

MISSION MHT-CET 2027

MATHEMATICS

PYQs BOOKLET

FOR MHT-CET

2027

MHTCET Yearwise PYQ (2020)

Marathi

Mathematics

1. If $\frac{\sin(A+B)}{\sin(A-B)}=\frac{\cos(C+D)}{\cos(C-D)}$, then tan A cot B =

(A) -cot C cot D (B) -tan C tan D
(C) cot C cot D (D) tan C tan D

2. tan 1° × tan 2° × tan 3° × ..... × tan 89° =

(A) 1 (B) $\sqrt{3}$
(C) 2 (D) $\sqrt{2}$

3. If A and B are two angles such that A, B, $\in(0, \pi)$ and they are not supplementary angles such that sin A - sin B = 0, then

(A) A - B = $\frac{\pi}{2}$ (B) A - B = $\frac{\pi}{3}$
(C) $A \neq B$ (D) A = B

4. If A and B are supplementary angles, then $\sin^2\frac{A}{2}+\sin^2\frac{B}{2}=$

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$
(C) 1 (D) 0

5. $\frac{\sin A + \sin 7A + \sin 13A}{\cos A + \cos 7A + \cos 13A}$ =

(A) cot 6A (B) cot 7A
(C) tan 7A (D) tan 6A

6. For $\theta \in (0, \frac{\pi}{2})$, tan 3θ · tan 2θ · tan θ + tan 2θ + tan θ = 1, then θ =

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{6}$
(C) $\frac{\pi}{12}$ (D) $\frac{\pi}{4}$

7. If tan A = $\frac{5}{6}$, tan B = $\frac{1}{11}$, then A + B =

(A) $\frac{\pi}{3}$ (B) $\frac{-\pi}{3}$
(C) $\frac{-\pi}{4}$ (D) $\frac{\pi}{4}$

8. sec 2θ - tan 2θ =

(A) cot($\frac{\pi}{4}$ -θ) (B) cot 2θ
(C) tan 2θ (D) tan($\frac{\pi}{4}$ -θ)

9. cosx·cos7x - cos5x·cos13x =

(A) 2cos²6x·cos12x (B) 2sin6x·sin12x
(C) 2sin6x·cos12x (D) 2sin²6x·cos6x

10. Which of the following have the same value?

(C) sin 120° (B) cos 930°
(C) tan 840° (D) cot(-1110°)
(A) only (C) and (D)
(B) only (A) and (B)
(C) only (A) and (C)
(D) All (A), (B), (C), (D)

11. sin 690° x sec 240° =

(A) $\frac{-1}{2}$ (B) 1
(C) -1 (D) $\frac{1}{2}$

12. If A + B + C = 180°, then the value of tan($\frac{A}{2}$)tan($\frac{B}{2}$) + tan($\frac{B}{2}$)tan($\frac{C}{2}$) + tan($\frac{C}{2}$)tan($\frac{A}{2}$) is

(A) 2 (B) 1
(C) -2 (D) -1

13. In a triangle ABC if $\frac{\sin A - \sin C}{\cos C - \cos A} = \cot B$, then A, B, C are in
(A) Harmonic progression
(B) Geometric progression
(C) Arithmetico-Geometric progression
(D) Arithmetic progression

14. $\tan A + 2\tan 2A + 4\tan 4A + 8\cot 8A =$
(A) $\tan A$ (B) $\tan 2A$
(C) $\cot A$ (D) $\cot 2A$

15. If $\sin\theta = \sin 15° + \sin 45°$, where $0° < \theta < 180°$, then $\theta =$
(A) 45° (B) 60°
(C) 75° (D) 150°

16. The line through the points (1, 4), (–5, 1) intersects the line $4x + 3y – 5 = 0$ in the point
(A) $\left(\frac{5}{3}, \frac{-5}{3}\right)$ (B) (–1, –3)
(C) (2, 1) (D) (–1, 3)

17. If the length of perpendicular drawn from the point (4, 1) on the line $3x – 4y + k = 0$ is 2 units, then the values of k are
(A) –2, 18 (B) –2, –18
(C) –2, 1 (D) 2, –18

18. The equation of a line passing through the point (7, –4) and perpendicular to the line passing through the points (2, 3) and (1,–2) is
(A) $x-2y-15=0$ (B) $x+2y+1=0$
(C) $x-5y-13=0$ (D) $x+5y+13=0$

19. If P_1 and P_2 are the lengths of perpendicular from the origin to the lines $x\sin\theta + y\cos\theta = 5\cos 2\theta$ and $x\,\text{cosec}\,\theta + y\sec\theta - 5 = 0$ respectively, then $p_1^2 + 4p_2^2 =$
(A) 25 (B) 5
(C) $\frac{1}{5}$ (D) $\frac{1}{25}$

20. The distance between the line given by $3x + 4y = 9$ and $6x + 8y = 15$ is
(A) 5 units (B) 0.5 units
(C) 0.3 units (D) 3 units

21. The equations of the lines which make intercepts on the axes whose sum is 8 and product is 15 are
(A) $3x - 5y + 15 = 0, 5x + 3y + 15 = 0$
(B) $3x + 5y - 15 = 0, 5x + 3y - 15 = 0$
(C) $3x + 5y + 15 = 0, 5x + 3y - 15 = 0$
(D) $5x - 3y + 15 = 0, 3x + 5y + 15 = 0$

22. The locus of a point of intersection of two lines $x\sqrt{3} - y = k\sqrt{3}$ and $\sqrt{3}kx + ky = \sqrt{3}$, $k \in R$, describes
(A) an ellipse (B) a hyperbola
(C) a pair of lines (D) a parabola

23. The lines cut X and Y axes at the points A and B respectively. The point (5, 6) divides the line segment AB internally in the ratio 3 : 1, then equation of line is
(A) $2x - 5y = -20$ (B) $2x + y = 16$
(C) $2x - y = 4$ (D) $2x + 5y = 40$

24. The acute angle included between the lines $x\sin\theta - y\cos\theta = 5$ and $x\sin\alpha - y\cos\alpha + 11 = 0$ is
(A) $|\theta - \alpha|$ (B) $\theta + \alpha$
(C) $\frac{\pi}{4}$ (D) $\frac{\pi}{3}$

25. The length of the perpendicular from the point $P(a, b)$ to the line $\frac{x}{a} + \frac{y}{b} = 1$ is
(A) $\left|\frac{\sqrt{a^2+b^2}}{ab}\right|$ units
(B) $\left|\frac{a^2}{\sqrt{a^2+b^2}}\right|$ units
(C) $\left|\frac{b^2}{\sqrt{a^2+b^2}}\right|$ units
(D) $\left|\frac{ab}{\sqrt{a^2+b^2}}\right|$ units

26. If the points A(5, k), B(-3, 1) and C(-7, -2) are collinear, then k =

(A) $\frac{1}{7}$ (B) 7

(C) -7 (D) $-\frac{1}{7}$

27. The acute angle between the lines given by $y-\sqrt{3}x + 1 = 0$ and $\sqrt{3}y-x + 7 = 0$ is

(A) 60° (B) 75°

(C) 45° (D) 30°

28. If (a, -2a), a > 0 is the midpoint of a line segment intercepted between the co-ordinate axes, then the equation of the line is

(A) 2x - y = 4a (B) x - 2y = 5a

(C) 2x - y + 4a = 0 (D) x - 2y + 4a = 0

29. The points A(-a, -b), B(0, 0), C(a, b) and D(a^2, ab) are

(A) vertices of a rectangle

(B) vertices of a parallelogram

(C) vertices of a square

(D) collinear

30. If θ is a parameter, then the parametric equations of the circle $x^2 + y^2 - 6x + 4y - 3 = 0$ are given by

(A) $x = 3 + 4\sin\theta$ and $y = 2 + 4\cos\theta$

(B) $x = 3 + 4\cos\theta$ and $y = -2 + 4\sin\theta$

(C) $x = -3 + 4\sin\theta$ and $y = -2 + 4\cos\theta$

(D) $x = 3 + 4\cos\theta$ and $y = 2 + 4\sin\theta$

31. The cartesian equation of the curve given by $x = 6\cos\theta$, $y = 6\sin\theta$ is

(A) $x^2 + y^2 = 6$ (B) $x^2 + y^2 = 5$

(C) $x^2 + y^2 = 16$ (D) $x^2 + y^2 = 36$

32. Two cards are drawn from a pack of well shuffled 52 playing cards one by one without replacement. Then the probability that both cards are queens is

(A) $\frac{3}{220}$ (B) $\frac{1}{221}$

(C) $\frac{1}{220}$ (D) $\frac{2}{221}$

33. If A and B are two independent events and $P(A)=\frac{3}{5}$, $P(B)=\frac{2}{3}$, then $P(A' \cap B')$ =

(A) $\frac{7}{15}$ (B) $\frac{4}{15}$

(C) $\frac{1}{15}$ (D) $\frac{2}{15}$

34. If A and B are independent events such that odds in favour of A is 2 : 3 and odds against B is 4 : 5, then $P(A \cap B)$ =

(A) $\frac{2}{9}$ (B) $\frac{1}{9}$

(C) $\frac{4}{5}$ (D) $\frac{3}{9}$

35. An urn contains 4 red and 5 white balls. Two balls are drawn one after the other without replacement, then the probability that both the balls are red is

(A) $\frac{4}{9}$ (B) $\frac{2}{9}$

(C) $\frac{5}{6}$ (D) $\frac{1}{6}$

36. If P(A') = 0.6, P(B)= 0.8 and P(B/A) = 0.3, then P(A/B) =

(A) $\frac{3}{20}$ (B) $\frac{3}{4}$

(C) $\frac{9}{20}$ (D) $\frac{7}{20}$

37. Two dice are thrown together. The probability that sum of the numbers is divisible by 2 or 3 is

(A) $\frac{1}{6}$ (B) $\frac{3}{4}$

(C) $\frac{1}{3}$ (D) $\frac{2}{3}$

38. If A and B are independent events and $P(A) = \frac{2}{3}$, $P(B)= \frac{3}{5}$, then $P(A' \cap B)$ =

(A) $\frac{2}{5}$ (B) $\frac{3}{5}$

(C) $\frac{4}{15}$ (D) $\frac{1}{5}$

39. In a single throw of three dice, the probability of getting a sum at least 5 is

(A) $\frac{51}{54}$ (B) $\frac{1}{54}$

(C) $\frac{2}{3}$ (D) $\frac{53}{54}$

40. The letters of the word 'LOGARITHM' are arranged at random. The probability that arrangements starts with vowel and end with consonant is

(A) $\frac{18}{9!}$ (B) $\frac{7!}{9!}$

(C) $\frac{1}{4}$ (D) $\frac{1}{9}$

41. The odds in favour of drawing a king from a pack of 52 playing cards is

(A) 1 : 4 (B) 1 : 12

(C) 4 : 1 (D) 12 : 1

42. Suppose that 5% of men and 0.25% of women have gray hair. A gray hair person is selected at random. If there are equal number of males and females, then the probability that the person selected being men is

(A) $\frac{11}{21}$ (B) $\frac{1}{21}$

(C) $\frac{20}{21}$ (D) $\frac{10}{21}$

43. The centre of the hyperbola $9x^2 - 36x - 16y^2 + 96y - 252 = 0$ is

(A) (–2, -3)

(B) (2, –3)

(C) (–2, 3)

(D) (2, 3)

44. The eccentricity of a rectangular hyperbola is

(A) 2 (B) $2\sqrt{2}$

(C) 1 (D) $\sqrt{2}$

45. If B is end point of minor axis of the ellipse $b^2x^2 + a^2y^2 = a^2b^2$ ($a > b$) and S and S' are foci of ellipse such that $\Delta SBS'$ is an equilateral triangle, then eccentricity e =

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$

(C) $\frac{3}{5}$ (D) $\frac{4}{5}$

46. The eccentricity of the ellipse $y^2 + 4x^2 - 12x + 6y + 14 = 0$ is

(A) $\frac{\sqrt{3}}{2}$ (B) $\frac{1}{\sqrt{3}}$

(C) $\frac{1}{2}$ (D) $\frac{1}{\sqrt{2}}$

47. The length of latus-rectum of the parabola $x^2 + 2y = 8x-7$ is

(A) 8 (B) 2

(C) 6 (D) 4

48. The cartesian co-ordinates of the point on the parabola $y^2 = x$ whose parameter is $-\frac{4}{3}$ are

(A) $(\frac{4}{9}, \frac{4}{3})$ (B) $(\frac{4}{3}, -\frac{4}{3})$

(C) $(\frac{4}{3}, \frac{4}{9})$ (D) $(\frac{4}{9}, -\frac{2}{3})$

49. If foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ ($b^2 < 16$) and the hyperbola $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$ coincide, then the value of b^2 is

(A) 4 (B) 9

(C) 14 (D) 7

50. If $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a joint equation of directrices of the hyperbola $16x^2 - 9y^2 = 144$, then g + f - c =

(A) -81 (B) -25

(C) 81 (D) 25

51. If P(θ) lies on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and S and S' are foci of the hyperbola, the SP . S'P =

(A) $a^2\tan^2\theta - b^2\sec^2\theta$

(B) $a^2\tan^2\theta + b^2\sec^2\theta$

(C) $a^2\sec^2\theta + b^2\tan^2\theta$

(D) $a^2\sec^2\theta - b^2\tan^2\theta$

52. The locus of a point of intersection of two lines $x\sqrt{3}-y=k\sqrt{3}$ and $\sqrt{3}kx+ky=\sqrt{3}$, $k\in R$, describes

(A) a parabola (B) a hyperbola

(C) an ellipse (D) a pair of lines

53. The eccentricity of the ellipse given by the equation $9x^2 + 16y^2 = 144$ is

(A) $\frac{\sqrt{7}}{4}$ (B) $\frac{1}{4}$

(C) $\frac{\sqrt{3}}{4}$ (D) $\frac{\sqrt{5}}{4}$

54. The length of latus rectum of the parabola whose focus is at (1, -2) and directrix is the line x + y + 3 = 0 is

(A) $8\sqrt{2}$ units (B) $2\sqrt{2}$ units

(C) $\sqrt{2}$ units (D) $4\sqrt{2}$ units

55. The co-ordinates of foci of the ellipse $16x^2 + 9y^2 = 144$ are

(A) (±7,0) (B) $\left(0,\pm\sqrt{7}\right)$

(C) $\left(\pm\sqrt{7},0\right)$ (D) (0, ±7)

56. The eccentricity of the hyperbola $16x^2 - 3y^2 - 32x - 12y - 44 = 0$ is

(A) $\sqrt{\frac{19}{3}}$ (B) $\sqrt{\frac{13}{19}}$

(C) $\frac{\sqrt{19}}{3}$ (D) $\frac{13}{\sqrt{19}}$

57. If $|3x - 2| \le 1/2$, then x ∈

(A) (1/2, 5/6) (B) [1/2, 5/6)

(C) [1/2, 5/6] (D) (1/2, 5/6]

58. If f: R → R is given by f(x) = 7x + 8 and $f^{-1}(12) = k/7$, then the value of k is

(A) 1 (B) 7

(C) 4 (D) 8

59. If a function f: R → R is defined by $f(x) = \frac{4x}{5}+3$, then $f^{-1}(x)$ =

(A) $\frac{5(x-3)}{4}$ (B) $\frac{4(x+3)}{5}$

(C) $\frac{4(x-3)}{5}$ (D) $\frac{5(x+3)}{4}$

60. The range of the function $f(x) = \frac{x-3}{5-x}$, $x \ne 5$ is

(A) R - {-1} (B) R - {1}

(C) R - {5} (D) R - {-5}

61. If R = {(a, b) / b = a - 1, a ∈ Z, 5 < a < 9}, then the range of R is

(A) {5, 6, 7}

(B) {5, 6, 7, 8, 9}

(C) {7, 8, 9}

(D) {6, 7, 8}

62. If $f(x) = ax^2 + bx + 2$ and f(1) = 4, f(3) = 38, then a - b =

(A) 8 (B) 2

(C) -2 (D) 15

63. If f: R → R, g: R → R defined by $f(x) = x^2 - 3x + 4$ and g(x) = 2x + 1, then the value of x for which f(x) = (fog) (x) is

(A) -1, $\frac{2}{3}$ (B) 1, $\frac{2}{3}$

(C) 1, $-\frac{2}{3}$ (D) –-1, $-\frac{2}{3}$

64. Domain of the real valued function f(x) = $\frac{x+2}{9-x^2}$ is

(A) R (B) R - {3}

(C) R - {-3, 3} (D) $-3 \le x \le 3$

65. If f(x) = $\frac{4x+7}{7x-4}$, then the value of f{f[f(2)]} =

(A) $\frac{35}{39}$ (B) $\frac{2}{3}$

(C) $\frac{3}{2}$ (D) $\frac{39}{35}$

66. If f: R→R, g: R→R are two functions defined by f(x) = 2x - 3, g(x) = $x^3 + 5$, then $(fog)^{-1}(x)$ =

(A) $\left(\frac{2x+3}{2}\right)^{1/2}$ (B) $\left(\frac{x-7}{2}\right)^{1/3}$

(C) $\left(\frac{x+7}{2}\right)^{1/3}$ (D) $\left(\frac{x-7}{2}\right)^{1/2}$

67. If f: R→R, such that f(x) = $\frac{e^x+e^{-x}}{e^x-e^{-x}}$, then f is

(A) an odd function

(B) a neither even nor odd function

(C) an even function

(D) a periodic function

68. If $[x]^2 - 5[x] + 6 = 0$, where [x] denotes greatest integer function then $x \in$

(A) (2, 4) (B) [2, 4]

(C) [2, 4) (D) (2, 4]

69. If f(x) = $\frac{3x+4}{5x-7}$, $x \ne \frac{7}{5}$ and g(x) = $\frac{7x+4}{5x-3}$, $x \ne \frac{3}{5}$, then (gof) (3) =

(A) 3 (B) -3

(C) $\frac{1}{3}$ (D) $-\frac{1}{3}$

70. Given A = {1, 2, 3, 4, 5}, B = {1, 4, 5}. If R is a relation from A to B such that (x, y) ∈ R with x > y, then range of R is

(A) {1, 4, 5} (B) {4, 5}

(C) {2, 4} (D) {1, 4}

71. The domain of a function f(y) = $\frac{\cos^{-1}(y-5)}{\sqrt{25-y^2}}$ is

(A) (4, 5] (B) (4, 6]

(C) (-5, 5) (D) [4, 5)

72. If $f(x) = 2x^2 + bx + c$, f(0) = 3 and f(2) = 1, then (fof) (1) =

(A) 0 (B) 2

(C) 1 (D) 3

73. If f(x) = $\frac{2x+3}{3x-2}$, $x \ne \frac{2}{3}$, then the function fof is

(A) a constant function

(B) an identity function

(C) an even function

(D) an exponential function

74. For f(x) = [x], where [x] is the greatest integer function, which of the following is true, for every $x \in R$.

(A) [x] + 1 = x

(B) [x] + 1 > x

(C) $[x] + 1 \le x$

(D) [x] + 1 < x

75. The domain of the function f(x) = $\sqrt{x}$ is

(A) R – {0} (B) $R^+ \cup \{0\}$

(C) R (D) R^+

76. The domain and range of the relation R given by R = $\{(x,y) / y=x+\frac{6}{x}, x,y \in N \text{ and } x < 6\}$

(A) Domain = {2, 3}, Range = {5}

(B) Domain = {1, 2}, Range = {5, 7}

(C) Domain = {1, 2, 3, 4}, Range = {5, 7}

(D) Domain = {1, 2, 3, 4, 5}, Range = {7, 5}

77. If $f(x) = \frac{3x+2}{5x-3}$, $x \in R - \{\frac{3}{5}\}$, then

(A) $f^{-1}(x) = -f(x)$

(B) $f[f(x)] = -x$

(C) $f^{-1}(x) = f(x)$

(D) $f^{-1}(x)$ does not exist

78. If $f(x) = \begin{cases} 6\beta - 3\alpha x, & \text{if } -4 \le x < -2 \\ 4x+1, & \text{if } -2 \le x \le 2 \end{cases}$ is continuous on [-4, 2], then $\alpha + \beta =$

(A) $\frac{-7}{6}$ (B) $\frac{-4}{7}$

(C) $\frac{4}{7}$ (D) $\frac{7}{6}$

79. If $f(x) = \begin{cases} \frac{4\sin \pi x}{5x}, & \text{for } x \neq 0 \\ 2k, & \text{for } x = 0 \end{cases}$ is continuous at $x = 0$, then the value of k is

(A) $\frac{4\pi}{5}$ (B) $\frac{\pi}{5}$

(C) $\frac{2\pi}{5}$ (D) $\frac{\pi}{10}$

80. If $f(x) = \begin{cases} \frac{|x-2|}{x-2}, & \text{for } x \neq 2 \\ 1, & \text{for } x = 2, \end{cases}$ Then which of the following statements is true?

(A) $\lim_{x \to 2^+} f(x) = \lim_{x \to 2^-} f(x)$

(B) f(x) is discontinuous at x = 2

(C) $\lim_{x \to 2} -f(x) = f(2)$

(D) f(x) is continuous at x = 2

81. The points of discontinuity of the function f(x) = $\begin{cases} \frac{1}{x-1}, & \text{if } 0 \le x \le 2 \\ \frac{x+5}{x+3}, & \text{if } 2 < x \le 4, \end{cases}$ in its domain are

(A) $x = 2$ only

(B) $x = 1$, x = 2

(C) $x = 0$, x = 2

(D) $x = 4$ only

82. If $f(x) = \frac{2^x - 1}{1 - 3^x}$, for $x \neq 0$ is continuous at x = 0, then f(0) =

(A) log 3

(B) -log 2

(C) $\frac{-(log 2)}{(log 3)}$

(D) $\frac{(log 2)}{(log 3)}$

83. The odds in favour of getting sum multiple of 3 , when pair of dice are thrown is

(A) 4 : 5 (B) 3 : 4

(C) 2 : 3 (D) 1 : 2

84. If X is *a.r.v.* with *c.d.f* $F(x)$ and its probability distribution is given by

$X = x$	–1.5	–0.5	0.5	1.5	2.5
$P(X = x)$	0.05	0.2	0.15	0.25	0.35

then, $F(1.5) - F(-0.5) =$

(A) 0.2 (B) 0.4

(C) 0.1 (D) 0.3

85. The odds in favour of drawing a king from a pack of 52 playing cards is

(A) 1 : 4 (B) 4 : 1

(C) 12 : 1 (D) 1 : 12

86. Out of 100 people selected at random, 10 have common cold. If five persons selected at random from the group, then the probability that at most one person will have common cold is

(A) 0.9851 (B) 0.9185

(C) 0.9245 (D) 0.9254

87. The pdf of a continuous r.v. X is given by $f(x) = \frac{x}{8}$, $0 < x < 4$, = 0, otherwise, then $P(X \le 2)$ is

(A) $\frac{1}{4}$ (B) $\frac{5}{16}$

(C) $\frac{9}{16}$ (D) $\frac{7}{16}$

88. If a die is thrown at random, then the expectation of the number on it is

(A) 2.4 (B) 3.3
(C) 2.1 (D) 3.5

89. The probability that bomb will miss the target is 0.2. Then, the probability that out of 10 bombs dropped exactly 2 will hit the target is

(A) $\frac{288}{5^9}$ (B) $\frac{144}{5^{10}}$
(C) $\frac{288}{5^{10}}$ (D) $\frac{144}{5^9}$

90. The letters of the word 'LOGARITHM' are arranged at random. The probability that arrangement starts with vowel and end with consonant is

(A) $\frac{1}{9}$ (B) $\frac{7!}{9!}$
(C) $\frac{18}{9!}$ (D) $\frac{1}{4}$

91. The *p.d.f* of *c.r.v* X is given by $f(x)=\begin{cases}\frac{x+2}{18} & , \text{ if } -2<x<4 \\ 0 & , \text{ otherwise}\end{cases}$ then $P(|x|<1)=$

(A) $\frac{2}{9}$ (B) $\frac{4}{9}$
(C) $\frac{1}{9}$ (D) $\frac{1}{18}$

92. If the *p.m.f* of *a. r.v. X* is given by $P(X=x)=\frac{^5C_x}{2^5}$ if $x=0,1,2,.....5=0$ otherwise, then which of the following is not true?

(A) $P(X\le 2)=P(X\ge 3)$
(B) $P(X\le 1)=P(X\ge 4)$
(C) $P(X\le 2)\ge P(X\ge 4)$
(D) $P(X\le 3)\le P(X\ge 3)$

93. If $f(x)=\frac{1-\sin x+\cos x}{1+\sin x+\cos x}$, for $x\neq\pi$ is continuous at $x=\pi$, then $f(\pi)=$

(A) 1 (B) -1
(C) 0 (D) 2

94. If $f(x)=\frac{|x|}{x}$, for $x\neq 0=1$, for $x=0$, then the function is

(A) differentiable but not continuous at x = 0
(B) continuous and differentiable at x = 0
(C) neither continuous nor differentiable at x = 0
(D) continuous but not differentiable at x = 0

95. The function $f(x)=\frac{x+1}{9x+x^3}$ is

(A) discontinuous at exactly two points
(B) discontinuous at exactly one point
(C) continuous for all real values of x
(D) discontinuous at exactly three points

96. The points of discontinuity of the function $f(x)=\frac{1}{x-1}$, if $0\le x\le 2$ $=\frac{x+5}{x+3}$, if $2<x\le 4$ in its domain are

(A) x = 1, x = 2 (B) x = 0, x = 2
(C) x = 2 only (D) x = 4 only

97. Let p: $\exists$ n $\in$ N such that n + 5 > 10 q: $\forall$n $\in$ N, n^2 + n is an even number while n^2-n is an odd number. The Truth values of p and q are respectively.

(A) T, F (B) T, T
(C) F, T (D) F, F

98. Which of the following statement pattern is a tautology?

$S_1 \equiv \sim p \rightarrow (q \leftrightarrow p)$,
$S_2 \equiv \sim p \vee \sim q$
$S_3 \equiv (p \rightarrow q) \wedge (q \rightarrow p)$,
$S_4 \equiv (p \rightarrow q) \vee (\sim p \leftrightarrow q)$

(A) S_2 (B) S_3
(C) S_1 (D) S_4

99. The negation of the statement 'If $5 < 7$ and $7 > 2$, then $5 > 2$' is
(A) $5 < 7$ and $7 > 2$ or $5 < 2$
(B) $5 < 7$ and $7 > 2$ and $5 > 2$
(C) $5 < 7$ and $7 > 2$ or $5 \leq 2$
(D) $5 < 7$ and $7 > 2$ and $5 \leq 2$

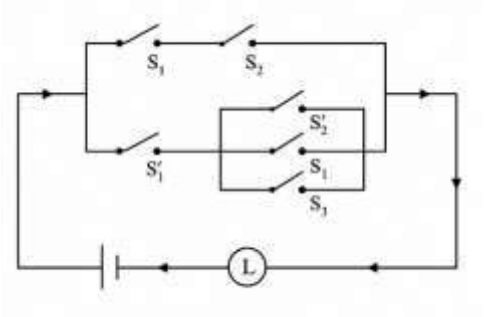
100. The dual of statement 'Mangoes are delicious but expensive' is
(A) Mangoes are delicious or Mangoes are expensive
(B) Mangoes are delicious or Mangoes are not expensive
(C) mangoes are not delicious and mangoes are not expensive
(D) mangoes are delicious and Mangoes are expensive

101. The negation of the statement pattern $\sim p \vee (q \rightarrow \sim r)$ is
(A) $p \wedge (q \wedge r)$ (B) $p \vee (q \wedge r)$
(C) $\sim p \wedge (q \wedge r)$ (D) $p \rightarrow (q \wedge \sim r)$

102. If $A = \{2, 3, 4, 5, 6\}$, then which of the following statement has truth value 'false'
(A) $\exists x \in A$, such that $x + 2$ is a prime number
(B) $\exists x \in A$, such that $x^2 + 1$ is an even number
(C) $\forall x \in A$, $x + 6$ is divisible by 2
(D) $\exists x \in A$, such that $(x - 2) \in N$

103. The logical expression $[p \wedge (q \vee r)] \vee [(\sim p \wedge q) \vee (\sim p \wedge r)]$ is equivalent to
(A) q (B) $p \wedge r$
(C) p (D) $q \vee r$

104. The symbolic from of the following circuit is



(where *p*, *q* and *r* represents switches S_1, S_2 and S_3 which are closed respectively)
(A) $(p \wedge q) \vee \sim p \vee [\sim p \vee p \vee r] = 1$
(B) $[(p \vee q) \wedge \sim p] \vee [\sim p \vee (q \vee r)] = 1$
(C) $(p \wedge q) \vee [\sim p \wedge (\sim q \vee p \vee r)] = 1$
(D) $(p \vee q) \vee [\sim p \vee (\sim q \wedge p \wedge r)] = 1$

105. The negation of the statement, $\exists x \in A$ such that $x + 5 > 8$ is
(A) $\forall x \in A$ such that $x + 5 \leq 8$
(B) $\forall x \in A$ such that $x + 5 > 8$
(C) $\exists x \in A$ such that $x + 5 < 8$
(D) $\forall x \in A$ such that $x + 5 \geq 8$

106. Which of the following statement pattern is a contradiction?
$S_1 = (p \rightarrow q) \wedge (p \wedge \sim q)$
$S_2 = [p \wedge (p \rightarrow q)] \rightarrow q$
$S_3 = (p \vee q) \rightarrow \sim p$
$S_4 = [p \wedge (p \rightarrow q)] \leftrightarrow q$
(A) S_3 (B) S_4
(C) S_2 (D) S_1

107. The contrapositive of the statement 'If Raju is courageous, then he will join Indian Army', is
(A) If Raju does not join Indian Army, then he is not courageous.
(B) If Raju join Indian Army, then he is not courageous
(C) If Raju join Indian Army, then he is courageous.
(D) If Raju does not join Indian Army, then he is courageous.

108. If the symbolic form of the switching circuit is $[(\sim p \vee (p \wedge \sim q)] \vee q$, then the current flows through the circuit only if
(A) both switches should be closed
(B) irrespective of status of the switches
(C) One switch should be open and other should be closed
(D) both switches should be open

109. The verbal statement of the same meaning, of the statement 'If the grass is green then it rains in July' is
(A) The grass is not green and it does not rains in July.
(B) The grass is not green or it rains in July
(C) The grass is not green if and only if it rains in July
(D) If the grass is not green, then it does not rain in July

110. If p : Seema is fat. q : She is happy, then the logical equivalent statement of 'If Seema is fat, then she is happy' is
(A) Seema is fat and she is happy.
(B) Seema is not fat or she is happy
(C) Seema is fat or she is happy
(D) Seema is not fat or she is unhappy

111. If ω is a complex cube root of unity and $A = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix}$, then $A^{-1} =$
(A) -A (B) 2A
(C) A^2 (D) A

112. If $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -1 \end{bmatrix}$, then $A^4A^{-1} =$

(A) $\begin{bmatrix} -4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (B) $\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & -1 \end{bmatrix}$

(C) $\begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 8 & 0 & 0 \\ 0 & -8 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

113. If $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ -\sin\theta & -\cos\theta \end{bmatrix}$, then $A^{-1} =$

(A) $\begin{bmatrix} -\cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ (B) $\begin{bmatrix} -\sin\theta & -\cos\theta \\ -\cos\theta & \sin\theta \end{bmatrix}$

(C) $\begin{bmatrix} \cos\theta & -\sin\theta \\ -\sin\theta & -\cos\theta \end{bmatrix}$ (D) $\begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & -\sin\theta \end{bmatrix}$

114. If $A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, then $[\text{adj}(\text{adj } A)]^{-1} =$
(A) I (B) 2A
(C) A^2 (D) A^{-1}

115. If the elements of matrix A are the reciprocals of elements of matrix $\begin{bmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{bmatrix}$, where ω is complex cube root of unity, then
(A) $A^{-1} = A^2$ (B) $A^{-1} = A$
(C) $A^{-1} = I$ (D) A^{-1} does not exist

116. The sum of the cofactors of the elements of second row of the matrix $\begin{bmatrix} 1 & 3 & 2 \\ -2 & 0 & 1 \\ 5 & 2 & 1 \end{bmatrix}$ is
(A) 23 (B) 5
(C) 3 (D) –23

117. In a triangle ABC with usual notations, $\frac{\cos A - \cos C}{a - c} + \frac{\cos B}{b} =$
(A) $\frac{1}{b}$ (B) $\frac{-1}{b}$
(C) $\frac{2}{b}$ (D) $\frac{-2}{b}$

118. In a triangle ABC with usual notations, if tan *A*, tan *B*, tan *C* are in H.P., then a^2, b^2, c^2 are in
(A) H.P. (B) A.P.
(C) G.P. (D) Not in A.P.

119. With usual notations, in triangle ABC, $a = \sqrt{3}+1$, $b = \sqrt{3}-1$ and $m\angle C = 60°$, then A-B=
(A) 60° (B) 45°
(C) 90° (D) 30°

120. The general solution of $\frac{1-\cos 2x}{1+\cos 2x}=3$ is

(A) $x=n\pi\pm\frac{\pi}{6}, n\in Z$

(B) $x=2n\pi\pm\frac{\pi}{3}, n\in Z$

(C) $x=2n\pi\pm\frac{\pi}{6}, n\in z$

(D) $x=n\pi\pm\frac{\pi}{3}, n\in z$

121. If $\vec{a}$ and $\vec{b}$ are non-collinear vectors. If $\vec{p}=(2x+1)\vec{a}-\vec{b}$ and $\vec{q}=(x-2)\vec{a}+\vec{b}$ are collinear vectors, then x =

(A) $\frac{1}{3}$ (B) 3

(C) $-\frac{1}{3}$ (D) -3

122. If $\vec{a}=2\hat{i}+3\hat{j}-\hat{k}, \vec{b}=-\hat{i}+2\hat{j}-4\hat{k}, \vec{c}=\hat{i}+\hat{j}+\hat{k}$, then $(\vec{a}\times\vec{b})\cdot(\vec{a}\times\vec{c})=$

(A) –74 (B) 64

(C) 74 (D) –64

123. If $\vec{u}=\hat{i}-2\hat{j}+\hat{k}, \vec{v}=3\hat{i}+\hat{k}$ and $\vec{w}=\hat{j}-\hat{k}$, then the volume of the parallelopiped with $\vec{u}\times\vec{v}, \vec{u}+\vec{w}$ and $\vec{v}+\vec{w}$ as coterminus edges is

(A) 12 cubic units (B) 24 cubic units

(C) 20 cubic units (D) 10 cubic units

124. If $\overline{AB}=3\hat{i}+5\hat{j}+4\hat{k}$, $\overline{AC}=5\hat{i}-5\hat{j}+2\hat{k}$ represent the sides of triangle ABC, then the length of median through A is

(A) 6 units (B) 5 units

(C) $\sqrt{6}$ units (D) $\sqrt{5}$ units

125. If $A(-1, 2, 3)$, $B(3, -2, 1)$, $C(2, 1, 3)$ and $D(-1, -2, 4)$ are the vertices of a tetrahedron, then its volume is

(A) $\frac{31}{6}$ cu. Units (B) $\frac{16}{31}$ cu. units

(C) $\frac{16}{3}$ cu. units (D) $\frac{13}{6}$ cu. units

126. If the vectors $\left(2\hat{i}-q\hat{j}+3\hat{k}\right)$ and $\left(4\hat{i}-5\hat{j}+6\hat{k}\right)$ are collinear, then the value of q is

(A) $-\frac{5}{2}$ (B) $\frac{5}{2}$

(C) $-\frac{2}{5}$ (D) $\frac{2}{5}$

127. If the vectors $\bar{a}, \bar{b}, \bar{c}$ are non-coplanar, then $\frac{\left[\bar{a}+2\bar{b}\ \ \bar{b}+2\bar{c}\ \ \bar{c}+2\bar{a}\right]}{\left[\bar{a}\ \bar{b}\ \bar{c}\right]}=$

(A) 3 (B) 9

(C) 8 (D) 6

128. The direction cosines of a line which lies in ZOX plane and makes an angle of 30° with Z-axis are

(A) $0, \frac{\sqrt{3}}{2}, \pm\frac{1}{2}$ (B) $\frac{\sqrt{3}}{2}, 0, \pm\frac{1}{2}$

(C) $0, \frac{1}{2}, \pm\frac{\sqrt{3}}{2}$ (D) $\pm\frac{1}{2}, 0, \frac{\sqrt{3}}{2}$

129. If $\left[\vec{a}\vec{b}\vec{c}\right]=3$, then the volume of the parallelopiped with $2\vec{a}+\vec{b}$, $2\vec{b}+\vec{c}$, $2\vec{c}+\vec{a}$ as coterminus edges is

(A) 15 cubic units (B) 25 cubic units

(C) 22 cubic units (D) 27 cubic units

130. If $\vec{a}=\hat{i}+5\hat{k}$, $\vec{b}=2\hat{i}+3\hat{k}$, $\vec{c}=4\hat{i}-\hat{j}+2\hat{k}$ and $\vec{d}=\hat{i}-\hat{j}$, then $(\vec{c}-\vec{a})\cdot(\vec{b}\times\vec{d})=$

(A) 12 (B) 30

(C) 10 (D) 20

131. If $\vec{a}=2\hat{i}-\hat{j}+\hat{k}$, $\vec{b}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{c}=3\hat{i}+\lambda\hat{j}+5\hat{k}$ are coplanar, then λ is the root of the equation

(A) $x^2+3x=6$

(B) $x^2+2x=4$

(C) $x^2+3x=4$

(D) $x^2+2x=6$

132. If the vectors $\hat{i}+\hat{j}+\hat{k}$, $\hat{i}-\hat{j}+\hat{k}$ and $2\hat{i}+3\hat{j}+m\hat{k}$ are coplanar, then m =

(A) -3 (B) 3

(C) 2 (D) -2

133. If $\left[\bar{a}\bar{b}\bar{c}\right]\neq 0$, then $\dfrac{\left[\bar{a}+\bar{b}\,\bar{b}+\bar{c}\,\bar{c}+\bar{a}\right]}{\left[\bar{b}\,\bar{c}\,\bar{a}\right]}=$

(A) 0 (B) 4

(C) 1 (D) 2

134. For any non-zero vectors $\bar{a}$ and $\bar{b}$, $\left[\bar{b}\,\bar{a}\times\bar{b}\,\bar{a}\right]=$

(A) $\left|\bar{a}\times\bar{b}\right|$ (B) 0

(C) $\left|\bar{a}\times\bar{b}\right|^2$ (D) $\bar{a}\times\bar{b}$

135. Let *G* be the centroid of a triangle *ABC* and *O* be any other point in that plane, then $\overline{OA}+\overline{OB}+\overline{OC}+\overline{OG}=$

(A) $\bar{O}$ (B) $4\overline{OG}$

(C) $3\overline{OG}$ (D) $2\overline{OG}$

136. If $\vec{a},\vec{b},\vec{c}$ are non-coplanar vectors and $\left(\vec{a}+\vec{b}+\vec{c}\right)\cdot\left[\left(\vec{a}\times\vec{b}\right)+\left(\vec{b}\times\vec{c}\right)+\left(\vec{c}\times\vec{a}\right)\right]=k\left[\vec{a}\vec{b}\vec{c}\right]$, then value of k is

(A) 3 (B) 2

(C) 4 (D) 1

137. The equation of the line passing through the point (1, 2, 3) and perpendicular to the lines $\dfrac{x-1}{1}=\dfrac{y-2}{2}=\dfrac{z-3}{3}$ and $\vec{r}=\lambda\left(-3\hat{i}+2\hat{j}+5\hat{k}\right)$ is

(A) $\vec{r}=\left(\hat{i}+2\hat{j}+3\hat{k}\right)+\lambda\left(2\hat{i}-7\hat{j}+4\hat{k}\right)$

(B) $\vec{r}=\left(\hat{i}+2\hat{j}+3\hat{k}\right)+\lambda\left(2\hat{i}+7\hat{j}-4\hat{k}\right)$

(C) $\vec{r}=\left(\hat{i}+2\hat{j}+3\hat{k}\right)+\lambda\left(2\hat{i}-7\hat{j}-4\hat{k}\right)$

(D) $\vec{r}=\left(\hat{i}+2\hat{j}+3\hat{k}\right)+\lambda\left(2\hat{i}+7\hat{j}+4\hat{k}\right)$

138. The distance of a point (1, 2, -1) from the plane x - 2y + 4z + 10 = 0 is

(A) $\dfrac{\sqrt{3}}{7}$ units (B) $\sqrt{\dfrac{7}{3}}$ units

(C) $\dfrac{3}{\sqrt{7}}$ units (D) $\sqrt{\dfrac{3}{7}}$ units

139. The co-ordinates of the point where the line $\dfrac{x-1}{2}=\dfrac{y-2}{-3}=\dfrac{z+3}{4}$ meets the plane 2x + 4y - z = 1 are

(A) (3, -1, 1) (B) (-2, 1, -1)

(C) (3, -1, -1) (D) (3, 1, -1)

140. The equations of planes parallel to the plane x + 2y + 2z + 8 = 0, which are at a distance of 2 units from the point (1, 1, 2) are

(A) x + 2y + 2z - 6 = 0 or x + 2y + 2z - 7 = 0

(B) x + 2y + 2z - 13 = 0 or x + 2y + 2z - 1 = 0

(C) x + 2y + 2z - 5 = 0 or x + 2y + 2z - 3 = 0

(D) x + 2y + 2z + 3 = 0 or x + 2y + 2z - 5 = 0

141. The equation of a plane containing the point (1, -1, 2) and perpendicular to the planes 2x + 3y - 2z = 5 and x + 2y - 3z = 8 is

(A) $\vec{r}.\left(5\hat{i}-4\hat{j}-\hat{k}\right)=7$

(B) $\vec{r}.\left(5\hat{i}+4\hat{j}-\hat{k}\right)=7$

(C) $\vec{r}.\left(5\hat{i}+4\hat{j}+2\hat{k}\right)=5$

(D) $\vec{r}.\left(4\hat{i}-5\hat{j}+3\hat{k}\right)=15$

142. The equation of a line passing through the point (2, 4, 6) and parallel to the line 3x + 4 = 4y - 1 = 1 - 4z is

(A) $\frac{x-2}{-4}=\frac{y-4}{3}=\frac{z-6}{-3}$

(B) $\frac{x-2}{4}=\frac{y-4}{3}=\frac{z-6}{3}$

(C) $\frac{x-2}{4}=\frac{y-4}{3}=\frac{z-6}{-3}$

(D) $\frac{x-2}{-4}=\frac{y-4}{-3}=\frac{z-6}{-3}$

143. The equation of a plane containing the point (1, -1, 1) and parallel to the plane 2x + 3y - 4z = 17 is

(A) $\vec{r}\cdot\left(2\hat{i}+3\hat{j}-4\hat{k}\right)=-5$

(B) $\vec{r}\cdot\left(3\hat{i}+4\hat{j}-2\hat{k}\right)=-3$

(C) $\vec{r}\cdot\left(4\hat{i}+3\hat{j}-4\hat{k}\right)=-3$

(D) $\vec{r}\cdot\left(2\hat{i}+3\hat{j}-4\hat{k}\right)=-15$

144. The equation of the plane passing through the points (2, 3, 1), (4, –5, 3) and parallel to Y-axis is

(A) x + z = 1

(B) x + z = 3

(C) z - x + 2 = 0

(D) x - z = 1

145. The length of the perpendicular to the plane $\vec{r}\cdot\left(\hat{i}-2\hat{j}+3\hat{k}\right)=14$ from the origin is

(A) 7 units (B) $\sqrt{14}$ units

(C) 14 units (D) $\sqrt{7}$ units

146. If the line $\vec{r}=\left(\hat{i}-2\hat{j}+3\hat{k}\right)+\lambda\left(2\hat{i}+\hat{j}+2\hat{k}\right)$ is parallel to the plane $\vec{r}\cdot\left(3\hat{i}-2\hat{j}-m\hat{k}\right)=5$, then value of m is

(A) 2 (B) -3

(C) 3 (D) -2

147. If the planes 2x – 5y + z = 8 and $2\lambda x-15y+\lambda z+6=0$ are parallel to each other, then value of λ is

(A) 2 (B) $\frac{1}{3}$

(C) 3 (D) -3

148. If the lines given by $\vec{r}=2\hat{i}+\lambda\left(\hat{i}+2\hat{j}+m\hat{k}\right)$ and $\vec{r}=\hat{i}+\mu\left(2\hat{i}+\hat{j}+6\hat{k}\right)$ are perpendicular, then the value of m is

(A) $\frac{2}{3}$ (B) $\frac{-2}{3}$

(C) $\frac{-3}{2}$ (D) $\frac{3}{2}$

149. If the lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$ intersect, then k =

(A) $\frac{2}{9}$ (B) $\frac{-2}{9}$

(C) $\frac{9}{2}$ (D) $\frac{-9}{2}$

150. The cartesian equation of the plane passing through the point (0, 7, –7) and containing the line $\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z+2}{1}$ is

(A) x + 2y + z = 7 (B) 2x + y + z = 0

(C) 2x + y - z = 14 (D) x + y + z = 0

151. The L.P.P. to maximize Z = x + y, subject to x + y ≤ 1, 2x + 2y ≥ 6, x ≥ 0, y ≥ 0 has

(A) no solution (B) one solution

(C) two solutions (D) infinite solutions

152. If L.P.P. has optimum solutions at two consecutive corner points of feasible region, then L.P.P. has

(A) no solution (B) two solutions

(C) unique solution (D) infinite solutions

153. The feasible region of L.P.P. Maximize Z = 70x + 50y subject to 8x + 5y ≤ 60, 4x + 5y ≤ 40 and x ≥ 0, y ≥ 0 is

(A) a triangle (B) a pentagon

(C) a square (D) a quadrilateral

154. The L.P.P. to minimize $Z = x + y$, subject to $x + y \le 30$, $x \le 15$, $y \le 20$, $x + y \ge 15$, $x, y \ge 0$ has
(A) unbounded solutions
(B) infinite solutions
(C) no solution
(D) a unique solution

155. The straight lines represented by the equation $9x^2 - 12xy + 4y^2 = 0$ are
(A) coincident (B) perpendicular
(C) intersect at 60° (D) parallel

156. If the equation $kxy + 5x + 3y + 2 = 0$ represents a pair of lines, then $k =$
(A) $\frac{15}{2}$ (B) 15
(C) $0, -\frac{15}{2}$ (D) $1, \frac{15}{2}$

157. If $(a, -2a), a > 0$ is the mid-point of a line segment intercepted between the co-ordinate axes, then the equation of the line is
(A) $x - 2y + 4a = 0$ (B) $2x - y = 4a$
(C) $x - 2y = 5a$ (D) $2x - y + 4a = 0$

158. If the angle between the lines given by the equation $x^2 - 3xy + \lambda y^2 + 3x - 5y + 2 = 0, \lambda \ge 0$, is $\tan^{-1}\left(\frac{1}{3}\right)$, then λ =
(A) $1, \frac{2}{5}$
(B) 10
(C) 2
(D) $\frac{2}{3}, 40$

159. If $x = \log t$, and $y + 1 = \frac{1}{t}$, then $e^{-x}\frac{d^2x}{dy^2} + \frac{dx}{dy} =$
(A) 2 (B) -1
(C) 0 (D) 1

160. If $\tan u = \sqrt{\frac{1-x}{1+x}}$, $\cos v = 4x^3 - 3x$, then $\frac{du}{dv} =$
(A) 2 (B) $\frac{1}{6}$
(C) 1 (D) $\frac{1}{2}$

161. If $y = \left(\frac{x^2}{x+1}\right)^x$ and $\frac{dy}{dx} = y\left[g(x) + \log\left(\frac{x^2}{x+1}\right)\right]$, then $g(x) =$
(A) $\frac{x^2}{x+1}$ (B) $\frac{x-1}{x+2}$
(C) $\frac{x+2}{x+1}$ (D) $x\log\left(\frac{x^2}{x+1}\right)$

162. If $x^2 + y^2 = t + \frac{1}{t}$, $x^4 + y^4 = t^2 + \frac{1}{t^2}$, then $\frac{dy}{dx} =$
(A) $\frac{-y}{x}$ (B) $-\frac{x}{2y}$
(C) $\frac{x}{2y}$ (D) $\frac{y}{x}$

163. If $y = \log\left[a^{3x}\left(\frac{5-x}{x+4}\right)^{3/4}\right]$, then $\frac{dy}{dx} =$
(A) $3 + \frac{3}{4(5-x)} - \frac{3}{4(x+4)}$
(B) $\frac{3}{\log a} - \frac{3}{4(5-x)} - \frac{3}{4(x+4)}$
(C) $\frac{3}{a} + \frac{3}{4(5-x)} - \frac{3}{4(x+4)}$
(D) $3\log a - \frac{3}{4(5-x)} - \frac{3}{4(x+4)}$

164. If $y = x^{xe^x}$, $\frac{dy}{dx} = y \cdot g(x)$, then g(x) =

(A) $\left[e^x + e^x.x.(1+\log x)\right]$

(B) $\left[e^x - e^x.x\,.\,(1+\log x)\right]$

(C) $\left[e^x(x+1)\log x\right]$

(D) $\left[e^x + e^x(x+1)\log x\right]$

165. If y = e^{4x} cos 5x, then $\frac{d^2y}{dx^2}$ at x = 0 is

(A) -8 (B) 8

(C) -9 (D) 9

166. If $y\sqrt{1-x^2} + x\sqrt{1-y^2} = 1$, then $\frac{dy}{dx}$ =

(A) $\sqrt{\frac{1-x^2}{1-y^2}}$ (B) $-\sqrt{\frac{1-y^2}{1-x^2}}$

(C) $\sqrt{\frac{1+y^2}{1+x^2}}$ (D) $\sqrt{\frac{1-x^2}{1-y^2}}$

167. If $y = \tan^{-1}\left[\sqrt{\frac{1+\cos\frac{x}{2}}{1-\cos\frac{x}{2}}}\right]$, then $\frac{dy}{dx}$ =

(A) $-\frac{1}{3}$ (B) $\frac{1}{4}$

(C) $-\frac{1}{4}$ (D) $\frac{1}{3}$

168. If $y = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\ldots$ then $\frac{dy}{dx}$ =

(A) y-1

(B) y+1

(C) y^2 - 1

(D) y

169. If y = 2^{ax} and $\left(\frac{dy}{dx}\right)_{x=1} = \log 256$, then a =

(A) 3 (B) 2

(C) 4 (D) 8

170. If f(x) = e^x g(x), g(0) = 4, g'(0) = 2, then f'(0) =

(A) 1 (B) 2

(C) 4 (D) 6

171. If f and g are differentiable functions satisfying $g'(a) = 2$, $g(a) = b$ and $fog = I$, where I is an identity function, then $f'(b)$ is equal to

(A) 2 (B) $\frac{2}{3}$

(C) $\frac{3}{2}$ (D) $\frac{1}{2}$

172. The derivative of $f(\tan x)$ with respect to g(sec x) at $x = \frac{\pi}{4}$, where $f'(1) = 2$ and $g'(\sqrt{2}) = 4$, is

(A) $\sqrt{2}$ (B) $\frac{1}{2}$

(C) 2 (D) $\frac{1}{\sqrt{2}}$

173. If $f(x) = |x-2|$, where $x \in [0, 4]$, then the Rolle's theorem cannot be applied to the function because

(A) $f(4) \neq f(0)$

(B) The function is not continuous at every point in the [0, 4]

(C) The function is not differentiable at every point in the (0, 4)

(D) Function is not well-defined in the domain

174. The displacement of a particle at the time t is given by $s = \sqrt{1+t}$, then its acceleration 'a' is proportional to

(A) square of the velocity

(B) $\sqrt[3]{S}$

(C) cube of the velocity

(D) $\sqrt{S}$

175. If the radius of a circle blot of oil is increasing at the rate of 2 cm/min, then the rate of change of its area when its radius is 3 cms is

(A) 12π cm²/min (B) 16 cm²/min
(C) 10π cm²/min (D) 14 cm²/min

176. The area of the square increases at the rate of 0.5 cm²/sec. The rate at which its perimeter is increasing when the side of the square is 10 cm long, is

(A) 0.4 cm/sec (B) 0.3 cm/sec
(C) 0.2 cm/sec (D) 0.1 cm/sec

177. The equation of the normal to the curve $2x^2 + y^2 = 12$ at the point (2, 2) is

(A) x-2y+2=0 (B) 2x-y+6=0
(C) x+2y+2=0 (D) 2x+y-6=0

178. The maximum volume of a right circular cylinder if the sum of its radius and height is 6m is

(A) $32\ \pi m^3$ (B) $4\ \pi m^3$
(C) $16\ \pi\ m^3$ (D) $64\ \pi m^3$

179. The function $f(x) = 3x^4 + 16x^3 - 30x^2 + 10$ is increasing for

(A) every real value of x
(B) $x \in (-5, 0) \cup (1, \infty)$
(C) $x \in [0, 1]$
(D) x = 0, x = 1 only

180. The equation of a normal to the curve $x = 4 \sec\theta$ and $y = 4\tan^2\theta$ at $\theta = \frac{\pi}{4}$ is

(A) $x + 2\sqrt{2}\, y = 12\sqrt{2}$
(B) $x+y\sqrt{2} = 7\sqrt{2}$
(C) $\sqrt{2}\, x+y=7\sqrt{2}$
(D) $2\sqrt{2}\, x+y=8\sqrt{2}$

181. A particle moves according to the law $s = t^3 - 6t^2 + 9t + 25$. The displacement of the particle at the time when its acceleration is zero, is

(A) – 27 units (B) 27 units
(C) 0 units (D) 9 units

182. A metal wire 108 meters long is bent to form a rectangle. If the area of the rectangle is maximum, then its dimensions are

(A) 28 m, 28 m (B) 28 m, 26 m
(C) 25 m, 25 m (D) 27 m, 27 m

183. The approximate value of $\log_{10}99$ is (Given $\log_{10}e = 0.4343$)

(A) 1.9857 (B) 1.9657
(C) 1.9757 (D) 1.9957

184. $\int \frac{x^2+1}{(x-3)(x-2)}dx = Px + Q\log|x-3| + R\log|x-2| + c$, where c is constant of integration, then the values of P, Q, R are, respectively

(A) 0, 10, -5 (B) 1, 10, 5
(C) 1, 10, -5 (D) 0, 10, 5

185. $\int \frac{\left(\sin^{-1}x\right)^{\frac{3}{2}}}{\sqrt{1-x^2}}dx =$

(A) $\frac{5}{2}\left(\cos^{-1}x\right)^{\frac{5}{2}} + c$ (B) $\frac{2}{5}\left(\sin^{-1}x\right)^{\frac{5}{2}} + c$
(C) $\frac{2}{5}\left(\cos^{-1}x\right)^{\frac{5}{2}} + c$ (D) $\frac{5}{2}\left(\sin^{-1}x\right)^{\frac{5}{2}} + c$

186. $\int\left[\frac{(1+\log x)}{\cos^2(x\log x)}\right]dx =$

(A) $\sin^2(x\log x) + c$ (B) $\tan(x\log x) + c$
(C) $\log(x\log x) + c$ (D) $\sin(x\log x) + c$

187. $\int \frac{5^x}{\sqrt{5^{-2x} - 5^{2x}}}dx =$

(A) $\sin^{-1}\left(5^{2x}\right) + c$
(B) $\tan^{-1}\left(5^x\right) + c$
(C) $\tan^{-1}\left(5^{2x}\right)\cdot\log 25 + c$
(D) $\frac{\sin^{-1}\left(5^{2x}\right)}{\log 25} + c$

188. $\int \frac{\sec x}{\sqrt{\log(\sec x+\tan x)}}dx =$

(A) $\sqrt{(\sec x+\tan x)}+c$

(B) $\sqrt{\log(\sec x+\tan x)}+c$

(C) $2\sqrt{\sec x+\tan x}+c$

(D) $2\sqrt{\log(\sec x+\tan x)}+c$

189. $\int \frac{e^x(1+2x)}{\sqrt{x}}dx =$

(A) $\frac{\sqrt{x}}{2}e^x+c$ (B) $\frac{1}{\sqrt{x}}e^x+c$

(C) $2\sqrt{x}e^x+c$ (D) $\sqrt{x}e^x+c$

190. If $\int x^x(1+\log x)dx = kx^x+c$, then k =

(A) $\log_e e$ (B) $\log_e\left(\frac{1}{e}\right)$

(C) $\log_e\left(\frac{1}{e^2}\right)$ (D) $\log_e\left(e^2\right)$

191. $\int \cot x\cdot\log\left[\log(\sin x)\right]dx =$

(A) $\log(\sin x)\left[\log(\sin x)-1\right]+c$

(B) $\log(\sin x)\left[\log(\sin x)+1\right]+c$

(C) $\log(\sin x)\left[\log(\log(\sin x))+1\right]+c$

(D) $\log(\sin x)\left[\log(\log(\sin x))-1\right]+c$

192. If $\int \frac{\sin\theta}{\sin 3\theta}d\theta = \frac{1}{2k}\log\left|\frac{k+\tan\theta}{k-\tan\theta}\right|+c$, then k =

(A) $\sqrt{3}$ (B) $\sqrt{5}$

(C) $\sqrt{7}$ (D) $\sqrt{2}$

193. If $\int\sqrt{x-\frac{1}{x}}\left(\frac{x^2+1}{x^2}\right)dx = \frac{2}{3}\left(x-\frac{1}{x}\right)^k+c$, then value of k is

(A) $\frac{2}{5}$ (B) $\frac{5}{2}$

(C) $\frac{3}{2}$ (D) $\frac{2}{3}$

194. $\int(1+x)\log x dx =$

(A) $\left(x+\frac{x^2}{2}\right)\log x-\left(x-\frac{x^2}{4}\right)+C$

(B) $\left(x+\frac{x^2}{2}\right)\log x-x-\frac{x^2}{4}+C$

(C) $\left(x+\frac{x^2}{2}\right)\log x+x+\frac{x^2}{4}+C$

(D) $\left(x+\frac{x^2}{2}\right)\log x+\left(x-\frac{x^2}{4}\right)+C$

195. $\int_0^\infty \frac{dx}{\left(x^2+4\right)\left(x^2+9\right)} =$

(A) $\frac{\pi}{80}$ (B) $\frac{\pi}{60}$

(C) $-\frac{\pi}{60}$ (D) $\frac{\pi}{120}$

196. $\int_{-5}^{5}\left[\frac{e^x+e^{-x}}{e^x-e^{-x}}\right]dx =$

(A) $3e^5$ (B) 0

(C) $2e^5$ (D) 1

197. $\int_{-a}^{a}x^2\left(\frac{e^{x^3}-e^{-x^3}}{e^{x^3}+e^{-x^3}}\right)dx =$

(A) $2\int_0^a x^2\left(\frac{e^{x^3}-e^{-x^3}}{e^{x^3}+e^{-x^3}}\right)dx$

(B) a

(C) a^2

(D) 0

198. $\int_{-8}^{8}\frac{x^5+x^3}{4-x^2}dx =$

(A) 16 (B) -8

(C) 8 (D) 0

199. $\int_0^{\frac{\pi}{2}} \frac{1-\cot x}{\operatorname{cosec} x+\cos x}dx =$

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$

(C) 0 (D) 1

200. If $\int_0^{\frac{\pi}{4}} \frac{\sin x+\cos x}{9+16\sin 2x}dx = k\log 3$, then $k =$

(A) $\frac{1}{40}$ (B) $\frac{1}{20}$

(C) $\frac{1}{10}$ (D) $\frac{1}{30}$

201. $\int_0^a (a-x)^{\frac{3}{2}} x^2 dx =$

(A) $\frac{16a^{\frac{9}{2}}}{315}$ (B) $\frac{-16a^{\frac{9}{2}}}{315}$

(C) $\frac{16a^{\frac{7}{2}}}{315}$ (D) $\frac{-16a^{\frac{7}{2}}}{315}$

202. $\int_{-2}^{1} [x+1]dx =$, (Where $[x]$ is greatest integer function not greater than x)

(A) –-1 (B) 0

(C) 1 (D) 2

203. $\int_0^{\frac{\pi}{2}} \log\sqrt{\frac{1-\cos 2x}{1+\cos 2x}}dx =$

(A) 1 (B) $\frac{\pi}{4}$

(C) 0 (D) $\frac{\pi}{8}$

204. $\int_0^a \sqrt{\frac{x}{a-x}}dx =$

(A) $\left(\frac{\pi}{4}\right)a$

(B) $2\pi a$

(C) $\frac{\pi a}{2}$

(D) πa

205. $\int_2^3 \frac{x}{x^2-1}dx =$

(A) $\left(\frac{1}{3}\right)log\left(\frac{8}{3}\right)$ (B) $\left(\frac{-1}{3}\right)log\left(\frac{8}{3}\right)$

(C) $\left(\frac{1}{2}\right)log\left(\frac{8}{3}\right)$ (D) $\left(\frac{-1}{2}\right)log\left(\frac{8}{3}\right)$

206. $\int_0^1 \left(\frac{x^2-2}{x^2+1}\right)dx =$

(A) $1+\frac{\pi}{4}$ (B) $1+\frac{3\pi}{4}$

(C) $1-\frac{3\pi}{4}$ (D) $1-\frac{\pi}{4}$

207. $\int_{\frac{\pi}{5}}^{\frac{3\pi}{10}} \left[\frac{\tan x}{\tan x+\cot x}\right]dx =$

(A) $\frac{\pi}{5}$ (B) $\frac{\pi}{2}$

(C) $\frac{\pi}{20}$ (D) $\frac{3\pi}{10}$

208. $\int_0^{\pi} \frac{e^{\cos x}}{\left(e^{\cos x}+e^{-\cos x}\right)}dx$

(A) $\frac{-\pi}{2}$ (B) $-\pi$

(C) $\frac{\pi}{2}$ (D) π

209. $\int_0^1 \left(1-\frac{x}{1!}+\frac{x^2}{2!}-\frac{x^3}{3!}+...\text{upto}\infty\right)e^{2x}dx =$

(A) e^2 (B) $e-1$
(C) e (D) $e+1$

210. If $\int_0^1 \left(5x^2-3x+k\right)dx = 0$, then $k =$

(A) $-\frac{1}{6}$ (B) $\frac{1}{3}$
(C) $-\frac{1}{3}$ (D) $\frac{1}{6}$

211. $\int_{-\pi}^{\pi} \frac{2x}{1+cos^2 x}dx =$

(A) 0 (B) $-\pi$
(C) 1 (D) π

212. $\int_0^1 tan^{-1}\left(\frac{2x}{1-x^2}\right)dx =$

(A) $\frac{\pi}{2}+log\,2$ (B) $\pi-log\,2$
(C) $\frac{\pi}{2}-log\,2$ (D) $\pi+log\,2$

213. The area of the region bounded by the parabola $x^2 = 16y$, y = 1, y = 4 and the Y-axis lying in the first quadrant is

(A) $\frac{56}{3}$ sq. units (B) $\frac{52}{3}$ sq. units
(C) $\frac{53}{3}$ sq. units (D) $\frac{55}{3}$ sq. units

214. The area of the region bounded by the curve y = sin x between $x = -\pi$ and $x = \frac{3\pi}{2}$ is

(A) 1 $(units)^2$ (B) 2 $(units)^2$
(C) 5 $(units)^2$ (D) 3 $(units)^2$

215. The area of the triangle formed by the lines joining vertex of the parabola $x^2 = 12y$ to the extremities of its latus rectum is

(A) 38 sq. units
(B) 28 sq. units
(C) 12 sq. units
(D) 18 sq. units

216. The area bounded by the parabola $x^2 = 4y$ and the lines y = 2, y = 4 is

(A) $\frac{4}{3}\left(8-2\sqrt{2}\right)$ sq. units
(B) $\left(8-2\sqrt{2}\right)$ sq. units
(C) $\frac{8}{3}\left(8-2\sqrt{2}\right)$ sq. units
(D) $\frac{8}{3}\left(8+2\sqrt{2}\right)$ sq. units

217. The area of the region bounded by the curve y = log x, X-axis and the lines x = 1, x = e is

(A) $\frac{1}{2}$ sq. units (B) $\frac{1}{e}$ sq. units
(C) 4 sq. units (D) 1 sq. units

218. The area bounded by the circle $x^2 + y^2 = 16$ and lines x = 0 and x = 2 is

(A) $\left[4\sqrt{3}+\frac{8\pi}{3}\right]$ sq. units
(B) $\frac{1}{2}\left[4\sqrt{3}-\frac{8\pi}{3}\right]$ sq. units
(C) $\frac{1}{2}\left[4\sqrt{3}+\frac{8\pi}{3}\right]$ sq. units
(D) $\left[4\sqrt{3}-\frac{8\pi}{3}\right]$ sq. units

219. The area of the region bounded by the curve $y = 4x^3 - 6x^2 + 4x + 1$ and the lines x = 1, x = 5 and X-axis is

(A) 400 sq. units (B) 378 sq. units
(C) 428 sq. units (D) 334 sq. units

220. If the body cools from 135°C to 80°C at room temperature of 25°C in 60 minutes, then the temperature of body after 2 hours is

(A) (52.5)°C (B) (10.75)°C
(C) (10.5)°C (D) (52.75)°C

221. The particular solution of the differential equation $\sin^2 y \frac{dx}{dy} + x = \cot y$, when x = 0 and $y = \frac{3\pi}{4}$ is

(A) $x = 1 + \cot y$ (B) $xy = \cot(x + y)$
(C) $xy = \cot(x + y)$ (D) $y = 1 + \cot x$

222. Degree of the differential equation $e^{\frac{dy}{dx}} + \left(\frac{dy}{dx}\right)^3 = x$ is

(A) 2
(B) 1
(C) 3
(D) not defined

223. The solution of differential equation $x^2 \frac{dy}{dx} = y^2 + xy$ is

(A) $\frac{y}{x} + log|x| = c$

(B) $\frac{x}{y} - log|x| = c$

(C) $\frac{y}{x} - log|x| = c$

(D) $\frac{x}{y} + log|x| = c$

224. The differential equation of the circles having their centres on the line y = 8 and touching the X-axis is

(A) $(y-8)^2\left[1-\left(\frac{dy}{dx}\right)^2\right] = 64$

(B) $(y-8)^2\left[1+\left(\frac{dy}{dx}\right)^2\right] = 64$

(C) $y^2\left[1+\left(\frac{dy}{dx}\right)\right] = 64$

(D) $(y-8)\left[1+\left(\frac{dy}{dx}\right)^2\right] = 64$

225. The bacteria increases at the rate proportional to the number of bacteria present. If the original number 'N' doubles in 4 hours, then the number of bacteria in 12 hours will be

(A) 4N (B) 8N
(C) 6N (D) 3N

226. The rate of disintegration of a ratio active element at time t is proportional to its mass at the time. Then the time during which the original mass of 1.5 gm. will disintegrate into its mass of 0.5 gm. is proportional to

(A) log 5
(B) log 2
(C) log 3
(D) log 4

227. Bismath has half life of 5 days. If sample originally has a mass of 800 mg. then the mass remaining after 30 days will be

(A) 12.5 mg
(B) 10.5 mg
(C) 12 mg
(D) 10 mg

228. The differential equation obtained by eliminating the arbitrary constants from the equation $y^2 = (2x+c)^5$ is

(A) $\left(\frac{dy}{dx}\right)^4 - 625y^4 = 0$

(B) $\left(\frac{dy}{dx}\right)^5 - 3125y^3 = 0$

(C) $\left(\frac{dy}{dx}\right)^3 - 125y^3 = 0$

(D) $xy\frac{dy}{dx} = 5$

229. If the population grows at the rate of 8% per year, then the time taken for the population to be doubled is (given log 2 = 0.6912)

(A) 10.27 years
(B) 6.8 years
(C) 8.64 years
(D) 4.3 years

230. $y = c^2 + \frac{c}{x}$ is the solution of the differential equation.

(A) $x^4\left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} + y = 0$

(B) $x^4\left(\frac{dy}{dx}\right)^2 + x\frac{dy}{dx} + y = 0$

(C) $x^4\left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} - y = 0$

(D) $x^4\left(\frac{dy}{dx}\right)^2 + x\frac{dy}{dx} - y = 0$

234. The general solution of the differential equation $\frac{dy}{dx} + \frac{1}{\sqrt{1-x^2}} = 0$ is

(A) $y + \sin^{-1}x = c$

(B) $x^2 + 2sin^2 y = c$

(C) $y^2 + 2sin^{-1}x = c$

(D) $x + sin^{-1}y = c$

232. Bacteria increases at the rate proportional to the number of bacteria present. If the original number N doubles in 4 hours, then the number of bacteria will be 4N in

(A) 2 hours

(B) 4 hours

(C) 8 hours

(D) 6 hours

233. If $\frac{d^2y}{dx^2} = sinx + e^x$; y(0) = 3 and $\frac{dy}{dx}$ at x = 0 is 4, then the equation of the curve is

(A) $y = 4 + 2x + e^x + \sin x$

(B) $y = 2 + 3x + e^x - \sin x$

(C) $y = 2 + 4x + e^x - \sin x$

(D) $y = 4 + 2x + e^x - \sin x$

234. Which of the following functions is not p.d.f. of a continuous random variable X? F_1 given by $f(x) = e^{-x}$ if $0 < x < \infty = 0$, otherwise F_2 given by $f(x) = \frac{1}{4} \times \frac{1}{\sqrt{x}}$ if $0 < x < 4 = 0$, otherwise F_3 given by $f(x) = 6x(1-x)$ if $0 < x < 1 = 0$, otherwise F_4 given by $f(x) = \frac{x}{2}$ if $-2 < x < 2 = 0$, otherwise

(A) F_1 (B) F_4

(C) F_2 (D) F_3

235. A random variable X takes the values 0, 1, 2. Its mean is 1.2, if P(X = 0) = 0.3, then P(X = 1) =

(A) 0.5 (B) 0.2

(C) 0.1 (D) 0.4

236. If the error involved in making a certain measurement is continuous random variable X with probability density function $f(x) = k(4-x^2)$ if $-2 \le x \le 2 = 0$, otherwise Then, P[-1 <x<1] =

(A) $\frac{1}{3}$ (B) $\frac{13}{16}$

(C) $\frac{11}{16}$ (D) $\frac{1}{2}$

237. The cumulative distribution function of a continuous random varibale X is given by $F(X = x) = \frac{\sqrt{x}}{2}$, then P[X > 1] is

(A) $\frac{1}{3}$ (B) $\frac{1}{4}$

(C) $\frac{1}{\sqrt{2}}$ (D) $\frac{1}{2}$

238. The p.d.f. of a continuous random variable X is given by $f(x) = \frac{1}{2}$ if 0<x<2 = 0 otherwise and if $a = P(X < \frac{1}{2})$, $b = P(X > \frac{3}{2})$, then relation between a and b is

(A) a-b=0 (B) 2a-b = 0

(C) 3a-b=0 (D) a-2b = 0

239. The probability that a person wins a prize on a lottery ticket is $\frac{1}{4}$. If he purchases 5 lottery tickets at random, then the probability that he wins at least one prize is

(A) $\frac{774}{1024}$ (B) $\frac{121}{1024}$

(C) $\frac{223}{1024}$ (D) $\frac{781}{1024}$

240. It is known that a box of 8 batteries contain 3 defective pieces and person randomly selects two batteries from the box. If X is the number of defective batteries selected, then $P(X \leq 1) =$

(A) $\frac{10}{28}$ (B) $\frac{55}{64}$

(C) $\frac{13}{28}$ (D) $\frac{14}{28}$

241. The probability that the person who undergoes certain operation will survive is 0.2. If 5 patients undergo similar operations, then the probability that exactly four will survive is

(A) 0.0032 (B) 0.0084

(C) 0.0064 (D) 0.0042

242. The probability that bomb will miss the target is 0.2. The probability that out of 10 bombs dropped exactly 2 will hit the target is

(A) $\frac{144}{5^{10}}$ (B) $\frac{288}{5^{10}}$

(C) $\frac{144}{5^{9}}$ (D) $\frac{288}{5^{9}}$

243. The probability that a basket-ball player makes the basket is 0.4. The probability that he makes exactly three baskets in four attempts is

(A) 0.064 (B) 0.256

(C) 0.0384 (D) 0.1536

244. A die is thrown 100 times, then the standard deviation of getting an even number is

(A) 5 (B) 15

(C) 20 (D) 10

MHTCET PYQ M (2020)
Marathi

Hints & Solutions

1. (A)

If $\frac{\sin(A+B)}{\sin(A-B)}=\frac{\cos(C+D)}{\cos(C-D)}$, find $\tan A\cot B$.

Using tangent forms,

$\frac{\tan A+\tan B}{\tan A-\tan B}=\frac{1-\tan C\tan D}{1+\tan C\tan D}$.

Cross-multiplication gives.

Hence

$\tan A\cot B=-\frac{1}{\tan C\tan D}=-\cot C\cot D.$

2. (A)

Eval ate $\tan1^\circ\tan2^\circ\cdots\tan89^\circ$.

Pair $\tan\theta\tan(90^\circ-\theta)=1$, and $\tan45^\circ=1$. The the product is 1.

3. (D)

For $A,B\in(0,\pi)$, $\sin A=\sin B$, and A,B are not supplementary. Determine the relation between A and B.

$\sin A=\sin B$ gives $A=B$ or $A+B=\pi$. The supplementary case is excluded, so $A=B$.

4. (C)

If $A+B=\pi$, evaluate $\sin^2\frac{A}{2}+\sin^2\frac{B}{2}$.

Since $B=\pi-A$,

$\sin^2\frac{A}{2}+\sin^2\frac{\pi-A}{2}=\sin^2\frac{A}{2}+\cos^2\frac{A}{2}=1.$

5. (C)

Evaluate $\frac{\sin A+\sin7A+\sin13A}{\cos A+\cos7A+\cos13A}$.

$\sin A+\sin13A=2\sin7A\cos6A,$

$\cos A+\cos13A=2\cos7A\cos6A.$

Thus, the ratio is $\tan7A$.

6. (C)

For $0<\theta<\frac{\pi}{2}$, solve

$\tan3\theta\tan2\theta\tan\theta+\tan2\theta+\tan\theta=1$.

For $\theta=\frac{\pi}{12}$,

$\tan\frac{\pi}{4}\tan\frac{\pi}{6}\tan\frac{\pi}{12}+\tan\frac{\pi}{6}+\tan\frac{\pi}{12}=1.$

This is the admissible listed value.

7. (D)

Given $\tan A=\frac{5}{6}$ and $\tan B=\frac{1}{11}$, find $A+B$.

$\tan(A+B)=\frac{\frac{5}{6}+\frac{1}{11}}{1-\frac{5}{66}}=1.$

For the intended acute angles, $A+B=\frac{\pi}{4}$.

8. (D)

Simplify $\sec2\theta-\tan2\theta$.

$\sec2\theta-\tan2\theta=\frac{1-\sin2\theta}{\cos2\theta}=\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}$

$=\tan\left(\frac{\pi}{4}-\theta\right)$

9. (D)

Simplify $\cos x\cos7x-\cos5x\cos13x$.

$\frac{1}{2}(\cos6x+\cos8x)-\frac{1}{2}(\cos8x+\cos18x)$

$=\frac{1}{2}(\cos6x-\cos18x)=\sin6x\sin12x.$

10. (A)

Determine which among $\sin120^\circ$, $\cos930^\circ$, $\tan840^\circ$, and $\cot(-1110^\circ)$ have the same value.

$\sin 120^\circ = \frac{\sqrt{3}}{2}, \cos 930^\circ = -\frac{\sqrt{3}}{2},$

$\tan 840^\circ = -\sqrt{3}, \cot\left(-1110^\circ\right) = -\sqrt{3}.$

Only the third and fourth expressions are equal.

11. (B)

Evaluate $\sin 690^\circ \sec 240^\circ$.

$\sin 690^\circ = \sin 330^\circ = -\frac{1}{2}, \sec 240^\circ = -2.$

The product is 1.

12. (B)

If $A + B + C = 180^\circ$, evaluate

$\tan\frac{A}{2}\tan\frac{B}{2} + \tan\frac{B}{2}\tan\frac{C}{2} + \tan\frac{C}{2}\tan\frac{A}{2}.$

Since $\frac{A}{2} + \frac{B}{2} + \frac{C}{2} = 90^\circ$, the standard tangent identity gives

$\tan\frac{A}{2}\tan\frac{B}{2} + \tan\frac{B}{2}\tan\frac{C}{2} + \tan\frac{C}{2}\tan\frac{A}{2} = 1.$

13. (D)

In $\triangle ABC$, if $\frac{\sin A - \sin C}{\cos C - \cos A} = \cot B$, identify the progression of A, B, C.

