ |1| Showing the step-by-step evaluation of the integral after substitution.

$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$ |1| Stating the final standard integral formula.

$x^2 + 2x + 2 = (x+1)^2 + 1$ |1| Completing the square for the quadratic expression in the denominator of a new integral.

$\int \frac{dx}{x^2+2x+2} = \tan^{-1}(x+1) + c$ |1| Applying the formula derived in the previous steps to solve the new integral.

48.



| 1 | Drawing an accurate diagram representing the ellipse.

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and identifying the area to be calculated.

$y=\frac{b}{a}\sqrt{a^2-x^2}$ or Area = 4 $\int_0^a ydx$ | 1 | Expressing y in terms of x and setting up the definite integral to calculate the area of the ellipse.

Area = 4 $\int_0^a \frac{b}{a}\sqrt{a^2-x^2}dx$ |1| Combining the previous steps into the integral setup.

Area = $\frac{4b}{a}\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]_0^a$ | 1 | Applying the standard integral formula for $\sqrt{a^2-x^2}$. $Area=\pi ab$ square units | 1 | Evaluating the definite integral to get the final area formula for the ellipse.

49. $\frac{dy}{dx}+\frac{2xy}{1+x^2}=\frac{1}{(1+x^2)^2}$ OR $P=\frac{2x}{1+x^2}$, $Q=\frac{1}{(1+x^2)^2}$ | 1 | Rearranging the given linear differential equation into its standard form, $y'+P(x)y=Q(x)$ and identifying P(x) and Q(x).

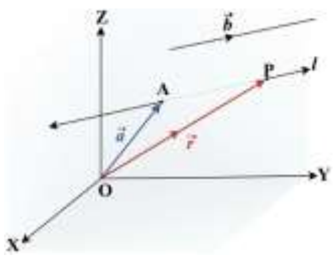
$I.F=e^{\int Pdx}=e^{\int\frac{2x}{1+x^2}dx}=e^{\log(1+x^2)}=1+x^2$

|1| Calculating the Integrating Factor (I.F.) for the linear differential equation.

$y(1+x^2)=\tan^{-1}x+c$ |1| Solving the integral and simplifying the expression to find the general solution.

$C=\frac{-\pi}{4}$ and Writing particular solution $y(1+x^2)=\tan^{-1}x-\frac{\pi}{4}$ Using the initial condition $y(0)=\frac{-\pi}{4}$ to find the value of the constant of integration (C) and stating the final particular solution.

50.



| 1 | Drawing an accurate 3D diagram showing the line passing through points *A* and *P*, the origin *O*, and the position vectors $\vec{a}$ and $\vec{r}$.

$\overrightarrow{AP}=\lambda\vec{b}$ where, λ is a scalar | 1 | Stating the vector equation for the line based on the position vector of point A and the direction vector $\vec{b}$.

$\vec{r}=\vec{a}+\lambda\vec{b}$ | 1 | Deriving the vector equation of the line in parametric form.

$\vec{r}=x\hat{i}+y\hat{j}+z\hat{k},\vec{a}=x_1\hat{i}+y_1\hat{j}+z_1\hat{k}$ and $\vec{b}=a\hat{i}+b\hat{j}+c\hat{k}$ |1|. Expressing the position vectors $\vec{r},\vec{a}$ and $\vec{b}$ in component form.

$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$ | 1 | Deriving the Cartesian equation of the line from its vector equation.

51. (a) Let $I=\int_0^a f(x)dx$ | 1 | Stating the definition of the definite integral to be evaluated.

Putting $x=a-t$, then $dx=-dt$. $x=0\Rightarrow t=a$ | 1 | Substituting $x=a-t$ to apply a property of definite integrals.

$I=-\int_a^0 f(a-t)dt$ | 1 | Rewriting the integral with new limits of integration after substitution.

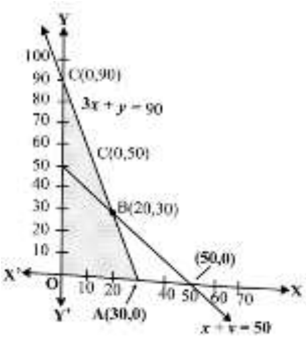
$I = \int_0^a f(a-x)dx$ | 1 | Applying the integral property $\int_a^b f(x)dx = -\int_b^a f(x)dx$ and changing the dummy variable back to x.

$I = \int_0^{\pi/2} \frac{\sin x - \cos x}{1+\sin x \cos x}dx$ and Getting $I = \int_0^{\pi/2} \frac{\cos x - \sin x}{1+\cos x \sin x}dx$ | 1 | Recognizing the given integral matches the form $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ and showing this step for two different expressions derived using this property.

$2I = \int_0^{\pi/2} \frac{\sin x - \cos x + \cos x - \sin x}{1+\sin x \cos x}dx$ | 1 | Adding the two expressions for I derived in the previous step.

$2I = 0 \Rightarrow I = 0$ | 1 | Simplifying the integrand to zero and solving for the value of the integral.

(b)



Corner points (0,0), (30,0), (20,30) and O(0,50)

|1| Correctly identifying the vertices of the feasible region from the graph.

Corner points | Z = -3x+4y | 1 | Creating a table to evaluate the objective function Z at each corner point.

(0,0) | 0 | Evaluated value.

(30,0) | 120 | Evaluated value.

(20,30) | 110 | Evaluated value.

(0,50) | 50 | Evaluated value.

Maximum of Z is 120 at ((30,0) OR showing it in the above second row | 1 | Identifying the maximum value of the objective function and the point at which it occurs.

52. **(a)** $A^2 = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$ | 1 | Calculating the square of matrix A.

$A^2 - 5A + 7I = 0$ | 1 | Showing that substituting matrix A into the given polynomial equation results in the zero matrix.

$7A^{-1} = 5I - A$ or $A^{-1} = \frac{1}{7}(5I - A)$ | 1 | Using the Cayley-Hamilton theorem result from the previous step to express the inverse matrix A^{-1}.

$A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$ OR $A^{-1} = \begin{bmatrix} \frac{2}{7} & -\frac{1}{7} \\ \frac{1}{7} & \frac{3}{7} \end{bmatrix}$

|1| Performing the scalar multiplication and subtraction to find the final form of the inverse matrix.

(b) $\lim_{x\to\pi^-} f(x) = \lim_{x\to\pi^+} f(x) = f(\pi)$ | 1 | Stating the condition for a function to be continuous at a point $(x = \pi)$.

$\lim_{x\to\pi^-} f(x) = k\pi + 1$ | 1 | Calculating the left-hand limit of the piecewise function at $x = \pi$.

$\lim_{x\to\pi^+} f(x) = \cos\pi = -1$ | 1 | Calculating the right-hand limit of the piecewise function at $x = \pi$.

$k = -\frac{2}{\pi}$. | 1 | Equating the left-hand and right-hand limits to solve for the value of constant k.