By sum-to-product,

$\frac{\sin A - \sin C}{\cos C - \cos A} = \cot\frac{A + C}{2} = \tan\frac{B}{2}.$

Thus $\tan\frac{B}{2} = \cot B$, giving $B = 60^\circ$. Hen $A + C = 120^\circ = 2B$, so A, B, C are in arithmetic progression.

14. (C)

Evaluate $\tan A + 2\tan 2A + 4\tan 4A + 8\cot 8A$.

Use $\tan x = \cot x - 2\cot 2x$. The terms telescope:

$\tan A + 2\tan 2A + 4\tan 4A + 8\cot 8A = \cot A.$.

15. (C)

If $\sin\theta = \sin 15^\circ + \sin 45^\circ$ and $0^\circ < \theta < 180^\circ$, find θ.

$\sin 15^\circ + \sin 45^\circ = 2\sin 30^\circ \cos 15^\circ = \cos 15^\circ = \sin 75^\circ.$

Hence $\theta = 75^\circ$ or 105°. Only 75° is listed.

16. (D)

Find the intersection of the line through $(1,4)$ and $(-5,1)$ with $4x + 3y - 5 = 0$.

The first line is $x - 2y + 7 = 0$. Solving with $4x + 3y - 5 = 0$ gives

$(x, y) = (-1, 3).$

17. (D)

The distance from $(4,1)$ to $3x - 4y + k = 0$ is 2. Find k.

$\frac{|12 - 4 + k|}{5} = 2 \Rightarrow |k + 8| = 10.$

Thus $k = 2$ or $k = -18$.

18. (D)

Find the line through $(7,-4)$ perpendicular to the line through $(2,3)$ and $(1,-2)$.

The given line has slope 5, so the required slope is $-\frac{1}{5}$:

$y + 4 = -\frac{1}{5}(x - 7) \Rightarrow x + 5y + 13 = 0.$

19. (A)

For the two stated lines, P_1, P_2 are perpendicular distances from the origin. Find $P_1^2 + 4P_2^2$.

$P_1 = 5|\cos 2\theta|, P_2 = \frac{5}{2}|\sin 2\theta|.$

Therefore

$P_1^2 + 4P_2^2 = 25\left(\cos^{22}\theta + \sin^{22}\theta\right) = 25.$

20. (C)

Find the distance between $3x + 4y = 9$ and the intended parallel line $6x + 8y = 15$.

Write the second line as $3x + 4y = \frac{15}{2}$. Hence

$$d=\frac{|9-15/2|}{\sqrt{3^2+4^2}}=\frac{3/2}{5}=0.3.$$

21. (B)

Find the lines whose positive axis intercepts have sum 8 and product 15.

The intercepts are roots of $t^2-8t+15=0$, namely 3 and 5. Thus

$\frac{x}{3}+\frac{y}{5}=1, \frac{x}{5}+\frac{y}{3}=1,$

or $5x+3y-15=0$ and $3x+5y-15=0$.

22. (B)

Eliminate k from $\sqrt{3}x-y=k\sqrt{3}$ and $\sqrt{3}kx+ky=\sqrt{3}$ and identify the locus.

From the first equation $k=x-\frac{y}{\sqrt{3}}$. Substitution gives

$$\left(x-\frac{y}{\sqrt{3}}\right)\left(\sqrt{3}x+y\right)=\sqrt{3}\Rightarrow 3x^2-y^2=3.$$

This is a hyperbola.

23. (D)

The point $(5,6)$ divides the intercept segment AB internally in the ratio $3:1$. Find the line.

Let $A=(a,0)$ and $B=(0,b)$. The section formula gives $(5,6)=(a/4,3b/4)$, so $a=20, b=8$. Hence

$$\frac{x}{20}+\frac{y}{8}=1\Rightarrow 2x+5y=40.$$

24. (A)

Find the acute angle between $x\sin\theta-y\cos\theta=5$ and $x\sin\alpha-y\cos\alpha+11=0$.

The normals make direction angles differing by $|\theta-\alpha|$; the same is true for the lines. Thus the acute angle is $|\theta-\alpha|$.

25. (D)

Find the perpendicular distance from $P(a,b)$ to $\frac{x}{a}+\frac{y}{b}=1$.

The line is $bx+ay-ab=0$. Therefore

$$d=\frac{|ab+ab-ab|}{\sqrt{a^2+b^2}}=\frac{|ab|}{\sqrt{a^2+b^2}}.$$

For positive a,b, this is $\frac{ab}{\sqrt{a^2+b^2}}$.

26. (B)

If $(5,k),(-3,1),(-7,-2)$ are collinear, find k.

Equating slopes,

$$\frac{k-1}{5-(-3)}=\frac{1-(-2)}{-3-(-7)}=\frac{3}{4}.$$

Thus $k-1=6$ and $k=7$.

27. (D)

Find the acute angle between $y-\sqrt{3}x+1=0$ and $\sqrt{3}y-x+7=0$.

Their slopes are $\sqrt{3}$ and $1/\sqrt{3}$, corresponding to inclinations 60° and 30°. The acute angle is 30°.

28. (A)

If $(a,-2a)$ is the midpoint of an intercept segment, find its line.

The intercepts are $(2a,0)$ and $(0,-4a)$. Thus

$$\frac{x}{2a}-\frac{y}{4a}=1\Rightarrow 2x-y=4a.$$

29. (D)

Classify $A(-a,-b),B(0,0),C(a,b),D(a^2,ab)$.

All position vectors are scalar multiples of (a,b):

$A=-1(a,b), B=0(a,b), C=1(a,b), D=a(a,b).$

Hence the points are collinear.

30. (B)

Find parametric equations of

$x^2+y^2-6x+4y-3=0$.

$x^2-6x+y^2+4y-3=0\Rightarrow(x-3)^2+(y+2)^2=16.$

Thus $x=3+4\cos\theta$, $y=-2+4\sin\theta$.

31. (D)

Eliminate θ from $x=6\cos\theta$, $y=6\sin\theta$.

$\frac{x^2}{36}+\frac{y^2}{36}=\cos^2\theta+\sin^2\theta=1.$

Therefore $x^2+y^2=36$.

32. (B)

Two cards are drawn without replacement. Find the probability both are queens.

$P=\frac{4}{52}\cdot\frac{3}{51}=\frac{1}{221}.$

33. (D)

Step 1: Recall De Morgan's Law

$P(A'\cap B')=P((A\cup B)')=1-P(A\cup B)$

Step 2 : For independent events:

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

Since A and B are independent:

$P(A\cap B)=P(A)\cdot P(B)=\frac{3}{5}\times\frac{2}{3}=\frac{2}{5}$

Step 3: Calculate $P(A\cup B)$:

$P(A\cup B)=\frac{3}{5}+\frac{2}{3}-\frac{2}{5}$

$=\left(\frac{3}{5}-\frac{2}{5}\right)+\frac{2}{3}=\frac{1}{5}+\frac{2}{3}$

$=\frac{3}{15}+\frac{10}{15}=\frac{13}{15}$

Step 4: Calculate P(A'∩ B'):

$P(A'\cap B')=1-\frac{13}{15}=\frac{2}{15}$

34. (A)

Odds in favour of A are $2:3$ and odds against B are $4:5$. Find $P(A\cap B)$ for independent events.

$P(A)=\frac{2}{5},P(B)=\frac{5}{9}.$

Thus $P(A\cap B)=\frac{2}{5}\cdot\frac{5}{9}=\frac{2}{9}.$

35. (D)

An urn has 4 red and 5 white balls. Two are drawn without replacement. Find the probability both are red.

$P=\frac{4}{9}\cdot\frac{3}{8}=\frac{1}{6}.$

36. (A)

Given $P(A')=0.6$, $P(B)=0.8$, *and* $P(B|A)=0.3$, *find* $P(A|B)$.

$P(A)=0.4$ and $P(A\cap B)=0.4(0.3)=0.12$.

Hence

$P(A|B)=\frac{0.12}{0.8}=0.15=\frac{3}{20}.$

37. (D)

Two dice are thrown. Find the probability that the sum is divisible by 2 or 3 .

There are 18 even-sum outcomes, 12 outcomes with sum divisible by 3 , and 6 common outcomes (sum 6 or 12). Thus

$P=\frac{18+12-6}{36}=\frac{2}{3}.$

38. (D)

Step 1: For independent events:

$P(A'\cap B)=P(A')\cdot P(B)$

Because A' and B are also independent (if A and B are independent, the nany combination of fA/A' and B/B' are independent)

Step 2: $P(A')$

$P(A')=1-P(A)=1-\frac{2}{3}=\frac{1}{3}$

Step 3: Calculate P(A' ∪ B)

$P(A'\cap B)=P(A')\cdot P(B)=\frac{1}{3}\times\frac{3}{5}=\frac{1}{5}$

39. (D)

Three dice are thrown. Find the probability that the sum is at least 5 .

The complement consists of sum 3 (one outcome) and sum 4 (three outcomes). Therefore

$$P = 1 - \frac{4}{216} = \frac{53}{54}.$$

40. (C)

Arrange the letters of LOGARITHM at random. Find the probability the arrangement starts with a vowel and ends with a consonant.

There are 3 vowels and 6 consonants among 9 distinct letters:

$$P = \frac{3}{9} \cdot \frac{6}{8} = \frac{1}{4}.$$

41. (B)

Find the odds in favour of drawing a king from a standard deck.

Favourable : unfavourable is $4:48 = 1:12$.

42. (C)

With equal numbers of men and women, 5% of men and 0.25% of women have grey hair. Find the probability a randomly selected grey-haired person is a man.

By Bayes' theorem,

$$P(M \mid G) = \frac{0.05(1/2)}{0.05(1/2) + 0.0025(1/2)} = \frac{20}{21}.$$

43. (D)

Find the centre of

$9x^2 - 36x - 16y^2 + 96y - 252 = 0$.

Completing squares gives

$9(x-2)^2 - 16(y-3)^2 = 144.$

The centre is $(2,3)$.

44. (D)

Find the eccentricity of a rectangular hyperbola.

For a rectangular hyperbola $a = b$, so

$$e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{2}.$$

45. (A)

For the stated ellipse, B and the two foci form an equilateral triangle. Find the eccentricity.

$BS = a$ and $SS' = 2c$. Equilateral condition gives $a = 2c$, hence

$$e = \frac{c}{a} = \frac{1}{2}.$$

46. (A)

Find the eccentricity of

$y^2 + 4x^2 - 12x + 6y + 14 = 0$.

$$4\left(x - \frac{3}{2}\right)^2 + (y+3)^2 = 4 \Rightarrow \frac{(x-3/2)^2}{1} + \frac{(y+3)^2}{4} = 1.$$

Thus $a^2 = 4, b^2 = 1$, and $e = \sqrt{1 - b^2/a^2} = \sqrt{3}/2$.

47. (B)

Find the latus-rectum length of $x^2 + 2y = 8x - 7$.

$$(x-4)^2 = -2\left(y - \frac{9}{2}\right).$$

Here $|4a| = 2$, which is the latus-rectum length.

48. (D)

Find the point on $y^2 = x$ with parameter $t = -\frac{4}{3}$.

For $y^2 = 4ax$ with $a = \frac{1}{4}$, the parametric point is $(at^2, 2at)$. Hence

$$\left(\frac{1}{4} \cdot \frac{16}{9}, \frac{1}{2} \cdot -\frac{4}{3}\right) = \left(\frac{4}{9}, -\frac{2}{3}\right).$$

49. (D)

The foci of the given ellipse and hyperbola coincide.

Find b^2 for the ellipse.

The hyperbola becomes

$$\frac{x^2}{144/25} - \frac{y^2}{81/25} = 1,$$

so $c^2 = 144/25 + 81/25 = 9$. For the ellipse, $c^2 = 16 - b^2$. Hence $16 - b^2 = 9$, giving $b^2 = 7$.

50. (C)

Find $g + f - c$ for the joint equation of the directrices of $16x^2 - 9y^2 = 144$.

The hyperbola is $x^2/9 - y^2/16 = 1$, with $c = 5$. Its directrices are $x = \pm a/e = \pm 9/5$, so their joint equation is

$25x^2 - 81 = 0.$

Thus $g = f = 0$, $c = -81$, and $g + f - c = 81$.

51. (B)

For $P(\theta) = (a\sec\theta, b\tan\theta)$ on a hyperbola, find $SP \cdot S'P$.

The focal distances are $a(e\sec\theta \pm 1)$. Therefore

$SP\,S'P = a^2\left(e^2\sec^2\theta - 1\right) = a^2\tan^2\theta + b^2\sec^2\theta.$

52. (B)

Eliminating k gives $3x^2 - y^2 = 3$, a hyperbola.

53. (A)

Find the eccentricity of $9x^2 + 16y^2 = 144$.

$\frac{x^2}{16} + \frac{y^2}{9} = 1, e = \sqrt{1 - \frac{9}{16}} = \frac{\sqrt{7}}{4}.$

54. (B)

A parabola has focus $(1,-2)$ and directrix $x + y + 3 = 0$. Find its latus-rectum length.

The focus-directrix distance is

$\frac{|1 - 2 + 3|}{\sqrt{2}} = \sqrt{2} = 2a.$

Hence the latus-rectum length is $4a = 2\sqrt{2}$.

55. (B)

Find the foci of $16x^2 + 9y^2 = 144$.

$\frac{x^2}{9} + \frac{y^2}{16} = 1, c^2 = 16 - 9 = 7.$

The foci are $\left(0, \pm\sqrt{7}\right)$.

56. (A)

Find the eccentricity of

$16x^2 - 3y^2 - 32x - 12y - 44 = 0$.

Completing squares,

$\frac{(x-1)^2}{3} - \frac{(y+2)^2}{16} = 1.$

Thus $e = \sqrt{1 + \frac{16}{3}} = \sqrt{\frac{19}{3}}.$

57. (C)

Solve $|3x - 2| \le \frac{1}{2}$.

$-\frac{1}{2} \le 3x - 2 \le \frac{1}{2} \Rightarrow \frac{1}{2} \le x \le \frac{5}{6}.$

58. (C)

For $f(x) = 7x + 8$, if $f^{-1}(12) = k/7$, find k.

$7x + 8 = 12$ gives $x = 4/7$. Therefore $k = 4$.

59. (A)

Find the inverse of $f(x) = \frac{4x}{5} + 3$.

From $y = 4x/5 + 3$,

$x = \frac{5(y-3)}{4}.$

Thus $f^{-1}(x) = \frac{5(x-3)}{4}$.

60. (A)

Find the range of $f(x) = \frac{x-3}{5-x}$.

Let $y = (x-3)/(5-x)$. Then

$x = \frac{5y+3}{y+1},$

which exists for every real y except $y = -1$. Hence the range consists of all real numbers except -1.

61. (A)

For $R = \{(a,b) : b = a - 1, a \in \mathbb{Z}, 5 < a < 9\}$, find the range.

$a = 6,7,8$, so $b = 5,6,7$. The range is $\{5,6,7\}$.

62. (A)

If $f(x)=ax^2+bx+2$, $f(1)=4$, and $f(3)=38$, find $a-b$.

$a+b=2, 9a+3b=36 \Rightarrow 3a+b=12.$

Thus $a=5, b=-3$, and $a-b=8$.

63. (A)

For $f(x)=x^2-3x+4$ and $g(x)=2x+1$, solve $f(x)=(f\circ g)(x)$.

$=4x^2-2x+2.$

Hence $3x^2+x-2=0$, so

$x=-1, \frac{2}{3}.$

64. (C)

Find the domain of $f(x)=\frac{x+2}{9-x^2}$. .

The denominator vanishes at $x=\pm 3$. Therefore the domain consists of all real numbers except -3 and 3 .

65. (C)

For $f(x)=\frac{4x+7}{7x-4}$, evaluate $f\left(f\left(f(2)\right)\right)$.

$f(2)=\frac{3}{2}, f\left(\frac{3}{2}\right)=2.$

Therefore $f\left(f\left(f(2)\right)\right)=f(2)=\frac{3}{2}$.

66. (B)

For $f(x)=2x-3$ *and* $g(x)=x^3+5$, *find* $(f\circ g)^{-1}(x)$.

$(f\circ g)(x)=2x^3+7.$

Thus

$(f\circ g)^{-1}(x)=\left(\frac{x-7}{2}\right)^{1/3}$.

67. (A)

Classify $f(x)=\frac{e^x+e^{-x}}{e^x-e^{-x}}$.

$f(x)=\coth x$, and $\coth(-x)=-\coth x$. Hence f is odd.

68. (C)

Solve $[x]^2-5[x]+6=0$, where $[x]$ is the greatest-integer function.

$([x]-2)([x]-3)=0$, so $[x]=2$ or 3. Therefore, $2\le x<4$.

69. (A)

For the stated rational functions f and g, evaluate $(g\circ f)(3)$.

$f(3)=\frac{13}{8}, g\left(\frac{13}{8}\right)=3.$

70. (D)

For $A=\{1,2,3,4,5\}$, $B=\{1,4,5\}$, and relation $x>y$, find the range.

For $y=1$ there are $x>1$, and for $y=4$ there is $x=5$. No $x\in A$ exceeds 5. Thus the range is $\{1,4\}$.

71. (D)

Find the domain of $f(y)=\frac{\cos^{-1}(y-5)}{\sqrt{25-y^2}}$.

$-1\le y-5\le 1$ gives $4\le y\le 6$, while the denominator requires $-5<y<5$. Their intersection is given by $4\le y<5$.

72. (D)

If $f(x)=2x^2+bx+c$, $f(0)=3$, and $f(2)=1$, find $(f\circ f)(1)$.

$c=3$ and $8+2b+3=1$, so $b=-5$. Then $f(1)=0$, and

$(f\circ f)(1)=f(0)=3.$

73. (B)

For $f(x)=\frac{2x+3}{3x-2}$, classify $f\circ f$.

Direct substitution gives

$(f\circ f)(x)=x.$

Thus it is the identity function.

74. (B)

For the greatest-integer function, choose the statement true for every real x.

By definition $[x] \le x < [x]+1$. Therefore $[x]+1 > x$ for every real x.

75. (B)

Find the domain of $f(x) = \sqrt{x}$.

The radicand must be non-negative, so the domain is given by $x \ge 0$.

76. ()

For $R = \{(x, y) : y = x + 6 / x, x, y \in \mathbb{N}, x < 6\}$, find domain and range.

Testing $x = 1,2,3,4,5$, integral y occurs for $x = 1,2,3$, giving $y = 7,5,5$. Thus

$\text{Dom}R = \{1,2,3\}, \text{Ran}R = \{5,7\}$.

77. (C)

For $f(x) = \dfrac{3x+2}{5x-3}$, determine its inverse relation.

Solving $y = (3x+2)/(5x-3)$ for x gives

$$x = \frac{3y+2}{5y-3} = f(y).$$

Hence $f^{-1} = f$.

78. (A)

The given piecewise function is continuous on $[-4,2]$. Find $\alpha + \beta$.

Continuity at $x = -2$ requires

$6\beta - 3\alpha(-2) = 4(-2) + 1 = -7.$

Thus $6(\alpha + \beta) = -7$, so $\alpha + \beta = -7/6$.

79. (C)

The stated piecewise function is continuous at $x = 0$. Find k.

$$\lim_{x \to 0} \frac{4\sin \pi x}{5x} = \frac{4\pi}{5} = 2k.$$

Hence $k = 2\pi / 5$.

80. (B)

For $f(x) = \dfrac{|x-2|}{x-2}$ when $x \ne 2$ and $f(2) = 1$, determine continuity at 2.

The left-hand limit is -1, whereas the right-hand limit is 1. The one-sided limits differ, so f is discontinuous at 2.

81. (A)

Find discontinuities in the domain of the stated piecewise rational function.

$x = 1$ is excluded from the domain. At $x = 2$, the left value is 1 whereas the right limit is $7/5$. Thus the only discontinuity in its domain is $x = 2$.

82. (C)

Choose $f(0)$ so $f(x) = \dfrac{2^x - 1}{1 - 3^x}$ is continuous at 0.

Using $a^x - 1 \sim x \log a$,

$$f(0) = \lim_{x \to 0} \frac{x\log 2}{-x\log 3} = -\frac{\log 2}{\log 3}.$$

83. (D)

Find the odds in favour that the sum of two dice is a multiple of 3.

There are 12 favourable and 24 unfavourable outcomes. Odds in favour are $12:24 = 1:2$.

84. (B)

For the given discrete distribution, find $F(1.5) - F(-0.5)$.

This equals $P(-0.5 < X \le 1.5)$, so

$0.15 + 0.25 = 0.40.$

85. (D)

Find the odds in favour of drawing a king from a standard deck.

The odds are $4:48 = 1:12$.

86. (B)

Out of 100 people, 10 have a cold. Five are selected; find the probability at most one has a cold.

Using the binomial model intended by the options, $X \sim B(5, 0.1)$:

$$P(X \le 1) = 0.9^5 + 5(0.1)(0.9)^4 = 0.91854.$$

87. (A)

For $f(x) = x/8$ *on* $0 < x < 4$, *find* $P(X \le 2)$.

$$P(X \le 2) = \int_0^2 \frac{x}{8} dx = \left[\frac{x^2}{16}\right]_0^2 = \frac{1}{4}.$$

88. (D)

Find the expectation of the number shown by a fair die.

$$E(X) = \frac{1+2+3+4+5+6}{6} = \frac{7}{2} = 3.5.$$

89. (D)

A bomb misses with probability 0.2. Find the probability exactly two of ten bombs hit.

Here hit probability is $0.8 = 4/5$:

$$P = \binom{10}{2}\left(\frac{4}{5}\right)^2\left(\frac{1}{5}\right)^8 = \frac{144}{5^9}.$$

90. (D)

Repeat of the LOGARITHM arrangement probability.

$$P = \frac{3}{9}\cdot\frac{6}{8} = \frac{1}{4}.$$

91. (A)

Given: $f(x) = \begin{cases} \dfrac{x+2}{18}, & -2 < x < 4 \\ 0, & \text{otherwise} \end{cases}$

We need

$$P(|X| < 1) = P(-1 < X < 1) = \int_{-1}^{1} \frac{x+2}{18} dx$$

Step 1: Integrate

$$= \frac{1}{18}\left[\frac{x^2}{2} + 2x\right]_{-1}^{1}$$

Step 2; Evaluate at limits

At x = 1:

$$\frac{1^2}{2} + 2(1) = \frac{1}{2} + 2 = \frac{5}{2}$$

At $x = -1$:

$$\frac{(-1)^2}{2} + 2(-1) = \frac{1}{2} - 2 = -\frac{3}{2}$$

Step 3 : subtract :

$$\frac{5}{2} - \left(-\frac{3}{2}\right) = \frac{5}{2} + \frac{3}{2} = \frac{8}{2} = 4$$

Step 4 : Multiply by $\frac{1}{18}$

$$P = \frac{1}{18} \times 4 = \frac{4}{18} = \frac{2}{9}$$

92. (D)

For $P(X = x) = \binom{5}{x} / 2^5$, identify the statement that is not true.

The distribution is symmetric about 2.5, so the first two equalities are true and $P(X \le 2) \ge P(X \ge 4)$ is true. But

$$P(X \le 3) = \frac{26}{32} > \frac{16}{32} = P(X \ge 3).$$

Thus the stated $\le$ relation is false.

93. (B)

Define the value at $x = \pi$ to make $f(x) = \dfrac{1 - \sin x + \cos x}{1 + \sin x + \cos x}$ continuous.

Factor with half-angles:

$$f(x) = \frac{\cos(x/2) - \sin(x/2)}{\cos(x/2) + \sin(x/2)}.$$

Hence the limiting value at $x = \pi$ is -1.

94. (C)

For $f(x) = |x|/x$ when $x \ne 0$ and $f(0) = 1$, classify behaviour at 0.

The left limit is -1 and the right limit is 1, so f is neither continuous nor differentiable at 0.

95. (B)

How many real discontinuities does $f(x)=\dfrac{x+1}{9x+x^3}$ *have?*

The denominator is $x\left(x^2+9\right)$, which has the only real zero $x=0$. Therefore there is exactly one discontinuity.

96. (C)

The only discontinuity in the domain is $x=2$.

97. (A)

Find the truth values of the stated quantified propositions p and q.

p is true because natural numbers greater than 5 exist. q is false because $n^2-n=n(n-1)$ is always even, not odd. Thus $(p,q)=(T,F)$.

98. (D)

Identify the tautology among the four statement patterns.

$S_4=(p\to q)\vee(\neg p\to q)=(\neg p\vee q)\vee(p\vee q)=T.$

99. (D)

Negate: If $5<7$ and $7>2$, then $5>2$.

The negation of $(P\wedge Q)\to R$ is $P\wedge Q\wedge\neg R$.

Therefore it is $5\langle 7 \text{ and } 7\rangle 2 \text{ and } 5\le 2.$

100. (A)

Find the dual of 'Mangoes are delicious but expensive.'

'But' represents conjunction. Its dual replaces conjunction by disjunction: Mangoes are delicious or mangoes are expensive.

101. (A)

Negate $\neg p\vee(q\to\neg r)$.

$\neg p\vee(q\to\neg r)=\neg p\vee\neg q\vee\neg r.$

Its negation is $p\wedge q\wedge r$.

102. (C)

Because for $x=3$, $x+6=9$ is not divisible by 2, so the statement "for all x" is false.

103. (D)

Simplify $\left[p\wedge(q\vee r)\right]\vee\left[(\neg p\wedge q)\vee(\neg p\wedge r)\right]$.

Distributing,

$pq\vee pr\vee\neg p\,q\vee\neg p\,r=q(p\vee\neg p)\vee r(p\vee\neg p)=q\vee r.$

104. (C)

Translate the shown switching circuit into symbolic form.

The upper branch has p and q in series, giving $p\wedge q$. The lower branch has $\neg p$ in series with three parallel switches $\neg q, p, r$, giving $\neg p\wedge(\neg q\vee p\vee r)$. Thus $(p\wedge q)\vee\left[\neg p\wedge(\neg q\vee p\vee r)\right]=1.$

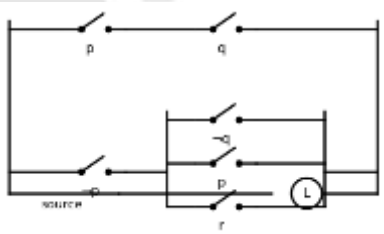


Fig: Switching circuit for Q104

105. (A)

Negate $\exists x\in A$ such that $x+5>8$.

$\neg\left(\exists x\in A: P(x)\right)\equiv\forall x\in A:\neg P(x).$

Hence $\forall x\in A$, $x+5\le 8$.

106. (D)

Identify the contradiction among the statement patterns.

$S_1=(p\to q)\wedge(p\wedge\neg q)=(\neg p\vee q)\wedge p\wedge\neg q,$

which is always false.

107. (A)

Find the contrapositive of 'If Raju is courageous, then he will join the Indian Army.'

The contrapositive of $p\to q$ is $\neg q\to\neg p$: If Raju does not join the Indian Army, then he is not courageous.

108. (B)

$(\neg p \vee (p \wedge \neg q)) \vee q$

Using distributive law:

$\neg p \vee (p \wedge \neg q) = (\neg p \vee p) \wedge (\neg p \vee \neg q)$

$= T \wedge (\neg p \vee \neg q) = \neg p \vee \neg q$

Now:

$(\neg p \vee \neg q) \vee q = \neg p \vee (\neg q \vee q) = \neg p \vee T = T$

So, expression is always TRUE $\rightarrow$ current always flows

109. (B)

Give an equivalent verbal statement for 'If the grass is green, then it rains in July.'

$p \rightarrow q \equiv \neg p \vee q$. Therefore: The grass is not green or it rains in July.

110. (B)

Give the logical equivalent of 'If Seema is fat, then she is happy.'

$p \rightarrow q \equiv \neg p \vee q$. Thus: Seema is not fat or she is happy.

111. (C)

For $A = \text{diag}(\omega, \omega)$, where $\omega^3 = 1$, find A^{-1}.

Since $\omega^{-1} = \omega^2$,

$A^{-1} = \text{diag}(\omega^2, \omega^2) = A^2$.

112. (D)

For $A = \text{diag}(2, -2, -1)$, find $A^4 A^{-1}$.

$A^4 A^{-1} = A^3$, hence

$A^3 = \text{diag}(8, -8, -1)$.

113. (C)

Find the inverse of the stated 2×2 trigonometric matrix A.

Direct multiplication gives $A^2 = I$. Therefore $A^{-1} = A$.

114. (D)

For the given 3×3 matrix A, find $\left[\text{adj}(\text{adj}A)\right]^{-1}$.

For a 3×3 matrix, $\text{adj}(\text{adj}A) = |A| A$. Here $|A| = 1$, so the expression is A^{-1}.

115. (D)

Using

$\frac{1}{\omega} = \omega^2, \quad \frac{1}{\omega^2} = \omega,$

$A = \begin{bmatrix} 1 & \omega^2 & \omega \\ \omega^2 & \omega & 1 \\ \omega & 1 & \omega^2 \end{bmatrix}.$

Now,

$|A| = 0$

So A is singular, hence A^{-1} does not exist.

116. (B)

Find the sum of cofactors of the entries in the second row of the given matrix.

The second-row cofactors are $1, -9, 13$. Their sum is $1 - 9 + 13 = 5$.

117. (B)

In a triangle, simplify $\frac{\cos A - \cos C}{a - c} + \frac{\cos B}{b}$.

Using the cosine rule for $\cos A, \cos B, \cos C$ and simplifying gives

$\frac{\cos A - \cos C}{a - c} + \frac{\cos B}{b} = -\frac{1}{b}.$

118. (B)

If $\tan A, \tan B, \tan C$ are in harmonic progression, determine the progression of a^2, b^2, c^2.

The H.P. condition gives $2\cot B = \cot A + \cot C$. Converting by the sine and cosine rules yields

$a^2 + c^2 = 2b^2$.

Hence a^2, b^2, c^2 are in arithmetic progression.

119. (C)

In $\triangle ABC$, $a = \sqrt{3} + 1$, $b = \sqrt{3} - 1$, *and* $C = 60^\circ$. *Find* $A - B$.

$\tan\frac{A - B}{2} = \frac{a - b}{a + b}\cot\frac{C}{2} = \frac{2}{2\sqrt{3}} \cdot \sqrt{3} = 1.$

Thus $(A-B)/2=45^\circ$, so $A-B=90^\circ$.

120. (D)

Solve $\frac{1-\cos 2x}{1+\cos 2x}=3$ generally.

The left side is $\tan^2 x$. Thus $\tan x=\pm\sqrt{3}$, so

$x=n\pi\pm\frac{\pi}{3}, n\in\mathbb{Z}.$

121. (A)

Two linear combinations of non-collinear vectors are collinear. Find x.

For $(2x+1)\vec{a}-\vec{b}$ and $(x-2)\vec{a}+\vec{b}$ to be collinear, the determinant of coefficients must vanish:

$(2x+1)(1)-(-1)(x-2)=0.$

Hence $3x-1=0$, so $x=1/3$.

122. (A)

For the given vectors, evaluate $(\vec{a}\times\vec{b})\cdot(\vec{a}\times\vec{c})$.

$\vec{a}\times\vec{b}=(-10,9,7), \vec{a}\times\vec{c}=(4,-3,-1).$

Their dot product is $-40-27-7=-74$.

123. (B)

Find the volume of the parallelepiped with edges $\vec{u}\times\vec{v}$, $\vec{u}+\vec{w}$, *and* $\vec{v}+\vec{w}$.

The scalar triple product of the three edge vectors is 24. Hence the volume is 24 cubic units.

124. (B)

Given $\overrightarrow{AB}$ and $\overrightarrow{AC}$, find the length of the median from A.

The median vector is

$\frac{\overrightarrow{AB}+\overrightarrow{AC}}{2}=\frac{(3,5,4)+(5,-5,2)}{2}=(4,0,3).$

Its length is 5.

125. (C)

Find the volume of the tetrahedron with the four given vertices.

$V=\frac{1}{6}\left|\det(B-A,C-A,D-A)\right|=\frac{32}{6}=\frac{16}{3}.$

126. (B)

If $(2,-q,3)$ and $(4,-5,6)$ are collinear, find q.

The second vector is twice the first, so $-5=-2q$.

Hence $q=5/2$.

127. (B)

Evaluate the ratio of scalar triple products shown in the question.

The coefficient determinant for $\vec{a}+2\vec{b}$, $\vec{b}+2\vec{c}$, $\vec{c}+2\vec{a}$ is

$\det\begin{pmatrix}1&0&2\\2&1&0\\0&2&1\end{pmatrix}=9.$

Thus, the ratio is 9.

128. (D)

Find direction cosines of a line in the ZOX plane making 30° with the Z-axis.

Since the line lies in ZOX, $m=0$. Also $n=\cos 30^\circ=\sqrt{3}/2$, and $l^2=1-n^2=1/4$. Hence

$(l,m,n)=\left(\pm\frac{1}{2},0,\frac{\sqrt{3}}{2}\right).$

129. (D)

Given $[\vec{a}\vec{b}\vec{c}]=3$, *find the volume with edges* $2\vec{a}+\vec{b}$, $2\vec{b}+\vec{c}$, $2\vec{c}+\vec{a}$.

The coefficient determinant is 9, so the new scalar triple product is $9\cdot 3=27$.

130. (A)

For the given vectors, evaluate $(\vec{c}-\vec{a})\cdot(\vec{b}\times\vec{d})$.

Direct cross and dot products give

$(\vec{c}-\vec{a})\cdot(\vec{b}\times\vec{d})=12.$

131. (C)

The three stated vectors are coplanar. Find the equation of which λ is a root.

Coplanarity gives

$\det\left(\vec{a},\vec{b},\vec{c}\right)=7\lambda+28=0,$

so $\lambda=-4$. It satisfies $x^2+3x=4$.

132. (C)

Find m so the three stated vectors are coplanar.

The determinant is $4-2m$. Setting it to zero gives $m=2$.

133. (D)

Evaluate the shown ratio of scalar triple products.

$\left[\vec{a}+\vec{b},\vec{b}+\vec{c},\vec{c}+\vec{a}\right]=2\left[\vec{a},\vec{b},\vec{c}\right],$

and $\left[\vec{b},\vec{c},\vec{a}\right]=\left[\vec{a},\vec{b},\vec{c}\right]$. The ratio is 2.

134. (C)

Evaluate the four-vector product $\left[\vec{b}\,\vec{a}\,\vec{b}\,\vec{a}\right]$.

With $\left[\vec{p}\vec{q}\vec{r}\vec{s}\right]=\left(\vec{p}\times\vec{q}\right)\cdot\left(\vec{r}\times\vec{s}\right)$,

$\left[\vec{b}\vec{a}\vec{b}\vec{a}\right]=\left|\vec{b}\times\vec{a}\right|^2=\left|\vec{a}\times\vec{b}\right|^2$.

135. (B)

If G is the centroid of $\triangle ABC$, simplify $\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}+\overrightarrow{OG}$.

For a centroid,

$\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}=3\overrightarrow{OG}.$

Adding $\overrightarrow{OG}$ gives $4\overrightarrow{OG}$.

136. (A)

Evaluate k in the stated vector identity involving cyclic cross products.

Expanding the dot product, the only nonzero terms are

$\vec{a}\cdot\left(\vec{b}\times\vec{c}\right)+\vec{b}\cdot\left(\vec{c}\times\vec{a}\right)+\vec{c}\cdot\left(\vec{a}\times\vec{b}\right)=3\left[\vec{a}\vec{b}\vec{c}\right].$

Thus $k=3$.

137. (A)

Find the line through $(1,2,3)$ perpendicular to the two given lines.

The direction vectors are $(1,2,3)$ and $(-3,2,5)$.

Their cross product is $(4,-14,8)=2(2,-7,4)$.

Hence

$\vec{r}=\left(\hat{i}+2\hat{j}+3\hat{k}\right)+\lambda\left(2\hat{i}-7\hat{j}+4\hat{k}\right).$

138. (D)

Find the distance of $(1,2,-1)$ from $x-2y+4z+10=0$.

$d=\dfrac{|1-4-4+10|}{\sqrt{1+4+16}}=\dfrac{3}{\sqrt{21}}=\dfrac{\sqrt{21}}{7}.$

139. (A)

Find where the given line meets $2x+4y-z=1$.

Put $\dfrac{x-1}{2}=\dfrac{y-2}{-3}=\dfrac{z+3}{4}=t$. Substitution in the plane gives $t=1$, hence

$(x,y,z)=(3,-1,1).$

140. (B)

Find planes parallel to $x+2y+2z+8=0$ at distance 2 from $(1,1,2)$.

Let the plane be $x+2y+2z+c=0$. Then

$\dfrac{|7+c|}{3}=2\Rightarrow c=-1$ or $-13.$

Thus, the planes are $x+2y+2z-1=0$ and $x+2y+2z-13=0$.

141. (A)

Find a plane through $(1,-1,2)$ perpendicular to the two given planes.

A normal is the cross product

$(2,3,-2)\times(1,2,-3)=(-5,4,1),$

or $(5,-4,-1)$. Through the point, the plane is

$\vec{r}\cdot\left(5\hat{i}-4\hat{j}-\hat{k}\right)=7.$

142. (C)

Find the line through $(2,4,6)$ parallel to $3x+4=4y-1=1-4z$.

The given line has direction ratios proportional to $(4,3,-3)$. Therefore

$$\frac{x-2}{4}=\frac{y-4}{3}=\frac{z-6}{-3}.$$

143. (A)

Find the plane through $(1,-1,1)$ parallel to $2x+3y-4z=17$.

The normal is $(2,3,-4)$. Substitution of the point gives $2-3-4=-5$, so

$$\vec{r}\cdot\left(2\hat{i}+3\hat{j}-4\hat{k}\right)=-5.$$

144. (D)

Find the plane through $(2,3,1)$ and $(4,-5,3)$ parallel to the Y-axis.

The direction between the points is $(2,-8,2)$, and another direction is $(0,1,0)$. A normal is $(1,0,-1)$.

Hence

$$x-z=1.$$

145. (B)

Find the distance from the origin to $\vec{r}\cdot\left(\hat{i}-2\hat{j}+3\hat{k}\right)=14$.

$$d=\frac{14}{\sqrt{1+4+9}}=\sqrt{14}.$$

146. (A)

The given line is parallel to a plane containing parameter m. Find m.

Line direction $(2,1,2)$ must be perpendicular to plane normal $(3,-2,-m)$:

$$6-2-2m=0\Rightarrow m=2.$$

147. (C)

Plane1: $2x-5y+z=8\rightarrow$ Normal $\vec{n}_1=(2,-5,1)$

Plane 2: $2\lambda x-15y+\lambda z+6=0\rightarrow$ Normal $\vec{n}_2=(2\lambda,-15,\lambda)$

For parallel planes:

$$\frac{2\lambda}{2}=\frac{-15}{-5}=\frac{\lambda}{1}$$

$$\lambda=3$$

148. (B)

The two given vector lines are perpendicular. Find m.

Their direction vectors are $(1,2,m)$ and $(2,1,6)$.

Thus

$$2+2+6m=0\Rightarrow m=-\frac{2}{3}.$$

149. (C)

The two stated lines intersect. Find k.

Solving the x- and z-coordinates gives parameters $t=-3/2$, $s=-5$. Equating y-coordinates then gives

$$-1+3\left(-\frac{3}{2}\right)=k+2(-5),$$

so $k=9/2$.

150. (D)

Find the plane through $(0,7,-7)$ containing the given line.

A point on the line is $(-1,3,-2)$ and its direction is $(-3,2,1)$. The vector to $(0,7,-7)$ is $(1,4,-5)$; their cross product is proportional to $(1,1,1)$. Thus

$$x+y+z=0.$$

151. (A)

Classify the L.P.P. with $x+y\le 1$ and $2x+2y\ge 6$, $x,y\ge 0$.

The second constraint is $x+y\ge 3$, which contradicts $x+y\le 1$. Hence the feasible set is empty.

152. (D)

If an L.P.P. has optimum solutions at two consecutive corner points, how many optimum solutions exist?

The objective line coincides with the edge joining those vertices, so every point on that edge is optimal. There are infinitely many optimum solutions.

153. (D)

Identify the shape of the feasible region for the stated two inequalities in the first quadrant.

The boundary vertices are $(0,0)$, $(15/2,0)$, the intersection $(5,4)$, and $(0,8)$. Hence the feasible region is a quadrilateral.

154. (B)

Classify the minimum solutions of the stated L.P.P.

The minimum possible value is $x+y=15$. A whole feasible segment of the line $x+y=15$ satisfies the remaining bounds, so there are infinitely many minimizers.

155. (A)

Classify the pair of lines $9x^2-12xy+4y^2=0$.

$9x^2-12xy+4y^2=(3x-2y)^2$.

It represents coincident lines.

156. (A)

Find k so $kxy+5x+3y+2=0$ represents a pair of lines.

The second-degree determinant condition gives $k(2k-15)=0$. Excluding $k=0$, which reduces the equation to a single first-degree line, gives $k=\frac{15}{2}$.

157. (B)

Repeat of the intercept-midpoint problem.

The line is $2x-y=4a$.

158. (C)

The angle between the pair of lines represented by the stated second-degree equation is $\tan^{-1}(1/3)$.

Find $\lambda \geq 0$.

For $ax^2+2hxy+by^2=0$,

$$\tan\theta=\frac{2\sqrt{h^2-ab}}{|a+b|}=\frac{\sqrt{9-4\lambda}}{|1+\lambda|}=\frac{1}{3}.$$

This gives $\lambda^2+38\lambda-80=0$, so $\lambda=2$ or -40. With $\lambda \geq 0$, $\lambda=2$.

159. (C)

Given $x=\log t$ and $y+1=1/t$, evaluate $e^{-x}\frac{d^2x}{dy^2}+\frac{dx}{dy}$.

Since $y+1=e^{-x}$, $x=-\log(y+1)$. Thus

$$\frac{dx}{dy}=-e^{x},\ \frac{d^2x}{dy^2}=e^{2x}.$$

The required expression is $e^{x}-e^{x}=0$.

160. (B)

If $\tan u=\sqrt{\frac{1-x}{1+x}}$ and $\cos v=4x^3-3x$, find du/dv.

Set $x=\cos 2u$. Then the first relation holds, and

$\cos v=4\cos^{32}u-3\cos 2u=\cos 6u.$

Hence $v=6u$ on the intended branch, so $du/dv=1/6$.

161. (C)

$$y=\left(\frac{x^2}{x+1}\right)^x$$

Taking log:

$$\ln y=x\ln\left(\frac{x^2}{x+1}\right)$$

Differentiate:

$$\frac{1}{y}\frac{dy}{dx}=\ln\left(\frac{x^2}{x+1}\right)+x\cdot\frac{x+2}{x(x+1)}$$

$$=\ln\left(\frac{x^2}{x+1}\right)+\frac{x+2}{x+1}$$

Compare with:

$$\frac{1}{y}\frac{dy}{dx}=g(x)+\log\left(\frac{x^2}{x+1}\right)$$

So:

$$g(x)=\frac{x+2}{x+1}$$

162. (A)

Given $x^2+y^2=t+1/t$ and $x^4+y^4=t^2+1/t^2$, find dy/dx.

Squaring the first equation and comparing with the second gives $x^2y^2=1$, so xy is constant. Differentiating,

$$y+x\frac{dy}{dx}=0\Rightarrow\frac{dy}{dx}=-\frac{y}{x}.$$

163. (D)

Differentiate the stated logarithmic expression involving a^{3x} and $\left[(5-x)/(x+4)\right]^{3/4}$.

Taking logarithms termwise,

$$\frac{dy}{dx}=3\log a-\frac{3}{4(5-x)}-\frac{3}{4(x+4)}.$$

164. (D)

For the printed function $y=x^{xe^x}$ and $dy/dx=y\,g(x)$, find $g(x)$.

$\log y=xe^x\log x.$

Therefore

$g(x)=e^x(x+1)\log x+e^x.$

165. (C)

For $y=e^{4x}\cos 5x$, find $y''(0)$.

$y''=e^{4x}\{(16-25)\cos 5x-40\sin 5x\}.$

Thus $y''(0)=-9$.

166. (B)

Implicitly differentiate $y\sqrt{1-x^2}+x\sqrt{1-y^2}=1.$

Let $x=\sin A$, $y=\sin B$. The equation gives $\sin(A+B)=1$, so $A+B=\pi/2$. Differentiating yields

$$\frac{dy}{dx}=-\sqrt{\frac{1-y^2}{1-x^2}}.$$

167. (C)

Differentiate $y=\tan^{-1}\sqrt{\frac{1+\cos(x/2)}{1-\cos(x/2)}}$.

The square root is $\cot(x/4)$ on the intended interval, so

$$y=\frac{\pi}{2}-\frac{x}{4},\frac{dy}{dx}=-\frac{1}{4}.$$

168. (D)

If $y=1+x+x^2/2!+x^3/3!+\cdots$, *find* dy/dx.

The series is e^x, so $dy/dx=e^x=y$.

169. (B)

If $y=2^{ax}$ and $y'(1)=\log 256$, find a.

$y'(1)=a(\log 2)2^a=8\log 2.$

Thus $a2^a=8$, giving $a=2$.

170. (D)

If $f(x)=e^xg(x)$, $g(0)=4$, $g'(0)=2$, *find* $f'(0)$.

$f'(x)=e^x(g+g'),\ f'(0)=4+2=6.$

171. (D)

If $f\circ g=I$, $g'(A)=2$, *and* $g(A)=B$, *find* $f'(B)$.

Differentiating $f(g(x))=x$ at A,

$$f'(B)g'(A)=1\Rightarrow f'(B)=\frac{1}{2}.$$

172. (D)

Find the derivative of $f(\tan x)$ with respect to $g(\sec x)$ at $x=\pi/4$.

$$\frac{df(\tan x)}{dg(\sec x)}=\frac{f'(\tan x)\sec^2 x}{g'(\sec x)\sec x\tan x}.$$

At $\pi/4$, this is $4/\left(4\sqrt{2}\right)=1/\sqrt{2}$.

173. (C)

Why can Rolle's theorem not be applied to $f(x)=|x-2|$ on $[0,4]$?

The function is continuous and $f(0)=f(4)$, but it is not differentiable at $x=2\in(0,4)$.

174. (C)

For $s=\sqrt{1+t}$, identify what acceleration is proportional to.

$$v=\frac{1}{2\sqrt{1+t}}, a=-\frac{1}{4(1+t)^{3/2}}=-2v^3.$$

Thus acceleration is proportional to the cube of velocity.

175. (A)

A circle's radius increases at 2 cm/min. Find dA/dt when $r=3$ cm.

$$\frac{dA}{dt}=2\pi r\frac{dr}{dt}=2\pi(3)(2)=12\pi \text{ cm}^2/\text{min}.$$

176. (D)

A square's area increases at $0.5\text{cm}^2/\text{s}$. Find the rate of change of perimeter when the side is 10cm .

$$2s\frac{ds}{dt}=0.5\Rightarrow\frac{ds}{dt}=0.025.$$

Since $P=4s$, $dP/dt=0.1$ cm/s.

177. (A)

Find the normal to $2x^2+y^2=12$ at $(2,2)$.

$$4x+2y\frac{dy}{dx}=0\Rightarrow m_t=-2.$$

Thus $m_n=1/2$, and

$$y=\frac{1}{2}(x-2)\Rightarrow x-2y+2=0.$$

178. (A)

Maximise the volume of a cylinder when $r+h=6$ m.

$V=\pi r^2(6-r), V'=3\pi r(4-r)$.

The maximum occurs at $r=4, h=2$, giving $V=32\pi\,\text{m}^3$.

179. (B)

Find where $f(x)=3x^4+16x^3-30x^2+10$ is increasing.

$(x)=12x(x+5)(x-1)$.

A sign chart gives $f'(x)>0$ on $(-5,0)\cup(1,\infty)$.

180. (A)

Find the normal to $x=4\sec\theta$, $y=4\tan^2\theta$. at $\theta=\pi/4$. .

At $\theta=\pi/4$, $(x,y)=(4\sqrt{2},4)$ and

$$\frac{dy}{dx}=2\sec\theta=2\sqrt{2}..$$

e normal slope is $-1/(2\sqrt{2})$, giving

$$x+2\sqrt{2}\,y=12\sqrt{2}.$$

181. (B)

For $s=t^3-6t^2+9t+25$, find displacement when acceleration is zero.

$a=s''=6t-12=0\Rightarrow t=2$.

Then $s(2)=8-24+18+25=27$.

182. (D)

A 108 m wire forms a rectangle of maximum area. Find its dimensions.

$2(l+b)=108$, so $l+b=54$. For fixed sum, product lb. is greatest at $l=b=27$. .

183. (D)

pximate $\log_{10}99$. using $\log_{10}e=0.4343$.

$\log_{10}99=2+\log_{10}(0.99)\approx 2-0.01\log_{10}e=1.995657\approx 1.9957$.

184. (C)

Find P,Q,R. in the stated partial-fraction integral.

$$\frac{x^2+1}{(x-3)(x-2)}=1+\frac{10}{x-3}-\frac{5}{x-2}.$$

us $(P,Q,R)=(1,10,-5)$.

185. (B)

Evaluate $\int \frac{\left(\sin^{-1}x\right)^{3/2}}{\sqrt{1-x^2}}dx$.

Let $u=\sin^{-1}x$. Then

$\int u^{3/2}\,du=\frac{2}{5}u^{5/2}+C=\frac{2}{5}\left(\sin^{-1}x\right)^{5/2}+C.$

186. (B)

Evaluate $\int \frac{1+\log x}{\cos^2(x\log x)}dx$.

Let $u=x\log x$, so $du=(1+\log x)dx$. Then

$\int \sec^2 u\,du=\tan u+C=\tan(x\log x)+C.$

187. (D)

Evaluate the stated integral involving 5^x and $\sqrt{5^{-2x}-5^{2x}}$.

It reduces to

$\int \frac{5^{2x}}{\sqrt{1-5^{4x}}}dx.$

With $u=5^{2x}$,

$I=\frac{\sin^{-1}\left(5^{2x}\right)}{\log 25}+C.$

188. (D)

Evaluate $\int \frac{\sec x}{\sqrt{\log(\sec x+\tan x)}}dx$.

Let $u=\log(\sec x+\tan x)$; then $du=\sec x\,dx$.

Hence

$I=2\sqrt{u}+C=2\sqrt{\log(\sec x+\tan x)}+C.$

189. (C)

Evaluate $\int \frac{e^x(1+2x)}{\sqrt{x}}dx$.

$\frac{d}{dx}\left(2\sqrt{x}\,e^x\right)=\frac{e^x}{\sqrt{x}}+2\sqrt{x}\,e^x=\frac{e^x(1+2x)}{\sqrt{x}}.$

Thus $I=2\sqrt{x}\,e^x+C$.

190. (A)

If $\int x^x(1+\log x)dx=kx^x+C$, *find* k.

Since $\frac{d}{dx}x^x=x^x(1+\log x)$, $k=1=\log_e e$.

191. (D)

Evaluate $\int \cot x\log\left[\log(\sin x)\right]dx$.

Let $u=\log(\sin x)$, so $du=\cot x\,dx$. Then

$\int \log u\,du=u(\log u-1)+C.$

Thus the answer is $\log(\sin x)\left[\log\log(\sin x)-1\right]+C$

192. (A)

In the stated antiderivative of $\sin\theta/\sin 3\theta$, find k.

With $t=\tan\theta$,

$\int \frac{\sin\theta}{\sin 3\theta}d\theta=\int \frac{dt}{3-t^2}=\frac{1}{2\sqrt{3}}\log\left|\frac{\sqrt{3}+t}{\sqrt{3}-t}\right|+C.$

Hence $k=\sqrt{3}$.

193. (C)

Let $t=x-\frac{1}{x}\Rightarrow dt=\left(1+\frac{1}{x^2}\right)dx=\frac{x^2+1}{x^2}dx$

Integral becomes:

$$\int \sqrt{t}dt=\frac{2}{3}t^{3/2}+c$$

So:

$$\frac{2}{3}\left(x-\frac{1}{x}\right)^{3/2}+c$$

Comparing:

$$k=\frac{3}{2}$$

194. (B)

Evaluate $\int (1+x)\log x\,dx$.

$I=\int \log x\,dx+\int x\log x\,dx$

$I=\left(x+\frac{x^2}{2}\right)\log x-x-\frac{x^2}{4}+C.$

195. (B)

Evaluate $\int_0^\infty \frac{dx}{(x^2+4)(x^2+9)}$.

$$\frac{1}{(x^2+4)(x^2+9)} = \frac{1}{5}\left(\frac{1}{x^2+4} - \frac{1}{x^2+9}\right).$$

Thus

$$I = \frac{1}{5}\left(\frac{\pi}{4} - \frac{\pi}{6}\right) = \frac{\pi}{60}.$$

196. (B)

Evaluate the symmetric integral of $\left(e^x + e^{-x}\right)/\left(e^x - e^{-x}\right)$ from -5 to 5.

The integrand coth$x$ is odd. Its Cauchy principal value over $[-5,5]$ is 0.

Intended option: (B). As an ordinary improper integral it is singular at $x = 0$ and does not converge.

197. (D)

Evaluate the symmetric integral shown in Q198.

The factor x^2 is even and

$$\frac{e^{x^3} - e^{-x^3}}{e^{x^3} + e^{-x^3}} = \tanh\left(x^3\right)$$

is odd. Hence the integrand is odd and the integral over $[-a,a]$ is 0.

198. (D)

Given $\int_{-8}^{8} \frac{x^5 + x^3}{4 - x^2} dx$

Step 1: Check if integer and is odd / even

Let $f(x) = \frac{x^5 + x^3}{4 - x^2}$

Step 2 :

$$f(-x) = \frac{(-x)^5 + (-x)^3}{4 - (-x)^2} = \frac{-x^5 - x^3}{4 - x^2} = -f(x)$$

So $f(x)$ is an odd function.

Step 3: For an odd function,

$$\int_{-a}^{a} f(x)dx = 0$$

199. (C)

$$I = \int_0^{\pi/2} \frac{\sin x - \cos x}{1 + \sin^2 x} dx$$

Let $x \to \frac{\pi}{2} - x$:

$$I = \int_0^{\pi/2} \frac{\cos x - \sin x}{1 + \cos^2 x} dx$$

Add both:

$$2I = \int_0^{\pi/2} (\sin x - \cos x)\left(\frac{1}{1+\sin^2 x} - \frac{1}{1+\cos^2 x}\right) dx = 0$$

So $I = 0$.

200. (B)

Let $t = \sin x - \cos x$

$dt = (\cos x + \sin x)dx, \sin 2x = 1 - t^2$

Limits: $x = 0 \to t = -1, x = \pi/4 \to t = 0$

$$I = \int_{-1}^{0} \frac{dt}{9 + 16(1 - t^2)} = \int_{-1}^{0} \frac{dt}{25 - 16t^2}$$

$$= \frac{1}{40}\left[\ln\left(\frac{5+4t}{5-4t}\right)\right]_{-1}^{0} = \frac{1}{40}(0 - \ln 9) = \frac{1}{20}\ln 3$$

So $k = \frac{1}{20}$

201. (A)

Let $x = a\sin^2\theta$, then $dx = 2a\sin\theta\cos\theta d\theta$, limits: $0 \to \pi/2$.

$$I = \int_0^{\pi/2} (a\cos^2\theta)^{3/2}(a^2\sin^4\theta)(2a\sin\theta\cos\theta)d\theta$$

$$= 2a^{9/2}\int_0^{\pi/2} \sin^5\theta\cos^4\theta d\theta$$

Using Beta function:

$$\int_0^{\pi/2} \sin^5\theta\cos^4\theta d\theta = \frac{8}{315}$$

So:

$$I = 2a^{9/2} \cdot \frac{8}{315} = \frac{16a^{9/2}}{315}$$

202. (B)

$$I = \int_{-2}^{1} [x+1] dx$$

Let $t = x + 1$, then limits: $t = -1$ to 2:

$$I=\int_{-1}^{2}|t|\,dx$$

$[-1,0):[t]=-1$

$[0,1):[t]=0$

$[1,2):[t]=1$

$$I=\int_{-1}^{0}(-1)dt+\int_{0}^{1}0dt+\int_{1}^{2}1dt$$

$$=-1+0+1=0$$

203. (C)

$$I=\int_{0}^{\pi/2}\log\left(\frac{1-\cos 2x}{\sqrt{1+\cos 2x}}\right)dx$$

Let $f(x)=\log\left(\frac{1-\cos 2x}{\sqrt{1+\cos 2x}}\right)$

Check:

$$f\left(\frac{\pi}{2}-x\right)=\log\left(\frac{1+\cos 2x}{\sqrt{1-\cos 2x}}\right)=-f(x)$$

So f is odd about $\pi/4$, hence integral over symmetric limits is 0.

204. (C)

Evaluate $\int_{0}^{a}\sqrt{\frac{x}{a-x}}\,dx$.

Let $x=a\sin^2\theta$. Then

$$I=2a\int_{0}^{\pi/2}\sin^2\theta\,d\theta=\frac{\pi a}{2}.$$

205. (C)

Evaluate $\int_{2}^{3}\frac{x}{x^2-1}dx$.

$$I=\frac{1}{2}\left[\log\left(3^2-1\right)-\log\left(2^2-1\right)\right]=\frac{1}{2}\log\frac{8}{3}.$$

206. (C)

Given:

$$I=\int_{0}^{1}\frac{x^2-2}{x^2+1}dx$$

Step1: Rewrite numerator:

$$\frac{x^2-2}{x^2+1}=\frac{x^2+1-3}{x^2+1}=1-\frac{3}{x^2+1}$$

Step 2: Integrate: $I=\int_{0}^{1}1dx-3\int_{0}^{1}\frac{1}{x^2+1}dx\ \$$

$$=[x]_0^1-3[\tan^{-1}x]_0^1$$

$$=(1-0)-3\left(\frac{\pi}{4}-0\right)$$

$$=1-\frac{3\pi}{4}$$

207. (C)

$$\frac{\tan x}{\tan x+\cot x}=\sin^2 x$$

So:

$$I=\int_{\pi/5}^{3\pi/10}\sin^2 x\,dx=\frac{1}{2}\int_{\pi/5}^{3\pi/10}(1-\cos 2x)dx$$

$$=\frac{1}{2}\left[x-\frac{\sin 2x}{2}\right]_{\pi/5}^{3\pi/10}$$

$\sin(3\pi/5)=\sin(2\pi/5)$, so they cancel:

$$I=\frac{1}{2}\left(\frac{3\pi}{10}-\frac{\pi}{5}\right)=\frac{\pi}{20}$$

208. (C)

Given: $I=\int_{0}^{\pi}\frac{e^{\cos x}}{e^{\cos x}+e^{-\cos x}}dx$

Step 1: Use property:

$$\int_0^a f(x)dx=\int_0^a f(a-x)dx$$

Here $a=\pi$:

$$I=\int_{0}^{\pi}\frac{e^{-\cos x}}{e^{\cos x}+e^{-\cos x}}dx$$

Step 2: Add both forms:

$$2I=\int_{0}^{\pi}\frac{e^{\cos x}+e^{-\cos x}}{e^{\cos x}+e^{-\cos x}}dx=\int_{0}^{\pi}1dx=\pi$$

Step 3: So $I=\frac{\pi}{2}$

209. (B)

Given:

$$I=\int_{0}^{1}\left(1-\frac{x}{1!}+\frac{x^2}{2!}-\frac{x^3}{3!}+\cdots\right)e^{2x}dx$$

Step 1 : Recognize the series:

$$1-\frac{x}{1!}+\frac{x^2}{2!}-\frac{x^3}{3!}+\cdots=e^{-x}$$

So:

$$I=\int_0^1 e^{-x}\cdot e^{2x}dx=\int_0^1 e^x dx$$

Step 2: Integrate:

$$I=[e^x]_0^1=e-1$$

210. (A)

$$\int_0^1(5x^2-3x+k)dx=0$$

Step 1 ;: Integrate :

$$\left[\frac{5x^3}{3}-\frac{3x^2}{2}+kx\right]_0^1=0$$

Step 2: Evaluate at limits:

$$\left(\frac{5}{3}-\frac{3}{2}+k\right)-0=0$$

Step 3 : Simplify:

$$\frac{5}{3}-\frac{3}{2}=\frac{10-9}{6}=\frac{1}{6}$$

So:

$$\frac{1}{6}+k=0\Rightarrow k=-\frac{1}{6}$$

211. (A)

Let $f(x)=\dfrac{2x}{1+\cos^2 x}$

$$f(-x)=\frac{-2x}{1+\cos^2 x}=-f(x)$$

So f is odd.

$$\int_{-\pi}^{\pi} f(x)dx=0$$

212. (C)

Evaluate $\int_0^1 \tan^{-1}\left(\dfrac{2x}{1-x^2}\right)dx$.

For, the inverse tangent equals $2\tan^{-1}x$. Therefore,

$$I=2\left[x\tan^{-1}x-\frac{1}{2}\log(1+x^2)\right]_0^1=\frac{\pi}{2}-\log 2$$

213. (A)

Given: Parabola $x^2=16y\Rightarrow x=4\sqrt{y}$

(first quadrant) boundaries: y = 1, y = 4, and Y-axis x = 0)

Step 1: Area in first quadrant:

$$\text{Area}=\int_{y=1}^4 x\,dy=\int_1^4 4\sqrt{y}\,dy$$

Step 2: Integrate:

$$=4\int_1^4 y^{1/2}dy=4\left[\frac{y^{3/2}}{3/2}\right]_1^4=4\cdot\frac{2}{3}\left[y^{3/2}\right]_1^4$$

$$=\frac{8}{3}\left(4^{3/2}-1^{3/2}\right)=\frac{8}{3}(8-1)=\frac{8}{3}\times 7=\frac{56}{3}$$

214. (C)

Find the area between $y=\sin x$ and the X-axis from $x=-\pi$ to $x=3\pi/2$.

Total geometric area is

$$\int_{-\pi}^{0}(-\sin x)dx+\int_0^{\pi}\sin x dx+\int_{\pi}^{3\pi/2}(-\sin x)dx=2+2+1=5.$$

215. (D)

Find the area of the triangle formed by the vertex and latus-rectum endpoints of $x^2=12y$.

Here $4a=12$, so $a=3$. The latus-rectum endpoints are $(\pm 6,3)$. The triangle has base 12 and height 3:

$$A=\frac{1}{2}(12)(3)=18.$$

216. (C)

Find the area bounded by $x^2=4y$, $y=2$, and $y=4$

The horizontal width is $4\sqrt{y}$. Hence

$$A=\int_2^4 4\sqrt{y}\,dy=\frac{8}{3}\left(8-2\sqrt{2}\right).$$

217. (D)

Find the area under $y=\log x$ from $x=1$ to $x=e$.

$$A=\int_1^e \log x\,dx=\left[x\log x-x\right]_1^e=1.$$

218. (A)

Find the area inside $x^2 + y^2 = 16$ between $x = 0$ and $x = 2$.

The vertical width is $2\sqrt{16 - x^2}$. Thus

$$A = 2\int_0^2 \sqrt{16 - x^2}\,dx = 4\sqrt{3} + \frac{8\pi}{3}.$$

219. (C)

Find the area under $y = 4x^3 - 6x^2 + 4x + 1$ from $x = 1$ to $x = 5$.

The function is positive on the interval, so

$$A = \left[x^4 - 2x^3 + 2x^2 + x\right]_1^5 = 430 - 2 = 428.$$

220. (A)

A body cools from 135° C to 80° C in one hour in a 25° C room. Find its temperature after two hours.

The excess temperature falls from 110 to 55, so it halves each hour. After two hours the excess is 27.5, giving

$$T = 25 + 27.5 = 52.5^\circ \text{C}.$$

221. (A)

Solve the stated differential equation with $x = 0$ when $y = 3\pi/4$.

Treating x as a function of y,

$$\frac{dx}{dy} + x\csc^2 y = \cot y \csc^2 y.$$

The solution satisfying the condition is

$$x = 1 + \cot y.$$

222. (D)

Find the degree of the differential equation containing $e^{dy/dx}$.

Because the derivative occurs inside an exponential function, the equation cannot be expressed as a polynomial in derivatives. Its degree is not defined.

223. (D)

Solve $x^2 \dfrac{dy}{dx} = y^2 + xy$.

Let $v = y/x$. Then $y' = v + xv'$ and

$$v + xv' = v^2 + v \Rightarrow \frac{dv}{v^2} = \frac{dx}{x}.$$

Thus $-1/v = \log x + C$, or

$$\frac{x}{y} + \log x = C.$$

224. (B)

Find the differential equation of circles centred on $y = 8$ and tangent to the X-axis.

The family is $(x - h)^2 + (y - 8)^2 = 64$. Since $x - h = -(y - 8)y'$, elimination gives

$$(y - 8)^2\left[1 + \left(\frac{dy}{dx}\right)^2\right] = 64.$$

225. (B)

$$2 = e^{0.08t} \Rightarrow t = \frac{\log 2}{0.08} = 8.64 \text{ years}.$$

A bacterial population doubles every 4 hours. Find the population after 12 hours.

Twelve hours contain three doubling pe $M = M_0 e^{-kt}$ riods, so

$$N(12) = N \cdot 2^3 = 8N.$$

226. (C)

A radioactive mass falls from 1.5 g to 0.5 g. To what logarithm is the time proportional?

For,

$$t = \frac{1}{k}\log\frac{M_0}{M} = \frac{1}{k}\log 3.$$

Thus it is proportional to $\log 3$.

227. (A)

Bismuth has half-life 5 days. Find the remaining mass from 800 mg after 30 days.

Thirty days are six half-lives:

$$M = 800\left(\frac{1}{2}\right)^6 = 12.5 \text{ mg}.$$

228. (B)

$$y^2 = (2x + c)^5$$

Differentiate:

$$2y\frac{dy}{dx}=10(2x+c)^4$$

$$y\frac{dy}{dx}=5(2x+c)^4$$

From original:

$$(2x+c)^4=y^{8/5}$$

So:

$$y\frac{dy}{dx}=5y^{8/5}\Rightarrow\frac{dy}{dx}=5y^{3/5}$$

Raise to 5:

$$\left(\frac{dy}{dx}\right)^5=3125y^3$$

229. (C)

Doubling time formula: $t=\dfrac{\ln 2}{r}=\dfrac{0.6912}{0.08}=8.64$

230. (C)

Identify the differential equation having $y=c^2+c/x$ as solution.

Let $p=dy/dx=-c/x^2$, so $c=-x^2p$. Substitution gives $y=x^4p^2-xp$,

Or $x^4\left(\dfrac{dy}{dx}\right)^2-x\dfrac{dy}{dx}-y=0.$

231. (A)

Given: $\dfrac{dy}{dx}+\dfrac{1}{\sqrt{1-x^2}}=0$

Step 1 : Rearrange :

$$\frac{dy}{dx}=-\frac{1}{\sqrt{1-x^2}}$$

Step 2 : Integrate both sides:

$$\int dy=-\int\frac{1}{\sqrt{1-x^2}}dx$$

$$y=-\sin^{-1}x+c$$

Step 3: Rearrange:

$$y+\sin^{-1}x=c$$

232. (C)

A bacterial population doubles in 4 hours. When will it become $4N$?

$4N=2^2N$, requiring two doubling periods: $2\times4=8$ hours.

233. (C)

Solve $y''=\sin x+e^x$, with $y(0)=3$ and $y'(0)=4$.

Integrating twice,

$y'=-\cos x+e^x+4,$

$y=-\sin x+e^x+4x+2.$

234. (B)

Which of the four given functions is not a p.d.f.?

F_1,F_2,F_3 are non-negative and integrate to 1. But $F_4(x)=x/2$ on $(-2,2)$ is negative for and integrates to 0. Therefore it is not a p.d.f.

235. (B)

A random variable takes $0,1,2$, has mean 1.2, and $P(X=0)=0.3$. Find $P(X=1)$.

Let $P(X=1)=p$, so $P(X=2)=0.7-p$. Then

$p+2(0.7-p)=1.2\Rightarrow p=0.2.$

236. (C)

For $f(x)=k\left(4-x^2\right)$ on $[-2,2]$, find $P(-1<X<1)$.

Normalization gives $k=3/32$. Therefore

$$P(-1<X<1)=\frac{3}{32}\int_{-1}^{1}\left(4-x^2\right)dx=\frac{11}{16}.$$

237. (D)

For the c.d.f. $F(x)=\sqrt{x}/2$, find $P(X>1)$.

$$P(X>1)=1-F(1)=1-\frac{1}{2}=\frac{1}{2}.$$

238. (A)

For the uniform p.d.f. $f(x)=1/2$ on $(0,2)$, compare $a=P(X<1/2)$ and $b=P(X>3/2)$.

$a=\frac{1}{2}\cdot\frac{1}{2}=\frac{1}{4}, b=\frac{1}{2}\cdot\frac{1}{2}=\frac{1}{4}.$

Hence $a-b=0$.

239. (D)

A lottery ticket wins with probability $1/4$. For five tickets, find the probability of at least one win.

$P(\text{at least one})=1-\left(\frac{3}{4}\right)^5=\frac{781}{1024}.$

240. (B)

Probability of selecting a defective = $\frac{3}{8}$

Probability of selecting non -defective = $\frac{5}{8}$

Step 2:

$P(X=0)=\left(\frac{5}{8}\right)^2=\frac{25}{64}\$$

$P(X=1)=2\cdot\frac{3}{8}\cdot\frac{5}{8}=\frac{30}{64}$

Step 3:

$P(X\le 1)=\frac{25}{64}+\frac{30}{64}=\frac{55}{64}$

241. (C)

Each patient survives an operation with probability 0.2. Find the probability exactly four of five survive.

$P=\binom{5}{4}(0.2)^4(0.8)=0.0064.$

242. (C)

Repeat of the bomb question: miss probability 0.2, exactly two hits out of ten.

$P=\binom{10}{2}\left(\frac{4}{5}\right)^2\left(\frac{1}{5}\right)^8=\frac{144}{5^9}.$

243. (D)

A basketball player scores with probability 0.4. Find the probability of exactly three scores in four attempts.

$P=\binom{4}{3}(0.4)^3(0.6)=0.1536.$

244. (A)

A die is thrown 100 times. Find the standard deviation of the number of even outcomes.

Here $X\sim B(100,1/2)$. Thus

$\sigma=\sqrt{npq}=\sqrt{100\cdot\frac{1}{2}\cdot\frac{1}{2}}=5.$

MHTCET *Y*earwise P*Y*Q (2021)
Marathi
Mathematics

1. If $\theta+\phi=\alpha$ and $\tan\theta=k\tan\phi$ (where K > 1), then the value of $\sin(\theta-\phi)$ is **[2021]**

(A) $k\tan\phi$ (B) $\sin\alpha$

(C) $\left(\frac{k-1}{k+1}\right)\sin\alpha$ (D) $k\cos\phi$

2. If $\sin(y+z-x)$, $\sin(z+x-y)$ and $\sin(x+y-z)$ are in Ap, then **[2021]**

(A) $\tan y = \tan x + \tan z$

(B) $\tan y = \tan x - \tan z$

(C) $2\tan y = \tan x + \tan z$

(D) $2\tan y = \tan x - \tan z$

3. The slope of the line through the origin which makes an angle of 30° with the positive direction of Y-axis measured anticlockwise is **[2021]**

(A) $\frac{-1}{\sqrt{3}}$ (B) $\frac{\sqrt{3}}{2}$

(C) $-\sqrt{3}$ (D) $\frac{-2}{\sqrt{3}}$

4. The x-intercept of a line passing through the point $\left(\frac{-1}{2},1\right)$ and (1,2) is **[2021]**

(A) –1 (B) –2

(C) 1 (D) 3

5. The equation of a line passing through (p cos α, p sin α) and making an angle (90 + α) with positive direction of *X*-axis is **[2021]**

(A) $x\cos\alpha - y\sin\alpha = 2p$

(B) $x\sin\alpha + y\cos\alpha = p$

(C) $x\cos\alpha + y\sin\alpha = p$

(D) $x\cos\alpha + y\sin\alpha = 3p$

6. The equation of perpendicular bisector of the line segment joining A(–2, 3) and B(6, –5) is **[2021]**

(A) $x + y = 3$ (B) $x + y = 1$

(C) $x - y = -1$ (D) $x - y = 3$

7. The equation of line, where length of the perpendicular segment from origin to the line is 4 and the inclination of this perpendicular segment with the positive direction of *X*–axis is 30°, is **[2021]**

(A) $x+\sqrt{3}y=8$ (B) $x-\sqrt{3}y=8$

(C) $\sqrt{3}x-y=8$ (D) $\sqrt{3}x+y=8$

8. The equation of a line with slope $-\frac{1}{\sqrt{2}}$ and make an intercept of $2\sqrt{2}$ units on negative direction of *Y*–axis is **[2021]**

(A) $\sqrt{2}y-x+4=0$ (B) $x+\sqrt{2}y+2\sqrt{2}=0$

(C) $\sqrt{2}y+x+4=0$ (D) $x+\sqrt{2}y-2\sqrt{2}=0$

9. If the lines $3x - 4y + 4 = 0$ and $6x - 8y - 7 = 0$ are tangents to a circle, then the radius of the circle is **[2021]**

(A) $\frac{7}{4}$ units (B) $\frac{3}{4}$ units

(C) $\frac{4}{3}$ units (D) $\frac{1}{4}$ units

10. The equation of the circle whose centre lies on the line $x - 4y = 1$ and which passes through the points (3, 7) and (5, 5) is **[2021]**

(A) $x^2 + y^2 + 6x - 2y + 90 = 0$

(B) $x^2 + y^2 - 6x - 2y - 25 = 0$

(C) $x^2 + y^2 - 6x + 2y - 30 = 0$

(D) $x^2 + y^2 + 6x + 2y - 90 = 0$

11. The equation of circle with centre at (2, –3) and the circumference 10π units is **[2021]**

(A) $x^2 + y^2 - 4x + 6y - 12 = 0$

(B) $x^2 + y^2 - 4x - 6y - 12 = 0$

(C) $x^2 + y^2 + 4x + 6y + 12 = 0$

(D) $x^2 + y^2 - 4x + 6y + 12 = 0$

12. Two circles centred at (2, 3) and (4, 5) intersects each other. If their radii are equal, then the equation of the common chord is **[2021]**

(A) $x + y + 1 = 0$ (B) $x + y - 1 = 0$

(C) $x + y - 7 = 0$ (D) $x + y + 7 = 0$

13. If a circle passes through the points (0, 0), (x, 0) and (0, y), then the coordinates of its centre are **[2021]**

(A) $\left(\frac{-x}{2}, \frac{y}{2}\right)$ (B) $\left(\frac{x}{2}, \frac{y}{2}\right)$

(C) $\left(\frac{-x}{2}, \frac{-y}{2}\right)$ (D) $\left(\frac{x}{2}, \frac{-y}{2}\right)$

14. The equation of common tangent to the circles $x^2 + y^2 - 4x + 10y + 20 = 0$ and $x^2 + y^2 + 8x - 6y - 24 = 0$ is **[2021]**

(A) $3x - 4y + 11 = 0$

(B) $3x - 4y - 11 = 0$

(C) $-3x - 4y + 11 = 0$

(D) $3x + 4y + 11 = 0$

15. Equation of the chord of the circle $x^2 + y^2 - 4x - 10y + 25 = 0$ having mid–point (1, 2) is **[2021]**

(A) $-x + 3y = 5$

(B) $x + 3y = 7$

(C) $5x + y = 7$

(D) $3x + y = 5$

16. If $y = 2x$ is a chord of circle $x^2 + y^2 - 10x = 0$, then the equation of circle with this chord as diameter is **[2021]**

(A) $x^2 + y^2 - 2x - 4y = 0$

(B) $x^2 + y^2 + 2x + 4y = 0$

(C) $x^2 + y^2 - 2x + 4y = 0$

(D) $x^2 + y^2 + 2x - 4y = 0$

17. Two dice are rolled simultaneously. The probability that the sum of the two numbers on the dice is a prime number, is **[2021]**

(A) $\frac{5}{11}$ (B) $\frac{5}{12}$

(C) $\frac{7}{12}$ (D) $\frac{7}{11}$

18. Two unbiased dice are thrown. Then the probability that neither a doublet nor a total of 10 will appear is **[2021]**

(A) $\frac{1}{12}$ (B) $\frac{1}{36}$

(C) $\frac{2}{9}$ (D) $\frac{7}{9}$

19. A coin is tossed and a die is thrown. The probability that the outcome will be head or a number greater than 4 or both, is **[2021]**

(A) $\frac{2}{3}$ (B) $\frac{1}{6}$

(C) $\frac{1}{2}$ (D) $\frac{1}{3}$

20. For two events A and B, $P(A \cup B) = \frac{5}{6}$, $P(A) = \frac{1}{6}$, $P(B) = \frac{2}{3}$, then A and B are **[2021]**

(A) independent

(B) mutually exhaustive

(C) mutually exclusive

(D) complementary

21. Rooms in a hotel are numbered from 1 to 19. Rooms are allocated at random as guests arrive. The first guest to arrive is given a room which is a prime number. The probability that the second guest to arrive is given a room which is a prime number is **[2021]**

(A) $\frac{8}{19} \times \frac{7}{18}$ (B) $\frac{8}{19}$

(C) $\frac{8}{19} \times \frac{7}{19}$ (D) $\frac{7}{18}$

22. If $P(A) = \frac{3}{10}$, $P(B) = \frac{2}{5}$, $P(A \cup B) = \frac{3}{5}$, then $P(A/B) \times P(B/A) =$ **[2021]**

(A) $\frac{1}{3}$ (B) $\frac{1}{12}$

(C) $\frac{1}{10}$ (D) $\frac{1}{4}$

23. The probability that at least one of the events E_1 and E_2 occurs is 0.6. If the simultaneous occurrence of E_1 and E_2 is 0.2, $P(E_1') + P(E_2') =$ **[2021]**

(A) 0.4 (B) 1.6
(C) 1.2 (D) 0.8

24. The value of $(1 + i)^5 (1 - i)^7$ is **[2021]**

(A) –64 (B) – 64i
(C) 64 i (D) 64

25. The complex number with argument $\frac{5\pi^c}{6}$ at a distance of 2 units from the origin is **[2021]**

(A) $\sqrt{3} - i$ (B) $\sqrt{3} + i$
(C) $-\sqrt{3} - i$ (D) $-\sqrt{3} + i$

26. If $z(2-i) = (3 + i)$, then $z^{38} =$ (where $z = x + iy$) **[2021]**

(A) $-(2^{19})i$ (B) $2^{19}i$
(C) $-(2^{19})$ (D) 2^{19}

27. If $x = 1 + 2i$, then the value of $x^3 + 7x^2 - x + 16$ is **[2021]**

(A) –17–24i
(B) –17 + 24i
(C) 17–24i
(D) 17+ 24i

28. If ω is the complex cube root of unity, then $(3 + 5\omega + 3\omega^2)^2 + (3 + 3\omega + 5\omega^2)^2 =$ **[2021]**

(A) –1 (B) 0
(C) 4 (D) –4

29. If amplitude of $(z - 2 - 3i)$ is $\frac{3\pi}{4}$, then locus of z is (where $z = x + iy$) **[2021]**

(A) $x + y = 1$
(B) $x+y=5$
(C) $x-y=-5$
(D) $x-y=1$

30. The square roots of the complex number $(-5 - 12i)$ are **[2021]**

(A) $\pm(2 - 3i)$ (B) $\pm(3 + 2i)$
(C) $\pm(2 + 3i)$ (D) $\pm(3 - 2i)$

31. If $\frac{3+2i}{1+i} = \frac{1}{2}(x + iy)$, then $x - y =$ **[2021]**

(A) 4 (B) 3
(C) 6 (D) 5

32. If the complex number $z = x + iy$, where $i = \sqrt{-1}$, satisfies the condition $|z + 1| = 1$, then z lies on **[2024, 2021]**

(A) X–axis.
(B) circle with centre (1,0) and radius 1 unit.
(C) circle with centre (–1,0) and radius 1 unit.
(D) Y–axis.

33. All the letters of the word 'ABRACADABRA' are arranged in different possible ways. Then the number of such arrangements in which the vowels are together is **[2021]**

(A) 1200
(B) 1240
(C) 1220
(D) 1260

34. The number of ways in which 8 different pearls can be arranged to form a necklace is **[2021]**

(A) 40320 (B) 5040
(C) 2520 (D) 1260

35. The numbers can be formed using the digits 1, 2, 3, 4, 3, 2, 1 so that odd digits always occupy odd places in _____ ways. **[2021]**

(A) 9 (B) 18
(C) 6 (D) 3

36. Out of 7 consonants and 4 vowels, the number of words consiting of 3 consonants and 2 vowels are **[2021]**

(A) 3300 (B) 210
(C) 120 (D) 25200

37. If ${}^{11}C_4 + {}^{11}C_5 + {}^{12}C_6 + {}^{13}C_7 = {}^{14}C_r$, then value of r is **[2021]**

(A) 11 (B) 14
(C) 7 (D) 3

38. The difference between the maximum values of ${}^6C_r$ and ${}^nC_r$ is 16, then n = **[2021]**

(A) 3 (B) 5
(C) 2 (D) 4

39. A committee of 5 is to be formed out of 6 men and 4 ladies. The number of ways this can be done, when at most 2 ladies are included, is **[2021]**

(A) 240 (B) 186
(C) 60 (D) 120

40. The domain of the function $f(x) = \frac{1}{\sqrt{x+|x|}}$ is **[2021]**

(A) $(-\infty, 0)$
(B) $(2, 5)$
(C) $(0, \infty)$
(D) $(-\infty, \infty)$

41. Let A = [a, b, c, d], B = [1, 2, 3]. Relation R_1, R_2, R_3, R_4 are as follows:
R_1 = [(a, 1), (b, 2), (c, 1), (d, 2)]
R_2 = [(a, 1), (b, 1), (c, 1), (d, 1)]
R_3 = [(a, 2), (b, 3), (c, 2), (d, 2)]
R_4 = [(a, 1), (b, 2), (a, 2), (d, 3)], then **[2021]**

(A) only R_3 and R_4 are not functions.
(B) only R_1 and R_2 are not functions.
(C) only R_3 is not a function.
(D) only R_4 is not a function.

42. If $f(x) = 3[x] + 5\{x + 1\}$, where $[x]$ is greatest integer function of x and $\{x\}$ is fractional part function of x, then $f(-1.32)$ = **[2021]**

(A) –4.6 (B) –2.6
(C) –7.4 (D) –3.4

43. Let A = {10, 11, 12, 14, 26} and let f : A→N be such that f(a) = highest prime factor of a, where a ∈ A, then range of f = **[2021]**

(A) {5, 7, 13}
(B) {5, 7, 11, 13}
(C) {3, 5, 7, 11, 13}
(D) {3, 7, 11, 13}

44. The domain of the function $f(x) = \sqrt{x-1} + \sqrt{6-x}$ is **[2021]**

(A) $[1, \infty)$
(B) $[1, 6]$
(C) $(-\infty, 1)$
(D) $(6, \infty)$

45. If $f(x) = [8x] - 3$, where $[x]$ is greatest integer function of x, then $f(\pi)$ = (where $\pi = 3.14$) **[2021]**

(A) 21 (B) 25
(C) 23 (D) 22

46. Range of the function $f(x) = 3 + 2^x + 4^x$ is **[2021]**

(A) $(3, \infty)$
(B) $(-\infty, \infty)$
(C) $[3, \infty)$
(D) $(-\infty, 3]$

47. The domain of the function $\log_{10}(x^2 - 5x + 6)$ is **[2021]**

(A) $(-\infty, \infty)$
(B) $(-\infty, 2) \cup (3, \infty)$
(C) $(2, 3)$
(D) None of these

48. If $f(x) = 2\{x\} + 5x$, where $\{x\}$ is fractional part function, then $f(-1.4)$ is **[2021]**

(A) 8.2 (B) –8.2
(C) –5.8 (D) –5

49. $\lim\limits_{x\to\infty}\left(\sqrt{x^2+5x-7}-x\right)$ = **[2021]**

(A) $\frac{7}{2}$ (B) 5
(C) $\frac{5}{2}$ (D) 6

50. If $a = \lim\limits_{n\to\infty}\frac{1+2+3+....+n}{n^2}$ and

$b = \lim\limits_{n\to\infty}\frac{1^2+2^2+3^2+....+n^2}{n^3}$, then **[2021]**

(A) a = b (B) 2a = 3b
(C) a = 2b (D) 3a = 2b

51. $\lim_{x\to1}\frac{(2x-3)\left(\sqrt{x}-1\right)}{2x^2+x-3}$ **[2021]**

(A) $\frac{1}{5}$ (B) $\frac{1}{10}$

(C) $\frac{-1}{10}$ (D) $\frac{-1}{5}$

52. $\lim_{x\to0}\frac{\cos(mx)-\cos(nx)}{x^2}=$ **[2021]**

(A) $\frac{m^2-n^2}{2}$ (B) m^2-n^2

(C) $\frac{n^2-m^2}{2}$ (D) n^2-m^2

53. $\lim_{x\to2}(x-1)^{\frac{1}{3x-6}}=$ **[2021]**

(A) e^2 (B) e^3

(C) $e^{\frac{1}{3}}$ (D) $e^{\frac{1}{2}}$

54. $\lim_{x\to1}\left[\frac{\sqrt{x}-1}{\log x}\right]=$ **[2021]**

(A) $\frac{1}{2}$ (B) 2

(C) –2 (D) $-\frac{1}{2}$

55. $\lim_{x\to1}\frac{ab^x-a^xb}{x^2-1}=$ **[2021]**

(A) $\frac{-ab}{2}\log\left(\frac{b}{a}\right)$ (B) $\frac{ab}{2}\log\left(\frac{b}{a}\right)$

(C) $ab\log\left(\frac{b}{a}\right)$ (D) $-ab\log\left(\frac{b}{a}\right)$

56. $\lim_{x\to0}\frac{\sqrt{1-\cos x^2}}{1-\cos x}=$ **[2021]**

(A) $\sqrt{2}$ (B) $\frac{1}{\sqrt{2}}$

(C) 0 (D) $\frac{1}{2}$

57. If f(x) = [x], for $x \in (-1, 2)$, then f is discontinuous at (where [x] represents floor function) **[2021]**

(A) $x = -1, 0, 1, 2$

(B) $x = -1, 0, 1$

(C) $x = 0, 1$

(D) $x = 2$

$f(x)=|x|+3, \quad if\ x\le -3$

58. Let $=-2x, \quad if\ -3<x<3$ **[2021]**

$=6x+2, \quad if\ x\ge 3, then$

(A) f(x) is discontinuous at both $x = -3$ as well as $x = 3$

(B) f(x) is continuous at $x = -3$ but discontinuous at $x = 3$

(C) f(x) is continuous at $x = -3$ as well as $x = 3$

(D) f(x) is discontinuous at $x = -3$ but f(x) is continuous at $x = 3$

59. If the function f(x) = 3ax + b, for $x < 1$ = 11, for $x = 1$ = 5ax – 2b, for $x > 1$ is continuous at $x = 1$. Then, the values of a and b are **[2021]**

(A) a = 2, b = 3 (B) a = 3, b = 3

(C) a = 2, b = 2 (D) a = 3, b = 2

60. The negation of a statement '$x \in A \cap B \rightarrow (x \in A$ and $x \in B)$' is **[2021]**

(A) $x \in A \cap B \rightarrow (x \in A$ or $x \in B)$

(B) $x \in A \cap B$ and $(x \notin A$ or $x \notin B)$

(C) $x \in A \cap B$ or $(x \in A$ and $x \in B)$

(D) $x \notin A \cap B$ and $(x \in A$ and $x \in B)$

61. p: It rains today

q: I am going to school

r: I will meet may friend

s: I will go to watch a movie.

Then symbolic form of the statement "If it does not rain today or I won't go to school then I will meet my friend and I will go to watch a movie" is **[2021]**

(A) $\sim(p \vee q) \rightarrow (r \vee s)$

(B) $(p \wedge q) \rightarrow (r \vee s)$

(C) $\sim(p \wedge q) \rightarrow (r \wedge s)$

(D) $(\sim p \wedge q) \rightarrow (r \wedge s)$

62. Negation of the satement $\forall\, x \in R$, $x^2 + 1 = 0$ is **[2021]**

(A) $\exists\, x \in R$ such that $x^2 + 1 < 0$.
(B) $\exists\, x \in R$ such that $x^2 + 1 \leq 0$.
(C) $\exists\, x \in R$ such that $x^2 + 1 \neq 0$.
(D) $\exists\, x \in R$ such that $x^2 + 1 = 0$

63. If p, q are true statements and r is false statement, then which of the following is correct? **[2021]**

(A) (p v q) v r has truth value F.
(B) $(p \wedge q) \rightarrow r$ has truth value T.
(C) $(p \rightarrow r) \rightarrow q$ has truth value F.
(D) $(p \leftrightarrow q) \rightarrow r$ has truth value F.

64. Given p : A man is judge, q : A man is honest If S_1 : If a man is a judge, then he is honest S_2 : If a man is a judge, then he is not honest S_3 : A man is not a judge or he is honest S_4 : A man is a judge and he is honest Then **[2021]**

(A) $S_2 \equiv S_3$ (B) $S_1 \equiv S_2$
(C) $S_2 \equiv S_4$ (D) $S_1 \equiv S_3$

65. S_1 : If –7 is an integer, then $\sqrt{-7}$ is a complex number S_2 : –7 is not an integer or $\sqrt{-7}$ is a complex number **[2021]**

(A) S_1 and S_2 are converse statements of each other
(B) S_1 and S_2 are negations of each other
(C) S_1 and S_2 are equivalent statements
(D) S_1 and S_2 are contrapositive of each other

66. "If two triangles are congruent, then their areas are equal." Is the given statement, then the contrapositive of the inverse of the given statement is (Where p : Two triangles are congruent, q : Their areas are equal) **[2021]**

(A) If two triangles are not congruent, then their areas are equal.
(B) If two triangles are not congruent, then their area are not equal.
(C) If areas of two triangles are equal, then they are congruent.
(D) If areas of two triangles are not equal, then they are congruent.

67. The negation of '$\forall\, x \in N$, $x^2 + x$ is even number' is **[2021]**

(A) $\forall\, x \in N$, $x^2 + x$ is not an even number.
(B) $\forall\, x \in N$, $x^2 + x$ is not an odd number.
(C) $\exists\, x \in N$ such that $x^2 + x$ is an even number.
(D) $\exists\, x \in N$ such that $x^2 + x$ is not an even number.

68. If p : It is raining. q : Weather is pleasant Then simplified form of the statement "It is not true, if it is raining then weather is not pleasant" is **[2021]**

(A) It is not raining or weather is pleasant.
(B) It is raining or weather is not pleasant.
(C) It is raining or weather is not pleasant.
(D) It is raining and the weather is pleasant.

69. The negation of the statement 7 is greater than 4 or 6 is less than 7 **[2021]**

(A) 7 is not greater than 4 and 6 is not less than 7
(B) 7 is not greater than 4 or 6 is not less than 7
(C) 7 is greater than 4 and 6 is less than 7
(D) None of the above

70. The contrapositive of the statement. 'If $2^2 = 5$, then I get first class' is **[2021]**

(A) If I do not get a first class, then $2^2 = 5$
(B) If I do not get a first class, then $2^2 \neq 5$
(C) If I get a first class, then $2^2 = 5$
(D) None of the above

71. The truth value of the statement 'Patna is in Bihar or 5 + 6 = 111' is **[2021]**

(A) True
(B) False
(C) Cannot say anything
(D) None of these

72. The negation of a statement '$x \in A \cap B \rightarrow (x \in A$ and $x \in B)$' is **[2021]**

(A) $x \in A \cap B \rightarrow (x \in A$ or $x \in B)$
(B) $x \in A \cap B$ or$(x \in A$ and $x \in B)$
(C) $x \in A \cap B$ and $(x \notin A$ or $x \notin B)$
(D) $x \notin A \cap B$ and $(x \in A$ and $x \in B)$

73. If inverse of $\begin{bmatrix} 1 & 2 & x \\ 4 & -1 & 7 \\ 2 & 4 & -6 \end{bmatrix}$ does not exist, then x= **[2021]**

(A) –3 (B) 2
(C) 3 (D) 0

74. $A(\alpha)=\begin{bmatrix} \cos\propto & \sin\propto \\ -\sin\propto & \cos\propto \end{bmatrix}$, then $\left[A^2(\alpha)\right]^{-1}$ = **[2021]**

(A) A(α) (B) A²(α)
(C) A(–2 α) (D) A(2 α)

75. The co–factors of the elements of second column of $\begin{bmatrix} 1 & -1 & 2 \\ 3 & 2 & 1 \\ -1 & 3 & 4 \end{bmatrix}$ are **[2021]**

(A) –13, 6, 5 (B) 13, 5, 6
(C) 13, –6, –5 (D) –13, –6, 5

76. If $A^{-1}=\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$ and $B^{-1}=\begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$, then $(AB)^{-1}$ = **[2021]**

(A) $\begin{bmatrix} 2 & 7 \\ 3 & -1 \end{bmatrix}$ (B) $\begin{bmatrix} 2 & -7 \\ -3 & 11 \end{bmatrix}$
(C) $\begin{bmatrix} 2 & -3 \\ -7 & 11 \end{bmatrix}$ (D) $\begin{bmatrix} 2 & 3 \\ 7 & -11 \end{bmatrix}$

77. If $A=\begin{bmatrix} 1 & 0 & 2 \\ -1 & 1 & -2 \\ 0 & 2 & 1 \end{bmatrix}$, adj $A=\begin{bmatrix} 5 & x & -2 \\ 1 & 1 & 0 \\ -2 & 2 & y \end{bmatrix}$, then value of $x + y$ is **[2021]**

(A) 6 (B) 3
(C) 4 (D) 5

78. If $A^{-1}=\frac{-1}{2}\begin{bmatrix} 1 & -4 \\ -1 & 2 \end{bmatrix}$, then $2A + I_2$ = , where I_2 is unit matrix of order 2 **[2021]**

(A) $\begin{bmatrix} 5 & 8 \\ 1 & 2 \end{bmatrix}$ (B) $\begin{bmatrix} 5 & 8 \\ 2 & 2 \end{bmatrix}$
(C) $\begin{bmatrix} 2 & 4 \\ 1 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 5 & 8 \\ 2 & 3 \end{bmatrix}$

79. The sum of three numbers is 6. Thrice the third number when added to the first number gives 7. On adding three times first number to the sum of second and third numbers we get 12. The product of these numbers is **[2021]**

(A) 20 (B) 3
(C) $\frac{20}{3}$ (D) $\frac{5}{3}$

80. If $A=\begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ and A^{-1} = KA, then K is **[2021]**

(A) 19 (B) $\frac{-1}{19}$
(C) –19 (D) $\frac{1}{19}$

81. If $A=\begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}$, then A (adj A) = **[2021]**

(A) $\begin{bmatrix} \frac{-1}{3} & 0 & 0 \\ 0 & \frac{-1}{3} & 0 \\ 0 & 0 & \frac{-1}{3} \end{bmatrix}$ (B) $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$
(C) $\begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & 2 \\ 3 & 2 & 4 \end{bmatrix}$

82. Which of the following matrices are invertible?

$A=\begin{bmatrix} 2 & 3 \\ 10 & 15 \end{bmatrix}$, $B=\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 3 \\ 1 & 2 & 3 \end{bmatrix}$, $C=\begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 4 & 6 & 8 \end{bmatrix}$

$D=\begin{bmatrix} 2 & 4 & 2 \\ 1 & 1 & 0 \\ 1 & 4 & 5 \end{bmatrix}$ **[2021]**

(A) both A and B (B) only C
(C) only A (D) only D

83. For a 3 × 3 matrix A, if A(adj A) = $\begin{bmatrix} -10 & 0 & 0 \\ 0 & -10 & 2 \\ 0 & 0 & -10 \end{bmatrix}$, then the value of determinant of A is **[2021]**

(A) 100 (B) –1000
(C) –10 (D) 20

84. If A = $\begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}$, and A (adj A) = kI, then the value of $(k + 1)^4$ is **[2021]**

(A) 256 (B) 81
(C) 16 (D) 625

85. If A = $\begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$ and A.adj A = AA^T, then 5a + b = **[2025, 2024, 2021]**

(A) 7 (B) 9
(C) 13 (D) 5

86. $sin^{-1}\left[sin(-600°)\right] + cot^{-1}\left(-\sqrt{3}\right) =$ **[2021]**

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$
(C) $\frac{\pi}{3}$ (D) $\frac{7\pi}{6}$

87. $tan^{-1}\left(tan\frac{5\pi}{6}\right) + cos^{-1}\left(cos\frac{13\pi}{6}\right) =$ **[2021]**

(A) 0 (B) 3π
(C) $-\frac{\pi}{6}$ (D) $\frac{\pi}{6}$

88. In ΔABC, with usual notations $\frac{b\sin B - c\sin C}{\sin(B-C)} =$ **[2021]**

(A) b
(B) C
(C) a
(D) a + b + c

89. With usual notations, in any ΔABC, if $a\cos B = b\cos A$, then the triangle is **[2021]**

(A) an isosceles triangle
(B) an equilateral triangle
(C) a right angled triangle
(D) a scalene triangle

90. In a triangle ABC, with usual notations a = 2, b = 3, c = 5, then $\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c} =$ **[2021]**

(A) $\frac{19}{30}$ (B) $\frac{19}{60}$
(C) $\frac{23}{60}$ (D) $\frac{38}{35}$

91. In any ΔABC, with usual notations, c(a cosB – b cosA) = **[2021]**

(A) $a^2 - b^2$ (B) $\frac{1}{a^2} - \frac{1}{b^2}$
(C) $a^2 + b^2$ (D) $\frac{1}{a^2} + \frac{1}{b^2}$

92. In a triangle ABC with usual notations, a = 2, b = 3, then value of $\frac{\cos 2A}{a^2} - \frac{\cos 2B}{b^2}$ is **[2021]**

(A) $\frac{5}{36}$ (B) $\frac{1}{4}$
(C) $\frac{1}{9}$ (D) $\frac{13}{36}$

93. The principal solutions, of the equation $\sqrt{3}sec\,x + 2 = 0$, are **[2024, 2021]**

(A) $\frac{2\pi}{3}, \frac{4\pi}{3}$
(B) $\frac{4\pi}{3}, \frac{5\pi}{3}$
(C) $\frac{5\pi}{6}, \frac{7\pi}{6}$
(D) $\frac{7\pi}{6}, \frac{11\pi}{6}$

94. If $cos^{-1}\left(\frac{12}{13}\right)+sin^{-1}\left(\frac{3}{5}\right)=sin^{-1}P$; then the value of P is **[2024, 2021]**

(A) $\frac{63}{65}$ (B) $\frac{56}{65}$

(C) $\frac{48}{65}$ (D) $\frac{36}{65}$

95. If $2\tan^{-1}(\cos x)=\tan^{-1}(2\operatorname{cosec} x)$, then the value of x is **[2025, 2021]**

(A) $-\frac{\pi}{4}$ (B) 0

(C) $\frac{\pi}{8}$ (D) $\frac{\pi}{4}$

96. The joint equation of the pair of lines through the origin and making an equilateral triangle with the line $x = 3$ is **[2021]**

(A) $3x^2-y^2=0$

(B) $\sqrt{3}x^2-2xy+y^2=0$

(C) $x^2-3y^2=0$

(D) $x^2+2xy-\sqrt{3}y^2=0$

97. If the equation $3x^2-kxy-3y^2=0$ represents the bisectors of angles between the lines $x^2-3xy-4y^2=0$, then value of k is **[2021]**

(A) –6 (B) –10

(C) 6 (D) 10

98. If the two lines given by $ax^2+2hxy+by^2=0$ make inclinations α and β, then tan(α + β) = **[2021]**

(A) $\frac{h}{a+b}$

(B) $\frac{2h}{a+b}$

(C) $\frac{h}{a-b}$

(D) $\frac{2h}{a-b}$

99. If $(m + 3n)(3m + n) = 4h^2$, then the acute angle between the lines represented by $mx^2 + 2hxy + ny^2 = 0$ is **[2021]**

(A) $\frac{\pi^c}{3}$ (B) $\frac{\pi^c}{6}$

(C) $tan^{-1}\left(\frac{3}{2}\right)$ (D) $tan^{-1}\left(\frac{1}{2}\right)$

100. Area of the triangle formed by the lines $y^2 - 9xy + 18x^2 = 0$ and $y = 9$ is **[2021]**

(A) $\frac{27}{3}$ sq. units (B) $\frac{27}{2}$ sq. units

(C) $\frac{27}{4}$ sq. units (D) 27 sq. units

101. If the angle between the lines is $\frac{\pi^c}{4}$ and slope of one of the lines is $\frac{1}{2}$, then slope of the other line is **[2021]**

(A) 3 or $-\frac{1}{3}$ (B) 4 or $-\frac{1}{4}$

(C) 2 or $-\frac{1}{2}$ (D) 3 or –3

102. If the lines represented by $(k^2 + 2)x^2 + 3xy - 6y^2 = 0$ are perpendicular to each other, then the values of k are **[2021]**

(A) ± 3 (B) ± 4

(C) ± 1 (D) ± 2

103. If P_1 and P_2 are perpendicular distances (in units) from point (2, –1) to the pair of lines $2x^2-5xy+2y^2=0$, then the value of P_1P_2 is **[2024, 2021]**

(A) 2 (B) 5

(C) 10 (D) 4

104. If the volume of a tetrahedron whose coterminus edges are $\bar{a}+\bar{b}$, $\bar{b}+\bar{c}$, $\bar{c}+\bar{a}$ is 24 cubic units, then the volume of parallelopiped whose coterminus edges are $\bar{a}$, $\bar{b}$, $\bar{c}$ is **[2021]**

(A) 72 cubic units (B) 144 cubic units

(C) 48 cubic units (D) 10 cubic units

105. If $\bar{e}_1, \bar{e}_2$ and $\bar{e}_1 + \bar{e}_2$ are unit vectors, then the angle between $\bar{e}_1$ and $\bar{e}_2$ is **[2021]**

(A) 135°
(B) 120°
(C) 90°
(D) 150°

106. If $\bar{a}, \bar{b}, \bar{c}$ are three vectors which are perpendicular to $\bar{b}+\bar{c}$, $\bar{c}+\bar{a}$ and $\bar{a}+\bar{b}$ respectively, such that $|\bar{a}|=2, |\bar{b}|=3, |\bar{c}|=4$, then $|\bar{a}+\bar{b}+\bar{c}|=$ **[2021]**

(A) 29 (B) $\sqrt{29}$
(C) 3 (D) 9

107. If $\left(2\hat{i}+6\hat{j}+27\hat{k}\right)\times\left(\hat{i}+\lambda\hat{j}+\mu\hat{k}\right)=\bar{0}$, then λ and μ are respectively **[2021]**

(A) $\frac{17}{2}, 3$ (B) $\frac{27}{2}, 3$
(C) $3, \frac{27}{2}$ (D) $3, \frac{17}{2}$

108. The angle between a line with direction ratio 2, 2, 1 and a line joining (3, 1, 4) and (7, 2, 12) is **[2021]**

(A) $\cos^{-1}\left(\frac{1}{3}\right)$ (B) $\cos^{-1}\left(\frac{2}{3}\right)$
(C) $\cos^{-1}\left(\frac{\sqrt{2}}{3}\right)$ (D) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

109. If $[\bar{a}\ \bar{b}\ \bar{c}]=4$, then the volume (in cubic units) of the parallelopiped with $\bar{a}+2\bar{b}$, $\bar{b}+2\bar{c}$ and $\bar{c}+2\bar{a}$ as coterminal edges, is **[2021]**

(A) 32 (B) 16
(C) 9 (D) 36

110. If area of the parallelogram with $\bar{a}$ and $\bar{b}$ as two adjacent sides is 20 square units, then the area of the parallelogram having $3\bar{a}+\bar{b}$ and $2\bar{a}+3\bar{b}$ as two adjacent sides in square units is **[2021]**

(A) 105 (B) 120
(C) 75 (D) 140

111. If vectors $\bar{a}=2\hat{i}+2\hat{j}+3\hat{k}$, $\bar{b}=-\hat{i}+2\hat{j}+\hat{k}$ and $\bar{c}=3\hat{i}+\hat{j}+2\hat{k}$ are such that, $\bar{a}+\lambda\bar{b}$ is perpendicular to $\bar{c}$, then $\lambda =$ **[2021]**

(A) –14 (B) 14
(C) 2 (D) –2

112. If $\frac{\pi}{2} < \theta \le \pi$ and $|\bar{a}|=5$, $|\bar{b}|=13$, $|\bar{a}\times\bar{b}|=25$, then the value of $\bar{a}\cdot\bar{b}$ is **[2021]**

(A) –12 (B) 60
(C) –60 (D) –13

113. The area of the parallelogram with vertices A(1, 2, 3), B(1, 3, a), C(3, 8, 6) and D(3, 7, 3) is $\sqrt{265}$ sq. units, then a = **[2021]**

(A) –5, 2 (B) 6
(C) –6, 0 (D) 6, 0

114. The area of triangle with vertices (1, 2, 0), (1, 0, a) and (0, 3, 1) is $\sqrt{6}$ sq. units, then the values of 'a' are **[2021]**

(A) –8, 1 (B) 2, –4
(C) –2, 4 (D) 8, –1

115. If $\hat{a}$ is a unit vector such that $\left(\vec{x}-\hat{a}\right)\cdot\left(\vec{x}+\hat{a}\right)=8$, then $|\vec{x}|=$ **[2021]**

(A) ±3 (B) $2\sqrt{2}$
(C) 3 (D) $\pm\sqrt{7}$

116. Let $\vec{v}=2\hat{i}+2\hat{j}-\hat{k}$ and $\vec{w}=\hat{i}+3\hat{k}$. If $\vec{u}$ is a unit vector, then the maximum value of the scalar triple product $[\vec{u}\,\vec{v}\,\vec{w}]$ is **[2021]**

(A) $\sqrt{6}$ (B) $\sqrt{10}$
(C) $\sqrt{13}$ (D) $\sqrt{89}$

117. If $|\vec{a}|=4, |\vec{b}|=5$, then the values of k for which $\vec{a}+k\vec{b}$ is perpendicular to $\vec{a}-k\vec{b}$ are **[2021]**

(A) $\pm\frac{5}{4}$ (B) $\pm\frac{2}{5}$
(C) $\pm\frac{16}{25}$ (D) $\pm\frac{4}{5}$

118. Let $\bar{a}=2\hat{i}+\hat{j}-2\hat{k}$ and $\bar{b}=\hat{i}+\hat{j}$. If $\bar{c}$ is a vector such that $\bar{a}\cdot\bar{c}=|\bar{c}|$, $|\bar{c}-\bar{a}|=2\sqrt{2}$ and the angle between $(\bar{a}\times\bar{b})$ and $\bar{c}$ is 60°, then the value of $|(\bar{a}\times\bar{b})\times\bar{c}|$ is **[2024, 2021]**

(A) $\frac{\sqrt{3}}{2}$

(B) $\frac{3\sqrt{3}}{2}$

(C) $\frac{5\sqrt{3}}{2}$

(D) $\frac{\sqrt{3}}{4}$

119. The cartesian equation of the plane passing through the point (0, 7, –7) and containing the line $\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z+2}{1}$ is **[2021]**

(A) $x + 2y + z = 7$

(B) $2x + y + z = 0$

(C) $2x + y – z = 14$

(D) $x + y + z = 0$

120. The parametric equations of a line passing through the points A(3, 4, –7) and B(1, –1, 6) are **[2021]**

(A) $x = 1 + 3\lambda, y = –1 + 4\lambda, z = 6 – 7\lambda$

(B) $x = 3 + \lambda, y = –1 + 4\lambda, z = –7 + 6\lambda$

(C) $x = 3 – 2\lambda, y = 4 – 5\lambda, z = –7 + 13\lambda$

(D) $x = –2 + 3\lambda, y = –5 + 4\lambda, z = 13 – 7\lambda$

121. The angle between a line with direction ratios 2, 2, 1 and a line joining (3, 1, 4) and (7, 2, 12) is **[2021]**

(A) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (B) $\cos^{-1}\left(\frac{1}{3}\right)$

(C) $\cos^{-1}\left(\frac{2}{3}\right)$ (D) $\cos^{-1}\left(\frac{\sqrt{2}}{3}\right)$

122. The cartesian equation of the line passing through the points A(2, 2, 1) and B(1, 3, 0) is **[2021]**

(A) $\frac{x+2}{1}=\frac{y+2}{-1}=\frac{z+1}{-1}$

(B) $\frac{x-2}{-1}=\frac{y-2}{1}=\frac{z-1}{-1}$.

(C) $\frac{x+2}{-1}=\frac{y+2}{1}=\frac{z+1}{-1}$

(D) None of these

123. The equation of the plane passing through (–2, 2, 2) and (2, –2, –2) and perpendicular to the plane $9x – 13y – 3z = 0$ is **[2021]**

(A) $5x – 3y + 2z = 12$

(B) $5x + 3y + 2z = 0$

(C) $5x + 3y – 2z + 8 = 0$

(D) $5x–3y + 2z + 12 = 0$

124. Equation of the plane passing through the point (2, 0, 5) and parallel to the vector $\hat{i}-\hat{j}+\hat{k}$ and $3\hat{i}+2\hat{j}-\hat{k}$ is **[2021]**

(A) $x – 4y – z + 3 = 0$

(B) $x + 4y + 5z – 27 = 0$

(C) $x – 4y – 5z + 23 = 0$

(D) $x – 4y + z – 7 = 0$

125. The line $\frac{x-2}{3}=\frac{y-1}{-5}=\frac{z+2}{2}$ lies in the plane $x + 3y – \alpha z + \beta = 0$, then value of $\alpha\beta$ is **[2021]**

(A) 42

(B) 1

(C) –42

(D) –2

126. The co–ordinates of the points on the line $\frac{x+2}{1}=\frac{y-1}{2}=\frac{z+1}{-2}$ at a distance of 12 units from the point A(–2, 1, –1) are *[2021]*

(A) (2, 9, –9), (–6, –7, 7)

(B) (2, 9, 7), (6, 5, –9)

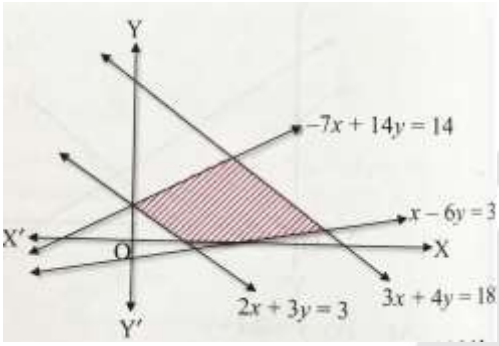
(C) (6, 9, –5), (–10, 9, –5)

(D) (6, –7, 3), (–10, 9, 3)

127. The cartesian equation of the plane passing through the point $A(7, 8, 6)$ and parallel to the XY plane is **[2025, 2021]**

(A) $z = 1$
(B) $y = 8$
(C) $x = 7$
(D) $z = 6$

128. The shaded figure given below is the solution set for the linear inequations. Choose the correct option. **[2021]**



(A) $3x + 4y \geq 18$; $x - 6y \leq 3$; $2x + 3y \geq 3$; $7x - 14y \leq 14$; $x \geq 0$; $y \geq 0$
(B) $3x + 4y \leq 18$; $x - 6y \leq 3$; $2x + 3y \leq 3$; $-7x + 14y \geq 14$; $x \geq 0$; $y \geq 0$
(C) $3x + 4y \leq 18$; $x - 6y \leq 3$; $2x + 3y \geq 3$; $-7x + 14y \leq 14$; $x \geq 0$; $y \geq 0$
(D) $3x + 4y \geq -18$; $x - 6y \leq 3$; $2x + 3y \leq 3$; $-7x + 14y \geq 14$; $x \geq 0$; $y \geq 0$

129. The objective function $Z = 4x + 5y$ subject to $2x + y \geq 7$; $2x + 3y \leq 15$; $y \leq 3$, $x \geq 0$; $y \geq 0$ has minimum value at the point. **[2021]**

(A) on the line $2x + 3y = 15$
(B) on X–axis
(C) on Y–axis
(D) origin

130. The common region of the solution of the inequations $x + y \geq 5$, $y \leq 4$, $x \geq 2$, $x, y \geq 0$ is **[2021]**

(A) unbounded and non–origin side
(B) unbounded and origin side
(C) bounded and origin side
(D) bounded and non–origin side

131. The common region of the solutions of the inequations $x + 2y \geq 4$, $2x - y \leq 6$ and $x, y > 0$ is **[2021]**

(A) bounded and origin side
(B) unbounded and non–origin side
(C) unbounded and origin side
(D) bounded and non–origin side

132. If $h(x) = \sqrt{4f(x)+3g(x)}$, $f(1) = 4$, $g(1) = 3$, $f'(1) = 3$, $g'(1) = 4$, then $h'(1) =$ **[2021]**

(A) $\frac{5}{12}$ (B) $\frac{12}{5}$
(C) $-\frac{5}{12}$ (D) $-\frac{12}{7}$

133. If $y = \sqrt{e^{\sqrt{x}}}$, then $\frac{dy}{dx} =$ **[2021]**

(A) $\frac{e^{\sqrt{x}}}{4\sqrt{x}}$ (B) $\frac{e^{\sqrt{x}}}{4x}$
(C) $\frac{e^{\frac{\sqrt{x}}{2}}}{4\sqrt{x}}$ (D) $\frac{e^{\sqrt{x}}}{2\sqrt{x}}$

134. If $e^{-y} \cdot y = x$, then $\frac{dy}{dx}$ is **[2021]**

(A) $\frac{y}{1-y}$
(B) $\frac{1}{xy(1-y)}$
(C) $\frac{1}{x(1-y)}$
(D) $\frac{y}{x(1-y)}$

135. If $y = \text{cosec}^{-1}\left[\frac{\sqrt{x}+1}{\sqrt{x}-1}\right] + cos^{-1}\left[\frac{\sqrt{x}-1}{\sqrt{x}+1}\right]$, then $\frac{dy}{dx} =$ **[2021]**

(A) 0 (B) 1
(C) $\frac{2}{\sqrt{x}+1}$ (D) $\frac{1}{2(\sqrt{x}-1)}$

136. The derivative of $(\log x)^x$ with respect to $\log x$ is **[2021]**

(A) $(\log x)^x\left[\frac{1}{\log x}-\log(\log x)\right]$

(B) $(\log x)^x\left[\log x+\frac{1}{\log(\log x)}\right]$

(C) $x(\log x)^x\left[\frac{1}{\log x}+\log(\log x)\right]$

(D) $x(\log x)^x\left[\log x+\frac{1}{\log(\log x)}\right]$

137. If $y=\tan^{-1}\left[\frac{1}{1+x+x^2}\right]+\tan^{-1}\left[\frac{1}{x^2+3x+3}\right], x>0$, then $\frac{dy}{dx}=$ **[2021]**

(A) $\frac{1}{1+x^2}-\frac{1}{1+(x+2)^2}$

(B) $\frac{-1}{1+x^2}+\frac{1}{1+(x+2)^2}$

(C) $\frac{1}{1+x^2}+\frac{1}{1+(x+2)^2}$

(D) $\frac{-1}{1+x^2}-\frac{1}{1+(x+2)^2}$

138. If $x^y\cdot y^x=16$, then $\frac{dy}{dx}$ at (2, 2) is **[2021]**

(A) –1 (B) 0
(C) 1 (D) 2

139. The derivative of the function $\cot^{-1}\left[(\cos 2x)^{\frac{1}{2}}\right]$ at $x=\frac{\pi}{6}$ is **[2021]**

(A) $\left(\frac{1}{3}\right)^{\frac{1}{2}}$ (B) $\left(\frac{2}{3}\right)^{\frac{1}{2}}$

(C) $\left(\frac{3}{2}\right)^{\frac{1}{2}}$ (D) $(3)^{\frac{1}{2}}$

140. If $y=\log_{10}x+\log_x 10+\log_x x+\log_{10}10$, then $\frac{dy}{dx}=$ **[2021]**

(A) $\frac{1}{x\log_e 10}+\frac{1}{x\log_{10}e}$

(B) $\frac{1}{x\log_e 10}+\frac{\log_e 10}{x(\log_{10}e)^2}$

(C) $\frac{1}{x\log_e 10}-\frac{1}{x\log_{10}e}$

(D) $\frac{1}{x\log_e 10}-\frac{\log_e 10}{x(\log_e x)^2}$

141. If $y=\tan^{-1}\left[\frac{\log\left(\frac{e}{x^2}\right)}{\log(ex^2)}\right]+\tan^{-1}\left[\frac{3+2\log x}{1-6\log x}\right]$, then $\frac{d^2y}{dx^2}=$ **[2021]**

(A) $\frac{2}{1+x^2}$ (B) $\frac{1}{1+x^2}$

(C) $\frac{3}{1+x^2}$ (D) 0

142. If $x = e^t(\sin t - \cos t)$ and $y = e^t(\sin t + \cos t)$, then $\frac{dy}{dx}$ at $t = \frac{\pi}{3}$ is **[2021]**

(A) $\sqrt{3}$

(B) $\frac{1}{\sqrt{3}}$

(C) $\frac{\sqrt{3}}{2}$

(D) $\frac{1}{2}$

143. If $h(x) = \sqrt{4f(x)+3g(x)}$, f(1) = 4, g(1) = 3, f'(1) = 3, g'(1) = 4, then h'(1) = **[2021]**

(A) $\frac{-5}{12}$ (B) $\frac{-12}{7}$

(C) $\frac{5}{12}$ (D) $\frac{12}{5}$

144. A wire of length 20 units is divided into two parts such that the product of one part and cube of the other part is maximum, then product of these parts is **[2021]**

(A) 5 (B) 75
(C) 15 (D) 70

145. A particle is moving on a straight line. The distance S travelled in time t is given by $S = at^2 + bt + 6$. If the particle comes to rest after 4 seconds at a distance of 16 m. from the starting point, then the acceleration of the particle is **[2021]**

(A) $\frac{-3}{4}$ m/sec² (B) $\frac{-1}{2}$ m/sec²
(C) –1 m/sec² (D) $\frac{-5}{4}$ m/sec²

146. The equation of the tangent to the curve $y = 4xe^x$ at $\left(-1, \frac{-4}{e}\right)$ is **[2021]**

(A) $x = -1$ (B) $x - \frac{e}{4}y = 0$
(C) $y = \frac{-4}{e}$ (D) $6x - \frac{e}{4}y = -5$

147. The surface area of a sperical balloon is increasing at the rate 2 cm²/sec. Then rate of increase in the volume of the balloon is……. when the radius of the balloon is 6 cm. **[2021]**

(A) 4 cm³/sec
(B) 16 cm³/sec
(C) 36 cm³/sec
(D) 6 cm³/sec

148. The curves $\frac{x^2}{a^2} + \frac{y^2}{4} = 1$ and $y^3 = 16x$ intersect each other orthogonally, then a^2 = **[2021]**

(A) 2 (B) $\frac{3}{4}$
(C) $\frac{1}{2}$ (D) $\frac{4}{3}$

149. A rectangle of maximum area is inscribed in an ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$, then its dimensions are **[2021]**

(A) $4\sqrt{2}, 6\sqrt{2}$
(B) $\sqrt{2}, 5\sqrt{2}$
(C) $4\sqrt{2}, 5\sqrt{2}$
(D) $4\sqrt{2}, \sqrt{2}$

150. If $f(x) = x^2 + ax + b$ has minima at $x = 3$ whose value is 5, then the values of a and b are respectively. **[2021]**

(A) –6, –14
(B) –6, 14
(C) 14, –6
(D) 6, 14

151. Function $f(x) = e^{\frac{-1}{x}}$ is strictly increasing for all x where **[2021]**

(A) x is only positive real number
(B) x is only negative real number
(C) x is a real number
(D) x is a non–zero real number

152. If $x = -2$ and $x = 4$ are the extreme points of $y = x^3 - \alpha x^2 - \beta x + 5$, then **[2021]**

(A) $\alpha = 3, \beta = 24$
(B) $\alpha = -24, \beta = -3$
(C) $\alpha = -3, \beta = -24$
(D) $\alpha = 24, \beta = 3$

153. The maximum area of the rectangle that can be incribed in a circle of radius r is **[2021]**

(A) $2r^2$ sq. units
(B) $\frac{\pi r^2}{4}$ sq. units
(C) πr^2 sq. units
(D) r^3 sq. units

154. The curve $y = ax^3 + bx^2 + cx + 5$ touches X–axis at P(–2, 0) and cuts Y–axis at a point Q, where its gradient is 3, then **[2021]**

(A) $a = \frac{1}{2}, b = \frac{3}{4}, c = 3$

(B) $a = \frac{1}{2}, b = \frac{-1}{4}, c = -3$

(C) $a = \frac{1}{2}, b = \frac{-3}{4}, c = -3$

(D) $a = \frac{-1}{2}, b = \frac{-3}{4}, c = 3$

155. $\int tan^{-1}(sec\,x + tan\,x)\,dx =$ **[2021]**

(A) $\frac{\pi x}{2} + \frac{x^2}{2} + c$

(B) $\frac{\pi x}{4} + \frac{x^2}{4} + c$

(C) $sin\,x + cos\,x + c$

(D) $sin\,x\,cos\,x + c$

156. If $\int \frac{1+x^2}{1+x^4}dx = \frac{1}{\sqrt{2}}tan^{-1}\left[\frac{f(x)}{\sqrt{2}}\right] + c$, then $f(x) =$ **[2021]**

(A) $x - \frac{1}{x}$

(B) $x + \frac{2}{x}$

(C) $x - \frac{1}{x^2}$

(D) $x + \frac{1}{x^2}$

157. $\int sec^4 x \cdot tan^4 x dx = \frac{tan^m x}{m} + \frac{tan^n x}{n} + c$ (where c is constant of integration), then $m + n =$ **[2021]**

(A) 8

(B) 12

(C) 10

(D) 16

158. $\int \frac{2x^2 - 1}{x^4 - x^2 - 20}dx =$ **[2021]**

(A) $\frac{1}{\sqrt{5}}log\left|\frac{x+\sqrt{5}}{x-\sqrt{5}}\right| + tan^{-1}\left(\frac{x}{2}\right) + c$

(B) $\frac{1}{2\sqrt{5}}log\left|\frac{x+\sqrt{5}}{x-\sqrt{5}}\right| + tan^{-1}\left(\frac{x}{2}\right) + c$

(C) $\frac{1}{2\sqrt{5}}log\left|\frac{x-\sqrt{5}}{x+\sqrt{5}}\right| + \frac{1}{2}tan^{-1}\left(\frac{x}{2}\right) + c$

(D) $\frac{1}{2}log\left|\frac{x-\sqrt{5}}{x+\sqrt{5}}\right| + \frac{1}{2}tan^{-1}\left(\frac{x}{2}\right) + c$

159. If $\int \frac{x^3}{\sqrt{1+x^2}}dx = a\left(1+x^2\right)^{3/2} + b\sqrt{1+x^2} + c$, then a + b =, (where c is constant of integration) **[2021]**

(A) $\frac{-2}{3}$ (B) $\frac{-1}{3}$

(C) $\frac{1}{3}$ (D) $\frac{2}{3}$

160. If $\int \frac{5tan\,x}{tan\,x - 2}dx = x + a\,log\left|sin\,x - 2cos\,x\right| + c$, then a = (where c is constant of integration) **[2021]**

(A) 1 (B) –2

(C) –1 (D) 2

161. $\int \frac{10^{x/2}}{\sqrt{10^{-x} - 10^x}}dx =$ **[2021]**

(A) $2\sqrt{10^{-x} + 10^x} + c$

(B) $\frac{2}{2\sqrt{10^{-x} + 10^x}} + c$

(C) $\frac{1}{log\,10}sin^{-1}\left(10^x\right) + c$

(D) $\frac{1}{log\,10}cos^{-1}(10x) + c$

162. $\int e^{\left(e^x + x\right)}dx =$ **[2021]**

(A) $e^x + x + c$ (B) $e^{\left(e^x\right)} \cdot x + c$

(C) $e^{\left(e^x\right)} + c$ (D) $e^{\left(e^x\right)}\left(e^x - 1\right) + c$

163. $\int sec^{-1}x dx =$ **[2021]**

(A) $xsec^{-1}x + log\left|x+\sqrt{x^2-1}\right| + c$

(B) $xsec^{-1}x - log\left|x+\sqrt{x^2-1}\right| + c$

(C) $xsec^{-1}x - log\left|x+\sqrt{x^2+1}\right| + c$

(D) $xsec^{-1}x + log\left|x+\sqrt{x^2+1}\right| + c$

164. If $\int \frac{\sqrt{x}}{x(x+1)} dx = k\,tan^{-1}m + c$, (where c is constant of integration), then **[2021]**

(A) k = 1, m = $\sqrt{x}$

(B) k = 2, m = $\sqrt{x}$

(C) k = 1, m = x

(D) k = 2, m = x

165. $\int \frac{dx}{32-2x^2} = A log(4-x) + B log(4+x) + c$, then the values of A and B are respectively (Where c is constant of integration) **[2021]**

(A) $\frac{-1}{8}, \frac{1}{8}$ (B) $\frac{1}{8}, \frac{-1}{8}$

(C) $\frac{-1}{16}, \frac{1}{16}$ (D) $\frac{1}{8}, \frac{1}{8}$

166. $\int cos^3 x \cdot e^{log(\sin x)} dx =$ **[2021]**

(A) $\frac{-e^{\sin x}}{4} + c$ (B) $\frac{-cos^4 x}{4} + c$

(C) $\frac{-sin^4 x}{4} + c$ (D) $\frac{e^{\sin x}}{4} + c$

167. If $\int \frac{(\cos x - \sin x)}{8 - \sin 2x} dx = \frac{1}{p} log\left[\frac{3+\sin x+\cos x}{3-\sin x-\cos x}\right] + c$, then p = (where c is a constant of integration) **[2021]**

(A) 12 (B) $\frac{1}{6}$

(C) 6 (D) 3

168. $\int cos^3 x e^{log(\sin x)^2} dx =$ **[2021]**

(A) $\frac{sin^3 x}{3} - sin^5 x + c$

(B) $\frac{sin^3 x}{3} - \frac{sin^5 x}{5} + c$

(C) $\frac{sin^3 x}{3} + \frac{sin^5 x}{5} + c$

(D) $sin^3 x + sin^5 x + c$

169. If $2f(x) - 3f(\frac{1}{x}) = x$, then $\int_1^e f(x) dx$ = **[2021]**

(A) $-\left(\frac{2+e^2}{5}\right)$ (B) $\frac{2+e}{5}$

(C) $\frac{2+e^2}{5}$ (D) $\frac{2-e^2}{5}$

170. If $f(x) = |x-1| + |x-2| + |x-3|$, $\forall\, x \in [1, 4]$, then $\int_1^4 f(x) dx$ = **[2021]**

(A) $\frac{1}{2}$ (B) 7

(C) $\frac{9}{2}$ (D) $\frac{19}{2}$

171. $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\text{cosec}x \cdot cot\, x}{1+\text{cosec}^2 x} dx$ **[2021]**

(A) $\frac{\pi}{4} - tan^{-1}2$ (B) $tan^{-1}1$

(C) $tan^{-1}2$ (D) $tan^{-1}\left(\frac{1}{3}\right)$

172. $\int_0^{\frac{\pi}{2}} \frac{\cos x}{3\cos x + \sin x} dx =$ **[2021]**

(A) $\frac{3\pi}{20} - \frac{log\,3}{100}$ (B) $\frac{3\pi}{10} - \frac{log\,3}{10}$

(C) $\frac{3\pi}{20} + \frac{log\,3}{10}$ (D) $\frac{3\pi}{20} - \frac{log\,3}{10}$

173. $\int_0^{\pi} x \sin x \cos^4 x dx =$ **[2021]**

(A) $\frac{\pi}{10}$ (B) $\frac{2\pi}{5}$

(C) $\frac{\pi}{5}$ (D) $\frac{\pi}{8}$

174. If $\int_0^{\frac{\pi}{2}} \frac{dx}{5+4\sin x} = A\tan^{-1} B$, then A + B = **[2021]**

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$

(C) 1 (D) 2

175. The area bounded by the parabola $y^2 = x$, the straight line $y = 4$ and Y–axis is **[2021]**

(A) $2\sqrt{7}$ sq. units (B) $\frac{64}{3}$ sq. units

(C) $\frac{16}{3}$ sq. units (D) $7\sqrt{2}$ sq. units

176. The area bounded between the curve $x^2 = y$ and the line $y = 4x$ is **[2021]**

(A) $\frac{32}{3}$ sq. units (B) $\frac{8}{3}$ sq. units

(C) $\frac{1}{3}$ sq. units (D) $\frac{16}{3}$ sq. units

177. The area bounded by the parabola $y^2 = x$ and the line $x + y = 2$ in the first quadrant is **[2021]**

(A) $\frac{7}{6}$ sq. units

(B) $\frac{1}{6}$ sq. units

(C) $\frac{2}{3}$ sq. units

(D) $\frac{6}{7}$ sq. units

178. The area of the region included between the parabolas $y^2 = 8x$ and $x^2 = 8y$, is **[2021]**

(A) $\frac{128}{3}$ sq. units (B) $\frac{64}{3}$ sq. units

(C) $\frac{32\sqrt{8}}{3}$ sq. units (D) $\frac{16\sqrt{8}}{3}$ sq. units

179. A differential equation for the temperature 'T' of a hot body as a function of time, when it is placed in a bath which is held at a constant temperature of 32° F, is given by (where k is a constant of proportionality) **[2021]**

(A) $\frac{dT}{dt} = kT + 32$

(B) $\frac{dT}{dt} = 32kT$

(C) $\frac{dT}{dt} = kT - 32$

(D) $\frac{dT}{dt} = -k(T - 32)$

180. The general solution of the differential equation $\frac{dy}{dx} = \frac{x+y+1}{x+y-1}$ is given by **[2021]**

(A) $y = x + \log(x + y) + c$

(B) $x - y = \log(x + y) + c$

(C) $x + y = \log(x + y) + c$

(D) $y = x \log(x + y) + c$

181. The general solution of the differential equation $x + y\frac{dy}{dx} = \sec\left(x^2 + y^2\right)$ is **[2021]**

(A) $\sin(x^2 + y^2) = 2x + c$

(B) $\sin(x^2 + y^2) + 2x = c$

(C) $\sin(x^2 + y^2) + x = c$

(D) $\cos(x^2 + y^2) = 2x + c$

182. An ice ball melts at the rate which is proportional to the amount of ice at that instant. Half the quantity of ice melts in 20 minutes. x_0 is the initial quantity of ice. If after 40 minutes the amount of ice left is Kx_0, then K = **[2021]**

(A) $\frac{1}{2}$

(B) $\frac{1}{8}$

(C) $\frac{1}{4}$

(D) $\frac{1}{3}$

183. If m is order and n is degree of the differential equation $y = \frac{dp}{dx} + \sqrt{a^2p^2 - b^2}$, where $p = \frac{dy}{dx}$, then the value of m + n is **[2021]**

(A) 2
(B) 3
(C) 4
(D) 5

184. The differential equation of an ellipse whose major axis is twice its minor axis, is **[2021]**

(A) $x + 4y\frac{dy}{dx} = 0$

(B) $x - 4y\frac{dy}{dx} = 0$

(C) $x + 2y\frac{dy}{dx} = 0$

(D) None of these

185. I: $y' = \frac{y+x}{x}$; II: $y' = \frac{x^2+y}{x^3}$; III: $y' = \frac{2xy}{y^2 - x^2}$

S1 : Differential equations given by I and II are homogeneous differential equations.
S2 : Differential equations given by II and III are homogeneous differential equations.
S3 : Differential equations given by I and III are homogeneous differential equations. **[2021]**

(A) only S1 is valid
(B) both S1 and S2 are valid
(C) only S3 is valid
(D) only S2 is valid

186. A spherical raindrop evaporates at a rate proportional to its surface area. If its radius originally is 3mm and 1hour later has been reduced to 2 mm, then the expression of radius r of the raindrop at any time t is (where $0 \le t \le 3$) **[2021]**

(A) r = t + 5
(B) r = t – 5
(C) r = 3 – t
(D) r = t + 3

187. The differential equation obtained by eliminating A and B from y = A cos ωt + B sin ωt **[2021]**

(A) $\frac{d^2y}{dt^2} + \omega^2 y = 0$ (B) $\frac{d^2y}{dt^2} + \omega y^2 = 0$

(C) $\frac{d^2y}{dt^2} - \omega^2 y = 0$ (D) $\frac{d^2y}{dt^2} - \omega y^2 = 0$

188. Rajesh has just bought a VCR from Maharashtra Electronics. Maharashtra Electronics offers after sales service contract for ₹1000.00 for the next five years. Considering the experience of VCR users, the following distribution of maintainance expenses for the next five years is formed.

Expenses	0	500	1000	1500	2000	2500	3000
Probability	0.35	0.25	0.15	0.10	0.08	0.05	0.02

The expected value of maintenance cost is **[2021]**

(A) ₹800/– (B) ₹700/–
(C) ₹770/– (D) ₹900/–

189. In a meeting 60% of the members favour and 40% oppose a certain proposal. A member is selected at random and we take X = 0 if he opposed and X = 1 if he is in favour, then Var(X) = **[2021]**

(A) 0.36 (B) 0.24
(C) 0.6 (D) 0.06

190. The probability distribution of the number of doublets in four throws of a pair of dice is given by **[2021]**

(A)

X	**0**	**1**	**2**	**3**	**4**
P(X)	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

(B)

X	**0**	**1**	**2**	**3**
P(X)	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

(C)

X	**1**	**2**	**3**	**4**
P(X)	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

(D)

X	**0**	**1**	**2**	**3**	**4**
P(X	$\frac{625}{1296}$	$\frac{125}{324}$	$\frac{25}{216}$	$\frac{5}{324}$	$\frac{1}{1296}$

191. The mean of the numbers obtained on throwing a die having written 1 on three faces, 2 on two faces and 5 on one face is **[2021]**

(A) 5 (B) $\frac{8}{3}$

(C) 1 (D) 2

192. It is observed that 25% of the cases related to child labour reported to the police station are solved. If 6 new cases are reported, then the probability that at least 5 of them will be solved is **[2021]**

(A) $\frac{19}{1024}$ (B) $\frac{19}{4096}$

(C) $\left(\frac{1}{4}\right)^6$ (D) $\frac{19}{2048}$

193. If $X \sim B(4, p)$ and $P(X = 0) = \frac{16}{81}$, then $P(X = 4) =$ **[2021]**

(A) $\frac{1}{27}$ (B) $\frac{1}{16}$

(C) $\frac{1}{81}$ (D) $\frac{1}{8}$

194. If the sum of the mean and the variance of a Binomial distribution for 5 trials is 1.8, then the value of p is **[2023, 2021, 2020, 2019]**

(A) 0.4

(B) 0.8

(C) 0.18

(D) 0.2

MHTCET PYQ M (2021)

Marathi

Mathematics

Hints & Solutions

1. **(C)**

Using

$$\frac{\sin(\theta-\phi)}{\sin(\theta+\phi)}=\frac{\tan\theta-\tan\phi}{\tan\theta+\tan\phi},$$

we get

$$\sin(\theta-\phi)=\frac{k-1}{k+1}\sin\alpha.$$

2. **(C)**

The A.P. condition gives

$$2\sin(x+z-y)=\sin(y+z-x)$$

$$+\sin(x+y-z)=2\sin y\cos(z-x).$$

After expansion and division by $\cos x\cos y\cos z$,

$$\tan x+\tan z=2\tan y.$$

3. **(C)**

Its inclination from the positive X-axis is

$$90^\circ+30^\circ=120^\circ.$$

Hence

$$m=\tan 120^\circ=-\sqrt{3}.$$

4. **(B)**

The slope is

$$m=\frac{2-1}{1+\frac{1}{2}}=\frac{2}{3}.$$

Thus

$$y-1=\frac{2}{3}\left(x+\frac{1}{2}\right).$$

Putting $y=0$ gives $x=-2$.

5. **(C)**

The required line is perpendicular to the radius vector $(\cos\alpha,\sin\alpha)$. Therefore its normal form is

$$x\cos\alpha+y\sin\alpha=p.$$

6. **(D)**

The midpoint is $(2,-1)$. Since the slope of AB is -1, the perpendicular slope is 1:

$$y+1=x-2\Rightarrow x-y=3.$$

7. **(D)**

Using the normal form,

$$x\cos 30^\circ+y\sin 30^\circ=4.$$

Therefore

$$\sqrt{3}x+y=8.$$

8. **(C)**

Slope $m=-\frac{1}{\sqrt{2}}$, y-intercept $c=-2\sqrt{2}$ (negative direction).

Equation:

$$y=-\frac{1}{\sqrt{2}}x-2\sqrt{2}$$

Multiply by $\sqrt{2}$: $\sqrt{2}y=-x-4$

9. **(B)**

Write the second line as $3x-4y-7/2=0$. Their separation is

$$d=\frac{\left|4+\frac{7}{2}\right|}{5}=\frac{3}{2}.$$

This is the diameter, so

$$r=\frac{d}{2}=\frac{3}{4}.$$

10. **(D)**

The perpendicular bisector of the two points is $y=x+2$. Solving with $x-4y=1$ gives the centre $(-3,-1)$. Hence

$$(x+3)^2+(y+1)^2=100,$$

or

$$x^2+y^2+6x+2y-90=0.$$

11. **(A)**

Since $2\pi r=10\pi$, $r=5$. Thus

$$(x-2)^2+(y+3)^2=25$$

which becomes

$$x^2+y^2-4x+6y-12=0.$$

12. **(C)**

The common chord is the perpendicular bisector of the line of centres. Its midpoint is $(3,4)$ and its slope is -1, so

$$x+y=7.$$

13. **(B)**

These points form a right triangle with hypotenuse joining $(x,0)$ and $(0,y)$. The circumcentre is the midpoint of the hypotenuse:

$$\left(\frac{x}{2},\frac{y}{2}\right).$$

14. **(B)**

The circles have centres $(2,-5)$, $(-4,3)$ and radii 3, 7. The contact point dividing the centres in the ratio $3:7$ is

$$\left(\frac{1}{5},-\frac{13}{5}\right).$$

The tangent is perpendicular to the line of centres, hence

$$3x-4y-11=0.$$

15. **(B)**

The centre is $(2,5)$. The chord is perpendicular to the line joining $(2,5)$ and $(1,2)$, so

$$x+3y=7.$$

16. **(A)**

The chord endpoints are obtained from

$$5x^2-10x=0,$$

namely $(0,0)$ and $(2,4)$. The required circle has centre $(1,2)$ and radius $\sqrt{5}$:

$$x^2+y^2-2x-4y=0.$$

17. **(B)**

Prime sums are $2,3,5,7,11$, with $1+2+4+6+2=15$ favourable outcomes. Therefore

$$P=\frac{15}{36}=\frac{5}{12}.$$

18. **(D)**

There are 6 doublets and 3 outcomes with sum 10, with one common outcome $(5,5)$. Thus the excluded outcomes are

$$6+3-1=8.$$

Hence

$$P=\frac{36-8}{36}=\frac{7}{9}.$$

19. **(A)**

$$P(H\cup N)=\frac{1}{2}+\frac{1}{3}-\frac{1}{6}=\frac{2}{3}.$$

20. **(C)**

$$P(A\cap B)=P(A)+P(B)-P(A\cup B)=0.$$

Therefore A and B are mutually exclusive.

21. **(D)**

There are 8 prime-numbered rooms. After one prime room is used, 7 prime rooms remain among 18 rooms:

$$P=\frac{7}{18}.$$

22. **(B)**

$$P(A\cap B)=\frac{3}{10}+\frac{2}{5}-\frac{3}{5}=\frac{1}{10}.$$

Hence

$$P(A|\ B)P(B|\ A)=\frac{1}{4}\cdot\frac{1}{3}=\frac{1}{12}.$$

23. **(C)**

$$P(E_1)+P(E_2)=0.6+0.2=0.8.$$

Therefore

$$P(E_1')+P(E_2')=2-0.8=1.2.$$

24. **(B)**

$$(1+i)(1-i)=1-i^2=2$$

So,

$$(1+i)^5(1-i)^7=(1+i)^5(1-i)^5(1-i)^2$$

$$=[(1+i)(1-i)]^5(1-i)^2$$

$$=2^5(1-2i+i^2)=32(1-2i-1)=32(-2i)=-64i$$

Thus, Option (B) is correct

25. **(D)**

Distance from origin = modulus = 2.

Argument $=\frac{5\pi}{6}$

So the complex number is:

$$2\left(\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}\right)$$

$$\cos\frac{5\pi}{6}=-\frac{\sqrt{3}}{2},\quad \sin\frac{5\pi}{6}=\frac{1}{2}$$

Thus:

$$2\left(-\frac{\sqrt{3}}{2}+i\cdot\frac{1}{2}\right)=-\sqrt{3}+i$$

26. **(A)**

$$z=\frac{3+i}{2-i}=1+i=\sqrt{2}\,\mathrm{cis}\,\frac{\pi}{4}.$$

Thus

$$z^{38}=2^{19}\mathrm{cis}\,\frac{19\pi}{2}=-2^{19}i.$$

27. **(B)**

$$x^2=-3+4i,\ x^3=-11-2i.$$

Therefore

$$x^3+7x^2-x+16=-17+24i.$$

28. **(D)**

Using $1+\omega+\omega^2=0$,

$$3+5\omega+3\omega^2=2\omega,\ 3+3\omega+5\omega^2=2\omega^2.$$

Hence

$$4\omega^2+4\omega^4=4\left(\omega^2+\omega\right)=-4.$$

29. **(B)**

The locus is the ray from $(2,3)$ with slope $\tan(3\pi/4)=-1$:

$$y-3=-(x-2)\Rightarrow x+y=5.$$

30. **(A)**

Since

$$(2-3i)^2=4-12i-9=-5-12i,$$

the square roots are

$$\pm(2-3i).$$

31. **(C)**

$$\frac{3+2i}{1+i}\times\frac{1-i}{1-i}=\frac{(3+2i)(1-i)}{1^2+1^2}=\frac{3-3i+2i-2i^2}{2}$$

$$=\frac{3-i+2}{2}=\frac{5-i}{2}$$

Given:

$$\frac{5-i}{2}=\frac{1}{2}(x+iy)\Rightarrow x=5,y=-1$$

So,

$$x-y=5-(-1)=6$$

32. **(C)**

$|z+1|=|(x+1)+iy|=1$

represents a circle with centre $(-1,0)$ and radius 1

33. **(D)**

The five vowels are all A's and form one block. With consonants B,B,R,R,C,D, there are seven objects, so

$\frac{7!}{2!2!}=1260.$

34. **(C)**

For a necklace, rotations and reflections are identical:

$\frac{(8-1)!}{2}=2520.$

35. **(B)**

Odd positions contain $1,1,3,3$ in

$\frac{4!}{2!2!}=6$

ways. Even positions contain $2,2,4$ in $3!/2!=3$ ways. Total:

$6\times 3=18.$

36. **(D)**

$\binom{7}{3}\binom{4}{2}5!=35\cdot 6\cdot 120=25200.$

37. **(C)**

Repeatedly using Pascal's identity,

$\binom{11}{4}+\binom{11}{5}=\binom{12}{5},$

$\binom{12}{5}+\binom{12}{6}=\binom{13}{6},$

$\binom{13}{6}+\binom{13}{7}=\binom{14}{7}.$

Thus $r=7$.

38. **(D)**

$Max:{}^{6}C_r={}^{6}C_3=20$

Given difference = 16 $\rightarrow$ max ${}^{n}C_r$ = 4.

For $n=4$, $\max {}^{4}C_2=6$, but if considering ${}^{n}C_r$ with $r=1$ or $n-1$, $\max=n$.

Set $n=4\rightarrow \max=4$ (if $r=1$ or 3).

So $n=4$.

39. **(B)**

$\binom{6}{5}+\binom{4}{1}\binom{6}{4}+\binom{4}{2}\binom{6}{3}=6+60+120=186.$

40. **(C)**

The denominator requires $x+|x|>0$. For $x>0$, it equals $2x>0$; for $x\le 0$, it is 0. Thus the domain is

$(0,\infty).$

41. **(D)**

In R_1,R_2,R_3, every element of A has exactly one image. In R_4, a has two images and c has none. Hence only R_4 is not a function.

42. **(B)**

$[-1.32]=-2,\ \{-0.32\}=0.68.$

Therefore

$f(-1.32)=3(-2)+5(0.68)=-2.6.$

43. **(C)**

The images are

$5,11,3,7,13.$

Hence the range is

$\{3,5,7,11,13\}.$

44. **(B)**

For $\sqrt{x-1}$ to be real, $x-1\ge 0\Rightarrow x\ge 1$.

For $\sqrt{6-x}$ to be real, $6-x\ge 0\Rightarrow x\le 6$.

Combining: $1\le x\le 6$.

So domain = [1, 6]

45. (D)

$8\pi = 25.12, [25.12] = 25.$

Thus

$f(\pi) = 25 - 3 = 22.$

46. (A)

For every real x, $2^x + 4^x > 0$, so $f(x) > 3$. As $x \to -\infty$, $f(x) \to 3$ without attaining it. Therefore the range is

$(3, \infty).$

47. (B)

The logarithm requires

$(x-2)(x-3) > 0.$

Hence

$x \in (-\infty, 2) \cup (3, \infty).$

48. (C)

$\{-1.4\} = 0.6.$

Therefore

$f(-1.4) = 2(0.6) + 5(-1.4) = -5.8.$

49. (C)

Rationalising,

$$\frac{5x-7}{\sqrt{x^2+5x-7}+x} \to \frac{5}{2}.$$

50. (B)

$a = \frac{1}{2},\ b = \frac{1}{3}.$

Thus

$2a = 3b.$

51. (C)

Since

$2x^2 + x - 3 = (2x+3)(x-1),$

$\sqrt{x} - 1 = \frac{x-1}{\sqrt{x}+1},$

the limit is

$\frac{-1}{5 \cdot 2} = -\frac{1}{10}.$

52. (C)

Using $\cos tx = 1 - 1/2t^2x^2 + o(x^2)$,

$\lim = \frac{n^2 - m^2}{2}.$

53. (C)

Let $h = x - 2$. Then

$(1+h)^{1/(3h)} \to e^{1/3}.$

54. (A)

By derivatives at $x = 1$,

$\lim = \frac{\frac{1}{2}}{1} = \frac{1}{2}.$

55. (B)

Applying L'Hospital's rule,

$\lim = \frac{ab\ln b - ab\ln a}{2} = \frac{ab}{2}\ln\left(\frac{b}{a}\right).$

56. (A)

Using $1 - \cos u \sim u^2/2$,

$\frac{\sqrt{x^4/2}}{x^2/2} = \sqrt{2}.$

57. (C)

The floor function is discontinuous at integers. Within $(-1, 2)$ these are

$x = 0, 1.$

58. (B)

At $x = -3$,

$f(-3)=|-3|+3=6, \lim_{x\to -3^+}(-2x)=6,$

so it is continuous. At $x=3$, the left limit is -6 while $f(3)=20$, so it is discontinuous.

59. (D)

Continuity requires

$3a+b=11,\ 5a-2b=11.$

Solving gives

$a=3, b=2.$

60. (B)

The negation of $p\to q$ is $p\wedge\neg q$. Hence

$x\in A\cap B$ and $(x\notin A$ or $x\notin B)$.

61. (C)

The antecedent is $\neg p\vee\neg q=\neg(p\wedge q)$ and the consequent is $r\wedge s$. Thus

$\neg(p\wedge q)\to(r\wedge s).$

62. (C)

The negation of a universal equality is an existential inequality:

$\exists x\in\mathbb{R}$ such that $x^2+1\neq 0.$

63. (D)

Since $p\leftrightarrow q$ is true,

$(p\leftrightarrow q)\to r=T\to F=F.$

64. (D)

$p\to q\equiv\neg p\vee q.$

Therefore $S_1\equiv S_3$.

65. (C)

For propositions p, q,

$p\to q\equiv\neg p\vee q.$

Thus S_1 and S_2 are equivalent.

66. (C)

The inverse is $\neg p\to\neg q$. Its contrapositive is $q\to p$,

i.e. if the areas are equal, then the triangles are congruent.

67. (D)

$\exists x\in\mathbb{N}$ such that x^2+x is not even.

68. (D)

The negation of $p\to\neg q$ is

$p\wedge q.$

Thus it is raining and the weather is pleasant.

69. (A)

By De Morgan's law,

$\neg(p\vee q)=\neg p\wedge\neg q.$

Thus 7 is not greater than 4 and 6 is not less than 7.

70. (B)

For $p\to q$, the contrapositive is $\neg q\to\neg p$. Therefore: if I do not get first class, then

$2^2\neq 5.$

71. (A)

The first component is true. A disjunction with one true component is true.

72. (C)

$x\in A\cap B$ and $(x\notin A$ or $x\notin B)$.

73. (A)

Its determinant is

$18(x+3).$

For non-existence of the inverse,

$18(x+3)=0\Rightarrow x=-3.$

74. (C)

The matrices satisfy

$A(\alpha)A(\beta)=A(\alpha+\beta), A(\alpha)^{-1}=A(-\alpha).$

Therefore

$\left[A^2(\alpha)\right]^{-1}=A(-2\alpha).$

75. **(A)**

The cofactors are

$C_{12}=-13, C_{22}=6, C_{32}=5.$

76. **(C)**

$$(AB)^{-1}=B^{-1}A^{-1}=\begin{bmatrix}1 & 0\\ -3 & 1\end{bmatrix}\begin{bmatrix}2 & -3\\ -1 & 2\end{bmatrix}$$

$$=\begin{bmatrix}2 & -3\\ -7 & 11\end{bmatrix}.$$

77. **(D)**

Direct calculation gives

$$\text{adj}A=\begin{bmatrix}5 & 4 & -2\\ 1 & 1 & 0\\ -2 & -2 & 1\end{bmatrix}.$$

Thus $x=4$, $y=1$, and $x+y=5$.

78. **(D)**

Inverting gives

$$A=\begin{bmatrix}2 & 4\\ 1 & 1\end{bmatrix}.$$

Hence

$$2A+I_2=\begin{bmatrix}5 & 8\\ 2 & 3\end{bmatrix}.$$

79. **(C)**

Let the numbers be a,b,c. The equations are

$a+b+c=6, a+3c=7, 3a+b+c=12.$

They give

$a=3, b=\frac{5}{3}, c=\frac{4}{3}.$

Thus the product is $20/3$.

80. **(D)**

Therefore

$$A^{-1}=\frac{1}{19}A,$$

so $K=1/19$.

81. **(B)**

Since

$A(\text{adj}A)=|A|I$

and $|A|=3$,

$A(\text{adj}A)=3I_3.$

82. **(D)**

Their determinants are respectively

$0,0,0,-4.$

Hence only D is invertible.

83. **(C)**

For every square matrix,

$A(\text{adj}A)=|A|I.$

The displayed matrix contains an off-diagonal entry 2, so the statement is internally inconsistent. If that entry is a typographical error and the intended matrix is $-10I$, then

$|A|=-10.$

84. **(A)**

$k=|A|=3.$

Therefore

$(k+1)^4=4^4=256.$

85. **(D)**

Equating the off-diagonal entry gives

$15a-2b=0.$

Equating the lower diagonal entry to $|A|$ gives

$13=10a+3b.$

Thus $a=2/5$, $b=3$, and

$5a+b=5.$

86. **(D)**

$\sin(-600^\circ)=-\sin(600^\circ)=-\sin(240^\circ)$

$=-\left(-\frac{\sqrt{3}}{2}\right)=\frac{\sqrt{3}}{2}$

So,

$$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)=\frac{\pi}{3}$$

And,

$$\cot^{-1}(-\sqrt{3})=\pi-\cot^{-1}(\sqrt{3})=\pi-\frac{\pi}{6}=\frac{5\pi}{6}$$

Now sum:

$$\frac{\pi}{3}+\frac{5\pi}{6}=\frac{2\pi}{6}+\frac{5\pi}{6}=\frac{7\pi}{6}$$

87. (A)

Using principal values,

$$\tan^{-1}\left(\tan 5\pi/6\right)=-\frac{\pi}{6},$$

$$\cos^{-1}\left(\cos 13\pi/6\right)=\frac{\pi}{6}.$$

The sum is 0.

88. (C)

By the sine rule, $b=2R\sin B$ and $c=2R\sin C$.

Thus

$$b\sin B-c\sin C=2R\left(\sin^2 B-\sin^2 C\right)$$

$$=2R\sin A\sin\left(B-C\right).$$

Since $a=2R\sin A$, the quotient is a.

89. (A)

Using the cosine rule and simplifying gives

$$a^2=b^2.$$

Hence $a=b$, so the triangle is isosceles.

90. (A)

The cosine rule gives

$$\cos A=1,\cos B=1,\cos C=-1.$$

Therefore

$$\frac{1}{2}+\frac{1}{3}-\frac{1}{5}=\frac{19}{30}.$$

91. (A)

By the cosine rule,

$$a\cos B-b\cos A=\frac{a^2-b^2}{c}.$$

Hence

$$c\left(a\cos B-b\cos A\right)=a^2-b^2.$$

92. (A)

The identity

$$\frac{\cos^2 A}{a^2}-\frac{\cos^2 B}{b^2}=\frac{1}{a^2}-\frac{1}{b^2}$$

gives

$$\frac{1}{4}-\frac{1}{9}=\frac{5}{36}.$$

93. (C)

$$\cos x=-\frac{\sqrt{3}}{2}.$$

In $\left[0,2\pi\right)$,

$$x=\frac{5\pi}{6},\frac{7\pi}{6}.$$

94. (B)

Let the two angles be A,B. Then

$$\sin\left(A+B\right)=\frac{5}{13}\cdot\frac{4}{5}+\frac{12}{13}\cdot\frac{3}{5}=\frac{56}{65}.$$

Thus $P=56/65$.

95. (D)

Taking tangents,

$$\frac{2\cos x}{1-\cos^2 x}=\frac{2}{\sin x}.$$

Thus $\cos x=\sin x$, and the listed solution is

$$x=\frac{\pi}{4}.$$

96. (C)

The lines meet $x=3$ at $y=\pm\sqrt{3}$, so their slopes are $\pm 1/\sqrt{3}$. Hence

$$y^2=\frac{x^2}{3}\Rightarrow x^2-3y^2=0.$$

97. (B)

The angle-bisector equation is

$3x^2+10xy-3y^2=0.$

Comparing with $3x^2-kxy-3y^2=0$ gives $k=-10.$

98. [D]

The slopes satisfy

$bm^2+2hm+a=0.$

Therefore

$$\tan(\alpha+\beta)=\frac{m_1+m_2}{1-m_1m_2}=\frac{2h}{a-b}.$$

99. (A)

For the pair of lines,

$$\tan\theta=\frac{2\sqrt{h^2-mn}}{|m+n|}.$$

The given condition yields

$4(h^2-mn)=3(m+n)^2,$

so $\tan\theta=\sqrt{3}$ and $\theta=\pi/3.$

100. (C)

The pair is

$(y-3x)(y-6x)=0.$

At $y=9$, the intersection abscissae are 3 and $3/2$. Thus

$$\text{Area}=\frac{1}{2}\left(3-\frac{3}{2}\right)9=\frac{27}{4}.$$

101. (A)

$$\left|\frac{m-\frac{1}{2}}{1+\frac{m}{2}}\right|=1.$$

Solving gives

$$m=3 \text{ or } m=-\frac{1}{3}.$$

102. (D)

For perpendicular lines, the sum of the coefficients of x^2 and y^2 is zero:

$k^2+2-6=0.$

Thus

$k=\pm 2.$

103. (D)

$2x^2-5xy+2y^2=(2x-y)(x-2y).$

The distances are $\sqrt{5}$ and $4/\sqrt{5}$, so their product is 4.

104. (A)

$$\left[\vec{a}+\vec{b},\vec{b}+\vec{c},\vec{c}+\vec{a}\right]=2\left[\vec{a},\vec{b},\vec{c}\right].$$

Thus

$$24=\frac{1}{6}\cdot 2\left|\left[\vec{a},\vec{b},\vec{c}\right]\right|,$$

so the required volume is 72.

105. (B)

$1=\left|\vec{e}_1+\vec{e}_2\right|^2=2+2\cos\theta.$

Hence $\cos\theta=-1/2$ and

$\theta=120^\circ.$

106. (B)

The three perpendicularity equations imply

$\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{c}=\vec{c}\cdot\vec{a}=0.$

Therefore

$$\left|\vec{a}+\vec{b}+\vec{c}\right|=\sqrt{2^2+3^2+4^2}=\sqrt{29}.$$

107. (C)

The vectors are parallel:

$$\frac{1}{2}=\frac{\lambda}{6}=\frac{\mu}{27}.$$

Thus

$$\lambda=3,\ \mu=\frac{27}{2}.$$

108. (B)

The second direction vector is $(4,1,8)$. Hence

$$\cos\theta = \frac{(2,2,1)\cdot(4,1,8)}{3\cdot 9} = \frac{18}{27} = \frac{2}{3}.$$

Thus $\theta = \cos^{-1}(2/3)$.

109. (D)

The coefficient determinant is

$$\begin{vmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 0 & 2 & 1 \end{vmatrix} = 9.$$

Thus the volume is

$9\times 4 = 36.$

110. (D)

$$\left(3\vec{a}+\vec{b}\right)\times\left(2\vec{a}+3\vec{b}\right) = 7\left(\vec{a}\times\vec{b}\right).$$

Hence the area is

$7\times 20 = 140.$

111. (A)

$$\left(\vec{a}+\lambda\vec{b}\right)\cdot\vec{c} = 14+\lambda = 0.$$

Therefore

$\lambda = -14.$

112. (C)

$$65\sin\theta = 25 \Rightarrow \sin\theta = \frac{5}{13}.$$

As θ is obtuse, $\cos\theta = -12/13$. Thus

$$\vec{a}\cdot\vec{b} = 65\left(-\frac{12}{13}\right) = -60.$$

113. (D)

Using adjacent sides

$\overrightarrow{AB} = (0,1,a-3), \overrightarrow{AD} = (2,5,0),$

we get

$$\left|\overrightarrow{AB}\times\overrightarrow{AD}\right|^2 = 29a^2 - 174a + 265.$$

Equating to 265 gives

$a = 0$ or 6.

114. (B)

Twice the area is the magnitude of the cross product of two side vectors. This gives

$2a^2 + 4a + 8 = 24.$

Hence

$a^2 + 2a - 8 = 0 \Rightarrow a = 2, -4.$

115. (C)

$$|\vec{x}|^2 - |\hat{a}|^2 = 8.$$

Since $|\hat{a}| = 1$,

$|\vec{x}|^2 = 9 \Rightarrow |\vec{x}| = 3.$

116. (D)

The maximum is $|\vec{v}\times\vec{w}|$. Here

$\vec{v}\times\vec{w} = (6,-7,-2),$

so

$\max\left|[\vec{u}\,\vec{v}\,\vec{w}]\right| = \sqrt{36+49+4} = \sqrt{89}.$

117. (D)

$$\left(\vec{a}+k\vec{b}\right)\cdot\left(\vec{a}-k\vec{b}\right) = 16 - 25k^2 = 0.$$

Thus

$$k = \pm\frac{4}{5}.$$

118. (B)

Let $r = |\vec{c}|$. Since $\vec{a}\cdot\vec{c} = r$ and $|\vec{c}-\vec{a}| = 2\sqrt{2}$ with $|\vec{a}| = 3$,

$r^2 + 9 - 2r = 8 \Rightarrow r = 1.$

Also $\left|\vec{a}\times\vec{b}\right| = 3$ and its angle with $\vec{c}$ is 60°. Therefore

$$\left|\left(\vec{a}\times\vec{b}\right)\times\vec{c}\right| = 3\cdot 1\cdot\sin 60^\circ = \frac{3\sqrt{3}}{2}.$$

119. (D)

A point on the line is $(-1,3,-2)$ and its direction is $(-3,2,1)$. Together with the vector $(1,4,-5)$ to

the given point, the normal is proportional to $(1,1,1)$. Hence

$x+y+z=0.$

120. (C)

A direction vector is

$B-A=(-2,-5,13).$

Therefore

$x=3-2\lambda, y=4-5\lambda, z=-7+13\lambda.$

121. (C)

As in Q108,

$\cos\theta=\frac{2}{3},$

so

$\theta=\cos^{-1}\left(\frac{2}{3}\right).$

122. (B)

A direction vector is $(-1,1,-1)$. Thus

$\frac{x-2}{-1}=\frac{y-2}{1}=\frac{z-1}{-1}.$

123. (B)

A normal satisfying both conditions is $(5,3,2)$.

Through $(-2,2,2)$,

$5(x+2)+3(y-2)+2(z-2)=0,$

so

$5x+3y+2z=0.$

124. (C)

The cross product of the parallel vectors is proportional to $(1,-4,-5)$. Hence

$(x-2)-4y-5(z-5)=0,$

or

$x-4y-5z+23=0.$

125. (C)

Orthogonality of the plane normal and line direction gives

$3-15-2\alpha=0\Rightarrow\alpha=-6.$

Using the point $(2,1,-2)$ gives $\beta=7$. Therefore

$\alpha\beta=-42.$

126. (A)

The line is

$(-2,1,-1)+t(1,2,-2).$

The direction magnitude is 3, so $3|t|=12$ and $t=\pm4$. The points are

$(2,9,-9),(-6,-7,7).$

127. (D)

A plane parallel to the XY-plane has equation $z=c$. Through the point,

$z=6.$

128. (C)

A test point inside the shaded polygon, for example $(2,1)$, satisfies

$3x+4y\le18, x-6y\le3, 2x+3y\ge3,$

$-7x+14y\le14, x\ge0, y\ge0.$

Therefore the correct system is option (C).

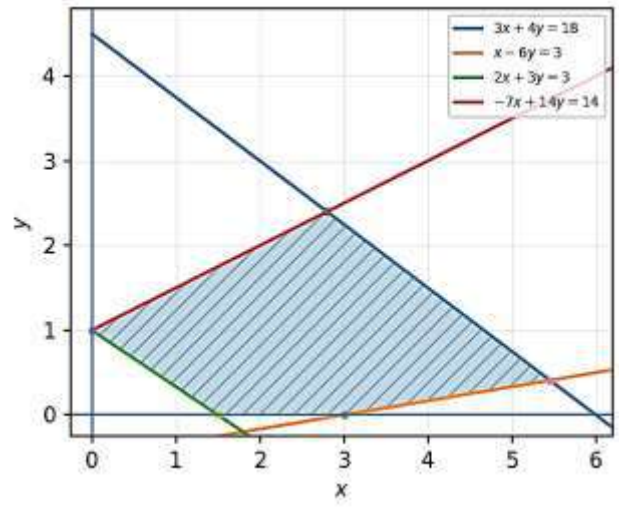


Fig: Feasible region corresponding to option (C)

129. (B)

On the X-axis, the feasible point of least x is $(7/2, 0)$, giving

$$Z = 4 \cdot \frac{7}{2} = 14.$$

Other feasible vertices give larger values. Hence the minimum occurs on the X-axis.

130. (A)

The region lies on the non-origin side of $x + y = 5$ and extends indefinitely to the right. It is unbounded and on the non-origin side.

131. (B)

Both inequalities require points above their boundary lines in the first quadrant. The common region extends indefinitely, and it lies on the non-origin side of $x + 2y = 4$.

132. (B)

$$h'(x) = \frac{4f'(x) + 3g'(x)}{2\sqrt{4f(x) + 3g(x)}}.$$

Thus

$$h'(1) = \frac{12 + 12}{2\sqrt{25}} = \frac{12}{5}.$$

133. (C)

$$y = e^{\sqrt{x}/2}.$$

Therefore

$$\frac{dy}{dx} = \frac{e^{\sqrt{x}/2}}{4\sqrt{x}}.$$

134. (D)

Differentiating,

$$e^{-y}(1 - y)\frac{dy}{dx} = 1.$$

Since $e^y = y / x$,

$$\frac{dy}{dx} = \frac{y}{x(1 - y)}.$$

135. (A)

Let

$$u = \sqrt{\frac{\sqrt{x} - 1}{\sqrt{x} + 1}}.$$

The first term is $\sin^{-1}u$ and the second is $\cos^{-1}u$. Their sum is $\pi / 2$, so

$$\frac{dy}{dx} = 0.$$

136. (C)

If $y = (\log x)^x$, then

$$\frac{dy}{dx} = y\left[\log(\log x) + \frac{1}{\log x}\right].$$

Since $d(\log x)/dx = 1/x$,

$$\frac{dy}{d(\log x)} = x(\log x)^x\left[\frac{1}{\log x} + \log(\log x)\right].$$

137. (B)

Use

$$\tan^{-1}\frac{1}{x^2 + x + 1} = \tan^{-1}(x + 1) - \tan^{-1}x,$$

$$\tan^{-1}\frac{1}{x^2 + 3x + 3} = \tan^{-1}(x + 2) - \tan^{-1}(x + 1).$$

Thus

$$\frac{dy}{dx} = -\frac{1}{1 + x^2} + \frac{1}{1 + (x + 2)^2}.$$

138. (A)

Taking logarithms,

$$y\log x + x\log y = \log 16.$$

Differentiation at $(2, 2)$ gives

$$y'(1 + \log 2) + (1 + \log 2) = 0,$$

so $y' = -1$.

139. (B)

$$\frac{d}{dx}\cot^{-1}\sqrt{\cos 2x} = \frac{\sin 2x}{\sqrt{\cos 2x}(1 + \cos 2x)}.$$

At $x=\pi/6$, this equals

$$\sqrt{\frac{2}{3}}.$$

140. (D)

The last two terms are constants. Hence

$$\frac{dy}{dx}=\frac{1}{x\ln 10}-\frac{\ln 10}{x(\ln x)^2}.$$

141. (D)

Let $L=\log x$. The first tangent argument is

$$\frac{1-2L}{1+2L}=\tan\left(\frac{\pi}{4}-\tan^{-1}(2L)\right),$$

and the second is

$$\frac{3+2L}{1-6L}=\tan\left(\tan^{-1}3+\tan^{-1}(2L)\right).$$

Their inverse tangents sum to a constant, so

$$\frac{d^2y}{dx^2}=0.$$

142. (B)

$$\frac{dx}{dt}=2e^t\sin t,\frac{dy}{dt}=2e^t\cos t.$$

Therefore

$$\frac{dy}{dx}=\cot t=\frac{1}{\sqrt{3}}.$$

143. (D)

$$h'(1)=\frac{4(3)+3(4)}{2\sqrt{4(4)+3(3)}}=\frac{12}{5}.$$

144. (B)

Maximise

$$P=x(20-x)^3.$$

Then

$$P'=(20-x)^2(20-4x)=0\Rightarrow x=5.$$

The parts are 5 and 15, and their product is 75.

145. (D)

$$v=2at+b,8a+b=0.$$

Also

$$16a+4b+6=16.$$

Solving gives $a=-5/8$, so acceleration $2a$ is

$$-\frac{5}{4}\text{ m s}^{-2}.$$

146. (C)

$$\frac{dy}{dx}=4e^x(x+1).$$

At $x=-1$, the slope is 0, so the tangent is

$$y=-\frac{4}{e}.$$

147. (D)

Since

$$S=4\pi r^2,V=\frac{4}{3}\pi r^3,$$

we have

$$\frac{dV}{dS}=\frac{r}{2}.$$

Thus

$$\frac{dV}{dt}=\frac{6}{2}\cdot 2=6\text{ cm}^3/\text{s}.$$

148. (D)

Their slopes are

$$m_1=-\frac{4x}{a^2y},m_2=\frac{16}{3y^2}.$$

Using $y^3=16x$ and $m_1m_2=-1$ gives

$$-\frac{4}{3a^2}=-1\Rightarrow a^2=\frac{4}{3}.$$

149. (C)

For an ellipse with semiaxes a,b, the maximum rectangle has dimensions $\sqrt{2}a$ and $\sqrt{2}b$. Here $5\sqrt{2}$ and $4\sqrt{2}$.

150. (B)

$f'(x)=2x+a,$

so $6+a=0$ and $a=-6$. Then

$f(3)=9-18+b=5\Rightarrow b=14.$

151. (D)

$$f'(x)=\frac{e^{-1/x}}{x^2}>0$$

for every $x\neq 0$. Thus it is strictly increasing on its domain of non-zero real numbers.

152. (A)

The derivative

$3x^2-2\alpha x-\beta$

has roots $-2,4$. Comparing sum and product gives

$\alpha=3,\beta=24.$

153. (A)

The maximum rectangle is a square whose diagonal is $2r$. Its side is $\sqrt{2}r$, so

$A_{\max}=2r^2.$

154. (D)

At the Y-axis, $c=y'(0)=3$. Touching at $x=-2$ gives

$-8a+4b-1=0,$

$12a-4b+3=0.$

Thus

$$a=-\frac{1}{2},b=-\frac{3}{4},c=3.$$

155. (B)

Using

$$\sec x+\tan x=\tan\left(\frac{\pi}{4}+\frac{x}{2}\right),$$

the integrand is $\pi/4+x/2$ on the principal interval. Therefore

$$\int=\frac{\pi x}{4}+\frac{x^2}{4}+C.$$

156. (A)

Let

$$f(x)=x-\frac{1}{x}.$$

Then

$$\frac{d}{dx}\left[\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{f(x)}{\sqrt{2}}\right)\right]=\frac{1+x^2}{1+x^4}.$$

157. (B)

Let $t=\tan x$. Then

$\sec^4 x\tan^4 x\,dx=\left(t^4+t^6\right)dt.$

Thus

$$\int=\frac{t^5}{5}+\frac{t^7}{7}+C,$$

so $m+n=12$.

158. (C)

$$\frac{2x^2-1}{\left(x^2-5\right)\left(x^2+4\right)}=\frac{1}{x^2-5}+\frac{1}{x^2+4}.$$

Hence

$$\int=\frac{1}{2\sqrt{5}}\ln\left|\frac{x-\sqrt{5}}{x+\sqrt{5}}\right|+\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+C.$$

159. (A)

With $u=1+x^2$,

$$\int\frac{x^3}{\sqrt{1+x^2}}dx=\frac{1}{3}u^{3/2}-u^{1/2}+C.$$

Thus

$$a+b=\frac{1}{3}-1=-\frac{2}{3}.$$

160. (D)

Differentiating

$x+a\ln|\sin x-2\cos x|$

and comparing with $5\tan x/(\tan x-2)$ gives

$a=2.$

161. (C)

The integrand simplifies to

$$\frac{10^x}{\sqrt{1-10^{2x}}}.$$

With $u=10^x$,

$$\int=\frac{1}{\ln 10}\sin^{-1}\left(10^x\right)+C.$$

162. (C)

Since $e^{e^x+x}=e^{e^x}e^x$, put $u=e^x$:

$$\int e^{e^x+x}dx=e^{e^x}+C.$$

163. (B)

By integration by parts,

$$\int \sec^{-1}x\,dx=x\sec^{-1}x-\ln\left|x+\sqrt{x^2-1}\right|+C.$$

164. (B)

Put $t=\sqrt{x}$, so $dx=2t\,dt$. Then

$$\int\frac{\sqrt{x}}{x(x+1)}dx=2\int\frac{dt}{1+t^2}=2\tan^{-1}\sqrt{x}+C.$$

Thus $k=2$, $m=\sqrt{x}$.

165. (C)

$$\int\frac{dx}{2\left(16-x^2\right)}=\frac{1}{16}\ln\left|\frac{4+x}{4-x}\right|+C.$$

Therefore

$$A=-\frac{1}{16},B=\frac{1}{16}.$$

166. (B)

Since $e^{\log(\sin x)}=\sin x$,

$$\int\cos^3 x\sin x\,dx=-\frac{\cos^4 x}{4}+C.$$

167. (C)

Let $u=\sin x+\cos x$. Then

$$\frac{d}{dx}\ln\frac{3+u}{3-u}=\frac{6(\cos x-\sin x)}{8-\sin 2x}.$$

Hence the integral coefficient is $1/6$, so $p=6$.

168. (B)

Since $e^{\log\left((\sin x)^2\right)}=\sin^2 x$, let $u=\sin x$:

$$\int\cos^3 x\sin^2 x\,dx=\int\left(u^2-u^4\right)$$

$$du=\frac{\sin^3 x}{3}-\frac{\sin^5 x}{5}+C.$$

169. (A)

Replacing x by $1/x$ and solving the two equations gives

$$f(x)=-\frac{1}{5}\left(2x+\frac{3}{x}\right).$$

Therefore

$$\int_1^e f(x)dx=-\frac{1}{5}\left[e^2-1+3\right]=-\frac{e^2+2}{5}.$$

170. (D)

Split at 2 and 3:

$$\int_1^2(4-x)dx+\int_2^3 x\,dx+\int_3^4(3x-6)$$

$$dx=\frac{5}{2}+\frac{5}{2}+\frac{9}{2}=\frac{19}{2}.$$

171. (D)

Let $u=\csc x$. Then

$$I=-\int_2^1\frac{du}{1+u^2}=\tan^{-1}2-\frac{\pi}{4}.$$

Using the tangent subtraction formula,

$$I=\tan^{-1}\left(\frac{1}{3}\right).$$

172. (D)

Write

$$\cos x=\frac{3}{10}(3\cos x+\sin x)+\frac{1}{10}(-3\sin x+\cos x).$$

Hence

$$I=\frac{3\pi}{20}-\frac{\ln 3}{10}.$$

173. (C)

For $f(x)=\sin x\cos^4 x$, $f(\pi-x)=f(x)$. Therefore

$$\int_0^{\pi} xf(x)\,dx = \frac{\pi}{2}\int_0^{\pi} f(x)\,dx.$$

Since $\int_0^{\pi} \sin x\cos^4 x\,dx = 2/5$, the value is $\frac{\pi}{5}$.

174. (C)

With $t = \tan(x/2)$,

$$I = 2\int_0^1 \frac{dt}{5t^2+8t+5} = \frac{2}{3}\tan^{-1}\left(\frac{1}{3}\right).$$

Thus $A = 2/3$, $B = 1/3$, and $A + B = 1$.

175. (B)

Using horizontal strips,

$$A = \int_0^4 \left(y^2 - 0\right)dy = \left[\frac{y^3}{3}\right]_0^4 = \frac{64}{3}.$$

176. (A)

They intersect at $x = 0, 4$. Hence

$$A = \int_0^4 \left(4x - x^2\right)dx = \frac{32}{3}.$$

177. (A)

The region consists of

$$\int_0^1 \sqrt{x}\,dx + \int_1^2 (2-x)\,dx = \frac{2}{3} + \frac{1}{2} = \frac{7}{6}.$$

178. (B)

The curves meet at $(0,0)$ and $(8,8)$. Using horizontal strips,

$$A = \int_0^8 \left(\sqrt{8y} - \frac{y^2}{8}\right)dy = \frac{64}{3}.$$

179. (D)

The cooling rate is proportional to the excess temperature:

$$\frac{dT}{dt} = -k(T - 32).$$

180. (A)

Let $v = x + y$. Then

$$\frac{dv}{dx} = \frac{2v}{v-1}, dx = \left(\frac{1}{2} - \frac{1}{2v}\right)dv.$$

Integration gives

$$y = x + \ln(x+y) + C.$$

181. (A)

Let $u = x^2 + y^2$. Then

$$\frac{du}{dx} = 2\left(x + y\frac{dy}{dx}\right) = 2\sec u.$$

Thus

$$\cos u\,du = 2dx \Rightarrow \sin\left(x^2+y^2\right) = 2x + C.$$

182. (C)

For exponential decay,

$$\frac{x(40)}{x_0} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}.$$

183. (C)

Since $dp/dx = d^2y/dx^2$, the order is 2. After isolating the radical and squaring, the highest derivative occurs to power 2, so the degree is 2. Hence

$$m + n = 4.$$

184. (A)

For

$$\frac{x^2}{4b^2} + \frac{y^2}{b^2} = 1,$$

differentiation gives

$$\frac{x}{2b^2} + \frac{2y}{b^2}\frac{dy}{dx} = 0.$$

Thus

$$x + 4y\frac{dy}{dx} = 0.$$

185. (C)

Equation I has right side $1+y/x$, so it is homogeneous. Equation II contains terms of unequal total degree and is not homogeneous. Equation III is a ratio of degree-2 homogeneous expressions and is homogeneous. Hence only statement S_3 is valid.

186. (C)

Since

$$4\pi r^2 \frac{dr}{dt} \propto -4\pi r^2,$$

dr/dt is constant. From the data,

$dr/dt = -1$, so

$r = 3 - t.$

187. (A)

Differentiating twice,

$$\frac{d^2y}{dt^2} = -\omega^2 y.$$

Therefore

$$\frac{d^2y}{dt^2} + \omega^2 y = 0.$$

188. (C)

$$E(X) = \sum xp(x) = 500(0.25) + 1000(0.15)$$
$$+1500(0.10) + 2000(0.08)$$
$$+2500(0.05) + 3000(0.02) = 770.$$

Thus the expected cost is 770,

189. (B)

For a Bernoulli variable,

$$\mathrm{Var}(X) = p(1-p) = 0.6(0.4) = 0.24.$$

190. (D)

A doublet has probability $1/6$, so

$$X \sim B\left(4, \frac{1}{6}\right).$$

Thus

$$P(X = k) = \binom{4}{k}\left(\frac{1}{6}\right)^k \left(\frac{5}{6}\right)^{4-k},$$

which gives the table in option (D).

191. (D)

$$E(X) = \frac{3(1) + 2(2) + 1(5)}{6} = 2.$$

192. (B)

For $X \sim B(6, 1/4)$,

$$P(X \geq 5) = \binom{6}{5}\left(\frac{1}{4}\right)^5 \frac{3}{4} + \left(\frac{1}{4}\right)^6 = \frac{19}{4096}.$$

193. (C)

$$(1-p)^4 = \frac{16}{81} = \left(\frac{2}{3}\right)^4,$$

so $p = 1/3$. Therefore

$$P(X = 4) = p^4 = \frac{1}{81}.$$

194. (D)

$$np + np(1-p) = 5p(2-p) = 1.8.$$

Thus

$$5p^2 - 10p + 1.8 = 0,$$

whose admissible root is

$p = 0.2.$

MHTCET Yearwise PYQ (2022)

Marathi

Mathematics

1. The value of $(\cos\alpha+\cos\beta)^2+(\sin\alpha+\sin\beta)^2$ is: **[2022]**

(A) $2\sin^2\left(\frac{\alpha-\beta}{2}\right)$ (B) $4\sin^2\left(\frac{\alpha-\beta}{2}\right)$

(C) $4\cos^2\left(\frac{\alpha-\beta}{2}\right)$ (D) $2\cos^2\left(\frac{\alpha-\beta}{2}\right)$

2. The value of $\cos^2 10°-\cos 10°.\cos 50°+\cos^2 50°$ is: **[2022]**

(A) $\frac{3}{2}$ (B) $\frac{3}{4}(1+\cos 20°)$

(C) $\frac{3}{4}$ (D) $\frac{3}{2}+\cos 20°$

3. $\frac{\sin^2(-160°)}{\sin^2 70°}+\frac{\sin(180°-\theta)}{\sin\theta}=$ **[2022]**

(A) $\tan^2(20°)$

(B) $\cot^2(20°)$

(C) $\sec^2(20°)$

(D) $\operatorname{cosec}^2(20°)$

4. If p = tan20°, then value of $\frac{\tan 160°-\tan 110°}{1+\tan 160°.\tan 110°}$ in terms of p is: **[2022]**

(A) $\frac{1+p^2}{2p^2}$ (B) $\frac{1+p^2}{2p}$

(C) $\frac{1-p^2}{2p}$ (D) $\frac{1-p^2}{2p^2}$

5. If $\cot(A+B)=0$, then $\sin(A+2B)$ is equal to: **[2022]**

(A) $\sin 2A$ (B) $\sin A$

(C) $\cos A$ (D) $\cos 2A$

6. If the lines $4x+3y-1=0$, $x-y+5=0$ and $kx+5y-3=0$ are concurrent, then k = **[2022]**

(A) 5 (B) 7

(C) 4 (D) 6

7. The equation of the line perpendicular to $2x-3y+5=0$ and making an intercept 3 with positive Y-axis is: **[2022]**

(A) $3x+2y-6=0$ (B) $3x+2y-12=0$

(C) $3x+2y-7=0$ (D) $3x+2y+6=0$

8. $N(3,-4)$ is the foot of the perpendicular drawn from the origin to a line L. Then the equation of the line L is: **[2022]**

(A) $4x+3y=0$ (B) $3x-4y-25=0$

(C) $x-y-7=0$ (D) $4x-3y-24=0$

9. The equation of a line, whose perpendicular distance from the origin is 5 units and the angle, which the perpendicular to the line from the origin makes is 210° with positive X-axis is: **[2022]**

(A) $x\sqrt{3}+y-10=0$

(B) $-x\sqrt{3}+y+10=0$

(C) $x\sqrt{3}-y+10=0$

(D) $x\sqrt{3}+y+10=0$

10. If the line through the points $(-2,6)$ and $(4,8)$ is perpendicular to the line passing through the points $(8,12)$ and $(x,24)$, then the value of x is: **[2022]**

(A) 12 (B) 4

(C) $\frac{1}{3}$ (D) –2

11. The set of all possible values of θ in the interval $(0,\pi)$ for which the points $(1,2)$ and $(\sin\theta,\cos\theta)$ lie on the same side of the line $x+y=1$, is _____. **[2022]**

(A) $\left(0,\frac{3\pi}{4}\right)$ (B) $\left(0,\frac{\pi}{4}\right)$

(C) $\left(0,\frac{\pi}{2}\right)$ (D) $\left(\frac{\pi}{4},\frac{3\pi}{4}\right)$

12. Let a, b, c and d be non-zero real numbers. If the point of intersection of the lines $4ax+2ay+c=0$ and $5bx+2by+d=0$ lies in the 4th quadrant and is equidistant from the two axes, then

(A) $2bc-3ad=0$ (B) $2ad-3bc=0$

(C) $2bc+3ad=0$ (D) $3bc+2ad=0$

13. The slopes of the lines, making angle of measures 45° with the line $2x-3y=5$ are: **[2022]**

(A) $\frac{-1}{5},-5$ (B) $5,\frac{1}{5}$

(C) $5,\frac{-1}{5}$ (D) $\frac{1}{5},-5$

14. The co-ordinates of the foot of the perpendicular from the point $(1,2)$ on the line $x-3y+7=0$ are: **[2022]**

(A) $\left(\frac{-13}{5},\frac{-2}{5}\right)$

(B) $\left(\frac{4}{5},\frac{13}{5}\right)$

(C) $(2,3)$

(D) $(-13,-2)$

15. The co-ordinates of a point on the line $x+y+3=0$, whose distance from the line $x+2y+2=0$ is $\sqrt{5}$ units, are **[2022]**

(A) $(-1,-4)$ (B) $(1,-4)$

(C) $(-1,4)$ (D) $(1,4)$

16. The equation of tangents to the circle $x^2+y^2=4$ which are parallel to $x+2y+3=0$ are **[2022]**

(A) $x+2y=\pm2\sqrt{3}$ (B) $x+2y=\pm2\sqrt{5}$

(C) $x-2y=\pm2\sqrt{5}$ (D) $x-2y=\pm2$

17. Given two circles $x^2+y^2+8x-6y-24=0$ and $x^2+y^2-4x+10y+20=0$. They are: **[2022]**

(A) disjoint

(B) concentric

(C) touching internally

(D) touching externally

18. The orthocenter and centroid of triangle are $A(-3,5)$ and $B(3,3)$ respectively. If C is the circumcentre of this triangle, then the radius of the circle having line segment AC as a diameter, is ____ units. **[2022]**

(A) $2\sqrt{10}$ (B) $3\sqrt{10}$

(C) $\frac{3\sqrt{10}}{2}$ (D) $\frac{2}{\sqrt{10}}$

19. The equation of circle passing through the points $(1,-2)$ and $(4,-3)$ and whose centre lies on the line $3x+2y=7$ is: **[2022]**

(A) $x^2+y^2+6x-2y-5=0$

(B) $x^2+y^2-6x+2y+5=0$

(C) $x^2+y^2+6x+2y-5=0$

(D) $x^2+y^2-6x-2y+5=0$

20. If the lines $3x-4y-7=0$ and $2x-3y-5=0$ pass through diameters of a circle of area 49π square units, then the equation of the circle is: **[2022]**

(A) $x^2+y^2+2x-2y-51=0$

(B) $x^2+y^2-2x+2y+51=0$

(C) $x^2+y^2+2x+2y+47=0$

(D) $x^2+y^2-2x+2y-47=0$

21. The parametric equations of the circle $x^2 + y^2 - ax - ay = 0$ are **[2024, 2022]**

(A) $x = \frac{a}{2} + \frac{\sqrt{a^2+b^2}}{2}\cos\theta,$
$y = \frac{b}{2} + \frac{\sqrt{a^2+b^2}}{2}\sin\theta$

(B) $x = \frac{-a}{2} + \frac{\sqrt{a^2+b^2}}{4}\sin\theta,$
$y = \frac{-b}{2} + \frac{\sqrt{a^2+b^2}}{4}\cos\theta$

(C) $x = \frac{a}{2} + \sqrt{\frac{a^2+b^2}{2}}\sin\theta,$
$y = \frac{b}{2} + \sqrt{\frac{a^2+b^2}{2}}\cos\theta$

(D) $x = \frac{a}{2} + \frac{\sqrt{a^2+b^2}}{4}\cos\theta,$
$y = \frac{b}{2} + \frac{\sqrt{a^2+b^2}}{4}\sin\theta$

22. A family with three children is chosen at random. The probability that the oldest and youngest children are of the same gender is: **[2022]**

(A) $\frac{2}{8}$ (B) $\frac{1}{2}$

(C) $\frac{1}{8}$ (D) $\frac{3}{8}$

23. Five letters are placed at random in five addressed envelopes. The probability that all the letters are not dispatched in the respective right envelopes is: **[2022]**

(A) $\frac{119}{120}$

(B) $\frac{1}{120}$

(C) $\frac{4}{5}$

(D) $\frac{1}{5}$

24. There are 2 shelves. One shelf has 5 Physics and 3 Biology books and other has 4 physics and 2 Biology books. Then the probability of drawing a Physics book is: **[2022]**

(A) $\frac{9}{38}$ (B) $\frac{9}{14}$

(C) $\frac{31}{48}$ (D) $\frac{1}{2}$

25. If three distinct numbers are chosen randomly form first 100 natural numbers, then the probability that all three of them are divisible by both 2 and 3 is: **[2022]**

(A) $\frac{4}{35}$ (B) $\frac{4}{1155}$

(C) $\frac{4}{55}$ (D) $\frac{80}{231}$

26. A bag contains 5 red balls and 3 green balls. A ball is selected at random and not replaced. A second ball is then selected. The probability of selecting one red ball and one green ball is: **[2022]**

(A) $\frac{15}{64}$ (B) $\frac{15}{112}$

(C) $\frac{15}{56}$ (D) $\frac{15}{28}$

27. A, B, C are three events, one of which must and only one can happen. The odds in favour A are 4 : 6, odds against B are 7 : 3, then odds against C are: **[2022]**

(A) 6 : 4 (B) 4 : 6

(C) 3 : 7 (D) 7 : 3

28. Three critics review a book. For the three critics, the odd in favour of the books are (5 : 2), (4 : 3) and (3 : 4) respectively. The probability that the majority is in favour of the book is: **[2022]**

(A) $\frac{209}{343}$ (B) $\frac{149}{343}$

(C) $\frac{185}{343}$ (D) $\frac{129}{343}$

29. Let ω be a complex cube root of unity with $\omega \neq 1$. A fair die is thrown three times. If r_1, r_2 and r_3 are the numbers obtained on the die, then the probability that $\omega^{r1} + \omega^{r2} + \omega^{r3} = 0$ is: **[2022]**

(A) $\frac{1}{8}$ (B) $\frac{2}{9}$

(C) $\frac{1}{9}$ (D) $\frac{1}{36}$

30. If $(x - iy)(3 + 5i)$ is the conjugate of $-6 - 24i$ (where $x, y, \in R$ and $i = \sqrt{-1}$), then the values of x and y are respectively. **[2022]**

(A) 5, 3 (B) 3, –3

(C) –3, 3 (D) 5, –3

31. If α and β are the complex cube roots of unity, then $\alpha^3 + \beta^3 + \alpha^{-2}\beta^{-2}$ is equal to **[2022]**

(A) 0 (B) 3

(C) –3 (D) 1

32. If $(x + iy)^{\frac{1}{3}} = a + ib$ where $x, y, a, b \in R$ and $i = \sqrt{-1}$, then $\frac{x}{a} - \frac{y}{b} =$ **[2022]**

(A) $a^2 + b^2$ (B) $2(a^2 - b^2)$

(C) $-2(a^2 + b^2)$ (D) $a^2 - b^2$

33. If $x = -2 - \sqrt{3}i$, where $i = \sqrt{-1}$, then the value of $2x^4 + 5x^3 + 7x^2 - x + 41$ is: **[2022]**

(A) 6 (B) 76

(C) –76 (D) –6

34. Let z be the complex number such that $|z| + z = 3 + i$ where $i = \sqrt{-1}$, then $|z| =$

(A) $\frac{\sqrt{34}}{3}$ (B) $\frac{5}{3}$

(C) $\frac{\sqrt{41}}{4}$ (D) $\frac{5}{4}$

35. A bag contains 5 red marbles, 4 black marbles and 3 white marbles. Then the number of ways in which 4 marbles can be drawn so that at most 2 of them are red is: **[2022]**

(A) 420 (B) 210

(C) 385 (D) 406

36. In a certain examination, a candidate has to pass in each of the 5 subjects. Hence, the number of ways he can fail is: **[2022]**

(A) 2^5 (B) $2^5 - 1$

(C) 5 (D) 5!

37. It is required to seat 5 men and 4 women in a row so that the men occupy odd places. Then the number of arrangements that are possible is: **[2022]**

(A) 2880 (B) 1440

(C) 144 (D) 362880

38. The number of ways in which the letters of the world MACHINE can be arranged such that the vowels may occupy only odd position, is: **[2022]**

(A) 576 (B) 625

(C) 288 (D) 1152

39. A man P has 7 friends, 4 of them are ladies and 3 are men. His wife Q also has 7 friends, 3 of them are ladies and 4 are men. Assume P and Q have no common friends. Then the total number of ways in which P and Q together can throw a party inviting 3 ladies and 3 men so that 3 friends of each of P and Q are in this party, is: **[2022]**

(A) 468 (B) 469

(C) 484 (D) 485

40. Number of ways, in which 6 men and 5 women can sit at a round table, if no two women sit together, are

(A) 7! × 5!

(B) 6! × 5!

(C) 30

(D) 5! × 4!

41. The total number of ways, of dividing 52 cards amongst 4 players, so that 3 players get 17 cards each and fourth player has just one card, are **[2022]**

(A) $\frac{52!}{17!}$

(B) $\frac{52!}{(17!)^3}\times 4$

(C) $\frac{52!}{17}$

(D) $\frac{52!}{(17!)^2}$

42. The number of different 9 digit numbers that can be formed, from the digits of the number 445577888 by rearranging its digits, so that the odd digits occupy even positions, are: **[2022]**

(A) 60 (B) 120

(C) 180 (D) 36

43. There are 6 periods in each working day of a school. The number of ways one can arrange 5 subjects such that each is allowed at least one period, is: **[2022]**

(A) 5 (B) 725

(C) 720 (D) 1800

44. If a question paper consists of 11 questions divided into two sections I and II. Section I consists of 6 questions and section II consists of 5 questions, then the number of different ways can student select 6 questions, taking at least 2 questions from each section, is: **[2022]**

(A) 275 (B) 350

(C) 425 (D) 225

45. If $f(x)=e^{|x|}, g(x)=\log x$, then $g\left[f(x)\right]$ = **[2022]**

(A) $-x^2$ (B) $|x|$

(C) $2x$ (D) 1

46. If $f(x)=\frac{a^x-a^{-x}}{a^x+a^{-x}}$, where a, x satisfy the necessary conditions, then $f^{-1}(x)=$ **[2022]**

(A) $\frac{1}{2}\log_a\left(\frac{1+x}{x}\right)$ (B) $\frac{1}{2}\log_a\left(\frac{x}{1-x}\right)$

(C) $\frac{1}{2}\log_a\left(\frac{1+x}{1-x}\right)$ (D) $\frac{1}{2}\log_a\left(\frac{2+x}{2-x}\right)$

47. The domain and range for the function $f(x)=e^{|x|\sin x}$ are: **[2022]**

(A) domain = R, range = R

(B) domain = R, range = [1, ∞)

(C) domain = R, range = (0, ∞)

(D) domain = R, range = [0, ∞)

48. If R denotes the set of all real numbers then the function $f:\mathrm{R}\to\mathrm{R}$ defined by $f(x)=|x|$ is: **[2022]**

(A) injective and surjective.

(B) injective.

(C) surjective

(D) neither injective nor surjective

49. The domain of the definition of the function $f(x)=\frac{1}{4-x^2}+\log_{10}\left(x^3-x\right)$ is: **[2022]**

(A) $(-1,0)\cup(1,2)\cup(3,\infty)$

(B) $(-2,-1)\cup(-1,0)\cup(2,\infty)$

(C) $(-1,0)\cup(1,2)\cup(2,\infty)$

(D) $(1,2)\cup(2,\infty)$

50. Let $\mathrm{A}=\{x\in\mathrm{R}/\,x \text{ is not a positive integer}\}$. Let a function f be defined as $f:\mathrm{A}\to\mathrm{R}$ so that $f(x)=\frac{2x}{x-1}$, then f is: **[2022]**

(A) surjective but not injective

(B) not injective

(C) injective but not surjective

(D) neither injective nor surjective

51. If the function $f:R-\{-1,1\}\to A$ defined by $f(x)=\frac{x^2}{1-x^2}$ is surjective, then A is equal to: **[2022]**

(A) $R-[-1,0)$ (B) $R-(-1,0)$

(C) $R-\{-1\}$ (D) $[0,\infty)$

52. For a suitable chosen real constant a, let a function $f:R-\{-a\}\to R$ be defined by $f(x)=\frac{a-x}{a+x}$. Further, suppose that for any real number $x\neq -a$ and $f(x)\neq -a$ (fof) $(x)=x$. Then $f\left(\frac{-1}{2}\right)$ is equal to: **[2022]**

(A) $\frac{1}{3}$ (B) 3

(C) –3 (D) $\frac{-1}{3}$

53. If $[x]$ is greatest integer function and $2[2x-5]-1=7$, then x lies in: **[2022]**

(A) $\left[\frac{9}{2},5\right]$ (B) $\left(\frac{9}{2},5\right)$

(C) $\left[\frac{9}{2},5\right)$ (D) $\left(\frac{9}{2},5\right]$

54. $\lim\limits_{x\to 0}\left(\frac{1+\tan x}{1+\sin x}\right)^{\operatorname{cosec} x}=$ **[2022]**

(A) 0 (B) e

(C) 1 (D) $\frac{1}{e}$

55. $\lim\limits_{x\to 1}\frac{2^{2x-2}-2^x+1}{\sin^2(x-1)}=$ **[2022]**

(A) $(\log 2)^2$ (B) $\frac{1}{2}(\log 2)^2$

(C) $2\log 2$ (D) $2(\log 2)^2$

56. $\lim\limits_{x\to\infty}\left[\frac{8x^2+5x+3}{2x^2-7x-5}\right]^{\frac{4x+3}{8x-1}}=$ **[2022]**

(A) 4 (B) $\sqrt{2}$

(C) 2 (D) $\frac{1}{2}$

57. $\lim\limits_{x\to 2}\left(\frac{5x-8}{8-3x}\right)^{\frac{3}{2x-4}}=$ **[2022]**

(A) e^6 (B) e^2

(C) $e^{\frac{5}{2}}$ (D) $e^{\frac{3}{2}}$

58. The quadratic equation whose roots are m and n, where $m=\lim\limits_{x\to 0}\frac{x\log(1+2x)}{x\tan x}$ and $n=\lim\limits_{x\to 0}\frac{\log x+\log\left(\frac{1+x}{x}\right)}{x}$, is **[2022]**

(A) $x^2+x+2=0$

(B) $x^2-x+2=0$

(C) $x^2-3x+2=0$

(D) $x^2+3x+2=0$

59. $\lim\limits_{n\to\infty}n\left(\sqrt{n^2+9}-n\right)=$ **[2022]**

(A) $\frac{9}{2}$ (B) $\frac{9}{4}$

(C) 9 (D) $\frac{9}{\sqrt{2}}$

60. $\lim\limits_{x\to 0}\frac{\sin\left(\pi\cos^2 x\right)}{x^2}$ is equal to: **[2024, 2022]**

(A) 1 (B) $-\pi$

(C) π (D) $\frac{\pi}{2}$

61. If $f(x)=\dfrac{e^{x^2}-\cos x}{x^2}$, for $x\neq 0$ is continuous at $x=0$, then value of $f(0)$ is: **[2022, 2018]**

(A) $\dfrac{2}{3}$ (B) $\dfrac{5}{2}$

(C) 1 (D) $\dfrac{3}{2}$

62. If $f(x)=\dfrac{x}{2}-1$, then on the interval $[0,\pi]$, where [·] represents greatest integer function, **[2022]**

(A) $\tan[f(x)]$ and $\dfrac{1}{f(x)}$ are both continuous.

(B) $\tan[f(x)]$ is discontinuous and $\dfrac{1}{f(x)}$ is continuous.

(C) $\tan[f(x)]$ and $\dfrac{1}{f(x)}$ are both discontinuous.

(D) $\tan[f(x)]$ is continuous but $\dfrac{1}{f(x)}$ is not continuous.

63. If the function $f(x)$ is continuous on its domain $[-2,2]$ where $f(x)=\dfrac{\sin ax}{x}+3,\quad -2\le x<0$

$= x+5\ 0\le x\le 1 = \sqrt{x^2+8}-b,\ 1<x\le 2$ then $7a+b+1$ is equal to: **[2022]**

(A) 11 (B) 10

(C) 12 (D) 14

64. Let f : R → R be a function defined as

$$f(x)=\begin{cases}5, & \text{if } x\le 1\\ a+bx, & \text{if } 1<x<3\\ b+5x, & \text{if } 3\le x<5\\ 30, & \text{if } x\ge 5\end{cases}$$, then f is: **[2022]**

(A) not continuous for any values of a and b.

(B) continuous if a = 0 and b = 5

(C) continuous if a = –5 and b = 10

(D) continuous if a = 5 and b = 5.

65. If $(p\wedge\sim r)\to(\sim p\vee q)$ has truth value 'F', then truth values of p, q and r are respectively. **[2022]**

(A) T, F, F

(B) F, F, F

(C) F, F, T

(D) T, T, T

66. Consider the statement "P(n) : n^2-n+37 is prime." Then, which one of the following is **true**? **[2022]**

(A) Both P(C) and P(5) are false.

(B) Both P(C) and P(5) are true.

(C) P(C) false, but P(5) is true.

(D) P(5) is false, but P(C) is true.

67. Negation of a statement 'If $\forall$ x, x is a complex number, then $x^2 < 0$ is: **[2022]**

(A) $\exists x$, x is not a complex number and $x^2\ge 0$.

(B) $\forall x$, x is a complex number and $x^2<0$.

(C) $\exists x$, x is not a complex number and $x^2<0$.

(D) $\forall x$, x is a complex number and $x^2\ge 0$.

68. The statement pattern $[p\to(q\to p)]\to[p\to(p\vee q)]$ is **[2022]**

(A) a contingency

(B) a tautology

(C) a contradiction

(D) equivalent to $p\leftrightarrow q$

69. Which of the following is **correct** statement?

(I) $S_1:(p\wedge q)\equiv\sim(p\to\sim q)$

(II) $S_2:(p\wedge q)\wedge(\sim p\vee\sim q)$ is tautology.

(III) $S_3:[p\wedge(p\to\sim q)\to q]$ is contradiction.

(IV) $S_4:p\to(q\to p)$ is contingency.

(A) statement S_1 is correct.

(B) statement S_4 is correct

(C) statement S_3 is correct

(D) statement S_1 and S_2 are correct.

70. If p : 25 is an odd prime number,
q : 14 is a composite number and
r : 64 is a perfect square number.
Then which of the following statement pattern is **true**?
(A) $\sim(q \wedge r) \vee p$ (B) $\sim p \vee (q \wedge r)$
(C) $(p \wedge q) \wedge r$ (D) $(p \vee q) \wedge (\sim r)$

71. If Statement I: if a quadrilateral ABCD is a square, then all of sides are equal.
Statement II: All the sides of a quadrilateral ABCD are equal, then ABCD is a square, then **[2022]**
(A) Statement II is a negation of statement I.
(B) Statement II is an inverse of statement I
(C) Statement II is a converse of statement I
(D) Statement II is a contrapositive of statement I.

72. For three simple statement p, q, and r, $p \rightarrow (q \vee r)$ is logically equivalent to: **[2022]**
(A) $(p \vee q) \rightarrow r$
(B) $(p \rightarrow \sim q) \wedge (p \rightarrow r)$
(C) $(p \rightarrow q) \vee (p \rightarrow r)$
(D) $(p \rightarrow q) \wedge (p \rightarrow \sim r)$

73. The negation of the statement.
"The payment will be made if and only if the work is finished in time." Is: **]2024, 2023, 2022[**
(A) The work is finished in time and the payment is not made.
(B) Either the work is finished in time and the payment is not made or the payment is made and the work is not finished in time
(C) The payment is made and the work is not finished in time.
(D) The work is finished in time and the payment is not made or the payment is made and the work is finished in time.

74. If $A = \begin{bmatrix} 0 & 1+2i & i-2 \\ -1-2i & 0 & K \\ 2-i & 7 & 0 \end{bmatrix}$ and A^{-1} does not exists, then K = , (where $i = \sqrt{-1}$) **[2022]**
(A) $1+2i$ (B) $1-2i$
(C) 7 (D) –7

75. If A is non-singular matrix of order 3 such that $(A-2I)\ (A-4I) = 0$, then $\frac{1}{6}A + \frac{4}{3}A^{-1}$ is (where I is a unit matrix of order 3 and 0 is a null matrix of order 3) **[2022]**
(A) 2I (B) 0
(C) 6I (D) I

76. If $(BA)^{-1} = C$ where $B = \begin{bmatrix} 2 & 6 & 4 \\ 1 & 0 & 1 \\ -1 & 1 & -1 \end{bmatrix}$ and $C = \begin{bmatrix} -1 & 0 & 1 \\ 1 & 1 & 3 \\ 2 & 0 & 2 \end{bmatrix}$, then A^{-1} is given by: **[2022]**
(A) $\begin{bmatrix} -3 & -5 & -5 \\ 0 & 9 & 2 \\ 2 & 14 & 6 \end{bmatrix}$
(B) $\begin{bmatrix} -3 & -5 & 5 \\ 0 & 9 & 14 \\ 2 & 2 & 6 \end{bmatrix}$
(C) $\begin{bmatrix} -3 & 5 & 5 \\ 0 & 9 & 2 \\ 2 & 14 & 6 \end{bmatrix}$
(D) $\begin{bmatrix} -3 & -3 & 5 \\ 0 & 9 & 2 \\ 2 & 14 & 6 \end{bmatrix}$

77. If matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$ and $A^{-1} = \alpha I + \beta A$ where I is a unit matrix of order 2 and α, β are constants, then the values of $\alpha + \beta + \alpha\beta$ is: **[2022]**
(A) 11 (B) –7
(C) 7 (D) –11

78. If $A = \begin{bmatrix} 1 & a & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 13 & 2 & -7 \\ -3 & b & 2 \\ -2 & 0 & 1 \end{bmatrix}$, then the values of *a* and *b* are respectively. **[2022]**
(A) 1, 2 (B) 2, –1
(C) 2, 1 (D) –1, 2

79. Given $A = \begin{bmatrix} x & 3 & 2 \\ 1 & y & 4 \\ 2 & 2 & z \end{bmatrix}$, if $xyz = 60$ and $8x + 4y + 3z = 20$, then A(adj A) is equal to:

[2022]

(A) $\begin{bmatrix} 60 & 0 & 0 \\ 0 & 60 & 0 \\ 0 & 0 & 60 \end{bmatrix}$ (B) $\begin{bmatrix} 108 & 0 & 0 \\ 0 & 108 & 0 \\ 0 & 0 & 108 \end{bmatrix}$

(C) $\begin{bmatrix} 20 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 20 \end{bmatrix}$ (D) $\begin{bmatrix} 68 & 0 & 0 \\ 0 & 68 & 0 \\ 0 & 0 & 68 \end{bmatrix}$

80. If $A = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$, then adj $(3A^2 + 12A)$ is equal to:

[2022]

(A) $\begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$ (B) $\begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$

(C) $\begin{bmatrix} 72 & -84 \\ -63 & 51 \end{bmatrix}$ (D) $\begin{bmatrix} 51 & 84 \\ 63 & 72 \end{bmatrix}$

81. If $A = \begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}$, then $(A^{-1})^3 =$ **[2022]**

(A) $\frac{1}{27}\begin{bmatrix} 1 & -26 \\ 0 & 27 \end{bmatrix}$ (B) $\frac{1}{27}\begin{bmatrix} 1 & -26 \\ 0 & -27 \end{bmatrix}$

(C) $\frac{1}{27}\begin{bmatrix} -1 & 26 \\ 0 & 27 \end{bmatrix}$ (D) $\frac{1}{27}\begin{bmatrix} -1 & 26 \\ 0 & -27 \end{bmatrix}$

82. If $\begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & -2 \\ 1 & 3 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 4 \end{bmatrix}$, then $2x - y + z =$

[2022]

(A) –3 (B) 1

(C) 2 (D) 3

83. If $A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -3 \\ 2 & 1 & 0 \end{bmatrix}$, B = adj A and C 5A, then $\frac{|\text{adj B}|}{|C|} =$ **[2025, 2022]**

(A) 2 (B) 4

(C) 1 (D) 5

84. $2\tan^{-1}\left(\frac{1}{3}\right) + \cos^{-1}\left(\frac{3}{5}\right) =$ **[2022]**

(A) $\frac{\pi}{2}$ (B) $\tan^{-1}\left(\frac{5}{4}\right)$

(C) 0 (D) $\frac{\pi}{4}$

85. The value of $\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{8}\right)$ is:

[2022]

(A) $\frac{2\pi}{3}$ (B) $\frac{4\pi}{3}$

(C) $\frac{\pi}{4}$ (D) $\frac{3\pi}{4}$

86. If the Cartesian co-ordinates of a point are $\left(\frac{-5\sqrt{3}}{2}, \frac{5}{2}\right)$, then its polar co-ordinates are:

[2022]

(A) $\left(5, \frac{13\pi}{18}\right)$ (B) $\left(5, \frac{11\pi}{18}\right)$

(C) $\left(5, \frac{2\pi}{3}\right)$ (D) $\left(5, \frac{5\pi}{6}\right)$

87. Considering only the Principal values of inverse function, the set

$A = \left\{x \geq 0 / \tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}\right\}$ **[2022]**

(A) is a singleton set.

(B) contains more than two elements.

(C) contains two elements.

(D) is an empty set.

88. The value of $\cos\left(2\cos^{-1}x+\sin^{-1}x\right)$ at $x=\frac{1}{5}$ where $0\le\cos^{-1}x\le\pi$ and $-\frac{\pi}{2}\le\sin^{-1}x\le\frac{\pi}{2}$, is:

(A) $\frac{2\sqrt{6}}{5}$ (B) $-\frac{2\sqrt{6}}{5}$

(C) $\frac{\sqrt{6}}{5}$ (D) $-\frac{\sqrt{6}}{5}$

89. If A = $2\tan^{-1}\left(\frac{1+x}{1-x}\right)$ and B = $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, where $x\in(0,1)$, then A –B = **[2022]**

(A) $4\tan^{-1}x$ (B) $\tan^{-1}x$

(C) $\frac{\pi}{4}$ (D) $\frac{\pi}{2}$

90. With usual notations, in ΔABC, $\angle C = 90°$, then the value of sin(A – B) is: **[2022]**

(A) $\frac{a^2-b^2}{b^2}$ (B) $\frac{a^2+b^2}{a^2}$

(C) $\frac{a^2+b^2}{a^2-b^2}$ (D) $\frac{a^2-b^2}{a^2+b^2}$

91. In a triangle ABC, with usual notations, if a : b : c = 7 : 8 : 9, then cos A : cosB : cosC =

(A) 3 : 4 : 5

(B) 5 : 6 : 7

(C) 7 : 8 : 9

(D) 14 : 11 : 6

92. The value of $\cos\left(\tan^{-1}\left(\sin\left(\cot^{-1}x\right)\right)\right)$ is: **[2022]**

(A) $\sqrt{\frac{x^2+1}{x^2+2}}$

(B) $\sqrt{\frac{x^2+1}{x^2-1}}$

(C) $\sqrt{\frac{1-x^2}{2+x^2}}$

(D) $\sqrt{\frac{1-x^2}{1+x^2}}$

93. The value of $\tan\left\{\frac{1}{2}\sin^{-1}\left(\frac{2x}{1+x^2}\right)+\frac{1}{2}\cos^{-1}\left(\frac{1-y^2}{1+y^2}\right)\right\}$ is: **[2022]**

(A) $\frac{x-y}{1+xy}$ (B) $\frac{x+y}{1-xy}$

(C) $\frac{x-y}{1-xy}$ (D) $\frac{x+y}{1+xy}$

94. The principal value of $\cot^{-1}\left(\frac{-1}{\sqrt{3}}\right)$ is:

(A) $\frac{\pi}{6}$ (B) $\frac{2\pi}{3}$

(C) $\frac{-\pi}{3}$ (D) $\frac{\pi}{3}$

95. The value of $\operatorname{cosec}^{-1}\left(\sqrt{2}\right)+\cos^{-1}\left(\frac{-1}{2}\right)-\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$ is equal to: **[2022]**

(A) $\frac{3\pi}{4}$ (B) $\frac{\pi}{6}$

(C) $\frac{\pi}{3}$ (D) $\frac{\pi}{4}$

96. The value of $\sin\left(2\sin^{-1}0.8\right)$ is equal to:

(A) 0.48 (B) 0.96

(C) 0.16 (D) 0.12

97. If $\tan^{-1}\left(\frac{1-x}{1+x}\right)=\frac{1}{2}\tan^{-1}x$, then x is: **[2023, 2022]**

(A) 1 (B) $\sqrt{3}$

(C) $\frac{1}{\sqrt{3}}$ (D) $\frac{1}{2\sqrt{3}}$

98. If $\sin\left(\cot^{-1}(x+1)\right)=\cos\left(\tan^{-1}x\right)$ then considering positive square roots, x has the value____. **[2024, 2022]**

(A) 0 (B) $\frac{9}{4}$

(C) $\frac{1}{2}$ (D) $-\frac{1}{2}$

99. The combined equation of the lines passing through the origin and making an acute angle α with the line $y = x$ is: **[2022]**

(A) $x^2 + 2xy\sec 2\alpha + y^2 = 0$

(B) $x^2 - 2xy\tan 2\alpha + y^2 = 0$

(C) $x^2 + 2xy\tan 2\alpha + y^2 = 0$

(D) $x^2 - 2xy\sec 2\alpha + y^2 = 0$

100. If the slope of one of the two lines $\frac{x^2}{a} + \frac{2xy}{h} + \frac{y^2}{b} = 0$ is twice that of the other, then $ab : h^2 =$ **[2022]**

(A) 1 : 2 (B) 2 : 1

(C) 8 : 9 (D) 9 : 8

101. The joint equation of two lines passing through the origin and perpendicular to the lines given by $2x^2 + 5xy + 3y^2 = 0$ is: **[2022]**

(A) $3x^2 - 5xy - 2y^2 = 0$

(B) $3x^2 + 5xy + 2y^2 = 0$

(C) $3x^2 - 5xy + 2y^2 = 0$

(D) $2x^2 - 5xy + 3y^2 = 0$

102. If the lines $x^2 - 4xy + y^2 = 0$ and $x + y = 10$ contain the sides of an equilateral triangle, then the area of equilateral triangle is: **[2022]**

(A) $\frac{5\sqrt{2}}{\sqrt{3}}$ sq. units (B) $\frac{25}{\sqrt{3}}$ sq. units

(C) $\frac{50}{\sqrt{3}}$ sq. units (D) $\frac{25\sqrt{2}}{\sqrt{3}}$ sq. units

103. If the slopes of the lines $kx^2 - 4xy + 5y^2 = 0$ differ by 2, then k = **[2022]**

(A) $\frac{4}{5}$ (B) $\frac{-21}{5}$

(C) $\frac{21}{5}$ (D) $\frac{5}{21}$

104. If θ is an actue angle between the lines $kx^2 - 4xy + y^2 = 0$ and $\tan\theta = \frac{1}{2}$, then value of k is: **[2022]**

(A) 3 (B) –3

(C) 21 (D) 4

105. The joined equation of lines passing through the origin and trisecting the first quadrant is: **[2022, 2019, 2016]**

(A) $x^2 + \sqrt{3}xy - y^2 = 0$

(B) $x^2 - \sqrt{3}xy - y^2 = 0$

(C) $\sqrt{3}x^2 - 4xy + \sqrt{3}y^2 = 0$

(D) $3x^2 - y^2 = 0$

106. The centroid of the triangle formed by the lines $x + 3y = 10$ and $6x^2 + xy - y^2 = 0$ is: **[2023, 2022]**

(A) $\left(\frac{1}{3}, \frac{-7}{3}\right)$

(B) $\left(\frac{-1}{3}, \frac{-7}{3}\right)$

(C) $\left(\frac{-1}{3}, \frac{7}{3}\right)$

(D) $\left(\frac{1}{3}, \frac{7}{3}\right)$

107. A vector with magnitude of 3 units, which is perpendicular to each other of the vectors $\vec{a} = 3\hat{i} + \hat{j} - 4\hat{k}$ and $\vec{b} = 6\hat{i} + 5\hat{j} - 2\hat{k}$, is given by: **[2022]**

(A) $\pm\left(2\hat{i} - 2\hat{j} - \hat{k}\right)$ (B) $\pm\left(2\hat{i} - 2\hat{j} + \hat{k}\right)$

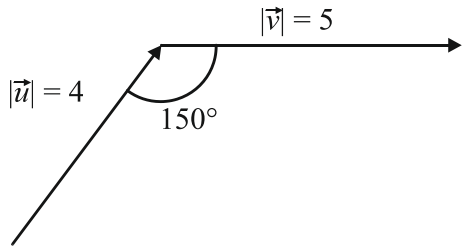
(C) $\pm\left(2\hat{i} + 2\hat{j} + \hat{k}\right)$ (D) $\pm\left(2\hat{i} + 2\hat{j} - \hat{k}\right)$

108. If $|\vec{a}| = \sqrt{26}, |\vec{b}| = 7$ and $|\vec{a} \times \vec{b}| = 35$, then $\vec{a} \cdot \vec{b} =$

(A) $\pm\sqrt{26}$ (B) $\pm 5\sqrt{26}$

(C) $\pm 7\sqrt{26}$ (D) ± 7

109. If $\vec{u},\vec{v}$ are two vectors represented in the following figure, then the value of $|\vec{u}\times\vec{v}|$ = **[2022]**



(A) 10 (B) $10\sqrt{3}$
(C) $5\sqrt{3}$ (D) 20

110. Let *a*, *b*, *c* be distinct non-negative numbers. If the vectors $a\hat{i}+a\hat{j}+c\hat{k}$, $\hat{i}+\hat{k}$ and $c\hat{i}+c\hat{j}+b\hat{k}$ lie in a plane, then *c* is: **[2022, 2019]**
(A) not arithmetic mean of a and b.
(B) the geometric mean of a and b.
(C) the arithmetic mean of a and b.
(D) the harmonic mean of a and b.

111. The vector projection of $\vec{b}$ on $\vec{a}$, where $\vec{a}=3\hat{i}+2\hat{j}+5\hat{k}$ and $\vec{b}=7\hat{i}-5\hat{j}-\hat{k}$ is: **[2022]**

(A) $\frac{3\left(3\hat{i}+2\hat{j}+5\hat{k}\right)}{\sqrt{38}}$ (B) $\frac{9\hat{i}+6\hat{j}+15\hat{k}}{19}$
(C) $\frac{3\left(3\hat{i}+2\hat{j}+5\hat{k}\right)}{38}$ (D) $\frac{6\left(3\hat{i}+2\hat{j}+5\hat{k}\right)}{\sqrt{38}}$

112. If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}|=|\vec{b}|=\sqrt{2}$ with $\vec{a}.\vec{b}=-1$, then the angle between $\vec{a}$ and $\vec{b}$ is: **[2022]**

(A) $\frac{2\pi}{3}$ (B) $\frac{5\pi}{6}$
(C) $\frac{5\pi}{9}$ (D) $\frac{3\pi}{4}$

113. If $\vec{a}=\hat{i}+\hat{j}+\hat{k},\vec{b}=\hat{i}-\hat{j}+2\hat{k},\vec{c}=x\hat{i}+(x-2)\hat{j}-\hat{k}$ and $\vec{c}$ is linear combination of $\vec{a}$ and $\vec{b}$ then *x* has the value. **[2022]**
(A) 0 (B) –2
(C) –4 (D) 1

114. If $\bar{a}+2\bar{b}+3\bar{c}=\bar{0}$ and $(\bar{a}\times\bar{b})+(\bar{b}\times\bar{c})(\bar{c}\times\bar{a})=\lambda(\bar{b}\times\bar{c})$, then λ has the value: **[2022]**
(A) 3 (B) 4
(C) 5 (D) 6

115. The position vectors of vertices of ΔABC are $4\hat{i}-2\hat{j},\hat{i}+4\hat{j}-3\hat{k}$ and $-\hat{i}+5\hat{j}+\hat{k}$ respectively, then m ∠ABC = **[2022]**

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$
(C) $\frac{\pi}{6}$ (D) $\frac{\pi}{3}$

116. Let $\bar{a}=2\hat{i}+\hat{j}-2\hat{k}$, $\bar{b}=\hat{i}+\hat{j}$ and $\bar{c}$ be a vector such that $|\bar{c}-\bar{a}|=3$. if $\bar{p}=\bar{a}\times\bar{b}$, then the angle between $\bar{p}$ and $\bar{c}$ is $\frac{\pi}{6}$ and $|\bar{p}\times\bar{c}|=3$. Thus $\bar{a}.\bar{c}$ is equal to: **[2022]**
(A) 1 (B) 4
(C) $\frac{1}{8}$ (D) 2

117. The position vector of a point that divides the line segment joining P ≡ (1, 2, –1) and Q ≡ (–1, 1, 1) externally in the ratio 1 : 2 is: **[2022]**
(A) $3\hat{i}+3\hat{j}-3\hat{k}$ (B) $-3\hat{i}+3\hat{k}$
(C) $3\hat{i}-3\hat{k}$ (D) $3\hat{i}+\hat{j}+3\hat{k}$

118. The sum of the lengths of projections of $p\hat{i}+q\hat{j}+r\hat{k}$ on the co-ordinates axes, where *p* = 4, *q* = –5, *r* = 7, is **[2022]**
(A) 28 units (B) 6 units
(C) 16 units (D) 20 units

119. If $\bar{a}$ and $\bar{b}$ are vectors such that $|\vec{a}|=3,|\vec{b}|=2$ and $\bar{a}.\bar{b}=5$, then $|\bar{a}-\bar{b}|=$ **[2022]**
(A) $\sqrt{23}$ (B) $\sqrt{3}$
(C) 5 (D) 3

120 The sum of the distinct real values of μ, for which the vectors $\mu\hat{i}+\hat{j}+\hat{k}, \hat{i}+\mu\hat{j}+\hat{k}, \hat{i}+\hat{j}+\mu\hat{k}$ are coplanar, is: **[2022]**

(A) 2 (B) –1

(C) 1 (D) 0

121. A unit vector $\hat{a}$ makes angles $\frac{\pi}{3}$ with $\hat{i}, \frac{\pi}{4}$ with $\hat{j}$ and $\theta \in (0,\pi)$ with $\hat{k}$, then a value of θ is: **[2022]**

(A) $\frac{5\pi}{12}$ (B) $\frac{\pi}{4}$

(C) $\frac{5\pi}{6}$ (D) $\frac{2\pi}{3}$

122. Let $\bar{A}, \bar{B}, \bar{C}$ be vectors of length 3 units, 4 units, 5 units respectively. Let $\vec{A}$ be perpendicular to $\bar{B}+\bar{C}, \bar{B}$ be perpendicular to $\bar{C}+\bar{A}$ and $\bar{C}$ be perpendicular to $\bar{A}+\bar{B}$, then the length of vectors $\bar{A}+\bar{B}+\bar{C}$ is: **[2024, 2022]**

(A) $2\sqrt{5}$ (B) $\sqrt{30}$

(C) $\sqrt{45}$ (D) $5\sqrt{2}$

123. Let $\bar{a}=\hat{i}+\hat{j}+\hat{k}, \bar{b}=\hat{i}-\hat{j}+\hat{k}$ and $\bar{c}=\hat{i}-\hat{j}-\hat{k}$ be three vectors. A vector $\vec{v}$ in the plane of $\bar{a}$ and $\bar{b}$, whose projection on $\bar{c}$ is $\frac{1}{\sqrt{3}}$, is given by:

[2024, 2022]

(A) $\hat{i}-3\hat{j}+3\hat{k}$ (B) $-3\hat{i}-3\hat{j}-\hat{k}$

(C) $3\hat{i}-\hat{j}+3\hat{k}$ (D) $\hat{i}+3\hat{j}-3\hat{k}$

124. The vector equation of plane containing the point (1, –1, 2) and perpendicular to planes $2x+3y-2z=5$ and $x+2y-3z=8$ is: **[2022]**

(A) $\vec{r}\cdot\left(-5\hat{i}+4\hat{j}-\hat{k}\right)=-7$

(B) $\vec{r}\cdot\left(-5\hat{i}+4\hat{j}-\hat{k}\right)=7$

(C) $\vec{r}\cdot\left(-5\hat{i}+4\hat{j}+\hat{k}\right)=-7$

(D) $\vec{r}\cdot\left(-5\hat{i}+4\hat{j}+\hat{k}\right)=7$

125. If the position vectors of the points A and B are $3\hat{i}+\hat{j}+2\hat{k}$ and $\hat{i}-2\hat{j}-4\hat{k}$ respectively, then the equation of the plane through B and perpendicular to AB is: **[2022]**

(A) $2x+3y+6z+28=0$

(B) $2x+3y+6z-11=0$

(C) $2x-3y-6z-32=0$

(D) $2x+3y+6z+9=0$

126. If the line passing through the points $(a,1,6)$ and $(3,4,b)$ crosses the yz-plane at the point $\left(0,\frac{17}{2},\frac{-13}{2}\right)$, then: **[2022]**

(A) $a=5, b=1$ (B) $a=-5, b=1$

(C) $a=-5, b=-1$ (D) $a=5, b=-1$

127. The ratio in which the plane $\vec{r}.\left(\hat{i}-2\hat{j}+3\hat{k}\right)$ = 17 divides the line joining the points $-2\hat{i}+4\hat{j}+7\hat{k}$ and $3\hat{i}-5\hat{j}+8\hat{k}$ is: **[2022]**

(A) 5 : 3 (B) 4 : 5

(C) 3 : 10 (D) 10 : 3

128. If the planes $\vec{r}.\left(2\hat{i}-\lambda\hat{j}+\hat{k}\right)=3$ and $\vec{r}.\left(4\hat{i}-\hat{j}+\mu\hat{k}\right)=5$ are parallel, then values of λ and μ are respectively. **[2022]**

(A) $\frac{1}{2},-2$ (B) $\frac{-1}{2},-2$

(C) $\frac{-1}{2},2$ (40 $\frac{1}{2},2$

129. If the foot of the perpendicular drawn from the origin to a plane is M (–1, –2, 2), then the vector equation of the plane is:

[2022]

(A) $\vec{r}\cdot\left(\hat{i}+2\hat{j}-2\hat{k}\right)=9$

(B) $\vec{r}\cdot\left(-\hat{i}-2\hat{j}-2\hat{k}\right)=9$

(C) $\vec{r}\cdot\left(\hat{i}+2\hat{j}+2\hat{k}\right)=9$

(D) $\vec{r}\cdot\left(-\hat{i}-2\hat{j}+2\hat{k}\right)=9$

130. Let S be the set of all real values of λ such that a plane passing through the points $(-\lambda^2, 1, 1)$, $(1, -\lambda^2, 1)$ and $(1, 1, -\lambda^2)$ also passes through the point $(-1, -1, 1)$. Then S is equal to: **[2022]**

(A) $\{-3,+3\}$

(B) $\{-1,1\}$

(C) $\{\sqrt{3}\}$

(D) $\{-\sqrt{3},\sqrt{3}\}$

131. Let $\bar{n}$ be a vector of magnitude $3\sqrt{3}$ such that it makes equal acute angles with the co-ordinate axes. Then the vector equation of a plane passing through $(1, -1, 2)$ and normal to $\bar{n}$ is: **[2022]**

(A) $\bar{r}.\left(3\hat{i}+3\hat{j}+3\hat{k}\right)=1$

(B) $\bar{r}.\left(3\hat{i}+3\hat{j}+3\hat{k}\right)=12$

(C) $\bar{r}.\left(\hat{i}+\hat{j}+\hat{k}\right)=3$

(D) $\bar{r}.\left(3\hat{i}+3\hat{j}+3\hat{k}\right)=6$

132. The maximum value of the objective function $Z=4x+5y$ subject to constraints $2x+3y\le 12$, $2x+y\le 8$ and $x\ge 0, y\ge 0$ is: **[2022]**

(A) 21

(B) 22

(C) 24

(D) 23

133. For the inequalities $x+y\le 3, 2x+5y\ge 10$ $x\ge 0, y\ge 0$, which of the following points lies in the feasible region? **[2022]**

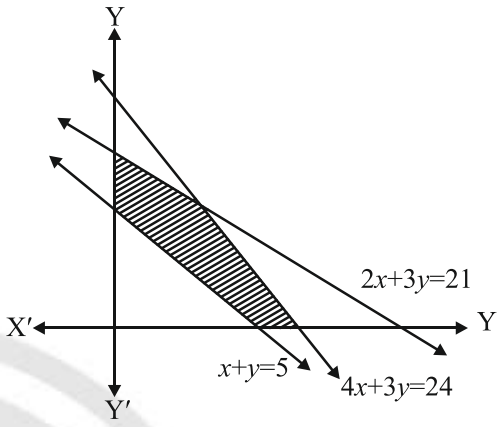
(A) $(2,2)$

(B) $(1,2)$

(C) $(4,2)$

(D) $(2,1)$

134. The function to be maximized is given by $Z=2x+y$. The feasible region for this function Z is the shaded region given below. The maximum value of Z is___ and occurs at the point ____. **[2022]**



(A) $21, (10.5, 0)$ (B) $9, (1.5, 6)$

(C) $10, (5, 0)$ (D) $12, (6, 0)$

135. The region represented by the inequations $2x+3y\le 18, x+y\ge 10, x\ge 0, y\ge 0$ is: **[2024, 2022]**

(A) unbounded

(B) bounded region, but not a singleton set

(C) singleton set

(D) null set

136. If $h(x)=\sqrt{4f(x)+3g(x)}$, $f(1)=4$, $g(1)=3$, $f'(1)=3, g'(1)=4$, then $h'(1)=$ **[2022]**

(A) $\frac{-5}{12}$ (B) $\frac{-12}{7}$

(C) $\frac{5}{12}$ (D) $\frac{12}{5}$

137. If $y=\sin^{-1}\left(\frac{5x+12\sqrt{1-x^2}}{13}\right)$, then $\frac{dy}{dx}$ = **[2022]**

(A) $\frac{x}{\sqrt{1-x^2}}$ (B) $\frac{2}{\sqrt{1-x^2}}$

(C) $\frac{-1}{\sqrt{1-x^2}}$ (D) $\frac{-x}{\sqrt{1-x^2}}$

138. If $y=\sec^{-1}\left(\frac{x+x^{-1}}{x-x^{-1}}\right)$, then $\frac{dy}{dx}=$ **[2022]**

(A) $\frac{-2}{1+x^2}$ (B) $\frac{-1}{1+x^2}$

(C) $\frac{2}{1-x^2}$ (D) $\frac{1}{1+x^2}$

139. If $x^y=e^{x-y}$, then $\frac{dy}{dx}=$ **[2022]**

(A) $\frac{\log x}{(1+\log x)^2}$ (B) $\frac{\log x}{1+\log x}$

(C) $\frac{x\log x}{(1+\log x)^2}$ (D) $\frac{\log x}{x(1+\log x)^2}$

140. If $\log(x+y)=\log xy+3$, then $\frac{dy}{dx}=$ **[2022]**

(A) $-\left(\frac{y}{x}\right)^2$ (B) $\left(\frac{y}{x}\right)^2$

(C) $-\left(\frac{x}{y}\right)^2$ (D) $\left(\frac{x}{y}\right)^2$

141. If $y=\cos^2\left(\frac{5x}{2}\right)-\sin^2\left(\frac{5x}{2}\right)$, then $\frac{d^2y}{dx^2}=$ **[2022]**

(A) $-\frac{25}{2}y$ (B) $\frac{25}{2}y$

(C) $25y$ (D) $-25y$

142. For $x>1$, if $(2x)^{2y}=4e^{2x-2y}$, then $(1+\log_e 2x)^2\frac{dy}{dx}$ is equal to **[2022]**

(A) $x\log_e 2x$

(B) $\log_e 2x$

(C) $\frac{x\log_e 2x+\log_e 2}{x}$

(D) $\frac{x\log_e 2x-\log_e 2}{x}$

143. For $x\in\left(0,\frac{1}{4}\right)$, if the derivative of $\tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right)$ is $\sqrt{x}.g(x)$ then $g(x)$ equals.

(A) $\frac{3x}{1-9x^3}$ (B) $\frac{3}{1+9x^3}$

(C) $\frac{9}{1+9x^3}$ (D) $\frac{3x\sqrt{x}}{1-9x^3}$

144. If $y=3\cos(\log x)+4\sin(\log x)$, then $x^2.y_2+xy_1=$ **[2022]**

(A) xy (B) $-y$

(C) y (D) $-xy$

145. If $y^{\frac{1}{m}}+y^{\frac{-1}{m}}=2x, x\neq 1$, then $\left(x^2-1\right)\left(\frac{dy}{dx}\right)^2$ is equal to: **[2022]**

(A) $\frac{my^2}{2}$ (B) my^2

(C) m^2y (D) m^2y^2

146. The domain of the derivative of the function $f(x)=\begin{cases}\tan^{-1}x, & \text{if } |x|\le 1\\ \frac{1}{2}(|x|-1) & \text{if } |x|>1\end{cases}$ is given by: **[2022]**

(A) $R-\{-1,1\}$ (B) $R-\{-1\}$

(C) $R-\{0\}$ (D) $R-\{1\}$

147. For $x\in R, f(x)=|\log 2-\sin x|$ and $g(x)=f(f(x))$, then **[2022]**

(A) $g'(0)=-\cos(\log 2)$

(B) g is not differentiable at $x=0$

(C) $g'(0)=\cos(\log 2)$

(D) g is differentiable at $x=0$ and $g'(0)=-\sin(\log 2)$

148. Derivative of $x^{\left(x^x\right)}$ is: **[2025, 2022]**

(A) $x^{\left(x^x\right)}\left(x^x+1+\log x\right)$

(B) $x^{\left(x^x\right)}\left(x^x+\log x\right)$

(C) $x^{\left(x^x\right)}\left(x^x+x^{x-1}\log x(1+\log x)\right)$

(D) $x^{\left(x^x\right)}\left(x^{x-1}+x^x\log x(1+\log x)\right)$

149. If $\sqrt{y-\sqrt{y-\sqrt{y-.....\infty}}}=\sqrt{x+\sqrt{x+\sqrt{x+....\infty}}}$ then $\frac{dy}{dx}=$ **[2025, 2022]**

(A) $\frac{y+x+1}{y-x+1}$ (B) $\frac{y-x-1}{y-x+1}$

(C) $\frac{y-x+1}{y-x-1}$ (D) 1

150. Two positive number x and y are such that $(x+y)=60$ and xy^3 is maximum, then the numbers are respectively. **[2022]**

(A) 45, 15 (B) 20, 40

(C) 40, 20 (D) 30, 30

151. If $f(x)=\frac{x}{\log x}$, then $f(x)$ is increasing in: **[2022]**

(A) $(0,\infty)$ (B) $[e,\infty)$

(C) $(-\infty,0)$ (D) (e,∞)

152. The function $f(x)=2x^3-9x^2+12x+29$ is monotonically increasing in the interval. **[2022]**

(A) $(-\infty,1)$ (B) $(-\infty,\infty)$

(C) $(-\infty,1)\cup(2,\infty)$ (D) $(2,\infty)$

153. If $y=4x-5$ is tangent to the curve $y^2=px^3+q$ at $(2,3)$, then **[2022]**

(A) $p=-2,q=-7$ (B) $p=2,q=-7$

(C) $p=2,q=7$ (D) $p=-2,q=7$

154. 20 meters of wire is available to fence of a flowerbed in the form of a circular sector. If the flowerbed is to have maximum surface area, then the radius of the circle is: **[2022]**

(A) 2 m (B) 5 m

(C) 4 m (D) 8 m

155. If the water is being poured at the rate 36 m^3/sec in a cylindrical vessel of base radius 3 m, then the rate at which water level is rising, is: **[2022]**

(A) $\frac{4}{\pi}$ m/sec (B) $\frac{\pi}{4}$ m/sec

(C) 4π m/sec (D) $\frac{3}{\pi}$ m/sec

156. The function, $f(x)=x\sqrt{1-x}$, where $x\in(0,1)$, has local maximum at $x=$ **[2022]**

(A) $\frac{3}{4}$ (B) $\frac{1}{3}$

(C) $\frac{2}{3}$ (D) $\frac{1}{4}$

157. The radius of a cylinder is increasing at the rate 2 cm/sec and its height is decreasing at the rate of 3 cm/sec, then the rate of change of volume, when radius is 3 cm and the height is 5 cm is: **[2022]**

(A) $33\pi\,cm^3$ / sec

(B) $11\pi\,cm^3$ / sec

(C) $23\pi\,cm^3$ / sec

(D) $44\pi\,cm^3$ / sec

158. For every value of $x\in[1,3]$, the function $f(x)=\frac{1}{8^x}$ is: **[2022]**

(A) increasing

(B) neither increasing nor decreasing

(C) decreasing

(D) increasing for $x>2$ and decreasing for $x\le 2$.

159. If the tangent to the curve $y=\frac{x}{x^2-3}$, $x\in R$, $\left(x\neq\pm\sqrt{3}\right)$ at a point $(\alpha,\beta)\neq(0,0)$ on it, is parallel to the line $2x+6y-11=0$, then **[2022]**

(A) $|6\alpha+2\beta|=9$ (B) $|2\alpha+6\beta|=11$

(C) $|6\alpha+2\beta|=19$ (D) $|2\alpha+6\beta|=19$

160. Local maximum and local minimum values respectively of the function $f(x)=(x-1)(x+2)^2$ are: **[2022]**

(A) $-4,0$ (B) $-4,4$

(C) $0,-4$ (D) $4,-4$

161. The tangent to the curve $y=x^3+ax-b$ at the point $(1,-5)$ is perpendicular to the line $y-x+4=0$, then which one of the following points lies on the curve? **[2022]**

(A) $(2,-1)$ (B) $(-2,1)$

(C) $(-2,2)$ (D) $(2,-2)$

162. If the curves $y^2=6x$ and $9x^2+by^2=16$ intersect each other at right angles, then the value of b is: **[2025, 2024, 2023, 2022]**

(A) 4 (B) $\frac{7}{2}$

(C) 6 (D) $\frac{9}{2}$

163. $\int(x-a)\left(x^{n-1}+x^{n-2}a+....+a^{n-1}\right)dx=$, (where C is a constant of integration) **[2022]**

(A) $\frac{x^{n+1}}{n+1}-a^nx+C$

(B) $na^{n-1}+C$

(C) x^n-a^n+C

(D) $\frac{x^{n+1}}{n+1}-a^n+C$

164. $\int(3-x)\sqrt{4-x}\,dx=$____, (where C is a constant of integration) **[2022]**

(A) $\frac{2}{3}(4-x)^{\frac{3}{2}}-\frac{2}{5}(4-x)^{\frac{5}{2}}+C$

(B) $\frac{2}{5}(4-x)^{\frac{5}{2}}-\frac{2}{3}(4-x)^{\frac{3}{2}}+C$

(C) $\frac{2}{3}(4-x)^{\frac{3}{2}}+\frac{2}{5}(4-x)^{\frac{5}{2}}+C$

(D) $-\frac{2}{5}(4-x)^{\frac{5}{2}}-\frac{2}{3}(4-x)^{\frac{3}{2}}+C$

165. $\int\frac{e^x}{\left(2+e^x\right)\left(e^x+1\right)}dx=$

(where C is a constant of integration) **[2022]**

(A) $\log\left(\frac{e^x+2}{e^x+1}\right)+C$ (B) $\log\left(\frac{e^x}{e^x+2}\right)+C$

(C) $\frac{e^x+1}{e^x+2}+C$ (D) $\log\left(\frac{e^x+1}{e^x+2}\right)+C$

166. If $\int\frac{2e^x+3e^{-x}}{3e^x+4e^{-x}}dx=Ax+B\log\left(3e^{2x}+4\right)+C$, then values of A and B are respectively (where C is a constant of integration.) **[2022]**

(A) $\frac{3}{4},\frac{1}{24}$ (B) $\frac{4}{3}.-24$

(C) $\frac{1}{4},\frac{1}{24}$ (D) $\frac{3}{4},\frac{-1}{24}$

167. $\int\frac{\sin 4x}{\sin x}dx=$ (where Cis a constant of integration) **{2022}**

(A) $\frac{2\sin 3x}{3}+2\sin x+C$

(B) $\frac{\sin 3x}{3}+4\sin x+C$

(C) $\frac{2}{3}\sin 3x-2\sin x+C$

(D) $\frac{1}{3}\sin 3x-\frac{2}{3}\sin x+C$

168. $\int \frac{x}{\sqrt{1-2x^4}}dx =$ (where C is a constant of integration) **[2022]**

(A) $\frac{1}{2\sqrt{2}}\sin^{-1}\left(2\sqrt{2}x^2\right)+C$

(B) $\frac{1}{2}\sin^{-1}\left(2x\right)+C$

(C) $\frac{1}{\sqrt{2}}\sin^{-1}\left(\sqrt{2}x\right)+C$

(D) $\frac{1}{2\sqrt{2}}\sin^{-1}\left(\sqrt{2}x^2\right)+C$

169. $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}dx =$_____+C, (where C is a constant of integration) **[2022]**

(A) $-2\sqrt{1-x}+\cos^{-1}\sqrt{x}+\sqrt{x(1-x)}$

(B) $-2\sqrt{1-x}+\cos^{-1}\sqrt{x}-\sqrt{x(1-x)}$

(C) $-2\sqrt{1-x}-\cos^{-1}\sqrt{x}+\sqrt{x(1-x)}$

(D) $2\sqrt{1-x}+\cos^{-1}\sqrt{x}+\sqrt{x(1-x)}$

170. If $\int \frac{d\theta}{\cos^2\theta(\tan 2\theta+\sec 2\theta)} = \lambda\tan\theta+2\log\left|f(\theta)\right|+C$

Where C is a constant of integration, then the ordered pair (λ, f(θ)) is equal to: **[2022]**

(A) $(1,1+\tan\theta)$ (B) $(1,1-\tan\theta)$

(C) $(-1,1+\tan\theta)$ (D) $(-1,1-\tan\theta)$

171. If $f(x)=\int \frac{x^2+\sin^2 x}{1+x^2}.\sec^2 x\,dx$ and $f(0)=0$, then $f(1)=$ **[2022]**

(A) $\tan 1-\frac{\pi}{4}$ (B) $\tan 1+\frac{\pi}{4}$

(C) $1-\frac{\pi}{4}$ (D) $\frac{\pi}{4}-1$

172. $\int \frac{xe^{2x}}{(1+2x)^2}dx =$_____, (where C is a constant of integration) **[2022]**

(A) $\frac{4e^{2x}}{1+2x}+C$

(B) $\frac{e^{2x}}{1+2x}+C$

(C) $\frac{e^{2x}}{2(1+2x)}+C$

(D) $\frac{e^{2x}}{4(1+2x)}+C$

173. $\int \frac{4x^2\cot^{-1}\left(x^3\right)}{1+x^6}dx =$ (whre C is a consant of integration) **[2022]**

(A) $\frac{-2}{3}\left(\cot^{-1}x^3\right)+C$

(B) $\frac{2}{3}\left(\cot^{-1}x^3\right)+C$

(C) $\frac{-2}{3}\left(\cot^{-1}x^3\right)^2+C$

(D) $\frac{2}{3}\left(\cot^{-1}x^3\right)^2+C$

174. $\int \frac{1}{\cos x\sqrt{\cos 2x}}dx =$ ___, (where C is a constant of integration.) **[2022]**

(A) $\tan^{-1}x+C$

(B) $\log\left(\tan x+\sqrt{\tan^2 x-1}\right)+C$

(C) $\sin^{-1}\left(\tan x\right)+C$

(D) $\log\left(\tan x+\sqrt{\tan^2 x+1}\right)+C$

175. The integral $\int \frac{3x^{13}+2x^{11}}{\left(2x^4+3x^2+1\right)}dx$ is equal to, (where C is a constant of integration.) **[2022]**

(A) $\frac{x^{12}}{6\left(2x^4+3x^2+1\right)^3}+C$

(B) $\frac{x^4}{6\left(2x^4+3x^2+1\right)^3}+C$

(C) $\frac{x^{12}}{\left(2x^4+3x^2+1\right)^3}+C$

(D) $\frac{x^4}{\left(2x^4+3x^2+1\right)^3}+C$

176. If $f(x)=\int \frac{5x^8+7x^6}{\left(x^2+1+2x^7\right)^2}dx$, $x\geq 0$ and $f(0)=0$, then value of $f(1)$ is: **[2022]**

(A) $-\frac{1}{4}$ (B) $-\frac{1}{2}$

(C) $\frac{1}{2}$ (D) $\frac{1}{4}$

177. If $\int \frac{\sqrt{1-x^2}}{x^4}dx=A(x)\left(\sqrt{1-x^2}\right)^m+C$ for a suitable chosen integer m and a function $A(x)$, where C is a constant of integration, then $\left(A(x)\right)^m$ equals. **[2022]**

(A) $\frac{1}{27x^6}$

(B) $-\frac{1}{27x^9}$

(C) $-\frac{1}{3x^3}$

(D) $\frac{1}{9x^4}$

178. The value of $\int \frac{2x^3-1}{x^4+x}dx$ is equal to, (where C is a constant of integration.) **[2022]**

(A) $\frac{1}{2}\log\frac{\left(x^3+1\right)^2}{x^3}+C$ (B) $\log\frac{\left(x^3+1\right)}{x}+C$

(C) $\log\frac{\left(x^3+1\right)}{x^2}+C$ (D) $\frac{1}{2}\log\frac{\left(x^3+1\right)}{x^2}+C$

179. If $\int e^{x^2}.x^3dx=e^{x^2}f(x)+C$, (where C is a constant of integration.) and $f(1)=0$, then value of $f(2)$ will be: **[2022]**

(A) $\frac{3}{2}$ (B) $\frac{1}{2}$

(C) $\frac{-3}{2}$ (D) $\frac{-1}{2}$

180. $\int \frac{3x-2}{(x+1)(x-2)^2}dx=$ ___, (where C is a constant of integration) **[2022]**

(A) $\frac{1}{9}\log(x+1)+\frac{5}{9}\log(x-2)-\frac{4}{3}\times\frac{1}{(x-2)}+C$

(B) $\frac{-5}{9}\log(x+1)+\frac{5}{9}\log(x-2)-\frac{1}{(x-2)}+C$

(C) $\frac{-5}{9}\log(x+1)+\frac{1}{9}\log(x-2)+\frac{1}{(x-2)}+C$

(D) $\frac{-5}{9}\log(x+1)+\frac{5}{9}\log(x-2)-\frac{4}{3}\times\frac{1}{(x-2)}+C$

181. If $\int \frac{x+1}{\sqrt{2x-1}}dx=f(x)\sqrt{2x-1}+c$, (where c is a constant of integration), then $f(x)$ is equal to: **[2024, 2022]**

(A) $\frac{1}{3}(x+1)$ (B) $\frac{1}{3}(x+4)$

(C) $\frac{2}{3}(x+2)$ (D) $\frac{2}{3}(x-4)$

182. $\int \frac{dx}{2+\cos x} =$ **[2025, 2022]**

(A) $2\tan^{-1}\left(\frac{1}{\sqrt{3}}\tan\frac{x}{2}\right)+c$, where c is the constant of integration.

(B) $\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{1}{\sqrt{3}}\tan\frac{x}{2}\right)+c$, where c is the constant of integration.

(C) $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{1}{\sqrt{3}}\tan\frac{x}{2}\right)+c$, where c is the constant of integration.

(D) $\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}\tan\frac{x}{2}\right)+c$, where c is the constant of integration.

183. $\int_0^{\frac{\pi}{4}} \sec^4 x dx =$ **[2022]**

(A) $\frac{1}{3}$ (B) 1

(C) $\frac{4}{3}$ (D) $\frac{2}{3}$

184. If $\int_a^b x^3 dx = 0$ and if $\int_a^b x^2 dx = \frac{2}{3}$, then a and b are respectively. **[2022]**

(A) 1, 1 (B) 1, –1

(C) –1, 1 (D) –1, –1

185. $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{e^x(x\sin x)}{e^{2x}-1}dx =$ **[2022]**

(A) 0 (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{2}$ (D) $\frac{\pi}{3}$

186. $\int_{-3}^{0} x\sqrt{x+4}dx$ **[2022]**

(A) $\frac{64}{15}$ (B) $\frac{-34}{15}$

(C) $\frac{94}{15}$ (D) $\frac{-94}{15}$

187. $\int_0^1 \frac{1}{\sqrt{3+2x-x^2}}dx =$ **[2022]**

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{6}$

(C) $\frac{\pi}{2}$ (D) $\frac{\pi}{4}$

188. The value of $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x}dx, a>0$ is: **[2022]**

(A) π (B) 2π

(C) $\frac{\pi}{2}$ (D) $a\pi$

189. $\int_0^2 [2x]dx =$ ____, (where [.] denotes the greatest integer function.) **[2022]**

(A) 2 (B) 3

(C) 4 (D) 5

190. If $\int_0^{\frac{\pi}{2}} \frac{\cot x}{\cot x+\operatorname{cosec} x}dx = m(\pi+n)$, then (m.n) equals

(A) $\frac{1}{2}$ (B) 1

(C) $-\frac{1}{2}$ (D) –1

191. $\int_0^2 [x]dx + \int_0^2 |x-1|dx =$ ____. (where [x] denotes the greatest integer function.) **[2022]**

(A) 4 (B) 3

(C) 1 (D) 2

192. $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} f(x)dx =$

Where $f(x)=\sin|x|+\cos|x|, x\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$. **[2022]**

(A) 0 (B) 2
(C) 4 (D) 8

193. Let [t] denote the greatest integer less than or equal to t.

Then the value of $\int_1^2 |2x-[3x]|dx$ is: **[2022]**

(A) 1 (B) $\frac{3}{2}$
(C) 2 (D) 0

194. $\int_0^{\frac{\pi}{2}} \sin^5\left(\frac{x}{2}\right).\sin x\,dx =$ **[2022]**

(A) $\frac{1}{14\sqrt{2}}$ (B) $\frac{1}{56\sqrt{2}}$
(C) $\frac{1}{28\sqrt{2}}$ (D) $\frac{1}{7\sqrt{2}}$

195. The area bounded by the curve, $y=-x^2$, X-axis $x=1$ and $x=4$, is: **[2022]**

(A) 10 sq. units (B) $\frac{21}{2}$ sq. units
(C) 20 sq. units (D) 21 sq. units

196. The area of the region bounded by the line $2y+x=8$, X-axis and the lines $x=2$ and $x=4$ is:

(A) 5 sq. units (B) 10 sq. units
(C) 4 sq. units (D) 6 sq. units

197. The area bounded by the curve $y^2=2x+1$ and the line $x-y=1$ is: **[2022]**

(A) $\frac{2}{3}$ sq. units (B) $\frac{16}{3}$ sq. units
(C) $\frac{8}{3}$ sq. units (D) $\frac{4}{3}$ sq. units

198. the area (in sq. units) of the region described by $A=\{(x,y)/x^2+y^2\le 1 \text{ and } y^2\le 1-x\}$ is: **[2022]**

(A) $\frac{\pi}{2}+\frac{4}{3}$ (B) $\frac{\pi}{2}-\frac{4}{3}$
(C) $\frac{\pi}{2}-\frac{2}{3}$ (D) $\frac{\pi}{2}+\frac{2}{3}$

199. The area (in sq. units) bounded by the curves $y=\sqrt{x}, 2y-x+3=0$, X-axis and lying in the first quadrant is: **[2024, 2022]**

(A) 36 (B) 18
(C) $\frac{27}{4}$ (D) 9

200. The differential equation $x^2(y+1)dx+y^2(x-1)dy=0$ has the general solution given by (where C is a constant of integration.) **[2022]**

(A) $(x+1)^2+(y+1)^2+2\log[(x-1)(y-1)]=C$
(B) $(x+1)^2+(y-1)^2+2\log[(x-1)(y+1)]=C$
(C) $(x-1)^2+(y-1)^2+2\log[(x+1)(y+1)]=C$
(D) $(x-1)^2+(y+1)^2+2\log[(x+1)(y-1)]=C$

201. The general solution of differential equation $xdy-ydx=0$ represent **[2022]**

(A) the circle whose centre is at the origin.
(B) a straight line passing through the origin.
(C) a rectangular hyperbola
(D) the parabola whose vertex is at the origin.

202 The general solution of differential equation $e^{\frac{1}{2}\left(\frac{dy}{dx}\right)}=3^x$ is:
(where C is a constant of integration.) **[2022]**

(A) $x=(\log 2)y^2+C$
(B) $y=x^2\log 3+C$
(C) $y=x\log 3+C$
(D) $y=2x\log 3+C$

203 The general solution of the differential equation $x^2+y^2-2xy\frac{dy}{dx}=0$ is:
(where C is a constant of integration.) **[2022]**

(A) $2\left(x^2-y^2\right)+x=C$

(B) $x^2+y^2=Cy$

(C) $x^2-y^2=Cx$

(D) $x^2+y^2=Cx$

204. If surrounding air is kept at 20°C and body cools from 80°C to 70°C in 5 minutes, then the temperature of the body after 15 minutes will be: **[2022]**

(A) 54.7°C (B) 51.7°C
(C) 52.7°C (D) 50.7°C

205. The particular solution of the differential equation $\frac{dy}{dx}-e^x=ye^x$, when $x=0$ and $y=1$ is: **[2022]**

(A) $\log\left(\frac{y+1}{2}\right)=e^x-1$

(B) $\log(y-1)=e^x-1$

(C) $\log 2(y-1)=e^x-1$

(D) $\log\left(\frac{y+1}{2}\right)=\frac{e^x}{2}-\frac{1}{2}$

206. For the differential equation $\left[1-\left(\frac{dy}{dx}\right)^2\right]^{\frac{5}{3}}=8\frac{d^2y}{dx^2}$ has the order and degree ____ respectively. **[2022]**

(A) 2 and 6 (B) 2 and 3
(C) 2 and 2 (D) 2 and 1

207. The particular solution of the differential equation $(2x-2y+3)dx-(x-y+1)dy=0$ when $x=0, y=1$ is: **[2022]**

(A) $2x+y-\log(x-y+2)-1=0$

(B) $x-y-\log(x-y+2)+1=0$

(C) $2x-y-\log(x-y+2)+1=0$

(D) $x-2y-\log(x-y+2)+2=0$

208. The differential equation $y'=\frac{y}{x+\sqrt{xy}}$ has general solution given by, (where C is a constant of integration) **[2022]**

(A) $y=Ce^{2\sqrt{xy}}$ (B) $y=Ce^{-2\sqrt{xy}}$

(C) $y=Ce^{2\left(\sqrt{\frac{y}{x}}\right)}$ (D) $y=Ce^{2\left(\sqrt{\frac{x}{y}}\right)}$

209. A curve passes through the point $\left(1,\frac{\pi}{6}\right)$. Let the slope of the curve at each point (x,y) be given by $\frac{y}{x}+\sec\left(\frac{y}{x}\right), x>0$, then the equation of the curve is: **[2022]**

(A) $\sec\left(\frac{2y}{x}\right)=\log x+2$

(B) $\text{cosec}\left(\frac{y}{x}\right)=\log x+2$

(C) $\cos\left(\frac{2y}{x}\right)=\log x+2$

(D) $\sin\left(\frac{y}{x}\right)=\log x+\frac{1}{2}$

210. The solution of the differential equation $\left(1+e^{-x}\right)\left(1+y^2\right)\frac{dy}{dx}=y^2$ which passes through the point $(0,1)$ is: **[2022]**

(A) $y^2+1=y\left(\log\left(\left(\frac{1+e^{-x}}{2}\right)+2\right)\right)$

(B) $y^2+1=y\left(\log\left(\left(\frac{1+e^{x}}{2}\right)+2\right)\right)$

(C) $y^2=1+y\log\left(\frac{1+e^{x}}{2}\right)$

(D) $y^2=1+y\log\left(\frac{1+e^{-x}}{2}\right)$

211. A firm is manufacturing 2000 items. It is estimated that the rate of change of production P with respect to additional number of workers x is given by $\frac{dP}{dx} = 100 - 12\sqrt{x}$.
If the firm employs 25 more workers, then the new level of production of items is:
(A) 2500 (B) 3000
(C) 3500 (D) 4500

212. If $y = y(x)$ and $\left(\frac{2+\sin x}{y+1}\right)\frac{dy}{dx} = -\cos x, y(0) = 1$, then $y\left(\frac{\pi}{2}\right) =$ **[2025, 2024, 2023, 2022]**
(A) $\frac{1}{3}$ (B) $\frac{2}{3}$
(C) $-\frac{1}{3}$ (D) 1

213. A random variable X assumes values 1, 2, 3…n with equal probabilities. If the ratio of variance of X to expected value of X is equal to 4, then the value of n is: **[2022]**
(A) 50 (B) 30
(C) 25 (D) 35

214. In pizza hut, the following distribution is found for the daily demand of:

No. of pizzas	5	6	7	8	9	10
Probability	0.07	0.2	0.3	0.3	0.07	0.06

Then expected daily demand and variance are respectively. **[2022]**
(A) 7.28 and 1.52
(B) 1.52 and 7.28
(C) 7.28 and 54.52
(D) 7.28 and 53

215. Two cards are drawn successively with replacement from a well shuffled pack of 52 cards. Then mean of number of kings is: **[2024, 2022]**
(A) $\frac{1}{13}$ (B) $\frac{1}{169}$
(C) $\frac{2}{13}$ (D) $\frac{4}{169}$

216. A bag contains 4 red and 3 black balls. One ball is drawn and then replaced in the bag and the process is repeated. Let X denote the number of times black ball is drawn in 3 draws. Assuming that at each draw each ball is equally likely to be selected, then probability distribution of x is given by:
[2024, 2022]
(A)

x	0	1	2	3
$P(x)$	$\left(\frac{4}{7}\right)^3$	$\frac{9}{7}\cdot\left(\frac{4}{7}\right)^2$	$\frac{12}{7}\cdot\left(\frac{3}{7}\right)^2$	$\left(\frac{3}{7}\right)^3$

(B)

x	0	1	2	3
$P(x)$	$\left(\frac{3}{7}\right)^3$	$\frac{12}{7}\cdot\left(\frac{3}{7}\right)^2$	$\frac{9}{7}\cdot\left(\frac{4}{7}\right)^2$	$\left(\frac{4}{7}\right)^3$

(C)

x	0	1	2	3
$P(x)$	$\left(\frac{3}{7}\right)^3$	$\frac{9}{7}\cdot\left(\frac{4}{7}\right)^2$	$\frac{12}{7}\cdot\left(\frac{3}{7}\right)^2$	$\left(\frac{4}{7}\right)^3$

(D)

x	0	1	2	3
$P(x)$	$\left(\frac{4}{7}\right)^3$	$\frac{12}{7}\cdot\left(\frac{4}{7}\right)^2$	$\frac{9}{7}\cdot\left(\frac{3}{7}\right)^2$	$\left(\frac{3}{7}\right)^3$

217. It is observed that 30% of the students appearing for a certain entrance test are science students. If 5 students are randomly selected from this group, the probability of having 2 science students among these students is: **[2022]**
(A) 0.3437
(B) 0.2547
(C) 0.3087
(D) 0.1087

218. For a Binomial variate X, mean is 2 and variance is 1. The odds in favour of X = 0 are **[2022]**
(A) 1:15 (B) 4:1
(C) 15:1 (D) 1:4

219. The incidence of occupational disease in an industry is such that the workmen have a 10% chance of suffering from it. The probability that out of 5 workmen, 3 or more will contact the disease is: **[2022]**

(A) 0.000856 (B) 0.0856
(C) 0.0000856 (D) 0.00856

220. For a Binomial distribution, $n = 6$, if $9P(x = 4) = P(X = 2)$, then q = **[2022]**

(A) $\frac{2}{5}$ (B) $\frac{3}{4}$
(C) $\frac{1}{4}$ (D) $\frac{1}{2}$

221. Let a random variable X have a Binomial distribution with mean 8 and variance 4. If $P(x \le 2) = \frac{k}{2^{16}}$, then k is equal to: **[2024, 2023, 2022]**

(A) 17
(B) 121
(C) 1
(D) 137

MHTCET Yearwise PYQ (2022)

Marathi

Mathematics

Hints & Solutions

1. (C)

ing the sum-to-product trigonometric identities:

$$\cos\alpha+\cos\beta=2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\sin\alpha+\sin\beta=2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

Squaring both expressions and adding them together:

$$(\cos\alpha+\cos\beta)^2+(\sin\alpha+\sin\beta)^2$$

$$=4\cos^2\left(\frac{\alpha+\beta}{2}\right)\cos^2\left(\frac{\alpha-\beta}{2}\right)$$

$$+4\sin^2\left(\frac{\alpha+\beta}{2}\right)\cos^2\left(\frac{\alpha-\beta}{2}\right)$$

Factoring out $4\cos^2\left(\frac{\alpha-\beta}{2}\right)$:

$$=4\cos^2\left(\frac{\alpha-\beta}{2}\right)\left[\cos^2\left(\frac{\alpha+\beta}{2}\right)+\sin^2\left(\frac{\alpha+\beta}{2}\right)\right]$$

Since $\cos^2\theta+\sin^2\theta=1$:

$$=4\cos^2\left(\frac{\alpha-\beta}{2}\right)(1)=4\cos^2\left(\frac{\alpha-\beta}{2}\right)$$

2. (C)

Using the trigonometric power-reduction and product-to-sum identities:

$$\cos^2 10^\circ=\frac{1+\cos20^\circ}{2}$$

$$\cos^2 50^\circ=\frac{1+\cos100^\circ}{2}$$

$$\cos10^\circ\cdot\cos50^\circ=\frac{1}{2}\left[\cos\left(50^\circ+10^\circ\right)+\cos\left(50^\circ-10^\circ\right)\right]$$

$$=\frac{1}{2}\left(\cos60^\circ+\cos40^\circ\right)$$

$$=\frac{1}{4}+\frac{1}{2}\cos40^\circ$$

Substitute these into the given expression:

$$\cos^2 10^\circ-\cos10^\circ\cdot\cos50^\circ+\cos^2 50^\circ$$

$$=\frac{1+\cos20^\circ}{2}-\left(\frac{1}{4}+\frac{1}{2}\cos40^\circ\right)+\frac{1+\cos100^\circ}{2}$$

$$=\left(\frac{1}{2}-\frac{1}{4}+\frac{1}{2}\right)+\frac{1}{2}\left(\cos100^\circ+\cos20^\circ-\cos40^\circ\right)$$

$$=\frac{3}{4}+\frac{1}{2}\left(2\cos60^\circ\cdot\cos40^\circ-\cos40^\circ\right)$$

Since $\cos60^\circ=\frac{1}{2}$:

$$2\cos60^\circ\cdot\cos40^\circ-\cos40^\circ$$

$$=\cos40^\circ-\cos40^\circ=0$$

Thus, the total value simplifies to:

$$\frac{3}{4}+\frac{1}{2}(0)=\frac{3}{4}$$

3. (C)

The given expression is:

$$E=\frac{\sin^2\left(-160^\circ\right)}{\sin^2 70^\circ}+\frac{\sin\left(180^\circ-\theta\right)}{\sin\theta}$$

Simplifying each term using standard trigonometric identities:

1. $\sin\left(-160^\circ\right)=-\sin\left(160^\circ\right)$

   $=-\sin\left(180^\circ-20^\circ\right)=-\sin20^\circ$

   $\Rightarrow\sin^2\left(-160^\circ\right)=(-\sin20^\circ)^2=\sin^2 20^\circ$

2. $\sin70^\circ=\sin\left(90^\circ-20^\circ\right)=\cos20^\circ$

   $\Rightarrow\sin^2 70^\circ=\cos^2 20^\circ$

3. $\sin\left(180^\circ-\theta\right)=\sin\theta$

Substituting these back into the expression:

$$E=\frac{\sin^2 20^\circ}{\cos^2 20^\circ}+\frac{\sin\theta}{\sin\theta}$$

$$E=\tan^2 20^\circ+1$$

Using the fundamental Pythagorean identity $1+\tan^2 x=\sec^2 x$:

$$E=\sec^2\left(20^\circ\right)$$

4. (C)

Calculation:

Given $p=\tan 20^\circ$.

Expressing $\tan 160^\circ$ and $\tan 110^\circ$ in terms of p:

$$\tan 160^\circ=\tan\left(180^\circ-20^\circ\right)=-\tan 20^\circ=-p$$

$$\tan 110^\circ=\tan\left(90^\circ+20^\circ\right)=-\cot 20^\circ=-\frac{1}{p}$$

Substituting these values into the given expression:

$$\frac{\tan 160^\circ-\tan 110^\circ}{1+\tan 160^\circ\cdot\tan 110^\circ}=\frac{-p-\left(\frac{1}{p}\right)}{1+(-p)\left(\frac{1}{p}\right)}$$

$$=\frac{-p+\frac{1}{p}}{1+1}$$

$$=\frac{\frac{1-p^2}{p}}{2}$$

$$=\frac{1-p^2}{2p}$$

5. (B)

Given that $\cot\left(A+B\right)=0$, we have:

$$A+B=\frac{\pi}{2}$$

$$B=\frac{\pi}{2}-A$$

Substituting $B=\frac{\pi}{2}-A$ into the expression $\mathrm{n}\left(A+2B\right)$:

$$\sin\left(A+2B\right)=\sin\left(A+2\left(\frac{\pi}{2}A\right)\right)$$

$$\sin\left(A+2B\right)=\sin\left(\pi-A\right)$$

6. (D)

The given equations of the lines are:

$$4x+3y-1=0 \text{--- (1)}$$

$$x-y+5=0 \text{--- (2)}$$

$$kx+5y-3=0 \text{--- (3)}$$

From equation (2), we have $y=x+5$.

Substituting this into equation (1):

$$4x+3\left(x+5\right)-1=0.$$

$$7x+14=0\Rightarrow x=-2$$

Substituting $x=-2$ into $y=x+5$ gives:

$$y=-2+5=3$$

So, the point of intersection of the first two lines is $\left(-2,3\right)$.

Since the three lines are concurrent, the point $\left(-2,3\right)$ must also lie on equation (3):

$$k\left(-2\right)+5\left(3\right)-3=0$$

$$-2k+12=0\Rightarrow k=6$$

7. (A)

The equation of the given line is:

$$2x-3y+5=0$$

Its slope m_1 is given by:

$$m_1=\frac{2}{3}$$

Since the required line is perpendicular to the given line, its slope m_2 is:

$$m_2=-\frac{1}{m_1}=-\frac{3}{2}$$

The line makes an intercept of 3 on the positive y-axis, so its y-intercept is $c=3$.

Using the slope-intercept form $y = mx + c$:

$y = -\frac{3}{2}x + 3$

Multiplying the entire equation by 2:

$2y = -3x + 6$

Rearranging the terms gives:

$3x + 2y - 6 = 0$

8. (B)

Let $O(0,0)$ be the origin and $N(3,-4)$ be the foot of the perpendicular on line L.

The slope of line segment ON is:

$m_{ON} = \frac{-4-0}{3-0} = -\frac{4}{3}$

Since line L is perpendicular to ON, the slope m of line L is:

$m = -\frac{1}{m_{ON}} = \frac{3}{4}$

Using the point-slope form, the equation of line L passing through $N(3,-4)$ with slope $m = \frac{3}{4}$ is:

$y - (-4) = \frac{3}{4}(x-3)$

$4(y+4) = 3(x-3)$

$4y + 16 = 3x - 9$

$3x - 4y - 25 = 0$

9. (D)

The normal form of the equation of a straight line is given by:

$x\cos\omega + y\sin\omega = p$

Here, $p = 5$ units and $\omega = 210^\circ$.

Calculating the trigonometric values:

$\cos 210^\circ = \cos(180^\circ + 30^\circ) = -\cos 30^\circ = -\frac{\sqrt{3}}{2}$

$\sin 210^\circ = \sin(180^\circ + 30^\circ) = -\sin 30^\circ = -\frac{1}{2}$

Substituting these values into the normal form:

$x\left(\frac{\sqrt{3}}{2}\right) + y\left(\frac{1}{2}\right) = 5$

$-\sqrt{3}x - y = 10$

$\sqrt{3}x + y + 10 = 0$

10. (B)

The slope (m_1) of the line passing through $(-2,6)$ and $(4,8)$ is:

$m_1 = \frac{8-6}{4-(-2)} = \frac{2}{6} = \frac{1}{3}$

The slope (m_2) of the line passing through $(8,12)$ and $(x,24)$ is:

$m_2 = \frac{24-12}{x-8} = \frac{12}{x-8}$

Since the two lines are perpendicular, the product of their slopes is -1:

$m_1 \cdot m_2 = -1$

$\frac{1}{3} \times \frac{12}{x-8} = -1$

$\frac{4}{x-8} = -1$

$x - 8 = -4$

$x = 4$

11. (C)

The given equation of the line is $x + y - 1 = 0$.

Let $L(x,y) = x + y - 1$.

For the points $(1,2)$ and $(\sin\theta, \cos\theta)$ to lie on the same side of the line, $L(1,2)$ and $L(\sin\theta, \cos\theta)$ must have the same sign.

Evaluating $L(x,y)$ at $(1,2)$:

$L(1,2) = 1 + 2 - 1 = 2 > 0$

Since $L(1,2) > 0$, we must also have $L(\sin\theta, \cos\theta) > 0$:

$\sin\theta + \cos\theta - 1 > 0$

$\sin\theta+\cos\theta>1$

Dividing both sides by $\sqrt{2}$:

$$\frac{1}{\sqrt{2}}\sin\theta+\frac{1}{\sqrt{2}}\cos\theta>\frac{1}{\sqrt{2}}$$

$$\sin\left(\theta+\frac{\pi}{4}\right)>\frac{1}{\sqrt{2}}$$

Since $\theta\in(0,\pi)$, we have $\left(\theta+\frac{\pi}{4}\right)\in\left(\frac{\pi}{4},\frac{5\pi}{4}\right)$.

For $\sin\left(\theta+\frac{\pi}{4}\right)>\frac{1}{\sqrt{2}}$ in this interval:

$$\frac{\pi}{4}<\theta+\frac{\pi}{4}<\frac{3\pi}{4}$$

Subtracting $\frac{\pi}{4}$ from all parts gives:

$$0<\theta<\frac{\pi}{2}$$

Thus, $\theta\in\left(0,\frac{\pi}{2}\right)$.

12. (B)

The given equations of the lines are:

$4ax+2ay+c=0$

$5bx+2by+d=0$

Using the cross-multiplication method to find the point of intersection (x,y):

$$\frac{x}{(2a)(d)-(2b)(c)}=\frac{y}{(c)(5b)-(d)(4a)}$$

$$=\frac{1}{(4a)(2b)-(5b)(2a)}$$

$$\frac{x}{2ad-2bc}=\frac{y}{5bc-4ad}=\frac{1}{-2ab}$$

Solving for x and y:

$$x=\frac{2ad-2bc}{-2ab}=\frac{bc-ad}{ab}$$

$$y=\frac{5bc-4ad}{-2ab}=\frac{4ad-5bc}{2ab}$$

Since the point lies in the 4th quadrant, $x>0$ and $y<0$.

Being equidistant from the coordinate axes means $|x|=|y|$. Since $x>0$ and $y<0$, we have:

$x=-y\Rightarrow x+y=0$

Substitute the values of x and y:

$$\frac{bc-ad}{ab}+\frac{4ad-5bc}{2ab}=0$$

$$\frac{2(bc-ad)+(4ad-5bc)}{2ab}=0$$

$$2bc-2ad+4ad-5bc=0$$

$$2ad-3bc=0$$

13. (C)

The equation of the given line is $2x-3y=5$, which can be rewritten in slope-intercept form as $y=\frac{2}{3}x-\frac{5}{3}$.

Thus, the slope of the line is $m_1=\frac{2}{3}$.

Let m be the slope of the required line making an angle $\theta=45^\circ$ with the given line. Using the angle formula between two lines:

$$\tan\theta=\left|\frac{m-m_1}{1+m_1m}\right|$$

Substituting $\theta=45^\circ$ and $m_1=\frac{2}{3}$:

$$1=\left|\frac{m-\frac{2}{3}}{1+\frac{2}{3}m}\right|=\left|\frac{3m-2}{3+2m}\right|$$

This gives two cases:

Case 1:

$$\frac{3m-2}{3+2m}=1\Rightarrow 3m-2=3+2m\Rightarrow m=5$$

Case 2:

$$\frac{3m-2}{3+2m}=-1\Rightarrow 3m-2=-3-2m$$

$$\Rightarrow 5m=-1\Rightarrow m=-\frac{1}{5}$$

Hence, the required slopes are $-\frac{1}{5}$ and 5.

14. (B)

The line equation is $x-3y+7=0$ and the given point is $(x_1, y_1)=(1,2)$.

Let the coordinates of the foot of the perpendicular be (h,k). Using the formula for the foot of perpendicular from a point (x_1,y_1) to a line $Ax+By+C=0$:

$$\frac{h-x_1}{A}=\frac{k-y_1}{B}=-\frac{Ax_1+By_1+C}{A^2+B^2}$$

Substituting $A=1$, $B=-3$, $C=7$, $x_1=1$, and $y_1=2$:

$$\frac{h-1}{1}=\frac{k-2}{-3}=-\frac{1(1)-3(2)+7}{1^2+(-3)^2}=-\frac{2}{10}=-\frac{1}{5}$$

Solving for h and k:

$$h-1=-\frac{1}{5}\Rightarrow h=\frac{4}{5}$$

$$k-2=-3\left(\frac{1}{5}\right)=\frac{3}{5}\Rightarrow k=\frac{13}{5}$$

Thus, the coordinates of the foot of the perpendicular are $\left(\frac{4}{5},\frac{13}{5}\right)$.

15. (B)

Let the coordinates of any point P lying on the line $x+y+3=0$ be $(t,-3-t)$.

The perpendicular distance d of the point $P(t,-3-t)$ from the line $x+2y+2=0$ is given by:

$$d=\frac{|t+2(-3-t)+2|}{\sqrt{1^2+2^2}}=\frac{|t4|}{\sqrt{5}}=\frac{|t+4|}{\sqrt{5}}$$

Given that the distance $d=\sqrt{5}$ units:

$$\frac{|t+4|}{\sqrt{5}}=\sqrt{5}$$

$$|t+4|=5$$

This gives two possible cases:

$t+4=5\Rightarrow t=1$

$t+4=-5\Rightarrow t=-9$

When $t=1$, the coordinates of the point are $(1,-3-1)=(1,-4)$.

When $t=-9$, the coordinates of the point are $(-9,-3-(-9))=(-9,6)$.

Thus, the point among the given options is (1, –4).

16. (B)

The equation of the given circle is:

$x^2+y^2=4$

This is a circle centered at the origin $(0,0)$ with radius $r=\sqrt{4}=2$.

Any line parallel to $x+2y+3=0$ is of the form:

$x+2y+c=0$

For this line to be tangent to the circle, the perpendicular distance from the center $(0,0)$ to the line must be equal to the radius r:

$$\frac{|1(0)+2(0)+c|}{\sqrt{1^2+2^2}}=2$$

$$\frac{|c|}{\sqrt{5}}=2$$

$$|c|=2\sqrt{5}\Rightarrow c=\pm 2\sqrt{5}$$

Substituting c back into the line equation gives:

$x+2y\pm 2\sqrt{5}=0\Rightarrow x+2y=\pm 2\sqrt{5}$

17. (D)

For the first circle $x^2+y^2+8x-6y-24=0$:

$C_1=(-4,3)$

$$r_1=\sqrt{(-4)^2+3^2-(-24)}=\sqrt{16+9+24}=7$$

For the second circle

$x^2+y^2-4x+10y+20=0$:

$C_2=(2,-5)$

$$r_2=\sqrt{2^2+(-5)^2-20}=\sqrt{4+25-20}=3$$

The distance between their centers C_1 and C_2 is:

$$C_1C_2 = \sqrt{()^2 + (-5-3)^2} = \sqrt{6^2 + (-8)^2}$$

$$= \sqrt{100} = 10$$

Since $C_1C_2 = r_1 + r_2 = 7 + 3 = 10$, the circles touch externally.

18. (C)

Step-by-step reasoning:

1. **Relation between Orthocentre, Centroid, and Circumcentre:**
   In any triangle, the centroid (G) divides the line segment joining the orthocentre (H) and circumcentre (O) internally in the ratio $2:1$.
   Here, orthocentre $H = A(-3,5)$, centroid $G = B(3,3)$, and let circumcentre $O = C(x,y)$.
2. Coordinates of Circumcentre C:
   Using the section formula for internal division in ratio $2:1$:

$$B = \left(\frac{2x+1(-3)}{2+1}, \frac{2y+1(5)}{2+1}\right)$$

$$(3,3) = \left(\frac{2x-3}{3}, \frac{2y+5}{3}\right)$$

   Equating the coordinates:

$$\frac{2x-3}{3} = 3 \Rightarrow 2x-3 = 9 \Rightarrow x = 6$$

$$\frac{2y+5}{3} = 3 \Rightarrow 2y+5 = 9 \Rightarrow y = 2$$

   So, $C = (6,2)$.
3. Length of Diameter AC:
   Using the distance formula between $A(-3,5)$ and C(6, 2):

$$AC = \sqrt{()^2 + (2-5)^2} = \sqrt{9^2 + (-3)^2}$$

$$= \sqrt{81+9} = \sqrt{90} = 3\sqrt{10}$$

4. **Radius of the Circle:**
   Since AC is the diameter of the circle, its radius r is:

$$r = \frac{AC}{2} = \frac{3\sqrt{10}}{2} \text{ units}$$

19. (B)

Substituting the given points $(-1,2)$ and $(-4,3)$ into the general equation of a circle $x^2 + y^2 + 2gx + 2fy + c = 0$:

For point $(-1,2)$:

$$(-1)^2 + 2^2 + 2g(-1) + 2f(2) + c = 0$$

$$\Rightarrow -2g + 4f + c = -5$$

For point $(-4,3)$:

$$(-4)^2 + 3^2 + 2g(-4) + 2f(3) + c = 0$$

$$\Rightarrow -8g + 6f + c = -25$$

Testing Option (2) $x^2 + y^2 + 6x - 2y + 5 = 0$:

- For $(-1,2)$:
  $(-1)^2 + 2^2 + 6(-1) - 2(2) + 5$
  $= 1 + 4 - 6 - 4 + 5 = 0$
- For $(-4,3)$:
  $(-4)^2 + 3^2 + 6(-4) - 2(3) + 5$
  $= 16 + 9 - 24 - 6 + 5 = 0$

Option (2) is the only equation that satisfies both points.

20. (D)

The point of intersection of two diameters gives the center of the circle.

Solving the given equations of the diameters:

$3x - 4y - 7 = 0$--- (1)

$2x - 3y - 5 = 0$--- (2)

Multiplying equation (1) by 3 and equation (2) by 4, we get:

$9x - 12y - 21 = 0$

$8x - 12y - 20 = 0$

Subtracting these equations gives

$x - 1 = 0 \Rightarrow x = 1$.

Substituting $x = 1$ in equation (2):

$2(1)-3y-5=0$

$\Rightarrow -3y=3 \Rightarrow y=-1$

Thus, the center of the circle is $(h,k)=(1,-1)$.

Given the area of the circle is 49π square units:

$\pi r^2=49\pi \Rightarrow r^2=49$

The equation of a circle with center (h,k) and radius r is:

$(x-h)^2+(y-k)^2=r^2$

$(x-1)^2+(y+1)^2=49$

$x^2-2x+1+y^2+2y+1=49$

$x^2+y^2-2x+2y-47=0$

21. (A)

$x=\frac{a}{2}+\frac{\sqrt{a^2+b^2}}{2}\cos\theta, y=\frac{b}{2}+\frac{\sqrt{a^2+b^2}}{2}\sin\theta$

The given equation of the circle is:

$x^2+y^2-ax-by=0$

Rewriting it in standard form by completing the square:

$\left(x\frac{a}{2}\right)^2+\left(y\frac{b}{2}\right)^2=\left(\frac{a}{2}\right)^2+\left(\frac{b}{2}\right)^2=\frac{a^2+b^2}{4}$

Comparing with $(x-h)^2+(y-k)^2=r^2$:

- Centre $(h,k)=\left(\frac{a}{2},\frac{b}{2}\right)$
- Radius $r=\sqrt{\frac{a^2+b^2}{4}}=\frac{\sqrt{a^2+b^2}}{2}$

The parametric equations of a circle are given by $x=h+r\cos\theta$ and $y=k+r\sin\theta$:

$x=\frac{a}{2}+\frac{\sqrt{a^2+b^2}}{2}\cos\theta$

$y=\frac{b}{2}+\frac{\sqrt{a^2+b^2}}{2}\sin\theta$

22. (B)

Let B represent a boy and G represent a girl. The order of children from oldest to youngest is represented as a triple (c_1,c_2,c_3), where c_1 is the oldest and c_3 is the youngest.

layout the sample space S for a family with 3 children:

$S=\{BBB,BBG,BGB,BGG,GBB,GBG,GGB,GGG\}$

The total number of equally likely outcomes is $n(S)=2^3=8$.

Let E be the event that the oldest and youngest children are of the same gender (i.e., $c_1=c_3$). The favorable outcomes are:

$E=\{BBB,BGB,GBG,GGG\}$

The number of favorable outcomes is $n(E)=4$.

Therefore, the required probability is:

$P(E)=\frac{n(E)}{n(S)}=\frac{4}{8}=\frac{1}{2}$

23. (A)

The total number of ways to put 5 letters into 5 addressed envelopes randomly is given by:

Total outcomes $=5!=120$

There is only 1 way in which all 5 letters are placed in their respective correct envelopes.

Thus, the probability that all letters are placed in the right envelopes is:

$P(\text{all in right envelopes})=\frac{1}{120}$

Therefore, the probability that all the letters are not dispatched in their respective right envelopes (i.e., the complement event) is:

$P(\text{not all in right envelopes})$

$=1-P(\text{all in right envelopes})$

$=1-\frac{1}{120}=\frac{119}{120}$

24. (C)

Let S_1 and S_2 denote the events of choosing the first shelf and the second shelf, respectively. Assuming both shelves are equally likely to be chosen:

$$P(S_1) = P(S_2) = \frac{1}{2}$$

- First shelf (S_1): Contains 5 Physics books and 3 Biology books (Total = 8 books).

The probability of choosing a Physics book from S_1 is:

$$P(\text{Physics} \mid S_1) = \frac{5}{8}$$

- Second shelf (S_2): Contains 4 Physics books and 2 Biology books (Total = 6 books).

The probability of choosing a Physics book from S_2 is:

$$P(\text{Physics} \mid S_2) = \frac{4}{6} = \frac{2}{3}$$

By the Theorem of Total Probability, the overall probability of drawing a Physics book $P(\text{Physics})$ is:

$$P(\text{Physics}) = P(S_1) \cdot P(\text{Physics} \mid S_1) + P(S_2) \cdot P(\text{Physics} \mid S_2)$$

$$P(\text{Physics}) = \left(\frac{1}{2} \times \frac{5}{8}\right) + \left(\frac{1}{2} \times \frac{2}{3}\right)$$

$$P(\text{Physics}) = \frac{5}{16} + \frac{1}{3} = \frac{15+16}{48} = \frac{31}{48}$$

25. (B)

The first 100 natural numbers are $1, 2, 3, \ldots, 100$.

The total number of ways to choose 3 distinct numbers from these 100 numbers is:

$$N(S) = \binom{100}{3} = \frac{100 \times 99 \times 98}{3 \times 2 \times 1}$$

A number is divisible by both 2 and 3 if and only if it is divisible by $\text{lcm}(2,3) = 6$.

among the first 100 natural numbers, the numbers divisible by 6 are $6, 12, 18, \ldots, 96$.

The total count of such numbers is:

$$n = \frac{100}{6} = 16$$

The number of ways to choose 3 numbers from these 16 numbers is:

$$N(E) = \binom{16}{3} = \frac{16 \times 15 \times 14}{3 \times 2 \times 1}$$

Therefore, the required probability P is:

$$P = \frac{\binom{16}{3}}{\binom{100}{3}} = \frac{16 \times 15 \times 14}{100 \times 99 \times 98}$$

Simplifying the fraction:

$$P = \left(\frac{16}{100}\right) \times \left(\frac{15}{99}\right) \times \left(\frac{14}{98}\right)$$

$$= \frac{4}{25} \times \frac{5}{33} \times \frac{1}{7} = \frac{4}{5 \times 33 \times 7} = \frac{4}{1155}$$

26. (D)

The total number of balls in the bag is $5 + 3 = 8$.

The total number of ways to select 2 balls out of 8 without replacement is:

$$\binom{8}{2} = \frac{8 \times 7}{2} = 28$$

The number of favorable outcomes for selecting 1 red ball from 5 red balls and 1 green ball from 3 green balls is:

$$\binom{5}{1} \times \binom{3}{1} = 5 \times 3 = 15$$

Therefore, the required probability is:

$$P(\text{one red and one green ball}) = \frac{15}{28}$$

27. (D)

Since one of the three events A, B, C must happen and only one can happen, A, B, and C are mutually exclusive and exhaustive events. Thus,

$$P(A) + P(B) + P(C) = 1$$

Given:

1. Odds in favour of A are $4:6$:

$$P(A) = \frac{4}{4+6} = \frac{4}{10}$$

2. Odds against B are $7:3$:

$$P(B)=\frac{3}{7+3}=\frac{3}{10}$$

Using the total probability formula:

$$P(C)=1-P(A)-P(B)=1-\frac{4}{10}-\frac{3}{10}=\frac{3}{10}$$

Therefore, $P(C')=1-P(C)=1-\frac{3}{10}=\frac{7}{10}$.

The odds against event C are:

$$\text{Odds against } C=P(C'):P(C)$$

$$=\frac{7}{10}:\frac{3}{10}=7:3$$

28. (A)

Let A, B, and C be the events that the three critics review the book in favour.

The probabilities of each critic being in favour and against are:

$$P(A)=\frac{5}{5+2}=\frac{5}{7}, P(A')=1-\frac{5}{7}=\frac{2}{7}$$

$$P(B)=\frac{4}{4+3}=\frac{4}{7}, P(B')=1-\frac{4}{7}=\frac{3}{7}$$

$$P(C)=\frac{3}{3+4}=\frac{3}{7}, P(C')=1-\frac{3}{7}=\frac{4}{7}$$

The majority is in favour if either exactly 2 critics are in favour or all 3 critics are in favour.

Probability that exactly 2 critics are in favour:

$$P(\text{exactly } 2)=P(A\cap B\cap C')$$

$$+P(A\cap B'\cap C)$$

$$+P(A'\cap B\cap C)$$

$$P(\text{exactly } 2)=\left(\frac{5}{7}\times\frac{4}{7}\times\frac{4}{7}\right)$$

$$+\left(\frac{5}{7}\times\frac{3}{7}\times\frac{3}{7}\right)+\left(\frac{2}{7}\times\frac{4}{7}\times\frac{3}{7}\right)$$

$$P(\text{exactly } 2)=\frac{80}{343}+\frac{45}{343}+\frac{24}{343}=\frac{149}{343}$$

Probability that all 3 critics are in favour:

$$P(\text{all } 3)=P(A\cap B\cap C)=\frac{5}{7}\times\frac{4}{7}\times\frac{3}{7}=\frac{60}{343}$$

Thus, the total probability that the majority is in favour is:

$$P(\text{majority in favour})=\frac{149}{343}+\frac{60}{343}=\frac{209}{343}$$

29. (B)

The possible outcomes $r\in\{1,2,3,4,5,6\}$ on a fair die can be partitioned into three remainder classes modulo 3:

- $r\equiv 1(\text{mod}3)\Rightarrow\omega^r=\omega$, for $r\in\{1,4\}$ (2 outcomes, probability $P=\frac{1}{3}$)
- $r\equiv 2(\text{mod}3)\Rightarrow\omega^r=\omega^2$, for $r\in\{2,5\}$ (2 outcomes, probability $P=\frac{1}{3}$)
- $r\equiv 0(\text{mod}3)\Rightarrow\omega^r=1$, for $r\in\{3,6\}$ (2 outcomes, probability $P=\frac{1}{3}$)

Since ω is a complex cube root of unity ($\omega\neq 1$), we have $1+\omega+\omega^2=0$.

The given sum $\omega^{r_1}+\omega^{r_2}+\omega^{r_3}=0$ is satisfied if and only if $\{\omega^{r_1},\omega^{r_2},\omega^{r_3}\}$ is a permutation of $\{1,\omega,\omega^2\}$. Thus, r_1,r_2,r_3 must leave distinct remainders when divided by 3.

The number of ways to assign the three distinct remainder classes to r_1,r_2,r_3 is $3!=6$.

Since the probability of each outcome falling into any specific remainder class is $\frac{1}{3}$, the required probability is:

$$P=3!\times\left(\frac{1}{3}\right)\times\left(\frac{1}{3}\right)\times\left(\frac{1}{3}\right)=6\times\frac{1}{27}=\frac{2}{9}$$

30. (B)

Let the given expression be $(x-iy)(3+5i)$.

The conjugate of $-6-24i$ is $-6+24i$.

According to the question:

$$(x-iy)(3+5i)=-6+24i$$

Expanding the left-hand side:

$3x+5xi-3yi-5yi^2=-6+24i$

Since $i^2=-1$:

$(3x+5y)+i(5x-3y)=-6+24i$

Equating the real and imaginary parts:

$3x+5y=-6$--- (1)

$5x-3y=24$--- (2)

Multiply equation (1) by 3 and equation (2) by 5:

$9x+15y=-18$

$25x-15y=120$

Adding both equations:

$34x=102\Rightarrow x=3$

Substituting $x=3$ into equation (1):

$3(3)+5y=-6\Rightarrow 9+5y=-6$

$\Rightarrow 5y=-15\Rightarrow y=-3$

Thus, $x=3$ and $y=-3$.

31. (B)

Since α and β are the complex cube roots of unity (i.e., ω and ω^2), we have:

$\alpha^3=1$

$\beta^3=1$

$\alpha\beta=1$

The given expression is:

$E=\alpha^3+\beta^3+\alpha^{-2}\beta^{-2}$

Using $\alpha\beta=1$:

$\alpha^{-2}\beta^{-2}=(\alpha\beta)^{-2}=1^{-2}=1$

Substituting these values into the expression:

$E=1+1+1=3$

32. (C)

Given:

$(x+iy)^{1/3}=a+ib$

Cubing both sides:

$x+iy=(a+ib)^3$

Expanding using

$(u+v)^3=u^3+3u^2v+3uv^2+v^3$:

$x+iy=a^3+3a^2(ib)+3a(ib)^2+(ib)^3$

$x+iy=a^3+3a^2bi-3ab^2-ib^3$

$x+iy=(a^3-3ab^2)+i(3a^2b-b^3)$

Equating the real and imaginary parts:

$x=a^3-3ab^2=a(a^2-3b^2)\Rightarrow\frac{x}{a}=a^2-3b^2$

$y=3a^2b-b^3=b(3a^2-b^2)\Rightarrow\frac{y}{b}=3a^2-b^2$

Now, calculating $\frac{x}{a}-\frac{y}{b}$:

$\frac{x}{a}-\frac{y}{b}=(a^2-3b^2)-(3a^2-b^2)$

$\frac{x}{a}-\frac{y}{b}=a^2-3b^2-3a^2+b^2$

$\frac{x}{a}-\frac{y}{b}=-2a^2-2b^2=-2(a^2+b^2)$

33. (A)

Given: $x=-2-\sqrt{-3}=-2-i\sqrt{3}$

Rearranging terms:

$x+2=-i\sqrt{3}$

Squaring both sides:

$(x+2)^2=(-i\sqrt{3})^2$

$x^2+4x+4=3i^2=-3$

$x^2+4x+7=0$

Now, dividing the given polynomial expression $2x^4+5x^3+7x^2-x+41$ by (x^2+4x+7):

$2x^4+5x^3+7x^2-x+41$

$=(x^2+4x+7)(2x^2-3x+5)+6$

Since $x^2+4x+7=0$:

$2x^4+5x^3+7x^2-x+41$

$=0\cdot(2x^2-3x+5)+6=6$

34. (B)

If $|z|+z=3+i$, find $|z|$.

Write $z = x + i$. Then $|z| + x = 3$ and $|z|^2 = x^2 + 1$.

From $|z| = 3 - x$

$$x^2 + 1 = (3 - x)^2 \Rightarrow x = \frac{4}{3}, \Rightarrow |z| = \frac{5}{3}.$$

35. (A)

The bag contains 5 red marbles and 7 non-red marbles (4 black +3 white), making a total of 12 marbles.

We need to draw 4 marbles such that at most 2 are red. The total number of ways can be calculated by subtracting the cases with more than 2 red marbles (3 or 4 red marbles) from the total possible ways to select 4 marbles from 12:

1. Total number of ways to draw 4 marbles from 12:
$${}^{12}C_4 = \frac{12 \times 11 \times 10 \times 9}{4 \times 3 \times 2 \times 1} = 495$$
2. Number of ways to draw exactly 3 red marbles and 1 non-red marble:
$${}^5C_3 \times {}^7C_1 = 10 \times 7 = 70$$
3. Number of ways to draw exactly 4 red marbles:
$${}^5C_4 \times {}^7C_0 = 5 \times 1 = 5$$

Subtracting the undesirable outcomes from the total outcomes:

Required number of ways $= 495 - (70 + 5) = 420$

36. (B)

1. For each of the 5 subjects, a candidate has 2 possible outcomes: either to pass or to fail.
2. The total number of possible outcomes for all 5 subjects together is:
$$2 \times 2 \times 2 \times 2 \times 2 = 2^5 = 32$$
3. To pass the examination, the candidate must pass in all 5 subjects, which can happen in only 1 way.
4. Therefore, the number of ways the candidate can fail (i.e., fail in at least one subject) is:

Number of ways to fail $= = 2^5 - 1 = 32 - 1 = 31$

37. (A)

The total number of seats in a row for 5 men and 4 women is $5 + 4 = 9$.

1. Out of the 9 positions, the odd places are 1, 3, 5, 7, and 9 (a total of 5 odd places).
2. The 5 men can be arranged in these 5 odd places in 5! ways.
3. The remaining 4 even places (2, 4, 6, and 8) can be occupied by the 4 women in 4! ways.

By the multiplication principle of counting, the total number of possible arrangements is:

$$\text{Total arrangements} = 5! \times 4! = 120 \times 24 = 2880$$

38. (A)

The word MACHINE consists of 7 distinct letters:

- Vowels (3): A, I, E
- Consonants (4): M, C, H, N

There are 7 positions in total, out of which 4 are odd positions ($1^{st}, 3^{rd}, 5^{th}, 7^{th}$) and 3 are even positions ($2^{nd}, 4^{th}, 6^{th}$).

1. The 3 vowels can be arranged in any 3 of the 4 odd positions in:
$${}^4P_3 = \frac{4!}{(4-3)!} = 24 \text{ ways}$$
2. The remaining 4 positions (the 1 remaining odd position and the 3 even positions) can be filled by the 4 consonants in:
$$4! = 24 \text{ ways}$$

By the fundamental principle of multiplication, the total number of arrangements is:

$$\text{Total ways} = 24 \times 24 = 576$$

39. (D)

Let P have 4 ladies (L_P) and 3 men (M_P) as friends, and Q have 3 ladies (L_Q) and 4 men (M_Q) as friends.

The party requires a total of 3 ladies and 3 men, such that 3 friends are chosen from P and 3 friends are chosen from Q.

Let x be the number of ladies selected from P's friends ($0 \le x \le 3$). The remaining selections are determined as follows:

- From P: x. ladies (L_P) and $(3x)$ men (M_P).
- From Q: $(3x)$ ladies (L_Q) and x men (M_Q).

The possible cases for x are:

1. Case $x = 0$:
$$\text{Ways} = \left({}^4C_0 \times {}^3C_3\right) \times \left({}^3C_3 \times {}^4C_0\right)$$
$$= (1\times 1)\times(1\times 1) = 1$$
2. Case $x = 1$:
$$\text{Ways} = \left({}^4C_1 \times {}^3C_2\right) \times \left({}^3C_2 \times {}^4C_1\right)$$
$$= (4\times 3)\times(3\times 4) = 144$$
3. Case $x = 2$:
$$\text{Ways} = \left({}^4C_2 \times {}^3C_1\right) \times \left({}^3C_1 \times {}^4C_2\right)$$
$$= (6\times 3)\times(3\times 6) = 324$$
4. Case $x = 3$:
$$\text{Ways} = \left({}^4C_3 \times {}^3C_0\right) \times \left({}^3C_0 \times {}^4C_3\right)$$
$$= (4\times 1)\times(1\times 4) = 16$$

Summing the total number of ways:

$$\text{Total ways} = 1+144+324+16 = 485$$

40. (B)

1. **Seating the 6 men:**

The number of ways to seat n persons around a circular table is given by $(n-1)!$.

Therefore, 6 men can be seated at a round table in:

$$(6-1)! = 5! \text{ ways}$$

2. **Seating the 5 women:**

When 6 men are seated around a circle, exactly 6 gaps (spaces) are created between them. To ensure no two women sit together, the 5 women must occupy these 6 gaps.

The number of ways to seat 5 women in 6 available gaps is given by:

$${}^6P_5 = \frac{6!}{(6-5)!} = 6! \text{ ways}$$

3. **Total number of arrangements:**

By the fundamental principal of counting, the total number of required seating arrangements is:

$$5! \times 6! = 6! \times 5!$$

41. (B)

1. **Selection of player:**

Out of 4 players, 1 player can be chosen to receive 1 card in $\binom{4}{1} = 4$ ways.

2. **Distribution of cards:**

- From 52 cards, 1 card can be given to this chosen player in $\binom{52}{1} = 52$ ways.
- From the remaining 51 cards, 17 cards can be given to the second player in $\binom{51}{17}$ ways.
- From the remaining 34 cards, 17 cards can be given to the third player in $\binom{34}{17}$ ways.
- The remaining 17 cards are given to the fourth player in $\binom{17}{17} = 1$ way.

3. **Total number of ways:**

$$\text{Total ways} = 4 \times 52 \times \binom{51}{17} \times \binom{34}{17} \times \binom{17}{17}$$
$$\text{Total ways} = 4 \times 52 \times \frac{51!}{17! \times 34!} \times \frac{34!}{17! \times 17!} \times 1$$
$$\text{Total ways} = 4 \times \frac{52 \times 51!}{(17!)^3} = \frac{4 \times 52!}{(17!)^3}$$

Alternatively, dividing 52 distinct cards into 4 groups of sizes 17, 17, 17, 1 gives $\dfrac{52!}{(17!)^3 \times 3!}$

ways. Distributing these 4 groups amongst 4 players in $4!$ ways yields:

$$\text{Total ways} = \frac{52!}{(17!)^3 \times 3!} \times 4! = \frac{4 \times 52!}{(17!)^3}$$

42. (A)

The given number is 445577888, which has a total of 9 digits.

Categorising the digits:

- Odd digits: $5,5,7,7$ (Total = 4 digits)
- Even digits: $4,4,8,8,8$ (Total = 5 digits)

In a 9-digit number, the available positions are $1,2,3,4,5,6,7,8,9$:

- Even positions: $2^{nd}, 4^{th}, 6^{th}, 8^{th}$ (Total = 4 positions)
- Odd positions: $1^{st}, 3^{rd}, 5^{th}, 7^{th}, 9^{th}$ (Total = 5 positions)

Since the 4 odd digits must occupy the 4 even positions, the number of ways to arrange the digits $5,5,7,7$ in these positions is:

$$\frac{4!}{2! \times 2!} = \frac{24}{4} = 6$$

The remaining 5 even digits $(4,4,8,8,8)$ must occupy the remaining 5 odd positions. The number of ways to arrange them is:

$$\frac{5!}{2! \times 3!} = \frac{120}{2 \times 6} = 10$$

By the multiplication principle of counting, the total number of required 9-digit numbers is:

$$\text{Total ways} = 6 \times 10 = 60$$

43. (D)

To arrange 5 subjects across 6 periods such that each subject is allowed at least one period, exactly one subject must be taught in 2 periods and the remaining 4 subjects must be taught in 1 period each.

1. Select 1 subject out of 5 to receive 2 periods:
$$\binom{5}{1} = 5 \text{ ways}$$
2. Select 2 periods out of 6 for this repeated subject:
$$\binom{6}{2} = \frac{6 \times 5}{2} = 15 \text{ ways}$$
3. Arrange the remaining 4 subjects in the remaining 4 periods:
$$4! = 24 \text{ ways}$$

Hence, the total number of ways is:

$$\text{Total ways} = 5 \times 15 \times 24 = 1800$$

44. (C)

Section I has 6 questions and Section II has 5 questions. A student needs to select a total of 6 questions, taking at least 2 questions from each section.

The possible cases for choosing (x,y) questions from Section I and Section II respectively are:

1. Selecting 2 questions from Section I and 4 questions from Section II:
$$\text{Number of ways} = {}^6C_2 \times {}^5C_4 = 15 \times 5 = 75$$
2. Selecting 3 questions from Section I and 3 questions from Section II:
$$\text{Number of ways} = {}^6C_3 \times {}^5C_3 = 20 \times 10 = 200$$
3. Selecting 4 questions from Section I and 2 questions from Section II:
$$\text{Number of ways} = {}^6C_4 \times {}^5C_2 = 15 \times 10 = 150$$

Total number of different ways to select the questions:

$$\text{Total ways} = 75 + 200 + 150 = 425$$

45. (B)

Given the functions:

$$f(x) = e^{|x|}$$

$$g(x) = \log x$$

To find the composite function $g[f(x)]$, substitute $f(x)$ into $g(x)$:

$$g[f(x)] = g\left(e^{|x|}\right)$$

Using the logarithmic identity $\log_e\left(e^k\right) = k$:

$$g\left[f(x)\right]=\log\left(e^{|x|}\right)=|x|$$

46. (C)

Let $y=f(x)$. Then,

$$y=\frac{a^x-a^{-x}}{a^x+a^{-x}}$$

Multiplying the numerator and the denominator by a^x:

$$y=\frac{a^{2x}-1}{a^{2x}+1}$$

Rearranging to solve for a^{2x}:

$$y\left(a^{2x}+1\right)=a^{2x}-1$$

$$y\cdot a^{2x}+y=a^{2x}-1$$

$$a^{2x}(1-y)=1+y$$

$$a^{2x}=\frac{1+y}{1-y}$$

Taking logarithm to base a on both sides:

$$2x=\log_a\left(\frac{1+y}{1-y}\right)$$

$$x=\frac{1}{2}\log_a\left(\frac{1+y}{1-y}\right)$$

Replacing y with x gives the inverse function:

$$f^{-1}(x)=\frac{1}{2}\log_a\left(\frac{1+x}{1-x}\right)$$

47. (C)

1. **Domain:**

The functions $|x|$ and $\sin x$ are defined for all real numbers $x\in\mathbb{R}$. The exponential function e^y is also defined for all real exponents $y\in\mathbb{R}$.

$$\text{Domain of } f(x)=\mathbb{R}$$

2. **Range:**

Let $y=|x|\sin x$.

- The function $y(x)=|x|\sin x$ is continuous for all $x\in\mathbb{R}$.
- As $x=2n\pi+\frac{\pi}{2}\to+\infty$ (where $n\in\mathbb{N}$), $y=\left(2n\pi+\frac{\pi}{2}\right)(1)\to+\infty$.
- As $x=2n\pi+\frac{3\pi}{2}\to+\infty$ (where $n\in\mathbb{N}$), $y=\left(2n\pi+\frac{3\pi}{2}\right)(-1)\to-\infty$.

By the Intermediate Value Theorem, the continuous function $y=|x|\sin x$ assumes every real value in $(-\infty,\infty)$, so its range is $\mathbb{R}$.

Since the exponential function $f(x)=e^y$ maps $(-\infty,\infty)$ onto $(0,\infty)$:

$$\text{Range of } f(x)=(0,\infty)$$

48. (D)

1. **Injectivity (One-one):**

A function f is injective if $f(x_1)=f(x_2)\Rightarrow x_1=x_2$.

Here, for $x_1=1$ and $x_2=-1$:

$$f(1)=|1|=1$$

$$f(-1)=|1|=1$$

Since $f(1)=f(-1)$ but $1\neq-1$, f is **not injective**.

2. **Surjectivity (Onto):**

A function $f:\mathbb{R}\to\mathbb{R}$ is surjective if its range equals its codomain $\mathbb{R}$.

Since $|x|\geq0$ for all $x\in\mathbb{R}$, the range of f is $[0,\infty)$.

For any negative real number in the codomain (e.g., $-1\in\mathbb{R}$), there exists no $x\in\mathbb{R}$ such that $f(x)=|x|=-1$.

Thus, Range ≠ Codomain, so f is **not surjective**.

Hence, f is **neither injective nor surjective**.

49. (C)

To find the domain of the function $f(x)=\frac{1}{4-x^2}+\log_{10}(x^3-x)$, we determine the set of real numbers x for which both terms are defined:

1. The term $\frac{1}{4-x^2}$ is defined when the denominator is non-zero:
   $4-x^2\neq 0\Rightarrow x^2\neq 4\Rightarrow x\neq\pm 2$
2. The term $\log_{10}(x^3-x)$ is defined when the argument is strictly positive:
   $x^3-x>0\Rightarrow x(x-1)(x+1)>0$

Analyzing the sign of the product $x(x-1)(x+1)$ across intervals separated by roots $x=-1,0,1$:

$x\in(-1,0)\cup(1,\infty)$

Combining both conditions:

$x\in((-1,0)\cup(1,\infty))\Rightarrow\{-2,2\}$

Since -2 is already outside $(-1,0)\cup(1,\infty)$, excluding $x=2$ splits the interval $(1,\infty)$ into $(1,2)\cup(2,\infty)$.

Thus, the domain of $f(x)$ is:

$(-1,0)\cup(1,2)\cup(2,\infty)$

50. (C)

1. Injectivity (One-one):

Let $x_1,x_2\in A$ such that $f(x_1)=f(x_2)$.

$\frac{2x_1}{x_1-1}=\frac{2x_2}{x_2-1}$

$2x_1(x_2-1)=2x_2(x_1-1)$

$2x_1x_2-2x_1=2x_1x_2-2x_2$

$-2x_1=-2x_2\Rightarrow x_1=x_2$

Thus, f is injective.

2. Surjectivity (Onto):

For $y\in\mathbb{R}$ (codomain), let

$y=f(x)=\frac{2x}{x-1}$.

$y(x-1)=2x\Rightarrow x(y-2)$

$=y\Rightarrow x=\frac{y}{y-2}(y\neq 2)$

- If $y=2$, $2=\frac{2x}{x-1}\Rightarrow -2=0$, which is impossible. So $y=2$ has no pre-image.
- If $y=4$, $x=\frac{4}{4-2}=2$. Since 2 is a positive integer, $2\notin A$. Thus, $y=4$ has no pre-image in A.

Since $\text{Range}(f)\neq\mathbb{R}$, f is not surjective.

Hence, f is **injective but not surjective**.

51. (A)

Since the function $f:\mathbb{R}-\{-1,1\}\to A$ is surjective, the set A must be equal to the range of $f(x)$.

Let $y=f(x)=\frac{x^2}{1-x^2}$.

Solving for x^2:

$y(1-x^2)=x^2$

$y-yx^2=x^2$

$x^2(1+y)=y$

$x^2=\frac{y}{1+y}$

Since $x\in\mathbb{R}$, we must have $x^2\geq 0$:

$\frac{y}{1+y}\geq 0\text{and}1+y\neq 0$

Using the critical points $y=-1$ and $y=0$ on a number line:

- For $y\geq 0$, $\frac{y}{1+y}\geq 0$ holds.
- For $-1<y<0$, $\frac{y}{1+y}<0$.

- For $y<-1$, $\frac{y}{1+y}>0$ holds.

Thus, the range of $f(x)$ is:

$$\text{Range}=(-\infty,-1)\cup[0,\infty)=\mathbb{R}-[-1,0)$$

Therefore, $A=\mathbb{R}-[-1,0)$.

52. (B)

The function $f:\mathbb{R}-\{-a\}\to\mathbb{R}$ is defined as:

$$f(x)=\frac{a-x}{a+x}$$

We are given that $(f\circ f)(x)=x$ for all valid x.

Computing $f(f(x))$:

$$(f\circ f)(x)=f\left(\frac{a-x}{a+x}\right)=\frac{a-\left(\frac{a-x}{a+x}\right)}{a+\left(\frac{a-x}{a+x}\right)}$$

$$=\frac{a(a+x)-(a-x)}{a(a+x)+(a-x)}$$

$$=\frac{(a^2-a)+(a+1)x}{(a^2+a)+(a-1)x}$$

Since $(f\circ f)(x)=x$:

$$\frac{(a^2-a)+(a+1)x}{(a^2+a)+(a-1)x}=x$$

$$(a^2-a)+(a+1)x=(a^2+a)x+(a-1)x^2$$

$$(a-1)x^2+(a^2-1)x-a(a-1)=0$$

For this equation to hold identically for all valid real numbers x, the coefficients must be zero:

$a-1=0\Rightarrow a=1$

Thus, the function is $f(x)=\frac{1-x}{1+x}$.

Since $f(f(x))=x$, the function is its own inverse, i.e., $f^{-1}(x)=f(x)$.

Therefore:

$$f\left(\frac{1}{2}\right)=\frac{1-\left(\frac{1}{2}\right)}{1+\left(\frac{1}{2}\right)}=\frac{\frac{3}{2}}{\frac{1}{2}}=3$$

53. (C)

Given equation:

$2[2x-5]-1=7$

Adding 1 to both sides:

$2[2x-5]=8$

$[2x-5]=4$

By the definition of the greatest integer function, if $[y]=n$ where $n\in\mathbb{Z}$, then $n\le y<n+1$.

Here, setting $y=2x-5$ and $n=4$:

$4\le 2x-5<5$

Adding 5 to all parts of the inequality:

$9\le 2x<10$

Dividing by 2:

$\frac{9}{2}\le x<5$

Therefore, $x\in\left[\frac{9}{2},5\right)$.

54. (C)

To evaluate the limit:

$$L=lim_{x\to 0}\left(\frac{1+\tan x}{1+\sin x}\right)^{\csc x}$$

This is of the indeterminate form 1^{∞}. Using the standard formula

$$lim_{x\to a}[f(x)]^{g(x)}=e^{lim_{x\to a}g(x)[f(x)-1]}$$

when $lim_{f(x)}=1$ and $lim_{g(x)}=\infty$:

$$L=\exp\left(lim_{x\to 0}\csc x\left[\frac{1+\tan x}{1+\sin x}1\right]\right)$$

$$L=\exp\left(lim_{x\to 0}\frac{1}{\sin x}\cdot\frac{\tan x-\sin x}{1+\sin x}\right)$$

$$L=\exp\left(lim_{x\to0}\frac{\frac{\sin x}{\cos x}-\sin x}{\sin x(1+\sin x)}\right)$$

$$L=\exp\left(lim_{x\to0}\frac{1-\cos x}{\cos x(1+\sin x)}\right)$$

Substituting $x=0$:

$$L=e^{\frac{1-1}{1\cdot(1+0)}}=e^0=1$$

55. (A)

The given limit is:

$$L=lim_{x\to1}\frac{2^{2x-2}-2^x+1}{\sin^2(x-1)}$$

Rewriting 2^{2x-2} as $\left(2^{x-1}\right)^2$ and 2^x as $2\cdot2^{x-1}$, the numerator forms a perfect square:

$$2^{2x-2}-2^x+1=(2^{x-1})^2-2\left(2^{x-1}\right)+1=(2^{x-1}-1)^2$$

Substitute this back into the limit:

$$L=lim_{x\to1}\left(\frac{2^{x-1}-1}{\sin(x-1)}\right)^2$$

Let $t=x-1$. As $x\to1$, $t\to0$:

$$L=lim_{t\to0}\left(\frac{2^t-1}{\frac{\sin t}{t}}\right)^2$$

Using the standard limit formulas $lim_{t\to0}\frac{2^t-1}{t}=\log2$ and $lim_{t\to0}\frac{\sin t}{t}=1$:

$$L=\left(\frac{\log2}{1}\right)^2=(\log2)^2$$

56. (C)

From the given question in the image, we evaluate the limit:

$$lim_{x\to\infty}\left[\frac{8x^2+5x+3}{2x^2-7x-5}\right]^{\frac{4x+3}{8x-1}}$$

First, evaluate the limit of the base as $x\to\infty$:

$$lim_{x\to\infty}\frac{8x^2+5x+3}{2x^2-7x-5}=lim_{x\to\infty}\frac{8+\frac{5}{x}+\frac{3}{x^2}}{2-\frac{7}{x}-\frac{5}{x^2}}=\frac{8}{2}=4$$

Next, evaluate the limit of the exponent as x→∞:

$$lim_{x\to\infty}\frac{4x+3}{8x-1}=lim_{x\to\infty}\frac{4+\frac{3}{x}}{8-\frac{1}{x}}=\frac{4}{8}=\frac{1}{2}$$

Combining these limits, we get:

$$\text{Required Limit}=4^{1/2}=\sqrt{4}=2$$

57. (A)

The given limit from Question 57 on the page is:

$$lim_{x\to2}\left(\frac{5x-8}{8-3x}\right)^{\frac{3}{2x-4}}$$

As $x\to2$, the base approaches $\frac{5(2)-8}{8-3(2)}=1$, and the exponent approaches $\frac{3}{0}=\infty$. This is of the indeterminate form 1^∞.

For a limit of the form $lim_{x\to a}[f(x)]^{g(x)}=1^\infty$, the value is e^L, where $L=lim_{x\to a}g(x)\left[f(x)-1\right]$.

First, simplify $f(x)-1$:

$$f(x)-1=\frac{5x-8}{8-3x}-1=\frac{(5x-8)-(8-3x)}{8-3x}$$

$$=\frac{8x-16}{8-3x}=\frac{8(x-2)}{8-3x}$$

Now, compute L:

$$L=lim_{x\to2}\frac{3}{2(x-2)}\cdot\frac{8(x-2)}{8-3x}$$

$$L=lim_{x\to2}\frac{12}{8-3x}=\frac{12}{8-3(2)}=\frac{12}{2}=6$$

Thus, the limit is e^6.

58. (C)

1. Evaluate m:

$$m = lim_{x\to 0}\frac{x\log(1+2x)}{x\tan x}$$

$$= lim_{x\to 0}\frac{\log(1+2x)}{\tan x}$$

Dividing the numerator and denominator by x:

$$m = lim_{x\to 0}\frac{\frac{\log(1+2x)}{2x}\cdot 2}{\frac{\tan x}{x}} = \frac{1\cdot 2}{1} = 2$$

2. Evaluate n:

$$n = lim_{x\to 0}\frac{\log x + \log\left(\frac{1+x}{x}\right)}{x}$$

Using the logarithm property $\log a + \log b = \log(a\cdot b)$:

$$\log x + \log\left(\frac{1+x}{x}\right) = \log\left(x\cdot\frac{1+x}{x}\right) = \log(1+x)$$

Thus:

$$n = lim_{x\to 0}\frac{\log(1+x)}{x} = 1$$

1. Form the quadratic equation:

$$x^2 - (m+n)x + mn = 0$$

$$x^2 - (2+1)x + (2\cdot 1) = 0$$

$$x^2 - 3x + 2 = 0$$

59. (A)

Rationalizing the expression by multiplying and dividing by $\left(\sqrt{n^2+9}+n\right)$:

$$lim_{n\to\infty} n\left(\sqrt{n^2+9}n\right)$$

$$= lim_{n\to\infty} n\cdot\frac{\left(\sqrt{n^2+9}n\right)\left(\sqrt{n^2+9}+n\right)}{\sqrt{n^2+9}+n}$$

$$lim_{n\to\infty} n\left(\sqrt{n^2+9}n\right) = lim_{n\to\infty}\frac{n\left(n^2+9-n^2\right)}{\sqrt{n^2+9}+n}$$

$$lim_{n\to\infty} n\left(\sqrt{n^2+9}n\right) = lim_{n\to\infty}\frac{9n}{\sqrt{n^2+9}+n}$$

Dividing the numerator and the denominator by n:

$$lim_{n\to\infty}\frac{9}{\sqrt{1+\frac{9}{n^2}}+1} = \frac{9}{\sqrt{1+0}+1} = \frac{9}{2}$$

60. (C)

To evaluate the limit:

$$lim_{x\to 0}\frac{\sin\left(\pi\cos^2 x\right)}{x^2}$$

Using the trigonometric identity $\cos^2 x = 1-\sin^2 x$, rewrite the numerator:

$$\sin\left(\pi\cos^2 x\right) = \sin\left(\pi\left(1-\sin^2 x\right)\right)$$

$$= \sin\left(\pi - \pi\sin^2 x\right) = \sin\left(\pi\sin^2 x\right)$$

Substitute this into the limit:

$$L = lim_{x\to 0}\frac{\sin\left(\pi\sin^2 x\right)}{x^2}$$

Multiply and divide by $\pi\sin^2 x$:

$$L = lim_{x\to 0}\left(\frac{\sin\left(\pi\sin^2 x\right)}{\pi\sin^2 x}\cdot\frac{\pi\sin^2 x}{x^2}\right)$$

Using the standard limits $lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1$ and $lim_{x\to 0}\frac{\sin x}{x} = 1$:

$$L = \left(lim_{\pi\sin^2 x\to 0}\frac{\sin\left(\pi\sin^2 x\right)}{\pi\sin^2 x}\right)\cdot\pi\cdot\left(lim_{x\to 0}\frac{\sin x}{x}\right)^2$$

$$L = 1\cdot\pi\cdot(1)^2 = \pi$$

61. (D)

Since $f(x)$ is continuous at $x = 0$, we have:

$$f(0) = lim_{x\to 0} f(x) = lim_{x\to 0}\frac{e^{x^2}-\cos x}{x^2}$$

Rewriting the expression:

$$f(0) = lim_{x\to 0}\left(\frac{e^{x^2}-1}{x^2}+\frac{1-\cos x}{x^2}\right)$$

Using the standard limit results $lim_{t\to 0}\frac{e^t-1}{t}=1$ (where $t=x^2$) and $lim_{x\to 0}\frac{1-\cos x}{x^2}=\frac{1}{2}$:

$f(0)=1+\frac{1}{2}=\frac{3}{2}$

62. (C)

$\tan\left[f(x)\right]$ and $\frac{1}{f(x)}$ are both discontinuous.

For $x\in[0,\pi]$, the range of $\frac{x}{2}$ is $\left[0,\frac{\pi}{2}\right]\approx[0,1.57]$.

Evaluating $f(x)=\left[\frac{x}{2}\right]-1$ piecewise on the interval $[0,\pi]$:

1. For $x\in[0,2)$:

$\frac{x}{2}\in[0,1)\Rightarrow\left[\frac{x}{2}\right]=0$

$\Rightarrow f(x)=0-1=-1$

2. For $x\in[2,\pi]$:

$\frac{x}{2}\in\left[1,\frac{\pi}{2}\right]\Rightarrow\left[\frac{x}{2}\right]=1\Rightarrow f(x)=1-1=0$

Thus,

$$f(x)=\begin{cases}-1, & 0\le x<2\\ 0, & 2\le x\le\pi\end{cases}$$

Now, checking the continuity of each given function:

- For $\tan\left[f(x)\right]$:

Since $f(x)$ takes integer values, $\left[f(x)\right]=f(x)$.

$lim_{x\to 2^-}\tan\left[f(x)\right]=\tan(-1)=-\tan(1)$

$lim_{x\to 2^+}\tan\left[f(x)\right]=\tan(0)=0$

Since LHL $\neq$ RHL at $x=2$, $\tan\left[f(x)\right]$ is **discontinuous** at $x=2$.

- For $\frac{1}{f(x)}$:

For $x\in[2,\pi]$, $f(x)=0$, making $\frac{1}{f(x)}=\frac{1}{0}$, which is undefined. Hence, $\frac{1}{f(x)}$ is **discontinuous** on $[0,\pi]$.

Therefore, both $\tan\left[f(x)\right]$ and $\frac{1}{f(x)}$ are discontinuous on $[0,\pi]$.

63. (C)

For $f(x)$ to be continuous on $[-2,2]$, it must be continuous at the transition points $x=0$ and x = 1.

1. Continuity at $x=0$:

$lim_{x\to 0^-}f(x)=f(0)$

$lim_{x\to 0^-}\left(\frac{\sin ax}{x}+3\right)=0+5$

$a+3=5\Rightarrow a=2$

2. Continuity at $x=1$:

$lim_{x\to 1^-}f(x)=lim_{x\to 1^+}f(x)$

$1+5=\sqrt{1^2+8}-b$

$6=3-b\Rightarrow b=-3$

3. Evaluating the required expression:

$7a+b+1=7(2)+(-3)+1$

$=14-3+1=12$

64. (A)

For $f(x)$ to be continuous everywhere on $\mathbb{R}$, it must be continuous at the transition points $x=1$, $x=3$, and $x=5$.

1. Continuity at $x=1$:

$lim_{x\to 1^-}f(x)=f(1)=5$

$lim_{x\to 1^+}f(x)=a+b$

$a+b=5$--- (1)

2. Continuity at $x=3$:

$lim_{x\to 3^-} f(x)=a+3b$

$lim_{x\to 3^+} f(x)=f(3)=b+15$

$a+3b=b+15\Rightarrow a+2b=15$--- (2)

3. Continuity at $x=5$:

$lim_{x\to 5^-} f(x)=b+25$

$lim_{x\to 5^+} f(x)=f(5)=30$

$b+25=30\Rightarrow b=5$--- (3)

Substituting $b=5$ from equation (3) into equation (1):

$a+5=5\Rightarrow a=0$

Now, substituting $a=0$ and $b=5$ into the left-hand side of equation (2):

$a+2b=0+2(5)=10\neq 15$

Since the three conditions are inconsistent, no real values of a and b can make $f(x)$ continuous everywhere.

65. (A)

A conditional statement $A\to B$ has the truth value 'F' if and only if A is 'T' and B is 'F'.

Here, for $[(p\wedge\sim r)\to(\sim p\vee q)]\equiv$ F:

1. $p\wedge\sim r\equiv$ T

   This implies that $p\equiv$ T and $\sim r\equiv$ T, which gives $r\equiv$ F.

2. $\sim p\vee q\equiv$ F

   This implies that $\sim p\equiv$ F (which is consistent with $p\equiv$ T) and $q\equiv$ F.

   Therefore, the truth values of p, q, and r are T, F, and F respectively.

66. (D)

Given statement: $P(n): n^2-n+37$ is prime.

1. For $n=3$:

   $P(3)=3^2-3+37=9-3+37=43$

   Since 43 is a prime number, $P(3)$ is true.

2. For $n=5$:

   $P(5)=5^2-5+37=25-5+37=57$

   Since $57=3\times 19$, it is divisible by 3 and 19, making it a composite number (not prime). Thus, $P(5)$ is false.

Hence, $P(5)$ is false, but $P(3)$ is true.

67. (D)

The given statement is an implication of the form $P\Rightarrow Q$, where:

- P: $\forall x, x$ is a complex number
- Q: $x^2<0$

The negation of a conditional statement $P\Rightarrow Q$ is logically equivalent to $P\wedge\sim Q$.

1. P remains: "$\forall x, x$ is a complex number"
2. The negation of Q ($\sim Q$) is: "$x^2\geq 0$"

   Therefore, the negation of the given statement $P\Rightarrow Q$ is $P\wedge\sim Q$, which is:

   "$\forall x, x$ is a complex number and $x^2\geq 0$"

68. (B)

Constructing the truth table for the given statement pattern $[(p\to q)\to p]\to[(p\to q)\vee p]$:

p	q	$p\to q$	$(p\to q)\to p$	$(p\to q)\vee p$	$[(p\to q)\to p]\to[(p\to q)\vee p]$
T	T	T	T	T	T
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	F	T	T

Alternative Logical Equivalence Method:

Using the laws of logic:

1. Antecedent:

   $[(p\to q)\to p]\equiv\sim(\sim p\vee q)\vee p$

   $\equiv(p\wedge\sim q)\vee p\equiv p$

2. Consequent:

$$\left[(p \to q) \vee p\right] \equiv (\sim p \vee q) \vee p$$
$$\equiv (\sim p \vee p) \vee q$$
$$\equiv \mathrm{T} \vee q \equiv \mathrm{T}$$

3. Given expression:
$$\left[(p \to q) \to p\right] \to \left[(p \to q) \vee p\right]$$
$$\equiv p \to \mathrm{T} \equiv \mathrm{T}$$

Since all truth values in the final column are T, the statement pattern is always true, which makes it a tautology.

69. (A)

1. Analysis of Statement S_1:

By the definition of conditional implication:
$p \to \sim q \equiv \sim p \vee \sim q$

By De Morgan's Law:
$\sim p \vee \sim q \equiv \sim (p \wedge q)$

Taking the negation on both sides:
$\sim (p \to \sim q) \equiv \sim (\sim (p \wedge q)) \equiv p \wedge q$

Thus, $p \wedge q \equiv \sim (p \to \sim q)$. Therefore, statement S_1 is correct.

2. Analysis of Statement S_2:

$(p \wedge q) \wedge (p \vee q) \equiv p \wedge q$

This expression evaluates to False when either p or q is False, so it is a contingency, not a tautology. Thus, S_2 is incorrect.

3. Analysis of Statement S_3:

$$\left[p \wedge (p \to q)\right] \to q \equiv \left[p \wedge (\sim p \vee q)\right] \to q$$
$$\equiv (p \wedge q) \to q$$
$$\equiv \sim (p \wedge q) \vee q \equiv \sim p \vee \sim q \vee q \equiv T$$

This statement is a tautology, not a contradiction. Thus, S_3 is incorrect.

70. (B)

First, determine the truth values of the given simple statements:

- p : "25 is an odd prime number" is False (F), because $25 = 5 \times 5$ is a composite number.
- q: "14 is a composite number" is True (T), because $14 = 2 \times 7$.
- r : "64 is a perfect square number" is True (T), because $64 = 8^2$.

Thus, the truth values are $p \equiv \mathrm{F}$, $q \equiv \mathrm{T}$, and $r \equiv \mathrm{T}$.

Now, evaluate the given statement patterns:

1. $\sim (q \wedge r) \vee p \equiv \sim (\mathrm{T} \wedge \mathrm{T}) \vee \mathrm{F} \equiv \mathrm{F} \vee \mathrm{F} \equiv \mathrm{F}$
2. $\sim p \vee (q \wedge r) \equiv \sim \mathrm{F} \vee (\mathrm{T} \wedge \mathrm{T}) \equiv \mathrm{T} \vee \mathrm{T} \equiv \mathrm{T}$
3. $(p \wedge q) \wedge r \equiv (\mathrm{F} \wedge \mathrm{T}) \wedge \mathrm{T} \equiv \mathrm{F} \wedge \mathrm{T} \equiv \mathrm{F}$
4. $(p \vee q) \wedge (\sim r) \equiv (\mathrm{F} \vee \mathrm{T}) \wedge \sim \mathrm{T} \equiv \mathrm{T} \wedge \mathrm{F} \equiv \mathrm{F}$

Therefore, statement pattern (2) evaluates to True (T).

71. (C)

Let p and q be two statements defined as:

- p : A quadrilateral $ABCD$ is a square.
- q : All sides of quadrilateral $ABCD$ are equal.

Statement I is a conditional statement of the form $p \Rightarrow q$ ("If p, then q").

The converse of a conditional statement $p \Rightarrow q$ is given by $q \Rightarrow p$ ("If q, then p").

Since Statement II is "If all the sides of a quadrilateral $ABCD$ are equal, then $ABCD$ is a square", it is of the form $q \Rightarrow p$.

Thus, Statement II is the converse of Statement I.

72. (C)

Using the conditional law of logic, $A \to B \equiv \sim A \vee B$.

The left-hand side statement expression can be written as:
$p \to (q \vee r) \equiv \sim p \vee (q \vee r)$

Now, evaluating option (3):
$(p \to q) \vee (p \to r) \equiv (\sim p \vee q) \vee (\sim p \vee r)$

By applying the associative, commutative, and idempotent laws of disjunction ($\vee$):

$(\sim p \vee q) \vee (\sim p \vee r) \equiv (\sim p \vee \sim p) \vee (q \vee r)$

$\equiv \sim p \vee (q \vee r)$

Since both expressions reduce to $\sim p \vee (q \vee r)$, we have:

$p \to (q \vee r) \equiv (p \to q) \vee (p \to r)$

73. (B)

Either the work is finished in time and the payment is not made or the payment is made and the work is not finished in time

Let:

p: The payment will be made.

q: The work is finished in time.

The given statement is a biconditional statement represented symbolically as $p \Leftrightarrow q$.

The negation of a biconditional statement $p \Leftrightarrow q$ is logically equivalent to:

$\sim (p \Leftrightarrow q) \equiv (q \wedge \sim p) \vee (p \wedge \sim q)$

Translating the components back into statements:

- $q \wedge \sim p$: The work is finished in time and the payment is not made.
- $p \wedge \sim q$: The payment is made and the work is not finished in time.

Therefore, the negation $\sim (p \Leftrightarrow q)$ is:

"Either the work is finished in time and the payment is not made or the payment is made and the work is not finished in time".

74. (D)

Find K so that the printed complex matrix is singular.

Its determinant simplifies to

$(4+3i)(K+7).$

Thus $K = -7$.

75. (D)

Given matrix equation:

$(A-2I)(A-4I) = 0$

Expanding the expression:

$A^2 - 4A - 2A + 8I = 0$

$A^2 - 6A + 8I = 0$

Since A is a non-singular matrix, A^{-1} exists. Multiplying both sides by A^{-1}:

$A^{-1}(A^2 - 6A + 8I) = 0$

$A - 6I + 8A^{-1} = 0$

$A + 8A^{-1} = 6I$

Dividing both sides by 6:

$\frac{1}{6}A + \frac{8}{6}A^{-1} = I$

$\frac{1}{6}A + \frac{4}{3}A^{-1} = I$

76. (A)

Using the reversal law for the inverse of a matrix product:

$(BA)^{-1} = A^{-1}B^{-1}$

Given that $(BA)^{-1} = C$, we have:

$A^{-1}B^{-1} = C$

Post-multiplying both sides by B:

$A^{-1}B^{-1}B = CB$

$A^{-1} = CB$

Evaluating the matrix multiplication CB:

$$A^{-1} = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 1 & 3 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & -6 & -4 & 3 & -5 & -5 \\ 1 & 0 & 1 & 0 & 9 & 2 \\ -1 & -1 & 1 & 2 & 14 & 6 \end{bmatrix}$$

Thus, option (1) is the correct choice.

77. (D)

By the Cayley-Hamilton Theorem, matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$ satisfies its characteristic equation $det(A - \lambda I) = 0$:

$(1-\lambda)(5-\lambda) - (2)(3) = 0$

$\lambda^2 - 6\lambda - 1 = 0$

Replacing λ by A:

$A^2 - 6A - I = O$

Multiplying both sides by A^{-1}:

$A-6I-A^{-1}=O$

$A^{-1}=-6I+A$

Comparing $A^{-1}=-6I+A$ with the given equation $A^{-1}=\alpha I+\beta A$:

$\alpha=-6 \text{and} \beta=1$

Now, calculating the required expression:

$\alpha+\beta+\alpha\beta=-6+1+(-6)(1)=-5-6=-11$

78. (B)

Determine a,b from the printed matrix and its inverse.

Imposing $AA^{-1}=I$ gives

$a=2, \Rightarrow b=-1.$

79. (D)

For any square matrix A of order 3, the property relating A and its adjoint is:

$$A(\text{adj } A)=|A|I_3=\begin{bmatrix}|A| & 0 & 0\\ 0 & |A| & 0\\ 0 & 0 & |A|\end{bmatrix}$$

Given matrix $A=\begin{bmatrix}3 & 2 & x\\ 1 & 4 & y\\ 2 & 2 & z\end{bmatrix}$.

Expanding the determinant $|A|$ along the first row:

$|A|=3(4z-2y)-2(z-2y)+x(2-8)$

$|A|=12z-6y-2z+4y-6x$

$|A|=10z-2y-6x$

Given the conditions, the real values satisfying $xyz=60$ are $x=4$, $y=3$, and $z=5$:

$xyz=4\times3\times5=60$

Substituting these values into the expression for $|A|$:

$|A|=10(5)-2(3)-6(4)=50-6-24=20$

Thus,

$$A(adj\,A)=|A|I=\begin{bmatrix}68 & 0 & 0\\ 0 & 68 & 0\\ 0 & 0 & 68\end{bmatrix}$$

80. (B)

Calculations:

Given the matrix $A=\begin{bmatrix}2 & 3\\ 4 & 1\end{bmatrix}$, we first calculate A^2:

$A^2=A\cdot A=\begin{bmatrix}2 & 3\\ 4 & 1\end{bmatrix}\begin{bmatrix}2 & 3\\ 4 & 1\end{bmatrix}$

$\begin{bmatrix}2(2)+3(4) & 2(3)+3(1)\\ 4(2)+1(4) & 4(3)+1(1)\end{bmatrix}\begin{bmatrix}16 & 9\\ 12 & 13\end{bmatrix}$

Now, compute $3A^2+12A$:

$3A^2=3\begin{bmatrix}16 & 9\\ 12 & 13\end{bmatrix}=\begin{bmatrix}48 & 27\\ 36 & 39\end{bmatrix}$

$12A=12\begin{bmatrix}2 & 3\\ 4 & 1\end{bmatrix}=\begin{bmatrix}24 & 36\\ 48 & 12\end{bmatrix}$

$3A^2+12A=\begin{bmatrix}48+24 & 27+36\\ 36+48 & 39+12\end{bmatrix}=\begin{bmatrix}72 & 63\\ 84 & 51\end{bmatrix}$

For a 2×2 matrix $P=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$, interchanging the primary diagonal elements gives:

$\text{adj}(3A^2+12A)=\begin{bmatrix}51 & 63\\ 84 & 72\end{bmatrix}$

81. (A)

To find $(A^{-1})^3$, we can use the matrix property $(A^{-1})^3=(A^3)^{-1}$.

First, calculate A^2:

$A^2=A\cdot A=\begin{bmatrix}3 & 2\\ 0 & 1\end{bmatrix}\begin{bmatrix}3 & 2\\ 0 & 1\end{bmatrix}=\begin{bmatrix}9 & 8\\ 0 & 1\end{bmatrix}$

Next, calculate A^3:

$$A^3 = A^2 \cdot A = \begin{bmatrix} 9 & 8 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 27 & 26 \\ 0 & 1 \end{bmatrix}$$

Now, find the inverse of A^3. For a 2×2 matrix $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, the inverse is given by $M^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

Here, $|A^3| = (27)(1) - (26)(0) = 27$.

$$(A^{-1})^3 = (A^3)^{-1} = \frac{1}{27}\begin{bmatrix} 1 & -26 \\ 0 & 27 \end{bmatrix} = \begin{bmatrix} \frac{1}{27} & -\frac{26}{27} \\ 0 & 1 \end{bmatrix}$$

82. (A)

Given the matrix equation:

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 3 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 4 \end{bmatrix}$$

Multiplying the matrices yields the system of linear equations:

$x + y + z = 0$--- (1)

$x + 2y + 2z = 3$--- (2)

$x + 3y + z = 4$--- (3)

Subtracting equation (1) from equation (2):

$(x + 2y + 2z) - (x + y + z) = 3 - 0 \Rightarrow y + z = 3$

Substituting $y + z = 3$ into equation (1):

$x + 3 = 0 \Rightarrow x = -3$

Subtracting equation (1) from equation (3):

$(x + 3y + z) - (x + y + z)$

$= 4 - 0 \Rightarrow 2y = 4 \Rightarrow y = 2$

Substituting $y = 2$ into $y + z = 3$:

$2 + z = 3 \Rightarrow z = 1$

Now, evaluating $2x + y + z$:

$2x + y + z = 2(-3) + 2 + 1 = -3$

83. (C)

First, calculate the determinant of the 3×3 matrix A:

$$|A| = \begin{vmatrix} 1 & -1 & 1 \\ 0 & 2 & -3 \\ 2 & 1 & 0 \end{vmatrix} = 1(0+3) - (-1)(0+6) + 1(0-4)$$

$= 3 + 6 - 4 = 5$

For a matrix A of order $n = 3$:

1. $B = \text{adj } A \Rightarrow |B| = |A|^{n-1} = |A|^2 = 5^2 = 25$
2. $|\text{adj } B| = |\text{adj}(\text{adj } A)| = |A|^{(n-1)^2} = |A|^4$
   $= 5^4 = 625$
3. $C = 5A \Rightarrow |C| = |5A| = 5^3|A|$
   $= 125 \times 5 = 625$

Now, evaluating the required ratio:

$$\frac{|\text{adj } B|}{|C|} = \frac{625}{625} = 1$$

84. (A)

Evaluate $2\tan^{-1}(1/3) + \cos^{-1}(3/5)$.

$2\tan^{-1}(1/3) = \tan^{-1}(3/4)$, while $\cos^{-1}(3/5) = \tan^{-1}(4/3)$. These complementary angles sum to $\frac{\pi}{2}$.

85. (C)

Using the formula $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$ for $xy < 1$:

First, group the terms pairwise:

$$\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{5}\right) = \tan^{-1}\left(\frac{\frac{1}{3}+\frac{1}{5}}{1-\frac{1}{3}\cdot\frac{1}{5}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{8}{15}}{\frac{14}{15}}\right) = \tan^{-1}\left(\frac{4}{7}\right)$$

$$\tan^{-1}\left(\frac{1}{7}\right)+\tan^{-1}\left(\frac{1}{8}\right)=\tan^{-1}\left(\frac{\frac{1}{7}+\frac{1}{8}}{1-\frac{1}{7}\cdot\frac{1}{8}}\right)=\tan^{-1}\left(\frac{\frac{15}{56}}{\frac{55}{56}}\right)$$

$$=\tan^{-1}\left(\frac{3}{11}\right)$$

Now, add the two resulting terms:

$$\tan^{-1}\left(\frac{4}{7}\right)+\tan^{-1}\left(\frac{3}{11}\right)=\tan^{-1}\left(\frac{\frac{4}{7}+\frac{3}{11}}{1-\frac{4}{7}\cdot\frac{3}{11}}\right)$$

$$=\tan^{-1}\left(\frac{\frac{65}{77}}{\frac{65}{77}}\right)=\tan^{-1}(1)=\frac{\pi}{4}$$

86. (D)

Convert $\left(-\frac{5\sqrt{3}}{2},\frac{5}{2}\right)$ to polar coordinates.

$r=5$. The point is in quadrant II with reference angle $\frac{\pi}{6}$, so $\theta=\frac{5\pi}{6}$.

87. (A)

To find the elements of set A, we solve the equation:

$$\tan^{-1}(2x)+\tan^{-1}(3x)=\frac{\pi}{4},\text{for } x\geq 0$$

Taking tangent on both sides using the identity $\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$:

$$\frac{2x+3x}{1-(2x)(3x)}=\tan\left(\frac{\pi}{4}\right)$$

$$\frac{5x}{1-6x^2}=1$$

$$6x^2+5x-1=0$$

$$(6x-1)(x+1)=0$$

This gives two roots: $x=\frac{1}{6}$ and $x=-1$.

Since the set specifies the condition $x\geq 0$:

- $x=-1$ is rejected as $-1<0$.
- For $x=\frac{1}{6}$, the product of terms $(2x)(3x)=6\left(\frac{1}{36}\right)=\frac{1}{6}<1$, which satisfies the principal value branch condition for $\tan^{-1}$.

Thus, set $A=\left\{\frac{1}{6}\right\}$, which contains exactly one element. Hence, A is a singleton set.

88. (B)

Using the identity $\cos^{-1}x+\sin^{-1}x=\frac{\pi}{2}$ for $x\in[-1,1]$, the given expression can be rewritten as:

$$\cos\left(2\cos^{-1}x+\sin^{-1}x\right)$$

$$=\cos\left(\cos^{-1}x+\left(\cos^{-1}x+\sin^{-1}x\right)\right)$$

$$\cos\left(2\cos^{-1}x+\sin^{-1}x\right)=\cos\left(\frac{\pi}{2}+\cos^{-1}x\right)$$

Using the trigonometric identity $\cos\left(\frac{\pi}{2}+\theta\right)=-\sin\theta$, we get:

$$\cos\left(\frac{\pi}{2}+\cos^{-1}x\right)=-\sin\left(\cos^{-1}x\right)=-\sqrt{1-x^2}$$

Substituting $x=\frac{1}{5}$:

$$-\sqrt{1-\left(\frac{1}{5}\right)^2}=-\sqrt{1-\frac{1}{25}}=-\sqrt{\frac{24}{25}}=-\frac{2\sqrt{6}}{5}$$

89. (D)

Calculation:

Given $x\in(0,1)$, we have $1\cdot x<1$. Using the addition formula for inverse tangent:

$$\tan^{-1}\left(\frac{1+x}{1-x}\right)=\tan^{-1}(1)+\tan^{-1}x=\frac{\pi}{4}+\tan^{-1}x$$

Thus, the value of A becomes:

$$A = 2\tan^{-1}\left(\frac{1+x}{1-x}\right) = 2\left(\frac{\pi}{4} + \tan^{-1}x\right)$$

$$= \frac{\pi}{2} + 2\tan^{-1}x$$

For $x \in (0,1)$, using the standard identity $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = 2\tan^{-1}x$:

$$B = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = 2\tan^{-1}x$$

Subtracting B from A:

$$A - B = \left(\frac{\pi}{2} + 2\tan^{-1}x\right) - 2\tan^{-1}x = \frac{\pi}{2}$$

90. (D)

In ΔABC, since $\angle C = 90^\circ$, c is the hypotenuse and by Pythagoras theorem:

$c^2 = a^2 + b^2$

Also, $A + B = 90^\circ$, so we have:

$$\sin A = \frac{a}{c}, \cos A = \frac{b}{c}$$

$$\sin B = \cos A = \frac{b}{c}, \cos B = \sin A = \frac{a}{c}$$

Using the compound angle formula for sine:

$$\sin(A-B) = \sin A\cos B - \cos A\sin B$$

$$\sin(A-B) = \left(\frac{a}{c}\right)\left(\frac{a}{c}\right) - \left(\frac{b}{c}\right)\left(\frac{b}{c}\right) = \frac{a^2-b^2}{c^2}$$

Substituting $c^2 = a^2 + b^2$:

$$\sin(A-B) = \frac{a^2-b^2}{a^2+b^2}$$

91. (D)

Let the sides of $\triangle ABC$ be $a = 7k$, $b = 8k$, and $c = 9k$, where $k > 0$.

Using the cosine rule:

$$\cos A = \frac{b^2+c^2-a^2}{2bc} = \frac{(8k)^2+(9k)^2-(7k)^2}{2(8k)(9k)}$$

$$= \frac{64+81-49}{144} = \frac{96}{144} = \frac{2}{3}$$

$$\cos B = \frac{a^2+c^2-b^2}{2ac} = \frac{(7k)^2+(9k)^2-(8k)^2}{2(7k)(9k)}$$

$$= \frac{49+81-64}{126} = \frac{66}{126} = \frac{11}{21}$$

$$\cos C = \frac{a^2+b^2-c^2}{2ab} = \frac{(7k)^2+(8k)^2-(9k)^2}{2(7k)(8k)}$$

$$= \frac{49+64-81}{112} = \frac{32}{112} = \frac{2}{7}$$

Multiplying each term by the LCM of the denominators, which is 21:

$$\cos A : \cos B : \cos C = \left(\frac{2}{3}\times 21\right) : \left(\frac{11}{21}\times 21\right) : \left(\frac{2}{7}\times 21\right)$$

$$= 14 : 11 : 6$$

92. (A)

Let $\theta = \cot^{-1}x$, which implies $\cot\theta = x$.

Using the trigonometric identity $\sin\theta = \frac{1}{\sqrt{1+\cot^2\theta}}$, we get:

$$\sin\left(\cot^{-1}x\right) = \frac{1}{\sqrt{x^2+1}}$$

Substituting this into the given expression yields:

$$\cos\left(\tan^{-1}\left(\frac{1}{\sqrt{x^2+1}}\right)\right)$$

Now, let $\phi = \tan^{-1}\left(\frac{1}{\sqrt{x^2+1}}\right)$, so

$$\tan\phi = \frac{1}{\sqrt{x^2+1}}.$$

Using the identity $\cos\phi = \frac{1}{\sqrt{1+\tan^2\phi}}$, we have:

$$\cos\phi = \frac{1}{\sqrt{1+\left(\frac{1}{\sqrt{x^2+1}}\right)^2}} = \frac{1}{\sqrt{1+\frac{1}{x^2+1}}}$$

$$= \frac{1}{\sqrt{\frac{x^2+2}{x^2+1}}} = \sqrt{\frac{x^2+1}{x^2+2}}$$

93. (B)

We know the standard inverse trigonometric identities:

$$\sin^{-1}\left(\frac{2x}{1+x^2}\right) = 2\tan^{-1}x$$

$$\cos^{-1}\left(\frac{1-y^2}{1+y^2}\right) = 2\tan^{-1}y$$

Substituting these identities into the given expression, we get:

$$\tan\left\{\frac{1}{2}\sin^{-1}\left(\frac{2x}{1+x^2}\right)+\frac{1}{2}\cos^{-1}\left(\frac{1-y^2}{1+y^2}\right)\right\}$$

$$= \tan\left\{\frac{1}{2}\left(2\tan^{-1}x\right)+\frac{1}{2}\left(2\tan^{-1}y\right)\right\}$$

simplifying the terms inside the bracket:

$$= \tan\left(\tan^{-1}x+\tan^{-1}y\right)$$

Using the addition formula for inverse tangent, $\tan^{-1}x+\tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$:

$$= \tan\left(\tan^{-1}\left(\frac{x+y}{1-xy}\right)\right)$$

$$= \frac{x+y}{1-xy}$$

Hence, the correct option is (2).

94. (B)

Let $y = \cot^{-1}\left(\frac{-1}{\sqrt{3}}\right)$.

The principal value branch of $\cot^{-1}x$ is $(0,\pi)$.

$$\cot y = -\frac{1}{\sqrt{3}}$$

$$\cot y = -\cot\left(\frac{\pi}{3}\right) = \cot\left(\pi\frac{\pi}{3}\right) = \cot\left(\frac{2\pi}{3}\right)$$

Since $\frac{2\pi}{3} \in (0,\pi)$, the principal value of $\cot^{-1}\left(\frac{1}{\sqrt{3}}\right)$ is $\frac{2\pi}{3}$.

95. (A)

Calculation:

Using the principal value branches of inverse trigonometric functions:

1. $\operatorname{cosec}^{-1}\left(\sqrt{2}\right) = \frac{\pi}{4}$
2. $\cos^{-1}\left(\frac{1}{2}\right) = \pi - \cos^{-1}\left(\frac{1}{2}\right) = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$
3. $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = \frac{\pi}{6}$

Substituting these values into the given expression:

$$\operatorname{cosec}^{-1}\left(\sqrt{2}\right)+\cos^{-1}\left(\frac{1}{2}\right)-\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$$

$$= \frac{\pi}{4}+\frac{2\pi}{3}-\frac{\pi}{6}$$

Taking the common denominator 12:

$$= \frac{3\pi+8\pi-2\pi}{12} = \frac{9\pi}{12} = \frac{3\pi}{4}$$

96. (B)

Let $\theta = \sin^{-1}(0.8)$, which gives $\sin\theta = 0.8$.

Since $\theta \in \left(0,\frac{\pi}{2}\right)$, $\cos\theta$ is positive:

$$\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-(0.8)^2} = \sqrt{0.36} = 0.6$$

Using the double-angle formula $\sin(2\theta) = 2\sin\theta\cos\theta$:

$$\sin\left(2\sin^{-1}0.8\right)=\sin\left(2\theta\right)=2\times0.8\times0.6=0.96$$

97. (C)

Solve $\tan^{-1}\frac{1-x}{1+x}=\frac{1}{2}\tan^{-1}x$.

Let $t=\tan^{-1}x$. Then $\frac{\pi}{4}-t=\frac{t}{2}$, so $t=\frac{\pi}{6}$ and $x=\frac{1}{\sqrt{3}}$.

98. (D)

Let $\cot^{-1}\left(x+1\right)=\theta$, then $\cot\theta=x+1$.

$$\sin\left(\cot^{-1}\left(x+1\right)\right)=\sin\theta=\frac{1}{\sqrt{1+\cot^2\theta}}$$

$$=\frac{1}{\sqrt{1+(x+1)^2}}=\frac{1}{\sqrt{x^2+2x+2}}$$

Let $\tan^{-1}x=\phi$, then $\tan\phi=x$.

$$\cos\left(\tan^{-1}x\right)=\cos\phi=\frac{1}{\sqrt{1+\tan^2\phi}}=\frac{1}{\sqrt{1+x^2}}$$

Equating both expressions as per the given condition:

$$\frac{1}{\sqrt{x^2+2x+2}}=\frac{1}{\sqrt{1+x^2}}$$

Squaring both sides:

$$x^2+2x+2=1+x^2$$

$$2x+2=1$$

$$2x=-1\Rightarrow x=-\frac{1}{2}$$

99. (D)

Calculation:

The given line is $y=x$, which has slope $m_0=1$ and inclination 45° with the positive x-axis.

Let $y=mx$ be the equation of any line passing through the origin and making an angle α with $y=x$.

Using the formula for the angle between two lines:

$$\tan\alpha=\left|\frac{m-1}{1+m}\right|$$

Squaring both sides:

$$\tan^2\alpha=\frac{\left(m-1\right)^2}{\left(m+1\right)^2}$$

$$\left(m-1\right)^2=\tan^2\alpha\left(m+1\right)^2$$

$$m^2-2m+1=\tan^2\alpha\left(m^2+2m+1\right)$$

$$m^2\left(1-\tan^2\alpha\right)-2m\left(1+\tan^2\alpha\right)+\left(1-\tan^2\alpha\right)=0$$

Dividing throughout by $\left(1\tan^2\alpha\right)$:

$$m^2-2m\left(\frac{1+\tan^2\alpha}{1-\tan^2\alpha}\right)+1=0$$

Since $\sec2\alpha=\frac{1+\tan^2\alpha}{1-\tan^2\alpha}$:

$$m^2-2m\sec2\alpha+1=0$$

Substituting $m=\frac{y}{x}$:

$$\left(\frac{y}{x}\right)^2-2\left(\frac{y}{x}\right)\sec2\alpha+1=0$$

Multiplying throughout by x^2:

$$x^2-2xy\sec2\alpha+y^2=0$$

100. (C)

Let m_1 and m_2 be the slopes of the two lines represented by the homogeneous quadratic equation $ax^2+2hxy+by^2=0$.

The sum and product of the slopes are given by:

$$m_1 + m_2 = -\frac{2h}{b}$$

$$m_1 m_2 = \frac{a}{b}$$

According to the given condition, $m_2 = 2m_1$.

Substituting $m_2 = 2m_1$ into the sum of slopes formula:

$$m_1 + 2m_1 = -\frac{2h}{b}$$

Now, substituting $m_2 = 2m_1$ into the product of slopes formula:

$$m_1(2m_1) = \frac{a}{b}$$

$$2m_1^2 = \frac{a}{b}$$

Substitute $m_1 = -\frac{2h}{3b}$:

$$2\left(\frac{2h}{3b}\right)^2 = \frac{a}{b}$$

$$2\left(\frac{4h^2}{9b^2}\right) = \frac{a}{b}$$

$$\frac{8h^2}{9b^2} = \frac{a}{b}$$

$$8h^2 = 9ab$$

$$\frac{ab}{h^2} = \frac{8}{9}$$

Thus, $ab : h^2 = 8 : 9$.

101. (C)

The general joint equation of a pair of lines passing through the origin is given by:

$$ax^2 + 2hxy + by^2 = 0$$

Comparing this with the given equation $2x^2 + 5xy + 3y^2 = 0$, we have $a = 2$, $2h = 5$, and $b = 3$.

The joint equation of two lines passing through the origin and perpendicular to the pair of lines $ax^2 + 2hxy + by^2 = 0$ is given by:

$$bx^2 - 2hxy + ay^2 = 0$$

Substituting $a = 2$, $2h = 5$, and $b = 3$:

$$3x^2 - 5xy + 2y^2 = 0$$

102. (C)

The pair of lines represented by $x^2 - 4xy + y^2 = 0$ passes through the origin $(0,0)$.

The angle θ between these lines is given by:

$$\tan\theta = \left|\frac{2\sqrt{H^2 - AB}}{A + B}\right| = \left|\frac{2\sqrt{(-2)^2 - (1)(1)}}{1+1}\right|$$

$$= \sqrt{3} \Rightarrow \theta = 60^\circ$$

Since the lines form an angle of 60° at the origin, the triangle bounded by these lines and $x + y = 10$ is an equilateral triangle.

The altitude h of this triangle is the perpendicular distance from the origin $(0,0)$ to the line $x + y - 10 = 0$:

$$h = \frac{|0 + 0 - 10|}{\sqrt{1^2 + 1^2}} = \frac{10}{\sqrt{2}} = 5\sqrt{2}$$

The area of an equilateral triangle with altitude h is given by:

$$\text{Area} = \frac{h^2}{\sqrt{3}} = \frac{\left(5\sqrt{2}\right)^2}{\sqrt{3}} = \frac{50}{\sqrt{3}} \text{ sq. units}$$

103. (B)

The given homogeneous equation representing a pair of lines is $kx^2 - 4xy + 5y^2 = 0$.

Comparing with the standard equation $ax^2+2hxy+by^2=0$, we get:

$a=k, 2h=-4, b=5$

Let m_1 and m_2 be the slopes of the lines. The sum and product of the slopes are given by:

$$m_1+m_2=-\frac{2h}{b}=\frac{4}{5}$$

$$m_1m_2=\frac{a}{b}=\frac{k}{5}$$

Given that the slopes differ by 2, we have $|m_1m_2|=2$. Squaring both sides:

$$(m_1-m_2)^2=4$$

$$(m_1+m_2)^2-4m_1m_2=4$$

Substitute the values of (m_1+m_2) and m_1m_2:

$$\left(\frac{4}{5}\right)^2-4\left(\frac{k}{5}\right)=4$$

$$\frac{16}{25}-\frac{4k}{5}=4$$

Dividing both sides by 4:

$$\frac{4}{25}-\frac{k}{5}=1$$

$$\frac{k}{5}=\frac{4}{25}-1=-\frac{21}{25}$$

$$k=-\frac{21}{5}$$

104. (A)

Given the acute angle between $kx^2-4xy+y^2=0$ has tangent $1/2$, find k.

Using $\tan\theta=\frac{2\sqrt{4-k}}{|k+1|}=\frac{1}{2}$,

the listed valid value is $k=3$.

105. (C)

Trisecting the first quadrant (angle of 90°) divides it into three equal angles of 30° each. Therefore, the two lines passing through the origin make inclinations of $\theta_1=30^\circ$ and $\theta_2=60^\circ$ with the positive x-axis.

Their respective slopes are:

$$m_1=\tan30^\circ=\frac{1}{\sqrt{3}}$$

$$m_2=\tan60^\circ=\sqrt{3}$$

Since both lines pass through the origin $(0,0)$, their individual equations are:

$$y=\frac{1}{\sqrt{3}}x\Rightarrow x-\sqrt{3}y=0$$

$$y=\sqrt{3}x\Rightarrow \sqrt{3}x-y=0$$

The combined (joint) equation of the pair of lines is given by the product of their individual equations:

$$\left(x-\sqrt{3}y\right)\left(\sqrt{3}x-y\right)=0$$

$$\sqrt{3}x^2-xy-3xy+\sqrt{3}y^2=0$$

$$\sqrt{3}x^2-4xy+\sqrt{3}y^2=0$$

106. (C)

Find the centroid of the triangle formed by $x+3y=10$ and $6x^2+xy-y^2=0$.

The pair factors into $y=3x$ and $y=-2x$. The vertices are $(0,0),(1,3),(-2,4)$, so the centroid is $\left(-\frac{1}{3},\frac{7}{3}\right)$.

107. (B)

Given vectors are $\vec{a} = 3\hat{i} + \hat{j} - 4\hat{k}$ and $\vec{b} = 6\hat{i} + 5\hat{j} - 2\hat{k}$.

A vector perpendicular to both $\vec{a}$ and $\vec{b}$ is given by their cross product $\vec{a} \times \vec{b}$:

$$\vec{a} \times \vec{b} = \hat{i}\left[(1)(-2) - (-4)(5)\right]$$
$$-\hat{j}\left[(3)(-2) - (-4)(6)\right]$$
$$+\hat{k}\left[(3)(5) - (1)(6)\right]$$

$$\vec{a} \times \vec{b} = 18\hat{i} - 18\hat{j} + 9\hat{k}$$

The magnitude of $\vec{a} \times \vec{b}$ is:

$$\left|\vec{a} \times \vec{b}\right| = \sqrt{18^2 + (-18)^2 + 9^2}$$
$$= \sqrt{324 + 324 + 81} = \sqrt{729} = 27$$

The unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ is:

$$\hat{n} = \pm\frac{\vec{a} \times \vec{b}}{\left|\vec{a} \times \vec{b}\right|} = \pm\frac{18\hat{i} - 18\hat{j} + 9\hat{k}}{27}$$
$$= \pm\left(\frac{2}{3}\hat{i} - \frac{2}{3}\hat{j} + \frac{1}{3}\hat{k}\right)$$

Therefore, a vector of magnitude 3 units perpendicular to both vectors is:

$$\vec{r} = \pm 3\hat{n} = \pm 3\left(\frac{2}{3}\hat{i} - \frac{2}{3}\hat{j} + \frac{1}{3}\hat{k}\right) = \pm\left(2\hat{i} - 2\hat{j} + \hat{k}\right)$$

108. (D)

We use the vector identity:

$$\left|\vec{a} \times \vec{b}\right|^2 + (\vec{a} \cdot \vec{b})^2 = |\vec{a}|^2 \left|\vec{b}\right|^2$$

Given $|\vec{a}| = \sqrt{26}$, $\left|\vec{b}\right| = 7$, and $\left|\vec{a} \times \vec{b}\right| = 35$, substituting these values gives:

$$35^2 + (\vec{a} \cdot \vec{b})^2 = (\sqrt{26})^2 \times 7^2$$

$$1225 + (\vec{a} \cdot \vec{b})^2 = 26 \times 49$$

$$1225 + (\vec{a} \cdot \vec{b})^2 = 1274$$

$$(\vec{a} \cdot \vec{b})^2 = 1274 - 1225 = 49$$

$$\vec{a} \cdot \vec{b} = \pm\sqrt{49} = \pm 7$$

109. (A)

The magnitude of the vector cross product of two vectors $\vec{u}$ and $\vec{v}$ is given by:

$$\left|\vec{u} \times \vec{v}\right| = |\vec{u}||\vec{v}|\sin\theta$$

where θ is the angle between the two vectors.
From the given figure:

$$\left|\vec{u}\right| = 4$$

$$\left|\vec{v}\right| = 5$$

- The angle between the directions of the two vectors is $\theta = 150^\circ$ (or its supplement 30°, for both of which $\sin\theta = \frac{1}{2}$).

Substituting these values into the formula:

$$\left|\vec{u} \times \vec{v}\right| = 4 \times 5 \times \sin 150^\circ = 20 \times \frac{1}{2} = 10$$

110. (B)

Three vectors $\vec{u} = a\hat{i} + a\hat{j} + c\hat{k}$, $\vec{v} = \hat{i} + 0\,\hat{j} + \hat{k}$, and $\vec{w} = c\hat{i} + c\hat{j} + b\hat{k}$ lie in the same plane if their scalar triple product is equal to zero:

$$\begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix} = 0$$

Expanding the determinant along the first row:

$$a(0 - c) - a(b - c) + c(c - 0) = 0$$

$$-ac - ab + ac + c^2 = 0$$

$$c^2 - ab = 0$$

$$c^2 = ab \Rightarrow c = \sqrt{ab}$$

Since a,b,c are distinct non-negative numbers, $c=\sqrt{ab}$ means c is the geometric mean of a and b.

111. (B)

The vector projection of $\vec{b}$ on $\vec{a}$ is given by the formula:

$$\text{Vector projection}=\left(\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|^2}\right)\vec{a}$$

Given:

$\vec{a}=3\hat{i}+2\hat{j}+5\hat{k}$

$\vec{b}=7\hat{i}-5\hat{j}-\hat{k}$

Calculate the scalar product $\vec{a}\cdot\vec{b}$:

$\vec{a}\cdot\vec{b}=(3)(7)+(2)(-5)+(5)(-1)=21-10-5=6$

Calculate $|\vec{a}|^2$:

$|\vec{a}|^2=3^2+2^2+5^2=9+4+25=38$

Substitute these values into the vector projection formula:

$$\text{Vector projection}=\frac{6}{38}\left(3\hat{i}+2\hat{j}+5\hat{k}\right)$$

$$=\frac{3}{19}\left(3\hat{i}+2\hat{j}+5\hat{k}\right)=\frac{9\hat{i}+6\hat{j}+15\hat{k}}{19}$$

112. (A)

Given that $|\vec{a}|=2.$, $|\vec{b}|=1$, and $\vec{a}\cdot\vec{b}=-1$.

The angle θ between two vectors $\vec{a}$ and $\vec{b}$ is given by:

$$\cos\theta=\frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$$

Substituting the given values into the formula:

$$\cos\theta=\frac{-1}{2\times1}=-\frac{1}{2}$$

Since $0\le\theta\le\pi$, we get:

$$\theta=\cos^{-1}\left(\frac{1}{2}\right)=\pi-\frac{\pi}{3}=\frac{2\pi}{3}$$

113. (B)

Since $\vec{c}$ is a linear combination of $\vec{a}$ and $\vec{b}$, there exist real scalars λ and μ such that:

$\vec{c}=\lambda\vec{a}+\mu\vec{b}$

Substituting $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=\hat{i}-\hat{j}+2\hat{k}$, and $\vec{c}=x\hat{i}+(x-2)\hat{j}-\hat{k}$:

$$x\hat{i}+(x-2)\hat{j}-\hat{k}=\lambda\left(\hat{i}+\hat{j}+\hat{k}\right)+\mu\left(\hat{i}-\hat{j}+2\hat{k}\right)$$

Equating the corresponding components of $\hat{i}$, $\hat{j}$, and $\hat{k}$:

$\lambda+\mu=x$--- (1)

$\lambda-\mu=x-2$--- (2)

$\lambda+2\mu=-1$--- (3)

Subtracting equation (2) from equation (1):

$(\lambda+\mu)-(\lambda-\mu)=x-(x-2)$

$2\mu=2\Rightarrow\mu=1$

Substituting $\mu=1$ into equation (3):

$\lambda+2(1)=-1\Rightarrow\lambda=-3$

Substituting the values of λ and μ into equation (1):

$x=-3+1=-2$

114. (D)

Given $\vec{a}+2\vec{b}+3\vec{c}=0$, simplify the sum of cyclic cross products.

$\vec{a}=-2\vec{b}-3\vec{c}$ gives $\vec{a}\times\vec{b}=3\vec{b}\times\vec{c}$ and $\vec{c}\times\vec{a}=2\vec{b}\times\vec{c}$. Thus the cyclic sum is $6\vec{b}\times\vec{c}$.

115. (B)

The position vectors of vertices A, B, and C are:

$\vec{a} = \hat{i} - 3\hat{j} - 5\hat{k}$

$\vec{b} = 3\hat{i} - 4\hat{j} - 4\hat{k}$

$\vec{c} = 2\hat{i} - \hat{j} + \hat{k}$

The vectors $\overrightarrow{BA}$ and $\overrightarrow{BC}$ representing the sides adjacent to angle B are:

$\overrightarrow{BA} = \vec{a} - \vec{b} = \left(\hat{i} - 3\hat{j} - 5\hat{k}\right) - \left(3\hat{i} - 4\hat{j} - 4\hat{k}\right) = -2\hat{i} + \hat{j} - \hat{k}$

$\overrightarrow{BC} = \vec{c} - \vec{b} = \left(2\hat{i} - \hat{j} + \hat{k}\right) - \left(3\hat{i} - 4\hat{j} - 4\hat{k}\right) = -\hat{i} + 3\hat{j} + 5\hat{k}$

Now, taking the scalar product:

$\overrightarrow{BA} \cdot \overrightarrow{BC} = (-2)(-1) + (1)(3) + (-1)(5) = 2 + 3 - 5 = 0$

Since $\overrightarrow{BA} \cdot \overrightarrow{BC} = 0$, the vectors $\overrightarrow{BA}$ and $\overrightarrow{BC}$ are perpendicular. Hence, $m\angle ABC = \frac{\pi}{2}$.

116. (D)

Step 1: Find vector $\vec{p} = \vec{a} \times \vec{b}$ and its magnitude

Given vectors $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$:

$$\vec{p} = \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 1 & 1 & 0 \end{vmatrix} = 2\hat{i} - 2\hat{j} + \hat{k}$$

$|\vec{p}| = \sqrt{2^2 + (-2)^2 + 1^2} = \sqrt{4 + 4 + 1} = 3$

Also, $|\vec{a}| = \sqrt{2^2 + 1^2 + (-2)^2} = 3$.

Step 2: Find the magnitude of vector $\vec{c}$

Given that $|\vec{p} \times \vec{c}| = 3$ and the angle between $\vec{p}$ and $\vec{c}$ is $\frac{\pi}{6}$:

$|\vec{p} \times \vec{c}| = |\vec{p}||\vec{c}| \sin\left(\frac{\pi}{6}\right)$

$3 = 3 \cdot |\vec{c}| \cdot \frac{1}{2}$

$|\vec{c}| = 2$

Step 3: Calculate $\vec{a} \cdot \vec{c}$

Given $|\vec{c}\vec{a}| = 3$, squaring both sides yields:

$|\vec{c}\vec{a}|^2 = 9$

$|\vec{c}|^2 + |\vec{a}|^2 - 2(\vec{a} \cdot \vec{c}) = 9$

Substituting $|\vec{c}| = 2$ and $|\vec{a}| = 3$:

$2^2 + 3^2 - 2(\vec{a} \cdot \vec{c}) = 9$

$13 - 2(\vec{a} \cdot \vec{c}) = 9$

$2(\vec{a} \cdot \vec{c}) = 4$

$\vec{a} \cdot \vec{c} = 2$

117. (A)

Find the point dividing $P(1,2,-1)$ and $Q(-1,1,1)$ externally in ratio $1:2$.

The external section formula gives

$\frac{1Q - 2P}{1-2} = 2P - Q = (3,3,-3)$.

118. (C)

Let the given vector be $\vec{v} = p\hat{i} + q\hat{j} + r\hat{k} = 4\hat{i} - 5\hat{j} + 7\hat{k}$.

The lengths of the projections of a vector $\vec{v} = x\hat{i} + y\hat{j} + z\hat{k}$ on the x-axis, y-axis, and z-axis are given by $|x|$, $|y|$, and $|z|$ respectively.

For the given values $p = 4$, $q = -5$, and $r = 7$:

- Length of projection on x-axis = $|4| = 4$ units
- Length of projection on y-axis = $|5| = 5$ units
- Length of projection on z-axis = $|7| = 7$ units

The sum of the lengths of the projections on the coordinate axes is:

$$S = |p| + |q| + |r| = 4 + 5 + 7 = 16 \text{ units}$$

119. (B)

We know the identity for the magnitude of the difference of two vectors:

$$\left|\vec{a}\vec{b}\right|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2\left(\vec{a}\cdot\vec{b}\right)$$

Given:

$$|\vec{a}| = 3$$

$$|\vec{b}| = 2$$

$$\vec{a}\cdot\vec{b} = 5$$

Substituting these values into the identity:

$$\left|\vec{a}\vec{b}\right|^2 = (3)^2 + (2)^2 - 2(5)$$

$$\left|\vec{a}\vec{b}\right|^2 = 9 + 4 - 10$$

$$\left|\vec{a}\vec{b}\right|^2 = 3$$

Taking the positive square root:

$$\left|\vec{a}\vec{b}\right| = \sqrt{3}$$

120. (B)

Three vectors are coplanar if their scalar triple product is equal to zero. Expanding the scalar triple product gives:

$$\mu\left(\mu^2 - 1\right) - 1(\mu - 1) + 1(1 - \mu) = 0$$

$$\mu(\mu - 1)(\mu + 1) - 2(\mu - 1) = 0$$

$$(\mu - 1)\left(\mu^2 + \mu - 2\right) = 0$$

$$(\mu - 1)^2(\mu + 2) = 0$$

Solving for μ, we get the real roots:

$$\mu = 1, -2$$

requirements state the sum of the *distinct* real values of μ:

$$\text{Sum} = 1 + (-2) = -1$$

121. (D)

Let $\alpha = \frac{\pi}{3}$, $\beta = \frac{\pi}{4}$, and $\gamma = \theta$ be the angles made by the unit vector $\hat{a}$ with the coordinate axes along $\hat{i}$, $\hat{j}$, and $\hat{k}$ respectively.

The direction cosines l, m, n of a unit vector satisfy:

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

Substituting the given values:

$$\cos^2\left(\frac{\pi}{3}\right) + \cos^2\left(\frac{\pi}{4}\right) + \cos^2\theta = 1$$

$$\left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \cos^2\theta = 1$$

$$\frac{1}{4} + \frac{1}{2} + \cos^2\theta = 1$$

$$\frac{3}{4} + \cos^2\theta = 1$$

$$\cos^2\theta = 1 - \frac{3}{4} = \frac{1}{4}$$

$$\cos\theta = \pm\frac{1}{2}$$

Given that $\theta \in (0, \pi)$:

$$\theta = \frac{\pi}{3} \text{ or } \theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

Among the given choices, $\theta = \frac{2\pi}{3}$ is present as option (4).

122. (D)

Given the magnitudes of the vectors:

$|\vec{A}|=3, |\vec{B}|=4, |\vec{C}|=5$

Since each vector is perpendicular to the sum of the other two, their dot products are zero:

$\vec{A}\cdot(\vec{B}+\vec{C})=0 \Rightarrow \vec{A}\cdot\vec{B}+\vec{A}\cdot\vec{C}=0$

$\vec{B}\cdot(\vec{C}+\vec{A})=0 \Rightarrow \vec{B}\cdot\vec{C}+\vec{B}\cdot\vec{A}=0$

$\vec{C}\cdot(\vec{A}+\vec{B})=0 \Rightarrow \vec{C}\cdot\vec{A}+\vec{C}\cdot\vec{B}=0$

Adding these three equations together gives:

$2\left(\vec{A}\cdot\vec{B}+\vec{B}\cdot\vec{C}+\vec{C}\cdot\vec{A}\right)=0$

Now, using the vector identity for the magnitude of the sum:

$\left|\vec{A}+\vec{B}+\vec{C}\right|^2=|\vec{A}|^2+|\vec{B}|^2+|\vec{C}|^2+2\left(\vec{A}\cdot\vec{B}+\vec{B}\cdot\vec{C}+\vec{C}\cdot\vec{A}\right)$

$\left|\vec{A}+\vec{B}+\vec{C}\right|^2=3^2+4^2+5^2+0=9+16+25=50$

$\left|\vec{A}+\vec{B}+\vec{C}\right|=\sqrt{50}=5\sqrt{2}$

123. (C)

Since $\vec{v}$ lies in the plane of $\vec{a}$ and $\vec{b}$, it can be expressed as a linear combination of $\vec{a}$ and $\vec{b}$:

$\vec{v}=x\vec{a}+y\vec{b}$

Substituting $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}-\hat{j}+\hat{k}$:

$\vec{v}=x\left(\hat{i}+\hat{j}+\hat{k}\right)+y\left(\hat{i}-\hat{j}+\hat{k}\right)=(x+y)\hat{i}+(x-y)\hat{j}+(x+y)\hat{k}$

The magnitude of $\vec{c}=\hat{i}-\hat{j}-\hat{k}$ is:

$|\vec{c}|=\sqrt{1^2+(-1)^2+(-1)^2}=\sqrt{3}$

The projection of $\vec{v}$ on $\vec{c}$ is given as $\frac{1}{\sqrt{3}}$:

$\frac{\vec{v}\cdot\vec{c}}{|\vec{c}|}=\frac{1}{\sqrt{3}}$

Evaluating the dot product $\vec{v}\cdot\vec{c}$:

$\vec{v}\cdot\vec{c}=(x+y)(1)+(x-y)(-1)+(x+y)(-1)=y-x$

Equating the projection:

$\frac{y-x}{\sqrt{3}}=\frac{1}{\sqrt{3}} \Rightarrow y-x=1 \Rightarrow y=x+1$

For $x=1$, we get $y=2$. Substituting these values into $\vec{v}$:

$\vec{v}=(1+2)\hat{i}+(1-2)\hat{j}+(1+2)\hat{k}=3\hat{i}-\hat{j}+3\hat{k}$

124. (C)

Find the plane through $(1,-1,2)$ perpendicular to both printed planes.

The required normal is the cross product $(2,3,-2)\times(1,2,-3)=(-5,4,1)$.

Hence $\vec{r}\cdot\left(-5\hat{i}+4\hat{j}+\hat{k}\right)=-7$.

125. (A)

Let the position vectors of points A and B be $\overrightarrow{OA}$ and $\overrightarrow{OB}$ respectively.

The normal vector $\vec{n}$ to the required plane is directed along $\overrightarrow{AB}$, which is given by:

$\vec{n}=\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}$

From the given vectors, the direction ratios of the normal vector $\overrightarrow{AB}$ are $(a,b,c)=(2,3,6)$, so:

$\vec{n}=2\hat{i}+3\hat{j}+6\hat{k}$

The equation of a plane passing through a point $B(x_B,y_B,z_B)=(-2,-4,-2)$ with normal vector $\vec{n}=a\hat{i}+b\hat{j}+c\hat{k}$ is given in vector form by:

$\left(\vec{r}-\overrightarrow{OB}\right)\cdot\overrightarrow{AB}=0$

Substituting $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$, $\overrightarrow{OB}=-2\hat{i}-4\hat{j}-2\hat{k}$, and $\overrightarrow{AB}=2\hat{i}+3\hat{j}+6\hat{k}$:

$\left[(x+2)\hat{i}+(y+4)\hat{j}+(z+2)\hat{k}\right]\cdot\left(2\hat{i}+3\hat{j}+6\hat{k}\right)=0$

Expanding the dot product gives:

$2(x+2)+3(y+4)+6(z+2)=0$

$2x+4+3y+12+6z+12=0$

$2x+3y+6z+28=0$

126. (A)

Let the point $P\left(0,\frac{17}{2},-\frac{13}{2}\right)$ divide the line segment joining $A(a,1,6)$ and $B(3,4,b)$ in the ratio $k:1$.

Using the section formula, the coordinates of P are given by:

$P\left(\frac{3k+a}{k+1},\frac{4k+1}{k+1},\frac{kb+6}{k+1}\right)$

Equating the y-coordinate to $\frac{17}{2}$:

$\frac{4k+1}{k+1}=\frac{17}{2}$

$2(4k+1)=17(k+1)$

$8k+2=17k+17$

$9k=-15\Rightarrow k=-\frac{5}{3}$

Equating the x-coordinate to 0:

$\frac{3k+a}{k+1}=0\Rightarrow 3k+a=0$

$3\left(\frac{5}{3}\right)+a=0\Rightarrow a=5$

Equating the z-coordinate to $-\frac{13}{2}$:

$\frac{kb+6}{k+1}=-\frac{13}{2}$

$\frac{-\frac{5}{3}b+6}{-\frac{5}{3}+1}=-\frac{13}{2}$

$\frac{-5b+18}{-2}=-\frac{13}{2}$

$-5b+18=13\Rightarrow 5b=5\Rightarrow b=1$

Thus, $a=5$ and $b=1$.

127. (C)

Let the position vectors of the given points A and B be:

$\vec{a}=2\hat{i}-4\hat{j}+7\hat{k}$

$\vec{b}=3\hat{i}-5\hat{j}+8\hat{k}$

The equation of the plane is given by:

$\vec{r}\cdot\vec{n}=d$

where $\vec{n}=2\hat{i}-3\hat{j}+\hat{k}$ and $d=17$.

Let the plane divide the line segment joining A and B in the ratio $k:1$. The position vector of the point of division P is:

$\vec{r}=\frac{k\vec{b}+\vec{a}}{k+1}$

Since P lies on the plane:

$\left(\frac{k\vec{b}+\vec{a}}{k+1}\right)\cdot\vec{n}=d$

$k\left(\vec{b}\cdot\vec{n}-d\right)=-\left(\vec{a}\cdot\vec{n}-d\right)$

$k=-\frac{\vec{a}\cdot\vec{n}-d}{\vec{b}\cdot\vec{n}-d}$

Calculating the dot products:

$\vec{a}\cdot\vec{n}=(2)(2)+(-4)(-3)+(7)(1)=4+12+7=23$

$\vec{b}\cdot\vec{n}=(3)(2)+(-5)(-3)+(8)(1)=6+15+8=29$

Substituting the values into the formula:

$$k = -\frac{23-17}{29-17} = -\frac{6}{12} = -\frac{1}{2}$$

Thus, the ratio in magnitude required for the division is $3:10$.

128. (D)

Find λ, μ so that the two printed planes are parallel.

Parallel normals require

$(4,-1,\mu) = 2(2,-\lambda,1)$,

so $\lambda = \frac{1}{2}, \mu = 2$.

129. (D)

Let $O(0,0,0)$ be the origin and $M(-1,-2,2)$ be the foot of the perpendicular from the origin to the plane.

The position vector of M on the plane is:

$\vec{a} = -\hat{i} - 2\hat{j} + 2\hat{k}$

Since OM is normal to the required plane, the normal vector $\vec{n}$ is given by:

$\vec{n} = \overrightarrow{OM} = -\hat{i} - 2\hat{j} + 2\hat{k}$

The vector equation of a plane passing through a point with position vector $\vec{a}$ and normal to vector $\vec{n}$ is:

$\vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$

Evaluating the dot product $\vec{a} \cdot \vec{n}$:

$\vec{a} \cdot \vec{n} = \left(-\hat{i} - 2\hat{j} + 2\hat{k}\right) \cdot \left(-\hat{i} - 2\hat{j} + 2\hat{k}\right) = (-1)^2 + (-2)^2 + 2^2 = 9$

Thus, the vector equation of the plane is:

$\vec{r} \cdot \left(-\hat{i} - 2\hat{j} + 2\hat{k}\right) = 9$

130. (D)

The equation of a plane passing through the three points $A(-\lambda^2,1,1)$, $B(1,-\lambda^2,1)$, and $C(1,1,-\lambda^2)$ is given by:

$$\begin{vmatrix} x-(-\lambda^2) & y-1 & z-1 \\ 1-(-\lambda^2) & -\lambda^2-1 & 1-1 \\ 1-(-\lambda^2) & 1-1 & -\lambda^2-1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x+\lambda^2 & y-1 & z-1 \\ 1+\lambda^2 & -(1+\lambda^2) & 0 \\ 1+\lambda^2 & 0 & -(1+\lambda^2) \end{vmatrix} = 0$$

Since $1+\lambda^2 \neq 0$ for all real λ, dividing the second and third rows by $(1+\lambda^2)$ gives:

$$\begin{vmatrix} x+\lambda^2 & y-1 & z-1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{vmatrix} = 0$$

Expanding the determinant along the first row:

$(x+\lambda^2)(1-0) - (y-1)(-1-0) + (z-1)(0-(-1)) = 0$

$(x+\lambda^2) + (y-1) + (z-1) = 0$

$x + y + z + \lambda^2 - 2 = 0$

Since this plane also passes through the point $(-1,-1,1)$, substituting these coordinates into the plane equation yields:

$-1 + (-1) + 1 + \lambda^2 - 2 = 0$

$\lambda^2 - 3 = 0$

$\lambda = \pm\sqrt{3}$

Thus, the set $S = \{-\sqrt{3}, \sqrt{3}\}$.

131. (D)

1. Find the direction cosines of $\vec{n}$:

   Since $\vec{n}$ makes equal acute angles ($\alpha = \beta = \gamma$) with the coordinate axes:

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$3\cos^2\alpha = 1 \Rightarrow \cos\alpha = \frac{1}{\sqrt{3}}$

Thus, the unit vector in the direction of $\vec{n}$ is:

$\hat{n} = \frac{1}{\sqrt{3}}\left(\hat{i} + \hat{j} + \hat{k}\right)$

2. Find the normal vector $\vec{n}$:
Given magnitude $|\vec{n}| = 3\sqrt{3}$:

$\vec{n} = |\vec{n}|\hat{n} = 3\sqrt{3}\cdot\frac{1}{\sqrt{3}}\left(\hat{i} + \hat{j} + \hat{k}\right) = 3\left(\hat{i} + \hat{j} + \hat{k}\right)$

3. **Formulate the vector equation of the plane:**
The position vector of the given point $(1,-1,2)$ is $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$.

The vector equation of a plane passing through $\vec{a}$ and normal to $\vec{n}$ is:

$(\vec{r} - \vec{a})\cdot\vec{n} = 0 \Rightarrow \vec{r}\cdot\vec{n} = \vec{a}\cdot\vec{n}$

Substituting $\vec{a}$ and $\vec{n}$:

$\vec{r}\cdot 3\left(\hat{i} + \hat{j} + \hat{k}\right) = \left(\hat{i} - \hat{j} + 2\hat{k}\right)\cdot 3\left(\hat{i} + \hat{j} + \hat{k}\right)$

$3\vec{r}\cdot\left(\hat{i} + \hat{j} + \hat{k}\right) = 3\left[(1)(1) + (-1)(1) + (2)(1)\right]$

$3\vec{r}\cdot\left(\hat{i} + \hat{j} + \hat{k}\right) = 3(2) = 6$

$\vec{r}\cdot\left(\hat{i} + \hat{j} + \hat{k}\right) = 2$

132. (B)

To find the maximum value of $Z = 4x + 5y$, we determine the corner points of the bounded feasible region defined by the system of linear inequalities:

$2x + 3y \le 12$

$2x + y \le 8$

$x \ge 0, y \ge 0$

1. Intersection of $2x + 3y = 12$ and the y-axis ($x = 0$) gives the corner point $(0,4)$.
2. Intersection of $2x + y = 8$ and the x-axis ($y = 0$) gives the corner point $(4,0)$.
3. Solving $2x + 3y = 12$ and $2x + y = 8$ simultaneously:

Subtracting the second equation from the first:

$2y = 4 \Rightarrow y = 2$

Substituting $y = 2$ into $2x + y = 8$:

$2x + 2 = 8 \Rightarrow x = 3$

This gives the corner point $(3,2)$.

4. The origin $(0,0)$.

Evaluating the objective function $Z = 4x + 5y$ at each corner point:

Corner Point	$Z = 4x + 5y$
$(0,0)$	$4(0) + 5(0) = 0$
$(4,0)$	$4(4) + 5(0) = 16$
$(0,4)$	$4(0) + 5(4) = 20$
$(3,2)$	$4(3) + 5(2) = 22$

The maximum value of Z is 22, which occurs at the point $(3,2)$.

133. (B)

Identify the feasible point for $x + y \le 3$, $2x + 5y \ge 10$, $x, y \ge 0$.

$(1,2)$ satisfies $1 + 2 = 3$ and $2 + 10 = 12 \ge 10$. The other listed points fail at least one inequality.

134. (D)

Use the shaded feasible region to maximise The vertices are $(0,5),(0,7),(1.5,6),(6,0),(5,0)$.

Evaluating Z gives the maximum

$Z_{max} = 12 \Rightarrow \text{at}(6,0).$

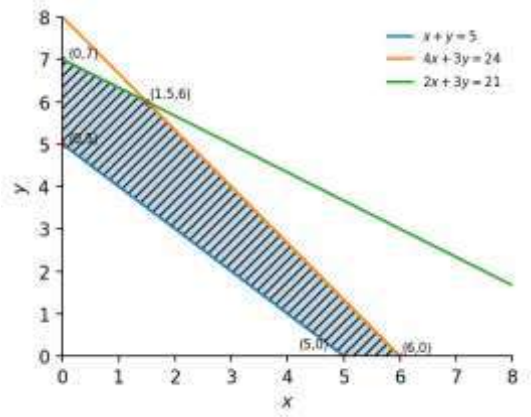


Fig: feasible region used for the linear-programming maximum

135. (D)

To find the region represented by the system of inequations $2x+3y\le 18$, $x+y\ge 10$, $x\ge 0$, and $y\ge 0$:

For non-negative variables $(x\ge 0, y\ge 0)$, the maximum value of $x+y$ subject to $2x+3y\le 18$ can be found at the vertices of the triangular region bounded by $(0,0)$, $(9,0)$, and $(0,6)$:

- At $(0,0)$: $x+y=0$
- At $(9,0)$: $x+y=9$
- At $(0,6)$: $x+y=6$

Thus, for all points satisfying $2x+3y\le 18$, $x\ge 0$, and $y\ge 0$, we have:

$x+y\le 9$

This directly contradicts the second inequation $x+y\ge 10$.

Alternatively, multiplying $x+y\ge 10$ by 2 gives:

$2x+2y\ge 20$

Since $y\ge 0$, it follows that:

$2x+3y=(2x+2y)+y\ge 20$

which contradicts $2x+3y\le 18$.

Since no point (x,y) satisfies all conditions simultaneously, the feasible region is empty (a null set).

136. (D)

Differentiate $h(x)=\sqrt{4f(x)+3g(x)}$ at $x=1$ using the printed data.

With $f(1)=4$, $g(1)=3$, $f'(1)=3$, $g'(1)=4$,

$$h'(1)=\frac{4f'(1)+3g'(1)}{2\sqrt{4f(1)+3g(1)}}=\frac{12}{5}.$$

137. (C)

Differentiate the printed inverse-sine expression.

After simplifying $1-u^2$ for $u=\left(5x+12\sqrt{1-x^2}\right)/13$, the principal-branch derivative is $\frac{dy}{dx}=-\frac{1}{\sqrt{1-x^2}}$.

138. (A)

The given function is:

$$y=\sec^{-1}\left(\frac{x+x^{-1}}{x-x^{-1}}\right)$$

Simplifying the term inside the function:

$$\frac{x+\frac{1}{x}}{x-\frac{1}{x}}=\frac{x^2+1}{x^2-1}$$

Since $\sec^{-1}(z)=\cos^{-1}\left(\frac{1}{z}\right)$, we can rewrite y as:

$$y=\cos^{-1}\left(\frac{x^2-1}{x^2+1}\right)=\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$

Using the property $\cos^{-1}(-u)=\pi-\cos^{-1}(u)$:

$$y=\pi-\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$

Substitute $x=\tan\theta$, so that $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)=2\tan^{-1}x$:

$$y=\pi-2\tan^{-1}x$$

Differentiating both sides with respect to x:

$$\frac{dy}{dx}=\frac{d}{dx}\left(\pi 2\tan^{-1}x\right)=0-2\cdot\frac{1}{1+x^2}=\frac{-2}{1+x^2}$$

139. (A)

Taking the natural logarithm on both sides of the given equation:

$$\log\left(x^y\right)=\log\left(e^{x-y}\right)$$

$$y\log x=x-y$$

Rearranging to express y in terms of x:

$$y\left(1+\log x\right)=x$$

$$y=\frac{x}{1+\log x}$$

Differentiating y with respect to x using the quotient rule:

$$\frac{dy}{dx}=\frac{\left(1+\log x\right)\cdot\frac{d}{dx}\left(x\right)-x\cdot\frac{d}{dx}\left(1+\log x\right)}{\left(1+\log x\right)^2}$$

$$\frac{dy}{dx}=\frac{\left(1+\log x\right)\cdot 1-x\cdot\frac{1}{x}}{\left(1+\log x\right)^2}.$$

$$\frac{dy}{dx}=\frac{1+\log x-1}{\left(1+\log x\right)^2}=\frac{\log x}{\left(1+\log x\right)^2}$$

140. (A)

Given the equation:

$$\log\left(x+y\right)=\log\left(xy\right)+3$$

Using the logarithmic property $\log\left(xy\right)=\log x+\log y$, we rewrite the equation as:

$$\log\left(x+y\right)=\log x+\log y+3$$

Differentiating both sides with respect to x:

$$\frac{1}{x+y}\left(1+\frac{dy}{dx}\right)=\frac{1}{x}+\frac{1}{y}\frac{dy}{dx}$$

Expanding the left side:

$$\frac{1}{x+y}+\frac{1}{x+y}\frac{dy}{dx}=\frac{1}{x}+\frac{1}{y}\frac{dy}{dx}$$

Rearranging the terms containing $\frac{dy}{dx}$ to one side:

$$\left(\frac{1}{x+y}\frac{1}{y}\right)\frac{dy}{dx}=\frac{1}{x}-\frac{1}{x+y}$$

Taking common denominators on both sides:

$$\left(\frac{y-\left(x+y\right)}{y\left(x+y\right)}\right)\frac{dy}{dx}=\frac{\left(x+y\right)-x}{x\left(x+y\right)}$$

$$\frac{-x}{y\left(x+y\right)}\frac{dy}{dx}=\frac{y}{x\left(x+y\right)}$$

Multiplying both sides by $\left(x+y\right)$ and solving for $\frac{dy}{dx}$:

$$\frac{dy}{dx}=\frac{y}{x}\cdot\left(\frac{y}{x}\right)=-\left(\frac{y}{x}\right)^2$$

141. (D)

Using the double-angle identity $\cos 2\theta=\cos^2\theta-\sin^2\theta$, the given function simplifies to:

$$y=\cos\left(2\cdot\frac{5x}{2}\right)=\cos\left(5x\right)$$

Differentiating with respect to x:

$$\frac{dy}{dx}=-5\sin\left(5x\right)$$

Differentiating again with respect to x:

$$\frac{d^2y}{dx^2}=-25\cos\left(5x\right)$$

Since $y=\cos\left(5x\right)$, we get:

$$\frac{d^2y}{dx^2}=-25y$$

142. (D)

The given equation is:

$$\left(2x\right)^{2y}=4e^{2x-2y}$$

Taking natural logarithm ($\log_e$) on both sides:

$$\log_e\left(()=\log_e\left(4e^{2x-2y}\right)\right.$$

Using logarithmic properties $\log_e\left(a^b\right)=b\log_e a$ and $\log_e\left(ab\right)=\log_e a+\log_e b$:

$$2y\log_e(2x) = \log_e 4 + \log_e\left(e^{2x-2y}\right)$$

$$2y\log_e(2x) = 2\log_e 2 + 2x - 2y$$

Dividing by 2 on both sides:

$$y\log_e(2x) = \log_e 2 + x - y$$

$$y(1+\log_e 2x) = x + \log_e 2$$

$$y = \frac{x+\log_e 2}{1+\log_e 2x}$$

Differentiating y with respect to x using the quotient rule:

$$\frac{dy}{dx} = \frac{(1+\log_e 2x)\cdot\frac{d}{dx}(x+\log_e 2) - (x+\log_e 2)\cdot\frac{d}{dx}(1+\log_e 2x)}{(1+\log_e 2x)^2}$$

Since $\frac{d}{dx}(x+\log_e 2) = 1$ and $\frac{d}{dx}(1+\log_e 2x) = \frac{1}{2x}\cdot 2 = \frac{1}{x}$:

$$\frac{dy}{dx} = \frac{(1+\log_e 2x)\cdot 1 - (x+\log_e 2)\cdot\frac{1}{x}}{(1+\log_e 2x)^2}$$

Multiplying both sides by $(1+\log_e 2x)^2$:

$$(1+\log_e 2x)^2\frac{dy}{dx} = 1+\log_e 2x - \left(1+\frac{\log_e 2}{x}\right)$$

$$(1+\log_e 2x)^2\frac{dy}{dx} = \log_e 2x - \frac{\log_e 2}{x}$$

$$= \frac{x\log_e 2x - \log_e 2}{x}$$

143. (C)

Find $g(x)$ when the derivative of the printed inverse tangent is $\sqrt{x}\,g(x)$.

The argument is $\tan\left(2\tan^{-1}\left(3x^{3/2}\right)\right)$. Hence the derivative is

$$\frac{9\sqrt{x}}{1+9x^3},$$

so $g(x) = \frac{9}{1+9x^3}$.

144. (B)

Given:

$$y = 3\cos(\log x) + 4\sin(\log x)$$

Differentiating both sides with respect to x:

$$y_1 = \frac{dy}{dx} = 3\left(-\sin(\log x)\cdot\frac{1}{x}\right) + 4\left(\cos(\log x)\cdot\frac{1}{x}\right)$$

$$xy_1 = -3\sin(\log x) + 4\cos(\log x)$$

Differentiating again with respect to x using the product rule on the left-hand side:

$$xy_2 + y_1 = -3\cos(\log x)\cdot\frac{1}{x} - 4\sin(\log x)\cdot\frac{1}{x}$$

Multiplying both sides by x:

$$x^2y_2 + xy_1 = -\left(3\cos(\log x) + 4\sin(\log x)\right)$$

$$x^2y_2 + xy_1 = -y$$

145. (D)

The given relation is $y^{1/m} + y^{-1/m} = 2x$.

Let $y^{1/m} = u$. Then:

$$u + \frac{1}{u} = 2x \Rightarrow u^2 - 2xu + 1 = 0$$

Solving for u using the quadratic formula:

$$u = x + \sqrt{x^2-1}$$

Since $u = y^{1/m}$, we have:

$$y = \left(x+\sqrt{x^2-1}\right)^m$$

Differentiating both sides with respect to x:

$$\frac{dy}{dx} = m\left(x+\sqrt{x^2-1}\right)^{m-1}\left(1+\frac{x}{\sqrt{x^2-1}}\right)$$

$$\frac{dy}{dx} = m\left(x+\sqrt{x^2-1}\right)^{m-1}\left(\frac{\sqrt{x^2-1}+x}{\sqrt{x^2-1}}\right)$$

$$\frac{dy}{dx} = \frac{m\left(x+\sqrt{x^2-1}\right)^m}{\sqrt{x^2-1}} = \frac{my}{\sqrt{x^2-1}}$$

Rearranging gives:

$$\sqrt{x^2-1}\frac{dy}{dx} = my$$

Squaring both sides:

$$\left(x^2 1\right)\left(\frac{dy}{dx}\right)^2 = m^2 y^2$$

146. (A)

The given function is:

$$f(x) = \begin{cases} \tan^{-1}x, & \text{if } |x| \le 1 \\ \frac{1}{2}\left(|x|-1\right), & \text{if } |x| > 1 \end{cases}$$

1. Continuity at $x = 1$:

$$lim_{x\to 1^-} f(x) = lim_{x\to 1^-} \tan^{-1}x = \frac{\pi}{4}$$

$$lim_{x\to 1^+} f(x) = lim_{x\to 1^+} \frac{1}{2}(x-1) = 0$$

Since $lim_{x\to 1^-} f(x) \neq lim_{x\to 1^+} f(x)$, the function $f(x)$ is discontinuous at $x = 1$.

Hence, $f'(1)$ does not exist.

2. Continuity at $x = -1$:

$$lim_{x\to -1^+} f(x) = lim_{x\to -1^+} \tan^{-1}x = -\frac{\pi}{4}$$

$$lim_{x\to -1^-} f(x) = lim_{x\to -1^-} \frac{1}{2}(-x-1) = 0$$

Since $lim_{x\to -1^+} f(x) \neq lim_{x\to -1^-} f(x)$, the function $f(x)$ is discontinuous at $x = -1$.

Hence, $f'(-1)$ does not exist.

3. Differentiability for $x \neq \pm 1$:

- For $x \in (-1,1)$, $f'(x) = \frac{1}{1+x^2}$, which is well-defined.
- For $x > 1$, $f'(x) = \frac{1}{2}$, which is well-defined.
- For $x < -1$, $f'(x) = -\frac{1}{2}$, which is well-defined.

Thus, the function is differentiable everywhere except at $x = 1$ and $x = -1$. Therefore, the domain of the derivative $f'(x)$ is $\mathbb{R} - \{-1,1\}$.

147. (C)

1. Behavior of $f(x)$ near $x = 0$:

Since $\log 2 \approx 0.693 > 0$ and $\sin 0 = 0$, there exists a neighborhood around $x = 0$ where $\log 2 - \sin x > 0$.

Therefore, in this neighborhood:

$$f(x) = |\log 2 \sin x| = \log 2 - \sin x$$

At $x = 0$:

$$f(0) = \log 2$$

$$f'(x) = -\cos x \Rightarrow f'(0) = -\cos 0 = -1$$

2. Behavior of $f(x)$ near $x = \log 2$:

Since

$$\sin(\log 2) \approx \sin(0.693) \approx 0.639 < \log 2,$$

we have $\log 2 - \sin(\log 2) > 0$.

Thus, in a neighborhood of $x = \log 2$:

$$f(x) = \log 2 - \sin x$$

$$f'(\log 2) = -\cos(\log 2)$$

3. Derivative of $g(x) = f(f(x))$ at $x = 0$:

By the chain rule:

$$g'(0) = f'(f(0)) \cdot f'(0)$$

$$g'(0) = f'(\log 2) \cdot (-1)$$

$$= (-\cos(\log 2)) \cdot (-1) = \cos(\log 2)$$

148. (D)

Differentiate x^{x^x}.

Logarithmic differentiation give

$$\frac{d}{dx} x^{x^x} = x^{x^x}\left[x^{x-1} + x^x \log x (1 + \log x)\right].$$

149. (C)

Calculation:

Let the given infinite series be equal to k:

$$k = \sqrt{y - \sqrt{y - \sqrt{y - \ldots \infty}}} = \sqrt{x + \sqrt{x + \sqrt{x + \ldots \infty}}}$$

From the left-hand side expression:

$k=\sqrt{y-k} \Rightarrow k^2=y-k \Rightarrow y=k^2+k$

From the right-hand side expression:

$k=\sqrt{x+k} \Rightarrow k^2=x+k \Rightarrow x=k^2-k$

Subtracting x from y:

$y-x=\left(k^2+k\right)-\left(k^2-k\right)=2k \Rightarrow 2k=y-x$

Differentiating y and x with respect to k:

$\frac{dy}{dk}=2k+1$

$\frac{dx}{dk}=2k-1$

Therefore, using the chain rule:

$\frac{dy}{dx}=\frac{\frac{dy}{dk}}{\frac{dx}{dk}}=\frac{2k+1}{2k-1}$

Substituting $2k=y-x$:

$\frac{dy}{dx}=\frac{y-x+1}{y-x-1}$

150. (A)

Given that $x+y=60$, we have $y=60-x$.

We need to maximize the product $P=x^3y$.

Substituting $y=60-x$:

$P=x^3(60-x)=60x^3-x^4$

Differentiating P with respect to x:

$\frac{dP}{dx}=180x^2-4x^3=4x^2(45-x)$

Setting $\frac{dP}{dx}=0$ to find critical points:

$4x^2(45-x)=0$

Since x is a positive number ($x>0$), we get:

$x=45$

To confirm that this yields a maximum, we find the second derivative:

$\frac{d^2P}{dx^2}=360x-12x^2$

At $x=45$:

$\frac{d^2P}{dx^2}=360(45)-12(45)^2=16200-24300=-8100<0$

Since the second derivative is negative, P is maximum at $x=45$.

The corresponding value of y is:

$y=60-45=15$

Therefore, the required numbers are 45 and 15 respectively.

151. (B)

Find where $f(x)=x/\log x$ is increasing.

$f'(x)=\frac{\log x-1}{(\log x)^2}$, which is nonnegative for $x \geq e$.

the function is increasing for $x \geq e$.

152. (C)

Given function:

$f(x)=2x^3-9x^2+12x+29$

Differentiating $f(x)$ with respect to x:

$f'(x)=6x^2-18x+12=6\left(x^2-3x+2\right)=6(x-1)(x-2)$

For $f(x)$ to be monotonically increasing, we require $f'(x)>0$:

$6(x-1)(x-2)>0$

By the sign scheme method (wavy curve method), $f'(x)>0$ when $x<1$ or $x>2$.

Therefore, the function $f(x)$ is monotonically increasing in the interval $(-\infty,1) \cup (2,\infty)$.

153. (B)

Since the point $(2,3)$ lies on the curve $y^2=px^3+q$, it must satisfy the equation:

$3^2=p(2)^3+q$

$8p+q=9$--- (1)

Differentiating the equation of the curve $y^2=px^3+q$ with respect to x:

$$2y\frac{dy}{dx}=3px^2$$

$$\frac{dy}{dx}=\frac{3px^2}{2y}$$

The slope of the tangent to the curve at the point $(2,3)$ is:

$$\frac{dy}{dx}|_{(2,3)}=\frac{3p(2)^2}{2(3)}=2p$$

The given tangent line is $y=4x-5$, which has a slope of 4. Equating the two slopes:

$2p=4\Rightarrow p=2$

Substituting $p=2$ into equation (1):

$8(2)+q=9$

$16+q=9\Rightarrow q=-7$

Thus, $p=2$ and $q=-7$.

154. (B)

Let r be the radius and l be the arc length of the circular sector.

financial perimeter available for fencing is:

$P=2r+l=20\Rightarrow l=20-2r$

The surface area A of the circular sector is:

$$A=\frac{1}{2}rl=\frac{1}{2}r(20-2r)=10r-r^2$$

To maximize the area, differentiate A with respect to r:

$$\frac{dA}{dr}=10-2r$$

Setting $\frac{dA}{dr}=0$:

$10-2r=0\Rightarrow r=5$ m

Since $\frac{d^2A}{dr^2}=-2<0$, the area A is maximum when the radius $r=5$ m.

155. (A)

The volume V of a cylindrical vessel with base radius r and height h of the water level is given by:

$V=\pi r^2h$

Since the vessel is cylindrical, its base radius $r=3$ m remains constant as the water level rises.

Substituting $r=3$ m:

$V=\pi(3)^2h=9\pi h$

Differentiating both sides with respect to time t:

$$\frac{dV}{dt}=9\pi\frac{dh}{dt}$$

Given that water is poured at the rate $\frac{dV}{dt}=36\ \text{m}^3/\text{sec}$:

$$36=9\pi\frac{dh}{dt}$$

$$\frac{dh}{dt}=\frac{36}{9\pi}=\frac{4}{\pi}\ \text{m/sec}$$

Thus, the rate at which the water level is rising is $\frac{4}{\pi}$ m/sec.

156. (C)

The given function is $f(x)=x\sqrt{1-x}$ for $x\in(0,1)$.

Differentiating $f(x)$ with respect to x using the product rule:

$$f'(x)=\sqrt{1-x}+x\cdot\frac{-1}{2\sqrt{1-x}}=\frac{2(1-x)-x}{2\sqrt{1-x}}=\frac{2-3x}{2\sqrt{1-x}}$$

To find the critical points, set $f'(x)=0$:

$$2-3x=0\Rightarrow x=\frac{2}{3}$$

By the first derivative test:

- For $x\in\left(0,\frac{2}{3}\right)$, $f'(x)>0$.
- For $x\in\left(\frac{2}{3},1\right)$, $f'(x)<0$.

Since $f'(x)$ changes sign from positive to negative at $x=\frac{2}{3}$, $f(x)$ has a local maximum at $x=\frac{2}{3}$.

157. (A)

The volume V of a cylinder of radius r and height h is given by:

$V=\pi r^2 h$

Differentiating both sides with respect to time t using the product rule:

$\frac{dV}{dt}=\pi\left(2rh\frac{dr}{dt}+r^2\frac{dh}{dt}\right)$

Given parameters:

- Radius, $r=3$ cm
- Height, $h=5$ cm
- Rate of increase of radius, $\frac{dr}{dt}=2$ cm/sec
- Rate of decrease of height, $\frac{dh}{dt}=-3$ cm/sec

Substituting these values into the derivative formula:

$\frac{dV}{dt}=\pi\left(2(3)(5)(2)+(3)^2(-3)\right)$

$\frac{dV}{dt}=\pi(60-27)=33\pi \text{ cm}^3/\text{sec}$

158. (C)

Determine monotonicity of $f(x)=1/8^x$ on $[1,3]$.

$f'(x)=-(\log 8)8^{-x}<0$, so the function is decreasing throughout.

159. (C)

The equation of the given curve is:

$y=\frac{x}{x^2-3}, x\neq\pm\sqrt{3}$

Differentiating with respect to x:

$\frac{dy}{dx}=\frac{(x^2-3)\cdot 1-x(2x)}{(x^2-3)^2}=\frac{-x^2-3}{(x^2-3)^2}$

The slope of the tangent at the point (α,β) is:

$m=\frac{dy}{dx}|_{(\alpha,\beta)}=\frac{-\alpha^2-3}{(\alpha^2-3)^2}$

The given line is $2x+6y-11=0$, whose slope is $-\frac{2}{6}=-\frac{1}{3}$. Since the tangent is parallel to this line, their slopes are equal:

$\frac{-\alpha^2-3}{(\alpha^2-3)^2}=-\frac{1}{3}$.

$\Rightarrow 3(\alpha^2+3)=(\alpha^2-3)^2$

$\Rightarrow 3\alpha^2+9=\alpha^4-6\alpha^2+9$

$\Rightarrow \alpha^4-9\alpha^2=0$

Since $(\alpha,\beta)\neq(0,0)$, $\alpha\neq 0$. Thus:

$\alpha^2-9=0\Rightarrow\alpha=\pm 3$

Since (α,β) lies on the curve:

$\beta=\frac{\alpha}{\alpha^2-3}$

For $\alpha=3$:

$\beta=\frac{3}{3^2-3}=\frac{3}{6}=\frac{1}{2}$

Substituting $\alpha=3$ and $\beta=\frac{1}{2}$ into $6\alpha+2\beta$:

$6\alpha+2\beta=6(3)+2\left(\frac{1}{2}\right)=18+1=19$

160. (C)

The given function is:

$f(x)=(x-1)(x+2)^2$

Differentiating $f(x)$ with respect to x:

$f'(x)=(1)(x+2)^2+(x-1)\cdot 2(x+2)$

$f'(x)=(x+2)(x+2+2x-2)=3x(x+2)$

Setting $f'(x)=0$ to find the critical points:

$3x(x+2)=0 \Rightarrow x=0$ or $x=-2$

Finding the second derivative:

$$f''(x)=\frac{d}{dx}(3x^2+6x)=6x+6$$

Testing the critical points:

1. At $x=-2$:

$$f''(-2)=6(-2)+6=-6<0$$

Therefore, $x=-2$ is a point of local maximum.

Local maximum value $=f(-2)=(-2-1)(-2+2)^2=0$

2. At $x=0$:

$$f''(0)=6(0)+6=6>0$$

Therefore, $x=0$ is a point of local minimum.

Local minimum value $=f(0)$

$=(0-1)(0+2)^2=-4$

Thus, the local maximum and local minimum values respectively are 0 and -4.

161. (D)

Determine which point lies on the cubic satisfying the printed tangent conditions.

From the point and perpendicular slope, $a=-4, b=2$, so $y=x^3-4x-2$. Substitution shows $(2,-2)$ lies on it.

162. (D)

The equations of the given curves are:

$y^2=6x$--- (i)

$9x^2+by^2=16$--- (ii)

Differentiating curve (i) with respect to x:

$$2y\frac{dy}{dx}=6 \Rightarrow m_1=\left(\frac{dy}{dx}\right)_1=\frac{3}{y}$$

Differentiating curve (ii) with respect to x:

$$18x+2by\frac{dy}{dx}=0 \Rightarrow m_2=\left(\frac{dy}{dx}\right)_2=-\frac{9x}{by}$$

Since the curves intersect each other at right angles (orthogonally), the product of their slopes at the point of intersection (x,y) must be -1:

$m_1 \cdot m_2=-1$

$$\left(\frac{3}{y}\right)\left(\frac{9x}{by}\right)=-1$$

$$\frac{27x}{by^2}=1 \Rightarrow by^2=27x$$

Substituting $y^2=6x$ from equation (i):

$b(6x)=27x$

Since $x \neq 0$ at the point of intersection:

$$6b=27 \Rightarrow b=\frac{27}{6}=\frac{9}{2}$$

163. (A)

Using the algebraic identity for the sum of a geometric progression, we simplify the integrand:

$$(x-a)(x^{n-1}+x^{n-2}a+\ldots+a^{n-1})=x^n-a^n$$

Now, integrating term by term with respect to x:

$$\int(x-a)(x^{n-1}+x^{n-2}a+\ldots+a^{n-1})dx=\int(x^n a^n)dx$$

$$=\int x^n dx-a^n\int 1dx$$

$$=\frac{x^{n+1}}{n+1}-a^n x+C$$

164. (A)

To evaluate the integral $I=\int(3-x)\sqrt{4-x}dx$:

Substitute $4-x=t$, which gives $dx=-dt$ and $3-x=t-1$.

Substituting these terms into the integral gives:

$$I=\int(t-1)\sqrt{t}(-dt)$$

$$I=\int(1-t)t^{\frac{1}{2}}dt$$

$$I=\int\left(t^{\frac{1}{2}}t^{\frac{3}{2}}\right)dt$$

$$I=\frac{t^{\frac{3}{2}}}{\frac{3}{2}}-\frac{t^{\frac{5}{2}}}{\frac{5}{2}}+C$$

$$I=\frac{2}{3}t^{\frac{3}{2}}-\frac{2}{5}t^{\frac{5}{2}}+C$$

Replacing t with $4-x$:

$$I=\frac{2}{3}(4-x)^{\frac{3}{2}}-\frac{2}{5}(4-x)^{\frac{5}{2}}+C$$

165. (D)

Let $I=\int\frac{e^x}{\left(2+e^x\right)\left(e^x+1\right)}dx$.

Substitute $e^x=t$, so that $e^xdx=dt$:

$$I=\int\frac{dt}{(t+2)(t+1)}$$

Using partial fractions:

$$\frac{1}{(t+1)(t+2)}=\frac{1}{t+1}-\frac{1}{t+2}$$

Integrating with respect to t:

$$I=\int\left(\frac{1}{t+1}\frac{1}{t+2}\right)dt=\log|t+1|-\log|t+2|+C$$

$$=\log\left|\frac{t+1}{t+2}\right|+C$$

Substituting $t=e^x$ back into the expression:

$$I=\log\left|\frac{e^x+1}{e^x+2}\right|+C$$

166. (D)

Multiply the numerator and denominator of the given integrand by e^x:

$$I=\int\frac{2e^x+3e^{-x}}{3e^x+4e^{-x}}dx=\int\frac{2e^{2x}+3}{3e^{2x}+4}dx$$

Express the numerator as:

$$2e^{2x}+3=A'\left(3e^{2x}+4\right)+B'\frac{d}{dx}\left(3e^{2x}+4\right)$$

$$2e^{2x}+3=A'\left(3e^{2x}+4\right)+B'\left(6e^{2x}\right)$$

Equating coefficients on both sides:

Constant term: $4A'=3\Rightarrow A'=\frac{3}{4}$

Coefficient of e^{2x} : $3A'+6B'$

$$=2\Rightarrow 3\left(\frac{3}{4}\right)+6B'$$

$$=2\Rightarrow 6B'=2-\frac{9}{4}=-\frac{1}{4}\Rightarrow B'=-\frac{1}{24}$$

Substituting back into the integral:

$$I=\int\left(\frac{3}{4}-\frac{1}{24}.\frac{6e^{2x}}{3e^{2x}+4}\right)dx$$

$$I=\frac{3}{4}x-\frac{1}{24}\log\left(3e^{2x}+4\right)+C$$

Comparing this result with $Ax+B\log\left(3e^{2x}+4\right)+C$:

$$A=\frac{3}{4},B=-\frac{1}{24}$$

167. (A)

The given integral is:

$$I=\int\frac{\sin4x}{\sin x}dx$$

Using the double-angle identity $\sin4x=2\sin2x\cos2x$ and $\sin2x=2\sin x\cos x$:

$$\frac{\sin4x}{\sin x}=\frac{2\sin2x\cos2x}{\sin x}$$

$$=\frac{4\sin x\cos x\cos2x}{\sin x}=4\cos2x\cos x$$

Using the product-to-sum trigonometric identity $2\cos A\cos B=\cos\left(A+B\right)+\cos\left(A-B\right)$:

$$4\cos2x\cos x=2\left[\cos\left(2x+x\right)+\cos\left(2x-x\right)\right]$$

$$=2\left(\cos3x+\cos x\right)$$

Integrating term by term:

$$I=\int2\left(\cos3x+\cos x\right)dx$$

$$I=2\left(\frac{\sin3x}{3}\right)+2\sin x+C=\frac{2\sin3x}{3}+2\sin x+C$$

168. (D)

The given integral is:

$$I=\int\frac{x}{\sqrt{1-2x^4}}dx=\int\frac{x}{\sqrt{1-\left(\sqrt{2}x^2\right)^2}}dx$$

Let $t=\sqrt{2}x^2$. Differentiating both sides with respect to x:

$$dt=2\sqrt{2}xdx\Rightarrow xdx=\frac{dt}{2\sqrt{2}}$$

Substituting this into the integral gives:

$$I=\frac{1}{2\sqrt{2}}\int\frac{dt}{\sqrt{1-t^2}}$$

Using the standard formula $\int\frac{dt}{\sqrt{1-t^2}}=\sin^{-1}(t)+C$:

$$I=\frac{1}{2\sqrt{2}}\sin^{-1}(t)+C$$

Substituting $t=\sqrt{2}x^2$ back:

$$I=\frac{1}{2\sqrt{2}}\sin^{-1}\left(\sqrt{2}x^2\right)+C$$

169. (A)

Let $I=\int\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}dx$.

Substitute $\sqrt{x}=\cos2\theta\Rightarrow x=\cos^2 2\theta$, so $dx=-4\cos2\theta\sin2\theta d\theta$.

$$I=\int\sqrt{\frac{1-\cos2\theta}{1+\cos2\theta}}\left(-4\cos2\theta\sin2\theta\right)d\theta$$

$$I=\int\left(\frac{\sin\theta}{\cos\theta}\right)\left(-4\cos2\theta\right)\left(2\sin\theta\cos\theta\right)d\theta$$

$$I=-8\int\sin^2\theta\cos2\theta d\theta$$

$$I=-4\int\left(1-\cos2\theta\right)\cos2\theta d\theta$$

$$=-4\int\left(\cos2\theta\frac{1+\cos4\theta}{2}\right)d\theta$$

$$I=-2\sin2\theta+2\theta+\frac{1}{2}\sin4\theta+C$$

Since $\cos2\theta=\sqrt{x}$:

$$\sin2\theta=\sqrt{1-x}$$

$$2\theta=\cos^{-1}\sqrt{x}$$

$$\sin4\theta=2\sin2\theta\cos2\theta=2\sqrt{x}\sqrt{1-x}=2\sqrt{x(1-x)}$$

Substituting these back yields:

$$I=-2\sqrt{1-x}+\cos^{-1}\sqrt{x}+\sqrt{x(1-x)}+C$$

170. (C)

Using trigonometric identities, express $\tan2\theta+\sec2\theta$ in terms of $\tan\theta$:

$$\tan2\theta+\sec2\theta=\frac{2\tan\theta}{1-\tan^2\theta}+\frac{1+\tan^2\theta}{1-\tan^2\theta}=\frac{1+2\tan\theta+\tan^2\theta}{1-\tan^2\theta}$$

$$=\frac{(1+\tan\theta)^2}{(1-\tan\theta)(1+\tan\theta)}=\frac{1+\tan\theta}{1-\tan\theta}$$

Substitute this into the given integral I:

$$I=\int\frac{d\theta}{\cos^2\theta\left(\tan2\theta+\sec2\theta\right)}$$

$$=\int\frac{\sec^2\theta}{\frac{1+\tan\theta}{1-\tan\theta}}d\theta=\int\frac{1-\tan\theta}{1+\tan\theta}\sec^2\theta d\theta$$

Let $u=\tan\theta$, so $du=\sec^2\theta d\theta$:

$$I=\int\frac{1-u}{1+u}du=\int\left(\frac{2}{1+u}1\right)du=2\log|1+u|-u+C$$

Substituting $u=\tan\theta$ back gives:

$$I=-\tan\theta+2\log|1+\tan\theta|+C$$

Comparing this result with $\lambda\tan\theta+2\log|f(\theta)|+C$:

$$\lambda=-1\text{and}f(\theta)=1+\tan\theta$$

Therefore, the ordered pair $\left(\lambda,f(\theta)\right)$ is $\left(-1,1+\tan\theta\right)$.

171. (A)

The given integral is:

$$f(x)=\int\frac{x^2+\sin^2x}{1+x^2}\cdot\sec^2xdx$$

Rewrite the numerator $x^2+\sin^2x$ as $\left(1+x^2\right)-\left(1-\sin^2x\right)=\left(1+x^2\right)-\cos^2x$:

$$f(x)=\int\frac{(1+x^2)-\cos^2 x}{1+x^2}\cdot\sec^2 x\,dx$$

$$f(x)=\int\left(1\frac{\cos^2 x}{1+x^2}\right)\sec^2 x\,dx$$

$$f(x)=\int\left(\sec^2 x\frac{1}{1+x^2}\right)dx$$

Integrating term by term:

$$f(x)=\tan x-\tan^{-1}x+C$$

Given that $f(0)=0$:

$$f(0)=\tan 0-\tan^{-1}0+C=0\Rightarrow C=0$$

So, the function is:

$$f(x)=\tan x-\tan^{-1}x$$

Substitute $x=1$:

$$f(1)=\tan 1-\tan^{-1}(1)=\tan 1-\frac{\pi}{4}$$

172. (D)

To evaluate the integral:

$$I=\int\frac{xe^{2x}}{(1+2x)^2}dx$$

Substitute $2x=t$, so that $x=\frac{t}{2}$ and $dx=\frac{dt}{2}$:

$$I=\int\frac{\frac{t}{2}e^t}{(1+t)^2}\cdot\frac{dt}{2}$$

$$I=\frac{1}{4}\int\frac{te^t}{(1+t)^2}dt$$

Rewrite the numerator t as $(1+t)-1$:

$$I=\frac{1}{4}\int e^t\left[\frac{(1+t)-1}{(1+t)^2}\right]dt$$

$$I=\frac{1}{4}\int e^t\left[\frac{1}{1+t}\frac{1}{(1+t)^2}\right]dt$$

Using the standard integral result $\int e^t\left[f(t)+f'(t)\right]dt=e^t f(t)+C$, where $f(t)=\frac{1}{1+t}$ and $f'(t)=-\frac{1}{(1+t)^2}$:

$$I=\frac{1}{4}\left(\frac{e^t}{1+t}\right)+C$$

Substituting $t=2x$ back:

$$I=\frac{e^{2x}}{4(1+2x)}+C$$

173. (C)

Let the given integral be:

$$I=\int\frac{4x^2\cot^{-1}(x^3)}{1+x^6}dx$$

Substitute $t=\cot^{-1}(x^3)$. Differentiating both sides with respect to x:

$$dt=\frac{-1}{1+(x^3)^2}\cdot 3x^2dx=\frac{-3x^2}{1+x^6}dx$$

$$\Rightarrow\frac{x^2}{1+x^6}dx=-\frac{1}{3}dt$$

Substituting t and dx into the integral:

$$I=4\int t\left(\frac{1}{3}\right)dt=-\frac{4}{3}\int tdt$$

$$I=-\frac{4}{3}\left(\frac{t^2}{2}\right)+C=-\frac{2}{3}t^2+C$$

Replacing t with $\cot^{-1}(x^3)$:

$$I=-\frac{2}{3}\left(\cot^{-1}x^3\right)^2+C$$

174. (C)

The given integral is:

$$I=\int\frac{1}{\cos x\sqrt{\cos 2x}}dx$$

Using the trigonometric identity $\cos 2x=\cos^2 x-\sin^2 x=\cos^2 x(1-\tan^2 x)$, we can rewrite the integrand as:

$$I=\int\frac{1}{\cos x\cdot\cos x\sqrt{1-\tan^2 x}}dx=\int\frac{\sec^2 x}{\sqrt{1-\tan^2 x}}dx$$

Substitute $t=\tan x$, so that $dt=\sec^2 x dx$:

$$I=\int\frac{dt}{\sqrt{1-t^2}}$$

Integrating with respect to t:

$$I=\sin^{-1}(t)+C$$

Substituting back $t=\tan x$:

$$I=\sin^{-1}(\tan x)+C$$

175. (A)

The given integral is:

$$I=\int\frac{3x^{13}+2x^{11}}{\left(2x^4+3x^2+1\right)^4}dx$$

Factor out x^4 from the term in the denominator:

$$\left(2x^4+3x^2+1\right)^4=\left[x^4\left(2+\frac{3}{x^2}+\frac{1}{x^4}\right)\right]^4$$

$$=x^{16}\left(2+\frac{3}{x^2}+\frac{1}{x^4}\right)^4$$

Dividing the numerator by x^{16}, the integral becomes:

$$I=\int\frac{\frac{3}{x^3}+\frac{2}{x^5}}{\left(2+3x^{-2}+x^{-4}\right)^4}dx$$

Let $t=2+3x^{-2}+x^{-4}$. Differentiating both sides with respect to x:

$$dt=\left(6x^{-3}4x^{-5}\right)dx=-2\left(3x^{-3}+2x^{-5}\right)dx$$

$$\Rightarrow\left(\frac{3}{x^3}+\frac{2}{x^5}\right)dx=-\frac{dt}{2}$$

Substituting this into I:

$$I=-\frac{1}{2}\int\frac{dt}{t^4}=-\frac{1}{2}\left(\frac{1}{3t^3}\right)+C=\frac{1}{6t^3}+C$$

Substituting $t=2+\frac{3}{x^2}+\frac{1}{x^4}=\frac{2x^4+3x^2+1}{x^4}$ back into the expression:

$$I=\frac{1}{6\left(\frac{2x^4+3x^2+1}{x^4}\right)^3}+C=\frac{x^{12}}{6(2x^4+3x^2+1)^3}+C$$

176. (D)

Divide the numerator and denominator of the integrand by x^{14}:

$$f(x)=\int\frac{5x^{-6}+7x^{-8}}{\left(2+x^{-5}+x^{-7}\right)^2}dx$$

Substitute $t=2+x^{-5}+x^{-7}$. Differentiating both sides gives $dt=-\left(5x^{-6}+7x^{-8}\right)dx$.

Now, integrate with respect to t:

$$f(x)=\int\frac{-dt}{t^2}=\frac{1}{t}+C=\frac{1}{2+x^{-5}+x^{-7}}+C$$

$$=\frac{x^7}{2x^7+x^2+1}+C$$

Using the given condition $f(0)=0$:

$$f(0)=\frac{0}{0+0+1}+C=0\Rightarrow C=0$$

So, the function is:

$$f(x)=\frac{x^7}{2x^7+x^2+1}$$

Substitute $x=1$:

$$f(1)=\frac{1^7}{2(1)^7+1^2+1}=\frac{1}{2+1+1}=\frac{1}{4}$$

177. (B)

Let the given integral be:

$$I=\int\frac{\sqrt{1-x^2}}{x^4}dx$$

Substitute $x=\sin\theta$, so $dx=\cos\theta d\theta$:

$$I=\int\frac{\cos\theta\cdot\cos\theta}{\sin^4\theta}d\theta=\int\cot^2\theta\csc^2\theta d\theta$$

Let $t=\cot\theta$, which gives $dt=-\csc^2\theta d\theta$:

$$I=-\int t^2dt=-\frac{t^3}{3}+C=-\frac{1}{3}\cot^3\theta+C$$

Since $\cot\theta = \frac{\sqrt{1-x^2}}{x}$, we get:

$$I = -\frac{1}{3x^3}\left(\sqrt{1-x^2}\right)^3 + C$$

Comparing this with $A(x)\left(\sqrt{1-x^2}\right)^m + C$:

$m = 3$

$$A(x) = -\frac{1}{3x^3}$$

Now, calculating $(\ )^m$:

$$(A(x))^m = \left(\frac{1}{3x^3}\right)^3 = -\frac{1}{27x^9}$$

178. (B)

The given integral is:

$$I = \int \frac{2x^3 - 1}{x^4 + x} dx$$

Dividing both the numerator and the denominator by x^2:

$$I = \int \frac{2x - \frac{1}{x^2}}{x^2 + \frac{1}{x}} dx$$

Let $u = x^2 + \frac{1}{x}$. Differentiating both sides with respect to x:

$$du = \left(2x - \frac{1}{x^2}\right)dx$$

Substituting u and du into the integral:

$$I = \int \frac{1}{u} du = \log|u| + C$$

Substituting back $u = x^2 + \frac{1}{x} = \frac{x^3+1}{x}$:

$$I = \log\left(\frac{x^3+1}{x}\right) + C$$

179. (A)

Calculation:

Given integral:

$$I = \int e^{x^2} \cdot x^3 dx$$

Let $t = x^2$, then $dt = 2xdx \Rightarrow xdx = \frac{dt}{2}$.

Substituting these into the integral gives:

$$I = \int e^t \cdot t \cdot \frac{dt}{2} = \frac{1}{2}\int te^t dt$$

Integrating by parts ($\int uvdt = u\int vdt - \int\left(u'\int vdt\right)dt$):

$$I = \frac{1}{2}\left(te^t e^t\right) + C = \frac{1}{2}e^t(t-1) + C$$

Substituting $t = x^2$ back:

$$I = e^{x^2}\left(\frac{x^2-1}{2}\right) + C$$

Comparing this with $I = e^{x^2} f(x) + C$, we get:

$$f(x) = \frac{x^2-1}{2}$$

Given that $f(1) = \frac{1^2-1}{2} = 0$, the function $f(x)$ is verified.

Now, calculating $f(2)$:

$$f(2) = \frac{2^2-1}{2} = \frac{3}{2}$$

180. (D)

Using partial fraction decomposition, let:

$$\frac{3x-2}{(\ )^2} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{D}{(x-2)^2}$$

Multiplying both sides by $(\ )^2$:

$$3x - 2 = A(x-2)^2 + B(x+1)(x-2) + D(x+1)$$

Substituting $x = -1$:

$$3(-1) - 2 = A(-1-2)^2$$

$$\Rightarrow -5 = 9A \Rightarrow A = -\frac{5}{9}$$

Substituting $x = 2$:

$$3(2) - 2 = D(2+1) \Rightarrow 4 = 3D \Rightarrow D = \frac{4}{3}$$

Equating the coefficients of x^2:

$$0 = A + B \Rightarrow B = -A = \frac{5}{9}$$

Now, substituting A, B, and D back into the integral:

$$\int \frac{3x-2}{(\quad)^2} dx = -\frac{5}{9}\int \frac{1}{x+1} dx + \frac{5}{9}\int \frac{1}{x-2} dx + \frac{4}{3}\int \frac{1}{(x-2)^2} dx$$

$$= -\frac{5}{9}\log(x+1) + \frac{5}{9}\log(x-2) - \frac{4}{3} \times \frac{1}{(x2)} + C$$

181. (B)

Find $f(x)$ from $\int \frac{x+1}{\sqrt{2x-1}} dx = f(x)\sqrt{2x-1} + C$.

Let $u = \sqrt{2x-1}$. Integration gives

$$\frac{x+4}{3}\sqrt{2x-1} + C,$$

so $f(x) = (x+4)/3$.

182. (B)

Correct option: (2) $\frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{1}{\sqrt{3}} \tan\frac{x}{2}\right) + c$,

where c is the constant of integration.

Let the given integral be:

$$I = \int \frac{dx}{2 + \cos x}$$

Substitute $\tan\frac{x}{2} = t$, which gives $dx = \frac{2dt}{1+t^2}$ and

$$\cos x = \frac{1-t^2}{1+t^2}.$$

Substituting these expressions into the integral:

$$I = \int \frac{\frac{2}{1+t^2}}{2 + \frac{1-t^2}{1+t^2}} dt$$

$$I = \int \frac{2}{2(1+t^2) + (1-t^2)} dt$$

$$I = \int \frac{2}{t^2+3} dt = 2\int \frac{dt}{t^2 + (\sqrt{3})^2}$$

Using the standard formula $\int \frac{dt}{t^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{t}{a}\right) + c$:

$$I = 2 \cdot \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{t}{\sqrt{3}}\right) + c$$

Replacing:

$$I = \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{1}{\sqrt{3}} \tan\frac{x}{2}\right) + c$$

183. (C)

The given integral is:

$$I = \int_0^{\frac{\pi}{4}} \sec^4 x dx$$

Rewriting $\sec^4 x$ in terms of $\tan x$:

$$I = \int_0^{\frac{\pi}{4}} (1 + \tan^2 x)\sec^2 x dx$$

Let $t = \tan x$, so that $dt = \sec^2 x dx$.

Changing the limits of integration:

- When $x = 0$., $t = \tan 0 = 0$
- When $x = \frac{\pi}{4}$., $t = \tan\left(\frac{\pi}{4}\right) = 1$

Substituting t into the integral:

$$I = \int_0^1 (1+t^2) dt$$

$$I = \left[t + \frac{t^3}{3}\right]_0^1$$

$$I = \left(1 + \frac{1}{3}\right) - 0 = \frac{4}{3}$$

184. (C)

Evaluating the first given integral:

$$\int_a^b x^3 dx = \left[\frac{x^4}{4}\right]_a^b = \frac{b^4 - a^4}{4} = 0 \Rightarrow b^4 - a^4 = 0$$

Evaluating the second given integral:

$$\int_a^b x^2 dx = \left[\frac{x^3}{3}\right]_a^b = \frac{b^3 - a^3}{3} = \frac{2}{3} \Rightarrow b^3 - a^3 = 2$$

From $b^4 - a^4 = 0$, we get $b^2 = a^2$, which implies $b = a$ or $b = -a$:

- If $b = a$, then $b^3 - a^3 = 0 \neq 2$, which is not possible.
- If $b = -a$, substituting into $b^3 - a^3 = 2$ gives:

$$(-a)^3 - a^3 = 2 \Rightarrow -2a^3 = 2 \Rightarrow a^3 = -1 \Rightarrow a = -1$$

Since $b = -a$, we get:

$$b = -(-1) = 1$$

Therefore, $a = -1$ and $b = 1$.

185. (A)

Let the given definite integral be:

$$I = \int_{-\pi/4}^{\pi/4} \frac{e^x (x\sin x)}{e^{2x} - 1} dx$$

Dividing the numerator and the denominator of the integrand by e^x, we get:

$$f(x) = \frac{x\sin x}{e^x - e^{-x}}$$

Now, replace x with $-x$ to check the symmetry of $f(x)$:

$$f(-x) = \frac{(-x)\sin(-x)}{e^{-x} - e^x} = \frac{x\sin x}{-(e^x - e^{-x})} = -f(x)$$

Since $f(-x) = -f(x)$, the integrand $f(x)$ is an odd function.

Using the property of definite integrals:

$$\int_{-a}^{a} f(x)dx = 0 \text{ when } f(x) \text{ is an odd function}$$

Thus,

$$I = 0$$

186. (D)

To evaluate the definite integral:

$$I = \int_{-3}^{0} x\sqrt{x+4}dx$$

Substitute $t = x + 4$, which gives $x = t - 4$ and $dx = dt$.

Change the limits of integration:

- When $x = -3$, $t = -3 + 4 = 1$
- When $x = 0$, $t = 0 + 4 = 4$

Substituting these into the integral:

$$I = \int_1^4 (t-4)\sqrt{t}dt$$

$$I = \int_1^4 \left(t^{3/2} 4t^{1/2}\right) dt$$

$$I = \left[\frac{2}{5}t^{5/2} \frac{8}{3}t^{3/2}\right]_1^4$$

Evaluating at the upper limit $t = 4$:

$$\frac{2}{5}(4)^{5/2} - \frac{8}{3}(4)^{3/2} = \frac{2}{5}(32) - \frac{8}{3}(8)$$

$$= \frac{64}{5} - \frac{64}{3} = -\frac{128}{15}$$

Evaluating at the lower limit $t = 1$:

$$\frac{2}{5}(1)^{5/2} - \frac{8}{3}(1)^{3/2} = \frac{2}{5} - \frac{8}{3} = -\frac{34}{15}$$

Subtracting the lower limit value from the upper limit value:

$$I = -\frac{128}{15} - \left(\frac{34}{15}\right) = -\frac{94}{15}$$

187. (B)

Calculation:

Given the definite integral:

$$I = \int_0^1 \frac{1}{\sqrt{3 + 2x - x^2}} dx$$

Completing the square in the quadratic expression under the square root:

$$3 + 2x - x^2 = 4 - (x^2 - 2x + 1) = 2^2 - (x-1)^2$$

Substituting this into the integral:

$$I = \int_0^1 \frac{dx}{\sqrt{2^2 - (x-1)^2}}$$

Using the standard integration formula $\int \frac{dx}{\sqrt{a^2 - u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C$:

$$I = \left[\sin^{-1}\left(\frac{x-1}{2}\right)\right]_0^1$$

Evaluating at the upper and lower limits:

$$I=\sin^{-1}\left(\frac{1-1}{2}\right)-\sin^{-1}\left(\frac{0-1}{2}\right)$$

$$I=\sin^{-1}(0)-\sin^{-1}\left(\frac{1}{2}\right)$$

$$I=0-\left(\frac{\pi}{6}\right)=\frac{\pi}{6}$$

188. (C)

Let the given integral be:

$$I=\int_{-\pi}^{\pi}\frac{\cos^2 x}{1+a^x}dx \text{---} (1)$$

Using the property of definite integrals $\int_a^b f(x)dx=\int_a^b f(a+b-x)dx$, substitute x with $-\pi+\pi-x=-x$:

$$I=\int_{-\pi}^{\pi}\frac{\cos^2(-x)}{1+a^{-x}}dx=\int_{-\pi}^{\pi}\frac{a^x\cos^2 x}{1+a^x}dx \text{---} (2)$$

Adding equations (1) and (2):

$$2I=\int_{-\pi}^{\pi}\frac{(1+a^x)\cos^2 x}{1+a^x}dx=\int_{-\pi}^{\pi}\cos^2 x dx$$

Since $\cos^2 x$ is an even function:

$$2I=2\int_0^{\pi}\cos^2 x dx$$

$$I=\int_0^{\pi}\frac{1+\cos 2x}{2}dx$$

$$I=\left[\frac{x}{2}+\frac{\sin 2x}{4}\right]_0^{\pi}=\frac{\pi}{2}$$

189. (B)

To evaluate the definite integral $\int_0^2[2x]dx$, we split the interval of integration at the points where the function $[2x]$ changes its value, which occurs when $2x$ is an integer ($x=0,\frac{1}{2},1,\frac{3}{2},2$):

$$[2x]=\begin{cases}0, & 0\le x<\frac{1}{2}\\ 1, & \frac{1}{2}\le x<1\\ 2, & 1\le x<\frac{3}{2}\\ 3, & \frac{3}{2}\le x<2\end{cases}$$

Splitting the integral into these sub-intervals:

$$\int_0^2[2x]dx=\int_0^{1/2}0dx+\int_{1/2}^{1}1dx+\int_1^{3/2}2dx+\int_{3/2}^{2}3dx$$

$$\int_0^2[2x]dx=0+1\cdot\left(1\frac{1}{2}\right)+2\cdot\left(\frac{3}{2}1\right)+3\cdot\left(2\frac{3}{2}\right)$$

$$\int_0^2[2x]dx=0+\frac{1}{2}+1+\frac{3}{2}=3$$

190. (D)

The given integral is:

$$I=\int_0^{\pi/2}\frac{\cot x}{\cot x+\csc x}dx$$

Expressing $\cot x$ and $\csc x$ in terms of $\sin x$ and $\cos x$:

$$I=\int_0^{\pi/2}\frac{\frac{\cos x}{\sin x}}{\frac{\cos x}{\sin x}+\frac{1}{\sin x}}dx=\int_0^{\pi/2}\frac{\cos x}{1+\cos x}dx$$

Rewriting the integrand:

$$\frac{\cos x}{1+\cos x}=\frac{1+\cos x-1}{1+\cos x}=1-\frac{1}{1+\cos x}=1-\frac{1}{2\cos^2\frac{x}{2}}=1-\frac{1}{2}\sec^2\frac{x}{2}$$

Integrating with respect to x:

$$I=\left[x-\tan\frac{x}{2}\right]_0^{\pi/2}$$

Applying the limits:

$$I=\left(\frac{\pi}{2}-\tan\frac{\pi}{4}\right)-(0-\tan 0)=\frac{\pi}{2}-1=\frac{1}{2}(\pi-2)$$

Comparing this with $m(\pi+n)$:

$$m(\pi+n)=\frac{1}{2}(\pi+(-2))$$

Thus, $m=\frac{1}{2}$ and $n=-2$.

Therefore:

$$m\cdot n=\frac{1}{2}\times(-2)=-1$$

191. (D)

The given expression is split into two integrals:

$$\int_0^2[x]dx+\int_0^2|x1|dx$$

1. For the first integral, using the definition of the greatest integer function $[x]$:

$$\int_0^2[x]dx=\int_0^1 0dx+\int_1^2 1dx=0+[x]_1^2=2-1=1$$

2. For the second integral, splitting at $x=1$:

$$\int_0^2|x1|dx=\int_0^1(1-x)dx+\int_1^2(x-1)dx$$

$$\int_0^2|x1|dx=\left[x\frac{x^2}{2}\right]_0^1+\left[\frac{x^2}{2}x\right]_1^2$$

$$\int_0^2|x1|dx=\left(1\frac{1}{2}\right)+\left((2-2)-\left(\frac{1}{2}1\right)\right)=\frac{1}{2}+\frac{1}{2}=1$$

Adding the two results:

$$\int_0^2[x]dx+\int_0^2|x1|dx=1+1=2$$

192. (C)

Since $f(x)=\sin|x|+\cos|x|$, we check for symmetry:

$$f(-x)=\sin|x|+\cos|x|=\sin|x|+\cos|x|=f(x)$$

Thus, $f(x)$ is an even function. Using the property of definite integrals for even functions, $\int_{-a}^{a}f(x)dx=2\int_0^a f(x)dx$:

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}f(x)dx=2\int_0^{\frac{\pi}{2}}(\sin|x|+\cos|x|)dx$$

For $x\in\left[0,\frac{\pi}{2}\right]$, $|x|=x$, so:

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}f(x)dx=2\int_0^{\frac{\pi}{2}}(\sin x+\cos x)dx$$

$$=2\left[-\cos x+\sin x\right]_0^{\frac{\pi}{2}}$$

$$=2\left[\left(-\cos\frac{\pi}{2}+\sin\frac{\pi}{2}\right)-(-\cos 0+\sin 0)\right]$$

$$=2\left[(0+1)-(-1+0)\right]=2(2)=4$$

193. (A)

Evaluate $\int_1^2|2x-[3x]|dx$.

Split at $x=4/3$ and $5/3$. The piecewise integration gives

194. (A)

Using the double-angle identity $\sin x=2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)$, the given integral is:

$$I=\int_0^{\frac{\pi}{2}}\sin^5\left(\frac{x}{2}\right)\cdot 2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)dx$$

$$I=2\int_0^{\frac{\pi}{2}}\sin^6\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)dx$$

Substitute $t=\sin\left(\frac{x}{2}\right)$, which gives $dt=\frac{1}{2}\cos\left(\frac{x}{2}\right)dx\Rightarrow\cos\left(\frac{x}{2}\right)dx=2dt$.

Change the limits of integration:

When $x=0$, $t=\sin(0)=0$.

When $x=\frac{\pi}{2}$, $t=\sin\left(\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}$.

Substituting t and the limits into the integral:

$$I=2\int_0^{\frac{1}{\sqrt{2}}}t^6(2dt)$$

$$I=4\int_0^{\frac{1}{\sqrt{2}}}t^6dt$$

$$I = 4\left[\frac{t^7}{7}\right]_0^{\frac{1}{\sqrt{2}}}$$

$$I = \frac{4}{7}\left(\frac{1}{\sqrt{2}}\right)^7$$

$$I = \frac{4}{7}\times\frac{1}{8\sqrt{2}}$$

$$I = \frac{1}{14\sqrt{2}}$$

195. (D)

The area bounded by the curve $y = x^2$, the X-axis, and the ordinates $x = 1$ and $x = 4$ is given by:

$$A = \int_1^4 y\,dx$$

Substituting $y = x^2$:

$$A = \int_1^4 x^2\,dx$$

$$A = \left[\frac{x^3}{3}\right]_1^4$$

$$A = \frac{4^3 - 1^3}{3} = \frac{64-1}{3} = \frac{63}{3} = 21 \text{ sq. units}$$

196. (A)

The given line is $x + 2y = 8$, which gives:

$$y = \frac{8-x}{2}$$

Since $y \geq 0$ for all $x \in [2,4]$, the required area bounded by the line, the X-axis, and the ordinates $x = 2$ and $x = 4$ is given by:

$$\text{Area} = \int_2^4 y\,dx$$

$$\text{Area} = \int_2^4 \frac{8-x}{2}\,dx$$

$$\text{Area} = \frac{1}{2}\left[8x - \frac{x^2}{2}\right]_2^4$$

$$\text{Area} = \frac{1}{2}\left[\left(8(4) - \frac{4^2}{2}\right)\left(8(2) - \frac{2^2}{2}\right)\right]$$

$$\text{Area} = \frac{1}{2}\left[(32-8)-(16-2)\right] = \frac{1}{2}(24-14)$$

$$= \frac{1}{2}\times 10 = 5 \text{ sq. units}$$

197. (B)

To find the points of intersection of the parabola $y^2 = 2x+1$ and the line $x - y = 1$, express x in terms of y:

$$x = \frac{y^2-1}{2} \text{ and } x = y+1$$

Equating the two expressions for x:

$$\frac{y^2-1}{2} = y+1$$

$$y^2 - 2y - 3 = 0$$

$$(y-3)(y+1) = 0 \Rightarrow y = 1, 3$$

The region is bounded between $y = -1$ and $y = 3$ with $x_{\text{line}} \geq x_{\text{curve}}$. The required area A is:

$$A = \int_{-1}^3 \left(x_{\text{line}} x_{\text{curve}}\right) dy$$

$$A = \int_{-1}^3 \left(y+1-\frac{y^2-1}{2}\right) dy$$

$$A = \frac{1}{2}\int_{-1}^3 \left(3+2y-y^2\right) dy$$

$$A = \frac{1}{2}\left[3y + y^2 - \frac{y^3}{3}\right]_{-1}^3$$

$$A = \frac{1}{2}\left[(9+9-9)\left(-3+1+\frac{1}{3}\right)\right]$$

$$A = \frac{1}{2}\left[9\left(\frac{5}{3}\right)\right] = \frac{1}{2}\left(\frac{32}{3}\right) = \frac{16}{3} \text{ sq. units}$$

198. (B)

The region A is bounded by the circle $x^2 + y^2 \leq 1$ and the parabola $y^2 \geq 1-x$, which gives $x \geq 1-y^2$.

To find the points of intersection of $x^2 + y^2 = 1$ and $x = 1-y^2$:

$(1-y^2)^2+y^2=1$

$1-2y^2+y^4+y^2=1$

$y^4-y^2=0\Rightarrow 0,\pm 1$

For $y\in[-1,1]$, x ranges from $1-y^2$ to $\sqrt{1-y^2}$. Due to symmetry about the x-axis, the required area is:

$$\text{Area}=2\int_0^1\left(\sqrt{1-y^2}-(1-y^2)\right)dy$$

Evaluating the integrals individually:

$$\int_0^1\sqrt{1-y^2}\,dy=\left[\frac{y}{2}\sqrt{1-y^2}+\frac{1}{2}\sin^{-1}y\right]_0^1=\frac{\pi}{4}$$

$$\int_0^1(1-y^2)dy=\left[y\frac{y^3}{3}\right]_0^1=\frac{2}{3}$$

Substituting these values back:

$$\text{Area}=2\left(\frac{\pi}{4}\frac{2}{3}\right)=\frac{\pi}{2}-\frac{4}{3}$$

199. (D)

To find the point of intersection of the curves $y=\sqrt{x}$ and $2y-x+3=0$:

Substitute $x=y^2$ into the line equation:

$2y-y^2+3=0\Rightarrow y^2-2y-3=0$

$(y-3)(y+1)=0$

Since the region lies in the first quadrant, $y\geq 0$, so $y=3$.

When $y=3$, $x=y^2=9$.

The region in the first quadrant bounded by $y=\sqrt{x}$ $(x=y^2)$, $2y-x+3=0$ $(x=2y+3)$, and the x-axis $(y=0)$ lies between $y=0$ and $y=3$.

Integrating along the y-axis:

$$\text{Area}=\int_0^3(x_{\text{line}}x_{\text{curve}})\,dy$$

$$\text{Area}=\int_0^3\left((2y+3)-y^2\right)dy$$

$$\text{Area}=\left[y^2+3y-\frac{y^3}{3}\right]_0^3$$

$$\text{Area}=\left(3^2+3(3)-\frac{3^3}{3}\right)-0=9+9-9=9\text{ sq. units}$$

200. (B)

The given differential equation is:

$x^2(y+1)dx+y^2(x-1)dy=0$

Separating the variables, we get:

$$\frac{x^2}{x-1}dx+\frac{y^2}{y+1}dy=0$$

Rewriting the numerators:

$$\left(x+1+\frac{1}{x-1}\right)dx+\left(y-1+\frac{1}{y+1}\right)dy=0$$

Integrating both sides with respect to their corresponding variables:

$$\int\left(x+1+\frac{1}{x-1}\right)dx+\int\left(y-1+\frac{1}{y+1}\right)dy=C_1$$

$$\left(\frac{x^2}{2}+x+\log(x-1)\right)+\left(\frac{y^2}{2}-y+\log(y+1)\right)=C_1$$

Multiplying the entire equation by 2:

$x^2+2x+y^2-2y+2\log\left[(x-1)(y+1)\right]=2C_1$

Completing the squares for x and y:

$(x+1)^2-1+(y-1)^2-1+2\log\left[(x-1)(y+1)\right]=2C_1$

$(x+1)^2+(y-1)^2+2\log\left[(x-1)(y+1)\right]=2C_1+2=C$

201. (B)

The given differential equation is:

$xdy-ydx=0$

Separating the variables:

$$\frac{dy}{y}=\frac{dx}{x}$$

Integrating both sides:

$$\int\frac{dy}{y}=\int\frac{dx}{x}$$

$\ln|y|=\ln|x|+\ln|c|$

$y=cx$

where c is an arbitrary constant.

This equation represents a family of straight lines passing through the origin $(0,0)$.

202. (B)

Solve $e^{\frac{1}{2}(dy/dx)} = 3^x$.

Taking logarithms gives $y' = 2x\log 3$. Hence

$y = x^2\log 3 + C.$

203. (C)

The given differential equation is:

$$x^2 + y^2 - 2xy\frac{dy}{dx} = 0$$

Rearranging the terms:

$$\frac{dy}{dx} = \frac{x^2 + y^2}{2xy}$$

This is a homogeneous differential equation. Substituting $y = vx$, we get $\frac{dy}{dx} = v + x\frac{dv}{dx}$:

$$v + x\frac{dv}{dx} = \frac{x^2 + v^2x^2}{2x(vx)} = \frac{1+v^2}{2v}$$

$$x\frac{dv}{dx} = \frac{1+v^2}{2v} - v = \frac{1-v^2}{2v}$$

Separating the variables gives:

$$\frac{2v}{1-v^2}dv = \frac{dx}{x}$$

Integrating both sides:

$$\int \frac{2v}{1-v^2}dv = \int \frac{dx}{x}$$

$$-\ln|1v^2| = \ln|x| - \ln|C|$$

$$\ln|1v^2| = \ln\left|\frac{C}{x}\right|$$

$$1 - v^2 = \frac{C}{x}$$

Substituting $v = \frac{y}{x}$ back into the equation:

$$1 - \frac{y^2}{x^2} = \frac{C}{x}$$

Multiplying both sides by x^2:

$$x^2 - y^2 = Cx$$

204. (A)

According to Newton's law of cooling:

$$\frac{T(t) - T_0}{T_i - T_0} = e^{-kt}$$

where $T_i = 80^\circ\text{C}$ is the initial temperature, $T_0 = 20^\circ\text{C}$ is the surrounding temperature, and $T(t)$ is the temperature at time t.

For $t = 5$ min, the body cools to $T(5) = 70^\circ\text{C}$:

$$\frac{70-20}{80-20} = e^{-5k}$$

$$e^{-5k} = \frac{50}{60} = \frac{5}{6}$$

To find the temperature $T(15)$ after $t = 15$ min:

$$\frac{T(15)-20}{80-20} = e^{-15k} = \left(e^{-5k}\right)^3$$

$$\frac{T(15)-20}{60} = \left(\frac{5}{6}\right)^3 = \frac{125}{216}$$

$$T(15) - 20 = 60 \times \frac{125}{216} = \frac{625}{18} \approx 34.72^\circ\text{C}$$

$$T(15) = 20 + 34.72^\circ\text{C} = 54.72^\circ\text{C} \approx 54.7^\circ\text{C}$$

205. (A)

The given differential equation is:

$$\frac{dy}{dx} - e^x = ye^x$$

Rearranging the terms:

$$\frac{dy}{dx} = e^x(y+1)$$

Separating the variables:

$$\frac{dy}{y+1} = e^x dx$$

Integrating both sides:

$$\int \frac{dy}{y+1} = \int e^x dx$$

$\log(y+1)=e^x+C$

Using the initial condition $x=0$ and $y=1$:

$\log(1+1)=e^0+C$

$\log 2=1+C\Rightarrow C=\log 2-1$

Substituting C back into the integrated equation:

$\log(y+1)=e^x+\log 2-1$

$\log(y+1)-\log 2=e^x-1$

$\log\left(\frac{y+1}{2}\right)=e^x-1$

206. (B)

The given differential equation is:

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{5}{3}}=8\frac{d^2y}{dx^2}$$

1. **Order:** The highest order derivative involved in the differential equation is $\frac{d^2y}{dx^2}$, so the **order = 2**.
2. **Degree:** To determine the degree, the equation must be made free from fractional exponents of derivatives. Cubing both sides gives:

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^5=512\left(\frac{d^2y}{dx^2}\right)^3$$

In this polynomial form in derivatives, the highest power raised to the highest order derivative $\frac{d^2y}{dx^2}$ is 3, so the **degree = 3**.

207. (C)

The given differential equation is:

$(2x-2y+3)dx-(x-y+1)dy=0$

Rearranging the terms:

$$\frac{dy}{dx}=\frac{2x-2y+3}{x-y+1}=\frac{2(x-y)+3}{(x-y)+1}$$

Let $x-y=v$. Differentiating both sides with respect to x:

$$1-\frac{dy}{dx}=\frac{dv}{dx}\Rightarrow\frac{dy}{dx}=1-\frac{dv}{dx}$$

Substituting v and $\frac{dy}{dx}$ into the differential equation:

$$1-\frac{dv}{dx}=\frac{2v+3}{v+1}$$

$$\frac{dv}{dx}=1-\frac{2v+3}{v+1}=\frac{(v+1)-(2v+3)}{v+1}=\frac{-v-2}{v+1}$$

Separating the variables:

$$\left(\frac{v+1}{v+2}\right)dv=-dx$$

$$\left(1\frac{1}{v+2}\right)dv=-dx$$

Integrating both sides:

$$\int\left(1\frac{1}{v+2}\right)dv=-\int dx$$

$v-\log(v+2)=-x+C$

Substituting $v=x-y$:

$(x-y)-\log(x-y+2)=-x+C$.

$2x-y-\log(x-y+2)=C$

Using the initial condition $x=0$ when $y=1$:

$2(0)-1-\log(0-1+2)=C$

$-1-\log(1)=C\Rightarrow C=-1$

Substituting $C=-1$ gives the particular solution:

$2x-y-\log(x-y+2)=-1$

$2x-y-\log(x-y+2)+1=0$

208. (D)

Solve $y'=y/\left(x+\sqrt{xy}\right)$.

Put $v=\sqrt{y/x}$. Separation gives

$\log y-2\sqrt{x/y}=C,$

so $y=Ce^{2\sqrt{x/y}}$.

209. (D)

The slope of the curve at any point (x,y) is given by:

$$\frac{dy}{dx}=\frac{y}{x}+\sec\left(\frac{y}{x}\right)$$

This is a homogeneous differential equation. Let $y=vx$, so $\frac{dy}{dx}=v+x\frac{dv}{dx}$.

Substituting these into the differential equation gives:

$$v+x\frac{dv}{dx}=v+\sec v$$

$$x\frac{dv}{dx}=\sec v$$

Separating the variables:

$$\frac{dv}{\sec v}=\frac{dx}{x}$$

$$\cos v\,dv=\frac{dx}{x}$$

Integrating both sides:

$$\int\cos v\,dv=\int\frac{dx}{x}$$

$$\sin v=\log x+C$$

Substituting $v=\frac{y}{x}$:

$$\sin\left(\frac{y}{x}\right)=\log x+C$$

Since the curve passes through the point $\left(1,\frac{\pi}{6}\right)$, substitute $x=1$ and $y=\frac{\pi}{6}$:

$$\sin\left(\frac{\pi}{6}\right)=\log 1+C$$

$$\frac{1}{2}=0+C\Rightarrow C=\frac{1}{2}$$

Therefore, the equation of the curve is:

$$\sin\left(\frac{y}{x}\right)=\log x+\frac{1}{2}$$

210. (C)

The given differential equation is:

$$\left(1+e^{-x}\right)\left(1+y^2\right)\frac{dy}{dx}=y^2$$

Separating the variables:

$$\frac{1+y^2}{y^2}dy=\frac{dx}{1+e^{-x}}$$

$$\left(\frac{1}{y^2}+1\right)dy=\frac{e^x}{e^x+1}dx$$

Integrating both sides:

$$\int\left(y^{-2}+1\right)dy=\int\frac{e^x}{e^x+1}dx$$

$$-\frac{1}{y}+y=\log\left|e^x+1\right|+C$$

Since the curve passes through the point $(0,1)$, substituting $x=0$ and $y=1$:

$$-\frac{1}{1}+1=\log\left|e^0+1\right|+C$$

$$0=\log 2+C\Rightarrow C=-\log 2$$

Substituting $C=-\log 2$ back into the general solution:

$$y-\frac{1}{y}=\log\left|e^x+1\right|-\log 2$$

$$y-\frac{1}{y}=\log\left|\frac{1+e^x}{2}\right|$$

Multiplying throughout by y:

$$y^2-1=y\log\left|\frac{1+e^x}{2}\right|$$

$$y^2=1+y\log\left|\frac{1+e^x}{2}\right|$$

211. (C)

Given the rate of change of production P with respect to the additional number of workers x:

$$\frac{dP}{dx}=100-12\sqrt{x}$$

Integrating both sides with respect to x:

$$P(x)=\int\left(10012x^{1/2}\right)dx=100x-12\cdot\frac{x^{3/2}}{\frac{3}{2}}+C=100x-8x^{3/2}+C$$

When no additional workers are employed ($x=0$), the initial level of production is $P(0)=2000$:

$C=2000$

Therefore, the production function is:

$P(x)=2000+100x-8x^{3/2}$

When the firm employs $x=25$ additional workers:

$P(25)=2000+100(25)-8(25)^{3/2}$

$P(25)=2000+2500-8(125)=4500-1000=3500$

Hence, the new level of production is 3500 items.

212. (A)

Given the differential equation:

$$\left(\frac{2+\sin x}{y+1}\right)\frac{dy}{dx}=-\cos x$$

Separating the variables gives:

$$\frac{1}{y+1}dy=-\frac{\cos x}{2+\sin x}dx$$

Integrating both sides:

$$\int\frac{1}{y+1}dy=-\int\frac{\cos x}{2+\sin x}dx$$

$\ln(y+1)=-\ln(2+\sin x)+\ln C$

$\ln\left[(y+1)(2+\sin x)\right]=\ln C$

$(y+1)(2+\sin x)=C$

Using the given condition $y(0)=1$:

$(1+1)(2+\sin 0)=C\Rightarrow 2\times 2=C\Rightarrow C=4$

Thus, the particular solution is:

$(y+1)(2+\sin x)=4$

Substitute $x=\frac{\pi}{2}$:

$$\left(y\left(\frac{\pi}{2}\right)+1\right)\left(2+\sin\frac{\pi}{2}\right)=4$$

$$\left(y\left(\frac{\pi}{2}\right)+1\right)(2+1)=4$$

$$3\left(y\left(\frac{\pi}{2}\right)+1\right)=4$$

$$y\left(\frac{\pi}{2}\right)+1=\frac{4}{3}$$

$$y\left(\frac{\pi}{2}\right)=\frac{4}{3}-1=\frac{1}{3}$$

213. (C)

For a random variable X. taking values $1,2,3,\ldots,n$. with equal probability $P(X=k)=\frac{1}{n}$:

1. Expected value $E(X)$:

$$E(X)=\sum_{k=1}^{n}k\cdot P(X=k)=\frac{1}{n}\sum_{k=1}^{n}k=\frac{1}{n}\cdot\frac{n(n+1)}{2}=\frac{n+1}{2}$$

2. Variance $Var(X)$:

$$E(X^2)=\frac{1}{n}\sum_{k=1}^{n}k^2=\frac{1}{n}\cdot\frac{n(n+1)(2n+1)}{6}=\frac{(n+1)(2n+1)}{6}$$

$$Var(X)=E(X^2)-[E(X)]^2=\frac{(n+1)(2n+1)}{6}-\left(\frac{n+1}{2}\right)^2=\frac{n^2-1}{12}$$

3. Ratio of variance to expected value:

$$\frac{Var(X)}{E(X)}=\frac{\frac{n^2-1}{12}}{\frac{n+1}{2}}=\frac{n-1}{6}$$

Given $\frac{Var(X)}{E(X)}=4$:

$$\frac{n-1}{6}=4$$

$n-1=24$

$n=25$

214. (A)

Let X denote the random variable for the daily demand of pizzas.

The expected daily demand $E(X)$ is given by:

$E(X)=\sum x_i p_i$

$E(X)=(5\times 0.07)+(6\times 0.2)+(7\times 0.3)+(8\times 0.3)$

$+(9\times0.07)+(10\times0.06)$

$E(X)=0.35+1.20+2.10+2.40+0.63+0.60=7.28$

Now, calculating $E(X^2)$:

$E(X^2)=\sum x_i^2 p_i$

$E(X^2)=(25\times0.07)+(36\times0.2)+(49\times0.3)$

$+(64\times03)+(81\times0.07)+(100\times9.96)$

$E(X^2)=1.75+7.20+14.70+19.20+5.67+6.00=54.52$

The variance $\text{Var}(X)$ is:

$\text{Var}(X)=E(X^2)-[E(X)]^2$

$\text{Var}(X)=54.52-(7.28)^2=54.52-52.9984=1.5216\approx1.52$

Therefore, the expected daily demand and variance are 7.28 and 1.52 respectively.

215. (C)

Let X denote the number of kings drawn in $n=2$ draws with replacement.

The probability of drawing a king in a single draw is:

$$p=\frac{4}{52}=\frac{1}{13}$$

Since drawing with replacement yields independent trials, X follows a Binomial distribution $B(n,p)$ with $n=2$ and $p=\frac{1}{13}$.

The mean of the number of kings is given by:

$$\text{Mean } E(X)=n\times p=2\times\frac{1}{13}=\frac{2}{13}$$

216. (A)

The total number of balls in the bag is $4+3=7$.

Let the success event be drawing a black ball. The probability of success in a single draw is:

$$p=\frac{3}{7}$$

The probability of failure (drawing a red ball) is:

$$q=1-p=\frac{4}{7}$$

Since the ball is replaced after each draw, the draws are independent, and X follows a Binomial Distribution with $n=3$ and $p=\frac{3}{7}$.

The probability mass function is given by:

$$P(X=x)=\binom{3}{x}p^x q^{3-x} \text{ for } x=0,1,2,3$$

Calculating the probabilities for each value of X:

1. For $x=0$:

$$P(X=0)=\binom{3}{0}\left(\frac{3}{7}\right)^0\left(\frac{4}{7}\right)^3=1\cdot1\cdot\left(\frac{4}{7}\right)^3=\left(\frac{4}{7}\right)^3$$

2. For $x=1$:

$$P(X=1)=\binom{3}{1}\left(\frac{3}{7}\right)^1\left(\frac{4}{7}\right)^2=3\cdot\left(\frac{3}{7}\right)\cdot\left(\frac{4}{7}\right)^2=\frac{9}{7}\left(\frac{4}{7}\right)^2$$

3. For $x=2$:

$$P(X=2)=\binom{3}{2}\left(\frac{3}{7}\right)^2\left(\frac{4}{7}\right)^1=3\cdot\left(\frac{3}{7}\right)^2\cdot\left(\frac{4}{7}\right)=\frac{12}{7}\left(\frac{3}{7}\right)^2$$

4. For $x=3$:

$$P(X=3)=\binom{3}{3}\left(\frac{3}{7}\right)^3\left(\frac{4}{7}\right)^0=1\cdot\left(\frac{3}{7}\right)^3\cdot1=\left(\frac{3}{7}\right)^3$$

This probability distribution matches table (1).

217. (C)

Let X be the number of science students among the 5 randomly selected students. Here, X follows a Binomial distribution $B(n,p)$ with:

- Number of trials, $n=5$
- Probability of selecting a science student, $p=30\%=0.3$
- Probability of not selecting a science student, $q=1-p=0.7$

The probability of selecting exactly 2 science students ($X=2$) is:

$$P(X=2)=\binom{5}{2}p^2q^{5-2}$$

$$P(X=2)=10\times(0.3)^2\times(0.7)^3$$

$$P(X=2)=10\times0.09\times0.343=0.3087$$

218. (A)

For a binomial variate $X \sim B(n, p)$:

$\text{Mean} = np = 2$

$\text{Variance} = npq = 1$

Dividing the variance by the mean gives:

$\frac{npq}{np} = \frac{1}{2} \Rightarrow q = \frac{1}{2}$

Since $p + q = 1$:

$p = 1 - \frac{1}{2} = \frac{1}{2}$

Substituting $p = \frac{1}{2}$ into $np = 2$:

$n\left(\frac{1}{2}\right) = 2 \Rightarrow n = 4$

The probability of $X = 0$ is:

$$P(X = 0) = \binom{4}{0} p^0 q^4 = 1 \times 1 \times \left(\frac{1}{2}\right)^4 = \frac{1}{16}$$

The probability against $X = 0$ is:

$$P(X \neq 0) = 1 - P(X = 0) = 1 - \frac{1}{16} = \frac{15}{16}$$

Therefore, the odds in favour of $X = 0$ are:

$$\text{Odds in favour} = \frac{P(X = 0)}{P(X \neq 0)} = \frac{1/16}{15/16} = \frac{1}{15} \text{ or } 1:15$$

219. (D)

Let X be the number of workmen suffering from the occupational disease out of $n = 5$ workmen.

This follows a Binomial distribution $B(n, p)$ with parameters:

$n = 5$

$p = 10\% = 0.1$

$q = 1 - p = 0.9$

The probability mass function is given by:

$$P(X = k) = \binom{n}{k} p^k q^{n-k}$$

We need to find the probability that 3 or more workmen contract the disease, i.e., $P(X \geq 3)$:

$$P(X \geq 3) = P(X = 3) + P(X = 4) + P(X = 5)$$

Calculating each probability term:

$$P(X = 3) = \binom{5}{3} (0.1)^3 (0.9)^2 = 10 \times 0.001 \times 0.81 = 0.0081$$

$$P(X = 4) = \binom{5}{4} (0.1)^4 (0.9)^1 = 5 \times 0.0001 \times 0.9 = 0.00045$$

$$P(X = 5) = \binom{5}{5} (0.1)^5 (0.9)^0 = 1 \times 0.00001 \times 1 = 0.00001$$

Adding the probabilities together:

$P(X \geq 3) = 0.0081 + 0.00045 + 0.00001 = 0.00856$

220. (B)

For a Binomial distribution $B(n, p)$, the probability of k successes is given by:

$$P(X = k) = \binom{n}{k} p^k q^{n-k}$$

Given $n = 6$, the probabilities $P(X = 4)$ and $P(X = 2)$ are:

$$P(X = 4) = \binom{6}{4} p^4 q^{6-4} = 15 p^4 q^2$$

$$P(X = 2) = \binom{6}{2} p^2 q^{6-2} = 15 p^2 q^4$$

Substituting these into the given relation $9P(X = 4) = P(X = 2)$:

$9 \cdot 15 p^4 q^2 = 15 p^2 q^4$

Dividing both sides by $15 p^2 q^2$ (since $p, q \neq 0$):

$9p^2 = q^2 \Rightarrow q = 3p$

Using the relation $p + q = 1$, we substitute $p = 1 - q$:

$q = 3(1 - q)$

$q = 3 - 3q$

$4q = 3 \Rightarrow q = \frac{3}{4}$

221. (D)

For a Binomial distribution $X \sim B(n, p)$, the mean and variance are given by:

$\text{Mean} = np = 8$

$\text{Variance} = npq = 4$

where $q = 1 - p$.

Dividing the variance by the mean:

$$q = \frac{npq}{np} = \frac{4}{8} = \frac{1}{2}$$

Thus, $p = 1 - q = \frac{1}{2}$.

Substituting $p = \frac{1}{2}$ into the mean equation:

$$n\left(\frac{1}{2}\right) = 8 \Rightarrow n = 16$$

The probability mass function of X is:

$$P(X = r) = \binom{16}{r}\left(\frac{1}{2}\right)^{r}\left(\frac{1}{2}\right)^{16-r} = \binom{16}{r}\left(\frac{1}{2}\right)^{16}$$

Now, calculating $P(X \le 2)$:

$$P(X \le 2) = P(X = 0) + P(X = 1) + P(X = 2)$$

$$P(X \le 2) = \left[\binom{16}{0} + \binom{16}{1} + \binom{16}{2}\right]\frac{1}{2^{16}}$$

$$P(X \le 2) = (1+16+120)\frac{1}{2^{16}} = \frac{137}{2^{16}}$$

Given $P(X \le 2) = \frac{k}{2^{16}}$, comparing both sides gives: $k = 137$

MHTCET Yearwise PYQ (2023)

Marathi

Mathematics

1. If $\cos 2B = \frac{\cos(A+C)}{\cos(A-C)}$. Then tan *A*, tan *B*, tan *C* are in- **[2023]**

(A) Geometric Progression.
(B) Arithmetic Progression.
(C) Harmonic Progression.
(D) Arithmetico–Geometric Progression.

2. If $\cos x + \cos y - \cos(x+y) = \frac{3}{2}$, then; **[2023]**

(A) $x + y = 0$ (B) $x = 2y$
(C) $x = y$ (D) $2x = y$

3. If $\tan\theta = \frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}$, $0 \le \alpha \le \frac{\pi}{2}$, then the value of cos 2θ is: **[2023]**

(A) cos 2α (B) sin α
(C) cos α (D) sin 2α

4. $\cos^2 48° - \sin^2 12° =$ _____, if $\sin 18° = \frac{\sqrt{5}-1}{4}$ **[2023]**

(A) $\frac{-\sqrt{5}+1}{8}$ (B) $\frac{\sqrt{5}-1}{8}$
(C) $\frac{\sqrt{5}+1}{8}$ (D) $\frac{-\sqrt{5}-1}{8}$

5. If $\left(1+\sqrt{1+x}\right)\tan x = 1+\sqrt{1-x}$, then sin 4*x* is: **[2023]**

(A) *x* (B) –*x*
(C) 4*x* (D) –4*x*

6. If sin(θ – α), sin θ and sin(θ + α) are in H.P., then the value of cos 2θ is: **[2023]**

(A) $1+4\cos^2\frac{\alpha}{2}$ (B) $1-4\cos^2\frac{\alpha}{2}$
(C) $-1-4\cos^2\frac{\alpha}{2}$ (D) $-1+4\cos^2\frac{\alpha}{2}$

7. The value of cos(18° – *A*)cos(18° + *A*) –cos(72° – *A*)cos(72° + *A*) is equal to*:* **[2024, 2023]**

(A) cos 54°
(B) cos 36°
(C) sin 54°
(D) sin 36°

8. Two sides of a square are along the lines $5x - 12y + 39 = 0$ and $5x - 12y + 78 = 0$, then area of the square is: **[2023]**

(A) 9 sq. units.
(B) $\frac{1}{3}$ sq. units.
(C) 18 sq. units.
(D) 3 sq. units.

9. If the vertices of a triangle are (–2, 3), (6, –1) and (4, 3), then the co–ordinates of the circumcentre of the triangle are:

(A) (1, 1) (B) (–1,–1)
(C) (–1, 1) (D) (1,–1)

10. The co–ordinates of the points on the line $2x - y = 5$ which are the distance of 1 unit from the line $3x + 4y = 5$ are: **[2023]**

(A) $\left(\frac{30}{11},\frac{-5}{11}\right),\left(\frac{20}{11},\frac{15}{11}\right)$
(B) $\left(\frac{-30}{11},\frac{5}{11}\right),\left(\frac{-20}{11},\frac{15}{11}\right)$
(C) $\left(\frac{30}{11},\frac{5}{11}\right),\left(\frac{20}{11},\frac{-15}{11}\right)$
(D) $\left(\frac{-30}{11},\frac{5}{11}\right),\left(\frac{-20}{11},\frac{-15}{11}\right)$

11. If k_i are possible values of k for which lines $kx + 2y + 2 = 0$, $2x + ky + 3 = 0$ and $3x + 3y + k = 0$ are concurrent, then Σk_i has the value- **[2023]**

(A) 0
(B) –2
(C) 2
(D) 5

12. The base of an equilateral triangle is represented by the equation $2x - y - 1 = 0$ and its vertex is (1, 2), then the length (in units) of the side of the triangle is: **[2023]**

(A) $\sqrt{\frac{20}{3}}$ (B) $\frac{2}{\sqrt{15}}$

(C) $\sqrt{\frac{8}{15}}$ (D) $\sqrt{\frac{15}{2}}$

13. A line is drawn through the point (1, 2) to meet the co–ordinate axes at P and Q such that it forms a ΔOPQ, where O is the origin. If the area of ΔOPQ is least, then the slope of the line PQ is: **[2023]**

(A) –2 (B) 2

(C) $\frac{-1}{2}$ (D) $\frac{1}{2}$

14. PS is the median of the triangle with vertices at $P(2, 2)$, Q(6, –1) and $R(7, 3)$, then the intercepts on the co–ordinate axes of the line passing through point (1, –1) and parallel to PS are respectively- **[2023]**

(A) $\frac{7}{2}, \frac{-7}{9}$ (B) $\frac{2}{7}, \frac{9}{7}$

(C) $\frac{-7}{2}, \frac{-7}{9}$ (D) –2, –9

15. p is the length of perpendicular from the origin to the line whose intercepts on the axes are a and b respectively, then $\frac{1}{a^2} + \frac{1}{b^2}$ equals- **[2023]**

(A) p^2 (B) $\frac{2}{p^2}$

(C) $\frac{1}{p^2}$ (D) $\frac{1}{2p^2}$

16. Let $P \equiv (-3, 0)$, $Q \equiv (0, 0)$ and $R \equiv (3, 3\sqrt{3})$ be three points. Then the equation of the bisector of the angle PQR is: **[2023]**

(A) $\frac{\sqrt{3}}{2}x + y = 0$ (B) $x + \sqrt{3}y = 0$

(C) $\sqrt{3}x + y = 0$ (D) $x + \frac{\sqrt{3}}{2}y = 0$

17. The sides of a rectangle are given by the equations $x = -2$, $x = 4$, $y = -2$ and $y = 5$ Then the equation of the circle, whose centre is the point of intersection of the diagonals, lying within the rectangle and touching only two opposite sides, is: **[2023]**

(A) $x^2 + y^2 + 2x + 3y + 9 = 0$

(B) $x^2 + y^2 - 2x + 3y + 9 = 0$

(C) $x^2 + y^2 + 2x - 3y - 9 = 0$

(D) $x^2 + y^2 - 2x - 3y - 9 = 0$

18. If a circle passes through points (4, 0) and (0, 2) and its centre lies on Y–axis. If the radius of the circle is r, then the value of $r^2 - r + 1$ is: **[2023]**

(A) 25 (B) 21

(C) 20 (D) 10

19. If λ is the perpendicular distance of a point P on the circle $x^2 + y^2 + 2x + 2y - 3 = 0$, from the line $2x + y + 13 = 0$, then maximum possible value of λ is: **[2023]**

(A) $2\sqrt{5}$ (B) $3\sqrt{5}$

(C) $4\sqrt{5}$ (D) $\sqrt{5}$

20. The circles $x^2 + y^2 + 2ax + c = 0$ and $x^2 + y^2 + 2by + c = 0$ touch each other externally, if **[2023]**

(A) $\frac{1}{a^2} - \frac{1}{b^2} = \frac{1}{c}$

(B) $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c}$

(C) $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$

(D) $\frac{1}{a^2} - \frac{1}{b^2} = \frac{1}{c^2}$

21. The centre of the circle whose radius is 3 units and touching internally the circle $x^2 + y^2 - 4x - 6y - 12 = 0$ at the point (–1, –1) is: **[2023]**

(A) $\left(\frac{4}{5}, \frac{7}{5}\right)$ (B) $\left(\frac{4}{5}, \frac{-7}{5}\right)$

(C) $\left(\frac{-4}{5}, \frac{-7}{5}\right)$ (D) $\left(\frac{-4}{5}, \frac{7}{5}\right)$

22. The abscissae of the two points A and B are the roots of the equation $x^2+2ax-b^2=0$ and their ordinates are roots of the equation $y^2+2py-q^2=0$. Then the equation of the circle with AB as diameter is given by **[2024, 2023]**

(A) $x^2+y^2-2ax-2py+\left(b^2+q^2\right)=0$

(B) $x^2+y^2-2ax-2py-\left(b^2+q^2\right)=0$

(C) $x^2+y^2+2ax+2py+\left(b^2+q^2\right)=0$

(D) $x^2+y^2+2ax+2py-\left(b^2+q^2\right)=0$

23. A problem in statistics is given to three students A, B and C. Their probabilities of solving the problem are $\frac{1}{2},\frac{1}{3}$ and $\frac{1}{4}$ respectively. If all of them try independently, then the probability, that problem is solved, is: **[2023]**

(A) $\frac{2}{3}$ (B) $\frac{3}{4}$

(C) $\frac{1}{3}$ (D) $\frac{1}{4}$

24. The three ships namely A, B and C sail from India to Africa. If the odds in favour of the ships reaching safely are 2 : 5, 3 : 7 and 6 : 11 respectively, then probability of all of them arriving safely is:**[2023]**

(A) $\frac{18}{595}$ (B) $\frac{11}{34}$

(C) $\frac{196}{217}$ (D) $\frac{1}{595}$

25. If A and B are two events such that $P(A)=\frac{1}{3}$, $P(B)=\frac{1}{5}$, $P(A\cup B)=\frac{1}{3}$, then the value of $P(A'/B')+P(B'/A')$ is: **[2023]**

(A) $\frac{5}{6}$ (B) 1

(C) $\frac{1}{6}$ (D) $\frac{11}{6}$

26. A and B are independent events with $P(A)=\frac{1}{4}$ and $P(A\cup B)=2P(B)-P(A)$, then $P(B)$ is: **[2023]**

(A) $\frac{1}{4}$ (B) $\frac{3}{5}$

(C) $\frac{2}{3}$ (D) $\frac{2}{5}$

27. A box contains 100 tickets numbered 1 to 100. A ticket is drawn at random from the box. Then the probability, that number on the ticket is a perfect square, is: **[2023]**

(A) $\frac{1}{10}$ (B) $\frac{3}{10}$

(C) $\frac{7}{100}$ (D) $\frac{9}{100}$

28. A card is drawn at random from a well shuffled pack of 52 cards. The probability that it is black card or face card is: **[2023]**

(A) $\frac{3}{13}$ (B) $\frac{5}{13}$

(C) $\frac{6}{13}$ (D) $\frac{8}{13}$

29. Two dice are rolled. If both dice have six faces numbered 1, 2, 3, 5, 7, 11, then the probability that the sum of the numbers on upper most face is prime is: **[2023]**

(A) $\frac{1}{4}$ (B) $\frac{3}{4}$

(C) $\frac{1}{9}$ (D) $\frac{2}{7}$

30. Three critics review a book. For the three critics the odds in favour of the book are 2 : 5, 3 : 4 and 4 : 3 respectively. The probability that the majority is in favour of the book, is given by- **[2023]**

(A) $\frac{183}{343}$ (B) $\frac{160}{343}$

(C) $\frac{209}{343}$ (D) $\frac{134}{343}$

31. If $Z_1 = 2 + i$ and $Z_2 = 3 - 4i$ and $\frac{\overline{Z_1}}{Z_2} = a + bi$, then the value of $-7a + b$ is: (where $i = \sqrt{-1}$ and $a, b \in R$) **[2023]**

(A) 1 (B) –1

(C) $\frac{-3}{25}$ (D) $\frac{-9}{25}$

32. The argument of $\frac{1+i\sqrt{3}}{\sqrt{3}+i}$, $i = \sqrt{-1}$ is: **[2023]**

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{6}$ (D) $\frac{\pi}{2}$

33. If $x = \frac{5}{1-2i}$, $i = \sqrt{-1}$, then the value of $x^3 + x^2 - x + 22$ is: **[2023]**

(A) 7 (B) 9

(C) 17 (D) 39

34. If $|z - 2 + i| \leq 2$, then the difference between the greatest and least value of $|z|$ is _____. ($i = \sqrt{-1}$) **[2023]**

(A) $2\sqrt{5} + 4$ (B) $2\sqrt{5}$

(C) 4 (D) 8

35. The value of $\frac{i^{248} + i^{246} + i^{244} + i^{242} + i^{240}}{i^{249} + i^{247} + i^{245} + i^{243} + i^{241}}$ ($i = \sqrt{-1}$) is: **[2023]**

(A) i (B) 1

(C) –1 (D) $-i$

36. If $(3x + 2) - (5y - 3)i$ and $(6x + 3) + (2y - 4)i$ are conjugates of each other, then the value of $\frac{x-y}{x+y}$ is: (where $i = \sqrt{-1}$, $x, y \in R$) **[2023]**

(A) –1 (B) 0

(C) 1 (D) 2

37. If $a > 0$ and $z = \frac{(1+i)^2}{a-i}$, $i = \sqrt{-1}$, has magnitude $\frac{2}{\sqrt{5}}$, then $\bar{z}$ is **[2023]**

(A) $-\frac{2}{5} - \frac{4}{5}i$ (B) $-\frac{2}{5} + \frac{4}{5}i$

(C) $\frac{2}{5} - \frac{4}{5}i$ (D) $\frac{2}{5} + \frac{4}{5}i$

38. Let z be the complex number with $\text{Im}(z) = 10$ and satisfying $\frac{2z-n}{2z+n} = 2i - 1$, where $i = \sqrt{-1}$, for some natural number 'n' then **[2025, 2023]**

(A) $n = 20$ and $\text{Re}(z) = 10$

(B) $n = 20$ and $\text{Re}(z) = -10$

(C) $n = 40$ and $\text{Re}(z) = 10$

(D) $n = 40$ and $\text{Re}(z) = -10$

39. A linguistic club of a certain Institute consists of 6 girls and 4 boys. A team of 4 members to be selected from this group including the selection of a Captain (from among these 4 members) for the team. If the team has to include atmost one boy, the number of ways of selecting the team is: **[2023]**

(A) 95 (B) 260

(C) 320 (D) 380

40. The value of $\frac{{}^{10}C_r}{{}^{11}C_r}$, when both the numerator and denominator are at their greatest values, is: **[2023]**

(A) $\frac{6}{11}$ (B) $\frac{1}{11}$

(C) $\frac{4}{11}$ (D) $\frac{3}{11}$

41. The teacher wants to arrange 5 students on the platform such that the boy B_1 occupies second position and the girls G_1 and G_2 are always adjacent to each other, then the number of such arrangements is: **[2023]**

(A) 24 (B) 12

(C) 8 (D) 16

42. If T_n denotes the number of triangles which can be formed using the vertices of regular polygon of n sides and $T_{n+1} - T_n = 21$, then n = ? **[2023]**

(A) 5 (B) 7
(C) 6 (D) 4

43. The number of words that can be formed by using the letters of the word CALCULATE such that each word starts and ends with a consonant, are: **[2023]**

(A) $5 \times 7!$ (B) $\frac{9!}{8}$
(C) $\frac{5 \times 7!}{2}$ (D) $20 \times 7!$

44. Five students are selected from n students such that the ratio of number of ways in which 2 particular students are selected to the number of ways 2 particular students not selected is 2 : 3. Then the value of n is: **[2023]**

(A) 5 (B) 6
(C) 11 (D) not possible

45. If in a regular polygon, the number of diagonals are 54, then the number of sides of the polygon are: **[2023]**

(A) 10 (B) 12
(C) 9 (D) 6

46. If $\log_2 x + \log_4 x + \log_8 x + \log_{16} x = \frac{25}{36}$ and $x = 2^k$, then k is: **[2023]**

(A) 1 (B) $\frac{1}{2}$
(C) $\frac{1}{3}$ (D) $\frac{1}{8}$

47. The domain of the function $f(x) = \sin^{-1}\left(\frac{|x|+5}{x^2+1}\right)$ is: $(-\infty, -a] \cup [a, \infty)$. Then a is equal to: **[2023]**

(A) $\frac{\sqrt{17}}{2} + 1$ (B) $\frac{\sqrt{17}-1}{2}$
(C) $\frac{1+\sqrt{17}}{2}$ (D) $\frac{\sqrt{17}}{2} - 1$

48. If $f(x) = \frac{3x+4}{5x-7}$ and $g(x) = \frac{7x+4}{5x-3}$, then $f(g(x))$ = **[2023]**

(A) $\frac{x^3+1}{x^2+2}$ (B) $41x$
(C) $g(f(x))$ (D) $\frac{5x-7}{41}$

49. If $g(x) = 1 + \sqrt{x}$ and $f(g(x)) = 3 + 2\sqrt{x} + x$, then $f(f(x))$ is: **[2023]**

(A) $x^2 + 4x + 6$ (B) $x^4 + x^2 + 6$
(C) $x^2 + x + 6$ (D) $x^4 + 4x^2 + 6$

50. If $f(x) = x^2 + 1$ and $g(x) = \frac{1}{x}$, then the value of $f(g(g(f(x))))$ at $x = 1$ is: **[2023]**

(A) 4 (B) 1
(C) 5 (D) 3

51. If $3f(x) - f\left(\frac{1}{x}\right) = 8\log_2 x^3$, $x > 0$, then $f(2)$, $f(4)$, $f(8)$ are in: **[2023]**

(A) A.P.
(B) G.P.
(C) H.P.
(D) Arithmetico Geometric Progression.

52. The range of the function $f(x) = \frac{x^2}{x^2+1}$ is: **[2023]**

(A) (0, 1) (B) [0, 1)
(C) (0, 1] (D) [0, 1]

53. If $f(x) = \frac{2x-3}{3x-4}$, $x \neq \frac{4}{3}$, then the value of $f^{-1}(x)$ is: **[2023]**

(A) $\frac{4x-3}{3x-2}$ (B) $\frac{3x-2}{4x+3}$
(C) $\frac{3x-4}{4x-2}$ (D) $\frac{2x+3}{4x-3}$

54. The domain of the definition of the function $f(x)$ is given by the equation $2^x + 2^y = 2$ is: **[2024, 2023]**

(A) $0 < x \leq 1$ (B) $0 \leq x \leq 1$
(C) $-\infty < x \leq 0$ (D) $-\infty < x < 1$

55. $\lim_{x\to 0}\frac{(1-\cos 2x)\cdot\sin 5x}{x^2\sin 3x}$ is: **[2023]**

(A) $\frac{10}{3}$ (B) $\frac{5}{3}$

(C) $\frac{5}{6}$ (D) $\frac{2}{3}$

56. $\lim_{x\to a}\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}=$ _____. **[2023]**

(A) $\frac{1}{3\sqrt{3}}$ (B) $\frac{2}{\sqrt{3}}$

(C) $\frac{2}{3\sqrt{3}}$ (D) $\frac{-2}{3\sqrt{3}}$

57. $\lim_{x\to 0}\frac{\cos 7x^\circ-\cos 2x^\circ}{x^2}$ is: **[2023]**

(A) $\frac{-45}{2}\pi^2$ (B) $\frac{-45}{2}\pi$

(C) $\frac{-\pi^2}{1440}$ (D) $\frac{-\pi^2}{2880}$

58. $\lim_{x\to 0}\frac{x\cot 4x}{\sin^2 x\cdot\cot^2(2x)}$ is equal to– **[2023]**

(A) 0 (B) 1

(C) 4 (D) $\frac{1}{4}$

59. If $f(x)=3x^{10}-7x^8+5x^6-21x^3+3x^2-7$, then $\lim_{\alpha\to 0}\frac{f(1-\alpha)-f(1)}{\alpha^3+3\alpha}=$ **[2023]**

(A) $\frac{53}{3}$ (B) $\frac{-53}{3}$

(C) $\frac{52}{3}$ (D) $\frac{-52}{3}$

60. $\lim_{x\to 0}\frac{\sqrt{1+x\sin x}-\sqrt{\cos x}}{\tan^2\frac{x}{2}}=$ **[2023]**

(A) 1 (B) 2

(C) 3 (D) –1

61. $\lim_{x\to\infty}x^3\left\{\sqrt{x^2+\sqrt{1+x^4}}-x\sqrt{2}\right\}=$ **[2023]**

(A) $\frac{1}{\sqrt{2}}$

(B) $\frac{1}{4\sqrt{2}}$

(C) $\frac{-1}{4\sqrt{2}}$

(D) $\frac{-1}{\sqrt{2}}$

62. If $f(a)=2, f'(a)=1, g(a)=-1, g'(a)=2$, then as x approaches a, $\frac{g(x)f(a)-g(a)f(x)}{(x-a)}$ approaches **[2023]**

(A) 3 (B) 5

(C) 0 (D) 2

63. The value of $\lim_{x\to 0}\frac{x}{|x|+x^2}$ is: **[2024, 2023]**

(A) 1

(B) –1

(C) 0

(D) does not exist.

64. $f(x)=\begin{cases}\frac{1-\cos kx}{x^2} & \text{, if } x\le 0\\ \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4} & \text{, if } x>0\end{cases}$ is continuous at x = 0, then the value of k is: **[2023]**

(A) 4 (B) 2

(C) –1 (D) –3

65. If $f(x)=\frac{4}{x^4}\left[1-\cos\frac{x}{2}-\cos\frac{x}{4}+\cos\frac{x}{2}\cdot\cos\frac{x}{4}\right]$ is continuous at x = 0, then $f(0)$ is: **[2023]**

(A) $\frac{1}{32}$ (B) $\frac{1}{16}$

(C) $\frac{1}{8}$ (D) $\frac{1}{64}$

66. If $f(x) = \begin{cases} \frac{x-3}{|x-3|}+a, & x<3 \\ a+b, & x=3 \\ \frac{|x-3|}{x-3}+b, & x>3 \end{cases}$ Is continuous at x=3, then the value of $a - b$ is: **[2023]**

(A) –1 (B) 0
(C) 1 (D) 2

67. The function $f(t) = \frac{1}{t^2+t-2}$ where $t = \frac{1}{x-1}$ is discontinuous at **[2023]**

(A) –2, 1 (B) $2, \frac{1}{2}$
(C) $\frac{1}{2}, 1$ (D) 2, 1

68. The function $f(x) = [x] \cdot \cos\left(\frac{2x-1}{2}\right)\pi$, where $[\cdot]$ denotes the greatest integer function, is discontinuous at **[2023]**

(A) all irrational numbers x.
(B) no x.
(C) all integer points.
(D) every rational x which is not an integer.

69. The function f defined on $\left(-\frac{1}{3}, \frac{1}{3}\right)$ by $f(x) = \begin{cases} \frac{1}{x}\log\left(\frac{1+3x}{1-2x}\right), & x \neq 0 \\ k, & x=0 \end{cases}$ is continuous at $x = 0$, then k is: **[2023]**

(A) 6 (B) 1
(C) 5 (D) –5

70. Let p, q, r be three statements, then $[p \to (q \to r)] \leftrightarrow [(p \wedge q) \to r]$ is: **[2023]**

(A) equivalent to $p \leftrightarrow q$.
(B) contingency.
(C) tautology.
(D) contradiction.

71. If truth value of logical statement $(p \leftrightarrow \sim q) \to (\sim p \wedge q)$ is false, then the truth values of p and q are respectively **[2023]**

(A) F, T
(B) T, T
(C) T, F
(D) F, F

72. The inverse of the statement "If the surface area increase, then the pressure decreases.", is **[2023]**

(A) If the surface area does not increase, then the pressure does not decrease.
(B) If the pressure decreases, then the surface area increases.
(C) If the pressure does not decrease, then the surface area does not increase.
(D) If the surface area does not increase, then the pressure decreases.

73. The contrapositive of "If x and y are integers such that xy is odd, then both x and y are odd" is: **[2023]**

(A) If both x and y are odd integers, then xy is odd.
(B) If both x and y are even integers, then xy is even.
(C) If x or y is an odd integer, then xy is odd.
(D) If both x and y are not odd integers, then the product xy is not odd.

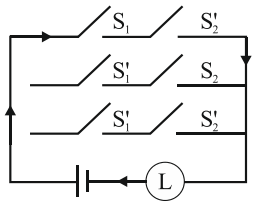
74. Let

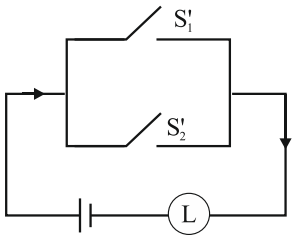
Statement 1 : If a quadrilateral is a square, then all of its sides are equal.

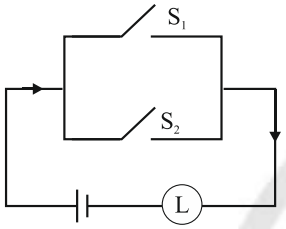
Statement 2 : All the sides of a quadrilateral are equal, then it is a square. **[2023]**

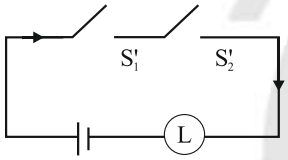
(A) Statement 2 is contrapositive of statement 1.
(B) Statement 2 is negation of statement 1.
(C) Statement 2 is inverse of statement 1.
(D) Statement 2 is the converse of statement 1.

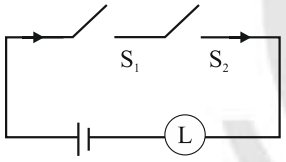
75. The given following circuit is equivalent to **[2023]**



(A)


(B)


(C)


(D)


76. If p and q are **true** statements and r and s are false statements, then the truth values of the statement patterns $(p \wedge q) \vee r$ and $(p \vee s) \leftrightarrow (q \wedge r)$ are respectively **[2023]**

(A) F, T (B) T, T
(C) F, F (D) T, F

77. The negation of the statement pattern $\sim s \vee (\sim r \wedge s)$ is equivalent to: **[2023]**

(A) $s \wedge r$ (B) $s \wedge (r \wedge \sim s)$
(C) $s \wedge \sim r$ (D) $s \vee (r \vee \sim s)$

78. Negation of inverse of the following statement pattern $(p \wedge q) \rightarrow (p \vee \sim q)$ is: **[2023]**

(A) p (B) $\sim q$
(C) $\sim p$ (D) q

79. Negation of contrapositive of statement pattern $(p \vee \sim q) \rightarrow (p \wedge \sim q)$ is **[2023]**

(A) $(\sim p \wedge q) \vee (p \wedge \sim q)$
(B) $(\sim p \vee q) \wedge (p \vee \sim q)$
(C) $(p \wedge \sim q) \vee (\sim p \wedge \sim q)$
(D) $(\sim p \vee \sim q) \wedge (p \vee q)$

80. If q is false and $p \wedge q \leftrightarrow r$ is **true**, then _____ is a tautology. **[2023]**

(A) $p \vee r$
(B) $(p \wedge r) \rightarrow p \vee r$
(C) $(p \vee r) \rightarrow p \wedge r$
(D) $p \wedge r$

81. The negation of the statement "The number is an odd number if and only if it is divisible by 3." **[2023]**

(A) The number is an odd number but not divisible by 3 or the number is divisible by 3 but not odd.
(B) The number is not an odd number *iff* it is not divisible by 3.
(C) The number is not an odd number but it is divisible by 3.
(D) The number is not an odd number or is not divisible by 3 but the number is divisible by 3 or odd.

82. The negation of the statement "The payment will be made if and only if the work is finished in time." is: **[2024, 2023, 2022]**

(A) The work is finished in time and the payment is not made.
(B) Either the work is finished in time and the payment is not made or the payment is made and the work is not finished in time.
(C) The payment is made and the work is not finished in time.
(D) The work is finished in time and the payment is not made or the payment is made and the work is finished in time.

83. The logical statement $\left[\sim(\sim p \vee q) \vee (p \wedge r) \wedge (\sim q \wedge r)\right]$ is equivalent to: **[2025, 2023]**

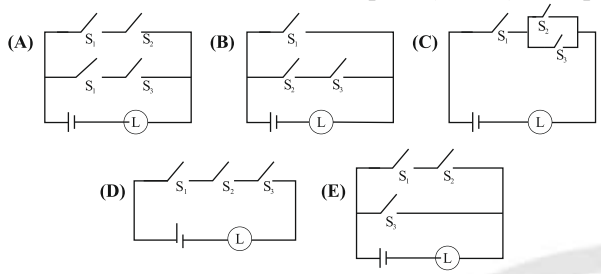
(A) $(p \wedge r) \wedge \sim q$ (B) $(\sim p \wedge \sim q) \wedge r$
(C) $\sim p \vee r$ (D) $(p \wedge \sim q) \vee r$

84. The statement pattern $[(p \to q) \wedge \sim q] \to r$ is a tautology when r is equivalent to **[2025, 2023]**

(A) $p \wedge \sim q$ (B) $q \vee p$

(C) $p \wedge q$ (D) $\sim q$

85. Which of the following are pairs of equivalent circuits **[2025, 2024, 2023]**



(A) (A) and (B) (B) (B) and (D)

(C) (C) and (E) (D) (A) and (C)

86. Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$ and $X = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$, if $AX = B$, then the value of $2a + b + 2c$ is: **[2023]**

(A) 10 (B) 8

(C) 6 (D) 12

87. If $B = \begin{bmatrix} 3 & \alpha & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$ is the adjoint of a 3×3 matrix A and $|A| = 4$, then α is equal to **[2023]**

(A) 1 (B) 0

(C) –1 (D) –2

88. If $A = \begin{bmatrix} i & 1 \\ 1 & 0 \end{bmatrix}$ where $i = \sqrt{-1}$ and $B = A^{2029}$, then $B^{-1} =$ **[2023]**

(A) –A (B) adj A

(C) –I (D) –adj A

89. If the matrix $A = \begin{bmatrix} 1 & 2 \\ -5 & 1 \end{bmatrix}$ and $A^{-1} = xA + yI$, when I is a unit matrix of order 2, then the value of $2x + 3y$ is **[2023]**

(A) $\frac{8}{11}$ (B) $\frac{4}{11}$

(C) $-\frac{8}{11}$ (D) $-\frac{4}{11}$

90. If $A = \begin{bmatrix} 2a & -3b \\ 3 & 2 \end{bmatrix}$ and $A \cdot \text{adj } A = AA^T$, then $2a + 3b$ is **[2023]**

(A) –1 (B) 1

(C) 5 (D) –5

91. Let $A = \begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix}$. If $B = I - {}^3C_1(\text{adj } A) + {}^3C_2(\text{adj } A)^2 - {}^3C_3(\text{adj}A)^3$, then the sum of all elements of the matrix B is: **[2023]**

(A) –1 (B) –3

(C) –4 (D) –5

92. If $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 4 & -1 \end{bmatrix}$, then $(A + B)^{-1}$ is **[2023]**

(A) $\begin{bmatrix} -\frac{1}{2} & 0 \\ -\frac{3}{2} & \frac{1}{2} \end{bmatrix}$ (B) $\begin{bmatrix} \frac{1}{2} & 0 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$

(C) $\begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{3}{2} & \frac{1}{2} \end{bmatrix}$ (D) $\begin{bmatrix} \frac{1}{2} & 0 \\ \frac{3}{2} & \frac{1}{2} \end{bmatrix}$

93. If $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$, then $A^T A^{-1} =$ **[2025, 2023]**

(A) $\begin{bmatrix} \cos 2x & -\sin 2x \\ -\sin 2x & \cos 2x \end{bmatrix}$

(B) $\begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$

(C) $\begin{bmatrix} -\cos 2x & \sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$

(D) $\begin{bmatrix} -\cos 2x & \sin 2x \\ -\sin 2x & \cos 2x \end{bmatrix}$

94. In ΔPQR, sin P, sin Q and sin R are in A.P., then **[2023]**

(A) its altitudes are in A.P.

(B) its altitudes are in H.P.

(C) its medians are in G.P.

(D) its medians are in A.P.

95. The value of $\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)$, $|x|<\frac{1}{2}, x\neq 0$ **[2023]**

(A) $\frac{\pi}{4}+\frac{1}{2}\cos^{-1}x^2$

(B) $\frac{\pi}{4}+\cos^{-1}x^2$

(C) $\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x^2$

(D) $\frac{\pi}{4}-\cos^{-1}x^2$

96. Let a, b, c be the lengths of sides of triangle ABC such that $\frac{a+b}{7}=\frac{b+c}{8}=\frac{c+a}{9}=k$. Then $\frac{\left(A(\Delta ABC)\right)^2}{k^4}=$ **[2023]**

(A) 36 (B) 32

(C) 38 (D) 40

97. In ΔABC, with usual notations, $m\angle C=\frac{\pi}{2}$, if $\tan\left(\frac{A}{2}\right)$ and $\tan\left(\frac{B}{2}\right)$ are the roots of the equation $a_1x^2+b_1x+c_1=0(a_1\neq 0)$, then **[2023]**

(A) $a_1+b_1=c_1$ (B) $b_1+c_1=a_1$

(C) $a_1+c_1=b_1$ (D) $b_1=c_1$

98. If $\begin{vmatrix}\cos(A+B) & -\sin(A+B) & \cos(2B)\\ \sin A & \cos A & \sin B\\ -\cos A & \sin A & \cos B\end{vmatrix}=0$, then the value of B is: **[2023]**

(A) $n\pi, n\in Z$

(B) $(2n+1)\frac{\pi}{2}, n\in Z$

(C) $(2n+1)\frac{\pi}{4}, n\in Z$

(D) $2n\frac{\pi}{3}, n\in Z$

99. If $\tan^{-1}a+\tan^{-1}b+\tan^{-1}c=\pi$, then which of the following statement is **true**? **[2023]**

(A) $a+b-c=abc$

(B) $a+b+c=2abc$

(C) $abc=1$

(D) $a+b+c=abc$

100. In a triangle ABC, with usual notations, if $c=4$, then value of $(a-b)^2\cos^2\frac{C}{2}+(a+b)^2\sin^2\frac{C}{2}$ is: **[2023]**

(A) 4 (B) 16

(C) 9 (D) 25

101. If the general solution of the equation $\frac{\tan 3x-1}{\tan 3x+1}=\sqrt{3}$ is $x=\frac{n\pi}{p}+\frac{7\pi}{q}$, $n, p, q, \in Z$, then $\frac{p}{q}$ is: **[2023]**

(A) 12 (B) $\frac{1}{12}$

(C) 3 (D) 36

102. If $\cos^{-1}\sqrt{p}+\cos^{-1}\sqrt{1-p}+\cos^{-1}\sqrt{1-q}=\frac{3\pi}{4}$, then q is **[2023]**

(A) $\frac{1}{2}$ (B) $\frac{1}{\sqrt{2}}$

(C) 1 (D) $\frac{1}{3}$

103. $\pi+\left(\sin^{-1}\frac{4}{5}+\sin^{-1}\frac{5}{13}+\sin^{-1}\frac{16}{65}\right)$ is equal to **[2023]**

(A) $\frac{\pi}{2}$ (B) $\frac{5\pi}{4}$

(C) $\frac{3\pi}{2}$ (D) $\frac{7\pi}{4}$

104. If $x=\text{cosec}\left(\tan^{-1}\left(\cos\left(\cot^{-1}\left(\sec\left(\sin^{-1}a\right)\right)\right)\right)\right)$, $a\in[0,1]$ **[2023]**

(A) $x^2-a^2=3$ (B) $x^2+a^2=3$

(C) $x^2-a^2=2$ (D) $x^2+a^2=2$

105. The solution set of $8\cos^2\theta + 14\cos\theta + 5 = 0$, in the interval $[0, 2\pi]$, is: **[2023]**

(A) $\left\{\frac{\pi}{3}, \frac{2\pi}{3}\right\}$ (B) $\left\{\frac{\pi}{3}, \frac{4\pi}{3}\right\}$

(C) $\left\{\frac{2\pi}{3}, \frac{4\pi}{3}\right\}$ (D) $\left\{\frac{2\pi}{3}, \frac{5\pi}{3}\right\}$

106. If general solution of $\cos^2\theta - 2\sin\theta + \frac{1}{4} = 0$ is $\theta = \frac{n\pi}{A} + (-1)^n \frac{\pi}{B}$, $n \in Z$, then $A + B$ has the value **[2023]**

(A) 7 (B) 6

(C) 1 (D) –7

107. In a triangle, the sum of lengths of two sides is x and the product of the lengths of the same two sides is y. If $x^2 - c^2 = y$, where c is the length of the third side of the triangle, then the circumradius of the triangle is: **[2023]**

(A) $\frac{c}{3}$ (B) $\frac{c}{\sqrt{3}}$

(C) $\frac{3}{2}y$ (D) $\frac{y}{\sqrt{3}}$

108. The value of $\sin(\cot^{-1}x)$ is: _____ **[2023]**

(A) $\frac{1}{\sqrt{1+x^2}}$ (B) $\sqrt{1+x^2}$

(C) $\frac{1}{x\sqrt{1+x^2}}$ (D) $x\sqrt{1+x^2}$

109. The value of x, for which $\sin(\cot^{-1}(x)) = \cos(\tan^{-1}(1 + x))$, is: **[2023]**

(A) 0 (B) 1

(C) $-\frac{1}{2}$ (D) $\frac{1}{2}$

110. The lengths of sides of a triangle are three consecutive natural numbers and its largest angle is twice the smallest one. Then the length of the sides of the triangle (in units) are: **[2023]**

(A) 3, 4, 5 (B) 4, 5, 6

(C) 5, 6, 7 (D) 2, 3, 4

111. If two angles of ΔABC are $\frac{\pi}{4}$ and $\frac{\pi}{3}$, then the ratio of the smallest and greatest sides are: **[2023]**

(A) $\left(\sqrt{3}-1\right):1$

(B) $\sqrt{3}:\sqrt{5}$

(C) $\sqrt{2}:\sqrt{3}$

(D) $\left(\sqrt{3}-1\right):4$

112. In ΔABC, $m\angle B = \frac{\pi}{3}$ and $m\angle C = \frac{\pi}{4}$. Let point D divide BC internally in the ratio 1 : 3, then $\frac{\sin(\angle BAD)}{\sin(\angle CAD)}$ has the value **[2023]**

(A) $\frac{1}{3}$ (B) $\frac{1}{\sqrt{3}}$

(C) $\frac{1}{\sqrt{6}}$ (D) $\sqrt{\frac{2}{3}}$

113. If $\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\tan^{-1}x$, then x is: **[2023, 2022]**

(A) 1 (B) $\sqrt{3}$

(C) $\frac{1}{\sqrt{3}}$ (D) $\frac{1}{2\sqrt{3}}$

114. In ΔABC with usual notation, $\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}$ and $a = \frac{1}{\sqrt{6}}$, then the area of triangle is _____ sq. units. **[2023]**

(A) $\frac{1}{8}$ (B) $\frac{1}{24\sqrt{3}}$

(C) $\frac{1}{24}$ (D) $\frac{1}{8\sqrt{3}}$

115. The value of $\cos(18° - A) \cdot \cos(18° + A) - \cos(72° - A)\cos(72° + A)$ is: **[2023]**

(A) cos 72°. (B) sin 54°.

(C) sin 18°. (D) cos 54°.

116. The general solution of the equation $3\sec^2\theta = 2\,\text{cosec}\theta$ is: **[2023]**

(A) $n\pi + \frac{\pi}{4}, n \in Z$

(B) $2n\pi + (-1)^n \frac{\pi}{12}, n \in Z$

(C) $n\pi + (-1)^n \frac{\pi}{6}, n \in Z$

(D) $n\pi + (-1)^n \frac{\pi}{3}, n \in Z$

117. The value of $\tan\left(\sin^{-1}\left(\frac{3}{5}\right) + \tan^{-1}\left(\frac{2}{3}\right)\right)$ is: **[2023]**

(A) $\frac{6}{17}$ (B) $\frac{17}{6}$

(C) $\frac{16}{7}$ (D) $\frac{7}{16}$

118. If $\left(\tan^{-1}x\right)^2 + \left(\cot^{-1}x\right)^2 = \frac{5\pi^2}{8}$, then the value of x is: **[2023]**

(A) –2 (B) –1

(C) 1 (D) 2

119. In ΔABC, with usual notations, if $\frac{b+c}{11} = \frac{c+a}{12} = \frac{a+b}{13}$, then the value of $\cos A + \cos B + \cos C$ is: **[2023]**

(A) $\frac{17}{35}$ (B) $\frac{51}{35}$

(C) $\frac{5}{7}$ (D) $\frac{19}{35}$

120. The principal value of $\sin^{-1}\left(\sin\frac{3\pi}{4}\right)$ is: **[2023]**

(A) $\frac{\pi}{4}$ (B) $\frac{3\pi}{4}$

(C) $-\frac{\pi}{4}$ (D) $\frac{5\pi}{4}$

121. If $\left(1+\sqrt{1+x}\right)\tan x = 1+\sqrt{1-x}$, then $\sin 4x$ is: **[2023]**

(A) x (B) $-x$

(C) $4x$ (D) $-4x$

122. The solutions of $\sin x + \sin 5x = \sin 3x$ in $\left(0, \frac{\pi}{2}\right)$ are: **[2023]**

(A) $\frac{\pi}{4}, \frac{\pi}{10}$ (B) $\frac{\pi}{6}, \frac{\pi}{3}$

(C) $\frac{\pi}{4}, \frac{\pi}{12}$ (D) $\frac{\pi}{8}, \frac{\pi}{16}$

123. If $\sin(\theta-\alpha)$, $\sin\theta$ and $\sin(\theta+\alpha)$ are in H.P., then the value of $\cos 2\theta$ is **[2023]**

(A) $1 + 4\cos^2\frac{\alpha}{2}$ (B) $1 - 4\cos^2\frac{\alpha}{2}$

(C) $-1 - 4\cos^2\frac{\alpha}{2}$ (D) $-1 + 4\cos^2\frac{\alpha}{2}$

124. The principal solutions of the equation $\sec x + \tan x = 2\cos x$ are: **[2023]**

(A) $\frac{\pi}{6}, \frac{5\pi}{6}$ (B) $\frac{\pi}{6}, \frac{\pi}{20}$

(C) $\frac{\pi}{6}, \frac{2\pi}{3}$ (D) $\frac{\pi}{6}, \frac{\pi}{12}$

125. If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = 3\pi$, then the value of $x^{2025} + x^{2026} + x^{2027}$ is: **[2023]**

(A) –1 (B) 0

(C) 1 (D) 3

126. If the angles A, B and C of a triangle are in an Arithmetic Progression and if a, b and c denote the lengths of the sides opposite to A, B and C respectively, then the value of the expression $\frac{a}{c}\sin 2C + \frac{c}{a}\sin 2A$ is: **[2023]**

(A) $\frac{1}{2}$ (B) $\frac{\sqrt{3}}{2}$

(C) 1 (D) $\sqrt{3}$

127. If $\sum_{r=1}^{50} \tan^{-1} \frac{1}{2r^2} = p$ then $\tan p$ is: **[2023]**

(A) $\frac{100}{101}$ (B) $\frac{51}{50}$

(C) $\frac{50}{51}$ (D) $\frac{101}{102}$

128. If $\cos^{-1} x - \cos^{-1} \frac{y}{3} = \alpha$, where $-1 \le x \le 1, -3 \le y \le 3, x \le \frac{y}{3}$, then for all x, y, $9x^2 - 6xy\cos\alpha + y^2$ is equal to **[2023]**

(A) $\sin^2\alpha$

(B) $3\sin^2\alpha$

(C) $9\sin^2\alpha$

(D) $\frac{4}{9}\sin^2\alpha$

129. In a triangle ABC, $m\angle A$, $m\angle B$, $m\angle C$ are in A.P. and lengths of two larger sides are 10 units, 9 units respectively, then the length (in units) of the third side is: **[2023]**

(A) $5+\sqrt{6}$ (B) $\sqrt{5}-1$

(C) $\sqrt{6}+1$ (D) $\sqrt{5}+1$

130. Given $0 \le x \le \frac{1}{2}$, then the value of $\tan\left(\sin^{-1}\left(\frac{x}{\sqrt{2}} + \frac{\sqrt{1-x^2}}{\sqrt{2}}\right) - \sin^{-1} x\right)$ is: **[2023]**

(A) 1 (B) $\sqrt{3}$

(C) –1 (D) $\frac{1}{\sqrt{3}}$

131. If the angles of a triangle are in the ratio 4 : 1 : 1, then the ratio of the longest side to the perimeter is: **[2024, 2023]**

(A) 1 : 6

(B) $\sqrt{3} : \left(2+\sqrt{3}\right)$

(C) $1 : \left(2+\sqrt{3}\right)$

(D) 2 : 3

132. If x, y, z are in Arithmetic Progression and $\tan^{-1}x$, $\tan^{-1}y$, $\tan^{-1}z$ are also in Arithmetic progression, where $x, z > 0$ and $xz < 1$, $y < 1$, then; **[2024, 2023]**

(A) $x = y = z$

(B) $2x = 3y = 6z$

(C) $6x = 3y = 2z$

(D) $6x = 4y = 3z$

133. In a triangle ABC, with usual notations, $2ac\sin\left(\frac{A-B+C}{2}\right)$ is equal to **[2024, 2023]**

(A) $a^2+b^2-c^2$ (B) $c^2+a^2-b^2$

(C) $b^2-a^2+c^2$ (D) $a^2-b^2-c^2$

134. If the slope of one of the lines represented by $ax^2 + (2a+1)xy + 2y^2 = 0$ is reciprocal of the slope of the other, then the sum of squares of slopes is: **[2023]**

(A) $\frac{17}{4}$ (B) $\frac{82}{9}$

(C) $\frac{97}{36}$ (D) 2

135. The joint equation of a pair of lines passing through the origin and making an angle of $\frac{\pi}{4}$ with the line $3x+2y-8=0$ is: **[2023]**

(A) $5x^2+24xy-5y^2=0$

(B) $5x^2-24xy+5y^2=0$

(C) $5x^2-24xy-5y^2=0$

(D) $5x^2+24xy+5y^2=0$

136. The joint equation of the lines pair of lines passing through the point (3, –2) and perpendicular to the lines $5x^2+2xy-3y^2=0$ is: **[2023]**

(A) $3x^2+2xy+5y^2+14x+26y+5=0$

(B) $3x^2+2xy-5y^2-14x-26y-5=0$

(C) $3x^2-2xy-5y^2-14x-26y+5=0$

(D) $3x^2-2xy+5y^2+14x+26y-5=0$

137. Let PQR be a right angled isosceles triangle, right angled at $P(2, 1)$. If the equation of the line QR is $2x + y = 3$, then the equation representing the pair of lines PQ and PR is: **[2023]**

(A) $3x^2 - 3y^2 + 8xy + 20x + 10y + 25 = 0$

(B) $3x^2 - 3y^2 + 8xy - 20x - 10y + 25 = 0$

(C) $3x^2 - 3y^2 + 8xy + 10x + 15y + 20 = 0$

(D) $3x^2 - 3y^2 - 8xy - 10x - 15y - 20 = 0$

138. If the pair of lines given by $(x\cos\alpha + y\sin\alpha)^2 = (x^2 + y^2)\sin^2\alpha$ are perpendicular to each other, then α is: **[2023]**

(A) 0 (B) $\frac{\pi}{2}$

(C) $\frac{\pi}{4}$ (D) $\frac{\pi}{6}$

139. If the angle between the lines given by $x^2 - 3xy + \lambda y^2 + 3x - 5y + 2 = 0;\ \lambda \geq 0$ is $\tan^{-1}\left(\frac{1}{3}\right)$, then the value of λ is: **[2023]**

(A) 1 (B) 2

(C) $\frac{9}{4}$ (D) –1

140. The perpendiculars are drawn to lines L_1 and L_2 from the origin making an angle $\frac{\pi}{4}$ and $\frac{3\pi}{4}$ respectively with positive direction of X–axis. If both the lines are at unit distance from the origin, then their joint equation is: **[2023]**

(A) $x^2 - y^2 + 2\sqrt{2}y + 2 = 0$

(B) $x^2 - y^2 - 2\sqrt{2}y - 2 = 0$

(C) $x^2 - y^2 + 2\sqrt{2}y - 2 = 0$

(D) $x^2 - y^2 - 2\sqrt{2}y + 2 = 0$

141. The centroid of the triangle formed by the lines $x + 3y = 10$ and $6x^2 + xy - y^2 = 0$ is: **[2023, 2022]**

(A) $\left(\frac{1}{3}, \frac{-7}{3}\right)$ (B) $\left(\frac{-1}{3}, \frac{-7}{3}\right)$

(C) $\left(\frac{-1}{3}, \frac{7}{3}\right)$ (D) $\left(\frac{1}{3}, \frac{7}{3}\right)$

142. The magnitude of the projection of the vector $2\hat{i} + \hat{j} + \hat{k}$ on the vector perpendicular to the plane containing the vectors $\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} + 3\hat{k}$ is: **[2023]**

(A) $\frac{2}{\sqrt{6}}$ (B) $\frac{1}{\sqrt{6}}$

(C) $\frac{5}{\sqrt{6}}$ (D) $\frac{7}{\sqrt{6}}$

143. Two adjacent sides of a parallelogram are $2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\hat{i} - 2\hat{j} - 3\hat{k}$, then the unit vector parallel to its diagonal is: **[2023]**

(A) $\frac{3}{7}\hat{i} - \frac{6}{7}\hat{j} + \frac{2}{7}\hat{k}$ (B) $\frac{2}{7}\hat{i} - \frac{6}{7}\hat{j} + \frac{3}{7}\hat{k}$

(C) $\frac{6}{7}\hat{i} - \frac{2}{7}\hat{j} + \frac{3}{7}\hat{k}$ (D) $\frac{1}{7}\hat{i} + \frac{1}{7}\hat{j} - \frac{3}{7}\hat{k}$

144. If D, E and F are the mid–points of the sides BC, CA and AB of triangle ABC respectively, then $\overline{AD} + \frac{2}{3}\overline{BE} + \frac{1}{3}\overline{CF} =$ **[2023]**

(A) $\frac{1}{2}\overline{AB}$ (B) $\frac{1}{2}\overline{AC}$

(C) $\frac{1}{2}\overline{BC}$ (D) $\frac{2}{3}\overline{AC}$

145. If $\left(\vec{a} \times \vec{b}\right) \times \vec{c} = -5\vec{a} + 4\vec{b}$ and $\vec{a} \cdot \vec{b} = 3$, then the value of $\vec{a} \times \left(\vec{b} \times \vec{c}\right)$ is: **[2023]**

(A) $3\vec{b} - 4\vec{c}$ (B) $4\vec{a} - 3\vec{b}$

(C) $4\vec{b} - 3\vec{c}$ (D) $3\vec{a} - 4\vec{c}$

146. If $\vec{p} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{q} = \hat{i} - 2\hat{j} + \hat{k}$. Then a vector of magnitude $5\sqrt{3}$ units perpendicular to the vector $\vec{q}$ and coplanar with $\vec{p}$ and $\vec{q}$ is **[2023]**

(A) $5\left(\hat{i} - \hat{j} + \hat{k}\right)$ (B) $5\left(\hat{i} + \hat{j} - \hat{k}\right)$

(C) $5\left(\hat{i} - \hat{j} - \hat{k}\right)$ (D) $5\left(\hat{i} + \hat{j} + \hat{k}\right)$

147. If $\vec{a} = m\vec{b} + n\vec{c}$, where $\vec{a} = 4\hat{i} + 13\hat{j} - 18\hat{k}$, $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{c} = 2\hat{i} + 3\hat{j} - 4\hat{k}$, then $m + n =$ **[2023]**

(A) 1 (B) 2

(C) 3 (D) –1

148. If the volume of tetrahedron, whose vertices are with position vectors $\hat{i}-6\hat{j}+10\hat{k}$, $-\hat{i}-3\hat{j}+7\hat{k}$, $5\hat{i}-\hat{j}+\lambda\hat{k}$ and $7\hat{i}-4\hat{j}+7\hat{k}$ is 11 cubic units, then value of λ is: **[2023]**

(A) 4 (B) 5
(C) 7 (D) 6

149. If $\bar{a}=\hat{i}+4\hat{j}+2\hat{k}$, $\bar{b}=3\hat{i}-2\hat{j}+7\hat{k}$, $\bar{c}=2\hat{i}-\hat{j}+4\hat{k}$, then a vector $\bar{d}$ which is parallel to vector $\bar{a}\times\bar{b}$ and which $\bar{c}\cdot\bar{d}=15$, is **[2023]**

(A) $30\hat{i}-\hat{j}-14\hat{k}$
(B) $90\hat{i}-3\hat{j}-42\hat{k}$
(C) $90\hat{i}+\hat{j}-7\hat{k}$
(D) $30\hat{i}-3\hat{j}+7\hat{k}$

150. The unit vector perpendicular to each of the vectors $\bar{a}+\bar{b}$ and $\bar{a}-\bar{b}$, where $\bar{a}=\hat{i}+\hat{j}+\hat{k}$ and $\bar{b}=3\hat{i}-2\hat{j}+5\hat{k}$ is **[2023]**

(A) $\dfrac{-14\hat{i}+4\hat{j}+10\hat{k}}{\sqrt{312}}$
(B) $\dfrac{14\hat{i}-4\hat{j}+10\hat{k}}{\sqrt{312}}$
(C) $\dfrac{14\hat{i}+4\hat{j}+10\hat{k}}{\sqrt{312}}$
(D) $\dfrac{-14\hat{i}-4\hat{j}+10\hat{k}}{\sqrt{312}}$

151. Let $\bar{a}=2\hat{i}+\hat{j}-2\hat{k}, \bar{b}=\hat{i}+\hat{j}$ and $\bar{c}$ be a vector such that $|\bar{c}-\bar{a}|=4, |(\bar{a}\times\bar{b})\times\bar{c}|=3$ and the angle between $\bar{c}$ and $\bar{a}\times\bar{b}$ is $\dfrac{\pi}{6}$, then $\bar{a}\cdot\bar{c}$ is equal to: **[2023]**

(A) -3 (B) $\dfrac{3}{2}$
(C) 3 (D) $\dfrac{-3}{2}$

152. If $\bar{a}=2\hat{i}+3\hat{j}+2\hat{k}$, $\bar{b}=2\hat{i}+\hat{j}-\hat{k}$ and $\bar{c}=3\hat{i}-\hat{j}$ are such that $\bar{a}+\lambda\bar{b}$ is perpendicular to $\bar{c}$, then the value of λ is: **[2023]**

(A) $\dfrac{-1}{5}$ (B) 3
(C) $\dfrac{3}{5}$ (D) $\dfrac{-3}{5}$

153. If $\bar{a}=\hat{i}+\hat{j}+\hat{k}, \bar{b}=4\hat{i}+3\hat{j}+4\hat{k}$ and $\bar{c}=\hat{i}+\alpha\hat{j}+\beta\hat{k}$ are linearly dependent vectors and $|\bar{c}|=\sqrt{3}$, then the values of α and β are respectively. **[2023]**

(A) 1, 1 (B) 2, 1
(C) 0, 1 (D) 1, 2

154. If $\bar{a},\bar{b},\bar{c}$ are three vectors, $|\bar{a}|=2, |\bar{b}|=4, |\bar{c}|=1, |\bar{b}\times\bar{c}|=\sqrt{15}$ and $\bar{b}=2\bar{c}+\lambda\bar{a}$, then the value of λ is: **[2023]**

(A) 2 (B) $2\sqrt{2}$
(C) 1 (D) 4

155. The centroid of tetrahedron with vertices at $A(-1, 2, 3)$, $B(3, -2, 1)$, $C(2, 1, 3)$ and $D(-1, -2, 4)$ is: **[2023]**

(A) $\left(\dfrac{3}{4},\dfrac{-1}{4},\dfrac{11}{4}\right)$ (B) $\left(\dfrac{5}{4},\dfrac{-3}{4},\dfrac{7}{4}\right)$
(C) $\left(\dfrac{-3}{4},\dfrac{-1}{4},\dfrac{11}{4}\right)$ (D) $\left(\dfrac{-5}{4},\dfrac{-3}{4},\dfrac{-7}{4}\right)$

156. If the area of the triangle with vertices (1, 2, 0), (1, 0, 2) and (0, x, 1) is $\sqrt{6}$ square units, then the value of x is: **[2023]**

(A) 1 (B) 2
(C) 3 (D) 4

157. A, B, C, D are four points in a plane with position vectors $\vec{a}$, $\vec{b}$, $\vec{c}$, $\vec{d}$ respectively such that $(\vec{a}-\vec{d})\cdot(\vec{b}-\vec{c})=(\vec{b}-\vec{d})\cdot(\vec{c}-\vec{a})=0$. The point D, then is the _____ of ΔABC **[2023]**

(A) centroid (B) circumcentre
(C) incentre (D) orthocentre

158. The unit vector which is orthogonal to the vector $3\hat{i}+2\hat{j}+6\hat{k}$ and coplanar with the vectors $2\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}+\hat{j}+\hat{k}$ is: **[2023]**

(A) $\frac{8\hat{i}-3\hat{j}+3\hat{k}}{\sqrt{82}}$ (B) $\frac{-8\hat{i}-3\hat{j}+3\hat{k}}{\sqrt{82}}$

(C) $\frac{-8\hat{i}+3\hat{j}+3\hat{k}}{\sqrt{82}}$ (D) $\frac{-\left(8\hat{i}+3\hat{j}+3\hat{k}\right)}{\sqrt{82}}$

159. If the volume of tetrahedron, whose vertices are $A(1, 2, 3)$, $B(-3, -1, 1)$, $C(2, 1, 3)$ and $D(-1, 2, x)$ is $\frac{11}{6}$ cubic units, then the value of x is **[2023]**

(A) 3 (B) –2

(C) 4 (D) –1

160. The scalar product of vectors $\vec{a}=\hat{i}+2\hat{j}+\hat{k}$ and a unit vector along the sum of vectors $\vec{b}=2\hat{i}-4\hat{j}+5\hat{k}$ and $\vec{c}=\lambda\hat{i}+2\hat{j}-3\hat{k}$ is one, then the value of λ is: **[2023]**

(A) 1 (B) –2

(C) –3 (D) 2

161. If $\bar{a}=2\hat{i}+3\hat{j}-4\hat{k}$ and $\bar{b}=\hat{i}-\hat{j}-\hat{k}$, then the projection of $\bar{b}$ in the direction of $\bar{a}$ is: **[2023]**

(A) $\frac{1}{\sqrt{29}}$ (B) $\frac{2}{\sqrt{3}}$

(C) $\frac{5}{\sqrt{3}}$ (D) $\frac{3}{\sqrt{29}}$

162. If $A(1, 4, 2)$ and $C(5, -7, 1)$ are two vertices of triangle ABC and $G\left(\frac{4}{3},0,-\frac{2}{3}\right)$ is centroid of the triangle ABC, then the mid point of side BC is: **[2023]**

(A) $\left(-2,-2,\frac{3}{2}\right)$ (B) $\left(2,2,\frac{3}{2}\right)$

(C) $\left(\frac{3}{2},2,-2\right)$ (D) $\left(\frac{3}{2},-2,-2\right)$

163. A vector $\bar{a}$ has components 1 and $2p$ with respect to a rectangular Cartesian system. This system is rotated through a certain angle about origin in the counter clock wise sense. If, with respect to the new system, $\bar{a}$ has components $p + 1$ and 1, then; **[2023]**

(A) $p = 1$ or $p = \frac{1}{3}$ (B) $p = -1$ or $p = -\frac{1}{3}$

(C) $p = -\frac{1}{3}$ or $p = 1$ (D) $p = \frac{1}{3}$ or $p = -1$

164. If θ is angle between the vectors $\vec{a}$ and $\vec{b}$ where $|\vec{a}|=4, |\vec{b}|=3$ and $\theta\in\left(\frac{\pi}{4},\frac{\pi}{3}\right)$, then $|\left(\vec{a}-\vec{b}\right)\times\left(\vec{a}+\vec{b}\right)|^2+4\left(\vec{a}\cdot\vec{b}\right)^2$ has the value **[2023]**

(A) 576 (B) 24

(C) 144 (D) 12

165. $\vec{a}=2\hat{i}+3\hat{j}+4\hat{k}, \vec{b}=\hat{i}-2\hat{j}+\hat{k}, \vec{c}=\hat{i}+\hat{j}-\hat{k}$ are three vectors. For a vector $\vec{r}$ with $\vec{r}\times\vec{a}=\vec{b}$ and $\vec{r}\cdot\vec{c}=3, |\vec{r}|$ is: **[2023]**

(A) $\sqrt{55}$

(B) $\sqrt{155}$

(C) $\sqrt{138}$

(D) $\sqrt{170}$

166. A tetrahedron has vertices at $P(2, 1, 3)$, $Q(-1, 1, 2)$, $R(1, 2, 1)$ and $O(0, 0, 0)$, then angle between the faces OPQ and PQR is: **[2023]**

(A) $cos^{-1}\left(\frac{5}{7\sqrt{59}}\right)$

(B) $cos^{-1}\left(\frac{\sqrt{25}}{\sqrt{59}\cdot\sqrt{35}}\right)$

(C) $cos^{-1}\left(\frac{5}{413}\right)$

(D) $cos^{-1}\left(\frac{25}{\sqrt{59}\sqrt{35}}\right)$

167. If ΔABC is right angled at A, where $A = (4, 2, x)$, $B = (3, 1, 8)$ and $C = (2, -1, 2)$, then the value of x is **[2023]**

(A) 4 (B) 2
(C) 3 (D) 1

168. If $\vec{a},\vec{b},\vec{c}$ are unit vectors and θ is angle between $\vec{a}$ and $\vec{c}$ and $\vec{a}+2\vec{b}+2\vec{c}=\vec{0}$, then $|\vec{a}\times\vec{c}|=$ **[2023]**

(A) $\frac{\sqrt{15}}{2}$ (B) $\frac{\sqrt{15}}{4}$
(C) $\sqrt{15}$ (D) $\frac{\sqrt{15}}{3}$

169. If $\vec{a},\vec{b},\vec{c}$ are three vectors with magnitudes $\sqrt{3}$, 1, 2 respectively, such that $\vec{a}\times(\vec{a}\times\vec{c})+3\vec{b}=\vec{0}$, if θ is the angle between $\vec{a}$ and $\vec{c}$, then $sec^2\theta$ is: **[2023]**

(A) 1 (B) $\frac{3}{2}$
(C) $\frac{4}{3}$ (D) $\frac{2}{\sqrt{3}}$

170. If $|\vec{a}|=2,|\vec{b}|=3,|\vec{c}|=5$ and each of the angles between the vectors $\vec{a}$ and $\vec{b}$, $\vec{b}$ and $\vec{c}$, $\vec{c}$ and $\vec{a}$ is 60°, then the value of $|\vec{a}+\vec{b}+\vec{c}|$ is: **[2023]**

(A) $\sqrt{69}$ (B) $\sqrt{70}$
(C) $\sqrt{80}$ (D) $\sqrt{39}$

171. Let $\vec{a}=\hat{i}+2\hat{j}-\hat{k}$ and $\vec{b}=\hat{i}+\hat{j}-\hat{k}$ be two vectors. If $\vec{c}$ is a vector such that $\vec{b}\times\vec{c}=\vec{b}\times\vec{a}$ and $\vec{c}\cdot\vec{a}=0$, then $\vec{c}\cdot\vec{b}$ is: **[2023]**

(A) $\frac{1}{2}$ (B) $\frac{3}{2}$
(C) $\frac{-3}{2}$ (D) $\frac{-1}{2}$

172. The incenter of the triangle ABC, whose vertices are $A(0,2,1)$, $B(-2,0,0)$ and $C(-2,0,2)$ is: **[2024, 2023]**

(A) $\left(\frac{3}{2},-\frac{1}{2},-1\right)$ (B) $\left(\frac{3}{2},\frac{1}{2},1\right)$
(C) $\left(-\frac{3}{2},\frac{1}{2},1\right)$ (D) $\left(-\frac{3}{2},-\frac{1}{2},-1\right)$

173. If $\bar{a}=\frac{1}{\sqrt{10}}\left(3\hat{i}+\hat{k}\right)$ and $\bar{b}=\frac{1}{7}\left(2\hat{i}+3\hat{j}-6\hat{k}\right)$, then the value of $\left(2\bar{a}-\bar{b}\right)\cdot\left(\left(\bar{a}\times\bar{b}\right)\times\left(\bar{a}+2\bar{b}\right)\right)=$ **[2025, 2024, 2023, 2020]**

(A) 3 (B) –3
(C) 5 (D) –5

174. Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|\vec{a}|=1,|\vec{b}|=4,\vec{a}\cdot\vec{b}=2$. If $\vec{c}=\left(2\vec{a}\times\vec{b}\right)-3\vec{b}$, then the angle between $\vec{b}$ and $\vec{c}$ is: **[2025, 2023]**

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{6}$
(C) $\frac{3\pi}{4}$ (D) $\frac{5\pi}{6}$

175. Two adjacent sides of a parallelogram $ABCD$ are given by $\overline{AB}=2\hat{i}+10\hat{j}+11\hat{k}$ and $\overline{AD}=-\hat{i}+2\hat{j}+2\hat{k}$. The side AD is rotated by an acute angle α in the plane of parallelogram so that AD becomes AD'. If AD' makes a right angle with side AB, then the cosine of the angle α is given by: **[2025, 2023]**

(A) $\frac{8}{9}$
(B) $\frac{\sqrt{17}}{9}$
(C) $\frac{1}{9}$
(D) $\frac{4\sqrt{5}}{9}$

176. $\bar{u},\bar{v},\bar{w}$ are three vectors such that $|\bar{u}|=1,|\bar{v}|=2,|\bar{w}|=3$. If the projection of $\bar{v}$ along $\bar{u}$ is equal to projection of $\bar{w}$ along $\bar{u}$ and $\bar{v},\bar{w}$ are perpendicular to each other, then $|\bar{u}-\bar{v}+\bar{w}|=$ **[2025, 2023]**

(A) 4 (B) $\sqrt{7}$

(C) $\sqrt{14}$ (D) 2

177. If a line L is the line of intersection of the planes $2x+3y+z=1$ and $x+3y+2z=2$. If line L makes an angle α with the positive X–axis, then the value of sec α is: **[2023]**

(A) $\sqrt{3}$ (B) 2

(C) 1 (D) $\sqrt{2}$

178. The co–ordinates of the point, where the line $\frac{x-1}{2}=\frac{y-2}{-3}=\frac{z+5}{4}$ meets the plane $2x+4y-z=3$, are: **[2023]**

(A) (3, –1, –1) (B) (3, 1, –1)

(C) (3, –1, 1) (D) (–3, –1, –1)

179. The equation of a plane, containing the line of intersection of the planes $2x-y-4=0$ and $y+2z-4=0$ and passing through the point (2, 1, 0), is: **[2023]**

(A) $3x-2y+z=4$

(B) $3x+2y+z=4$

(C) $3x-2y-z=4$

(D) $3x+2y-z=-4$

180. The plane through the intersection of planes $x+y+z=1$ and $2x+3y-z+4=0$ and parallel to Y–axis also passes through the point **[2023]**

(A) (3, 3, –1)

(B) (–3, 0, 1)

(C) (3, 2, 1)

(D) (–3, 0, –1)

181. Two lines $\frac{x-3}{1}=\frac{y+1}{3}=\frac{z-6}{-1}$ and $\frac{x+5}{7}=\frac{y-2}{-6}=\frac{z-3}{4}$ intersect at the point R. Then reflection of R in the XY–plane has co–ordinates **[2023]**

(A) (2, –4, –7) (B) (2, –4, 7)

(C) (–2, 4, 7) (D) (2, 4, 7)

182. If the line $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-2}{4}$ meets the plane $x+2y+3z=15$ at the point P, then the distance of P from the origin is: **[2023]**

(A) $\frac{7}{2}$ units (B) $\frac{9}{2}$ units

(C) $\frac{\sqrt{5}}{2}$ units (D) $2\sqrt{5}$ units

183. The equation of line passing through the point (1, 2, 3) and perpendicular to the lines $\frac{x-2}{3}=\frac{y-1}{2}=\frac{z+1}{-2}$ and $\frac{x}{2}=\frac{y}{-3}=\frac{z}{1}$ is: **[2023]**

(A) $\vec{r}=\left(\hat{i}+2\hat{j}+3\hat{k}\right)+\lambda\left(4\hat{i}+7\hat{j}-13\hat{k}\right)$

(B) $\vec{r}=\left(\hat{i}+2\hat{j}+3\hat{k}\right)+\lambda\left(-4\hat{i}+7\hat{j}-13\hat{k}\right)$

(C) $\vec{r}=\left(\hat{i}+2\hat{j}+3\hat{k}\right)+\lambda\left(-4\hat{i}-7\hat{j}-13\hat{k}\right)$

(D) $\vec{r}=\left(\hat{i}+2\hat{j}+3\hat{k}\right)+\lambda\left(4\hat{i}-7\hat{j}-13\hat{k}\right)$

184. Let $\vec{A}$ be a vector parallel to line of intersection of planes P_1 and P_2 through origin. P_1 is parallel to the vectors $2\hat{j}+3\hat{k}$ and $4\hat{j}-3\hat{k}$ and P_2 is parallel to $\hat{j}-\hat{k}$ and $3\hat{i}+3\hat{j}$, then the angle between $\vec{A}$ and $2\hat{i}+\hat{j}-2\hat{k}$ is: **[2023]**

(A) $\frac{\pi}{3}$

(B) $\frac{\pi}{2}$

(C) $\frac{\pi}{6}$

(D) $\frac{3\pi}{4}$

185. A plane is parallel to two lines whose direction ratios are 1, 0, –1 and –1, 1, 0 and it contains the point (1, 1, 1). If it cuts the co–ordinate axes at *A*, *B*, *C*, then the volume of the tetrahedron *OABC* (in cubic units) is: **[2023]**

(A) $\frac{9}{4}$ (B) $\frac{9}{2}$

(C) 9 (D) 27

186. The equation of the plane through (–1, 1, 2) whose normal makes equal acute angles with co–ordinate axes is: **[2023]**

(A) $x+y+z-3=0$

(B) $x+y+z-2=0$

(C) $x+y-z-2=0$

(D) $x-y+z-3=0$

187. The distance of the point $P(-2, 4, -5)$ from the line $\frac{x+3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$ is: **[2023]**

(A) $\frac{\sqrt{37}}{10}$ (B) $\sqrt{\frac{37}{10}}$

(C) $\frac{37}{\sqrt{10}}$ (D) $\frac{37}{10}$

188. If the line $\frac{1-x}{3}=\frac{7y-14}{2p}=\frac{z-3}{2}$ and $\frac{7-7x}{3p}=\frac{y-5}{1}=\frac{6-z}{5}$ are at right angles, then p = **[2023]**

(A) $\frac{70}{11}$ (B) $\frac{11}{70}$

(C) $\frac{-70}{11}$ (D) $\frac{-11}{70}$

189. The equation of the line, passing through (1, 2, 3) and parallel to planes $x-y+2z=5$ and $3x+y+z=6$, is: **[2023]**

(A) $\frac{x-1}{-3}=\frac{y-2}{5}=\frac{z-3}{4}$

(B) $\frac{x-1}{-3}=\frac{y-2}{-5}=\frac{z-3}{4}$

(C) $\frac{x-1}{4}=\frac{y-2}{5}=\frac{z-3}{3}$

(D) $\frac{x-1}{5}=\frac{y-2}{7}=\frac{z-3}{1}$

190. The shortest distance (in units) between the lines $\frac{x+1}{3}=\frac{y+2}{1}=\frac{z+1}{2}$ and $\vec{r}=\left(2\hat{i}-2\hat{j}+3\hat{k}\right)+\lambda\left(\hat{i}+2\hat{j}\right)$ is: **[2023]**

(A) $\frac{8}{3\sqrt{5}}$

(B) $\frac{1}{3\sqrt{5}}$

(C) $\frac{7}{3\sqrt{5}}$

(D) $\frac{2}{3\sqrt{5}}$

191. A line drawn from the point $A(1, 3, 2)$ parallel to the line $\frac{x}{2}=\frac{y}{4}=\frac{z}{1}$, intersects the plane $3x+y+2z=5$ in point *B*, then co–ordinates of point *B* are: **[2023]**

(A) $\left(\frac{1}{6},\frac{4}{3},\frac{19}{12}\right)$

(B) $\left(-\frac{1}{6},-\frac{4}{3},\frac{19}{12}\right)$

(C) $\left(\frac{1}{6},\frac{4}{3},-\frac{19}{12}\right)$

(D) $\left(-\frac{1}{6},-\frac{4}{3},-\frac{19}{12}\right)$

192. A line L_1 passes through the point, whose p. v. (position vector) $3\hat{i}$, is parallel to the vector $-\hat{i}+\hat{j}+\hat{k}$. Another line L_2 passes through the point having p.v. $\hat{i}+\hat{j}$ is parallel to vector $\hat{i}+\hat{k}$, then the point of intersection of lines L_1 and L_2 has p.v. **[2023]**

(A) $2\hat{i}+2\hat{j}+\hat{k}$

(B) $2\hat{i}+\hat{j}+\hat{k}$

(C) $2\hat{i}-\hat{j}-\hat{k}$

(D) $2\hat{i}-2\hat{j}+\hat{k}$

193. The acute angle between the line joining the points (2, 1, –3), (–3, 1, 7) and a line parallel to $\frac{x-1}{3}=\frac{y}{4}=\frac{z+3}{5}$ through the point (–1, 0, 4) is: **[2023]**

(A) $cos^{-1}\left(\frac{1}{\sqrt{10}}\right)$ (B) $cos^{-1}\left(\frac{5}{7\sqrt{10}}\right)$

(C) $cos^{-1}\left(\frac{7}{5\sqrt{10}}\right)$ (D) $cos^{-1}\left(\frac{3}{5\sqrt{10}}\right)$

194. If the shortest distance between the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{\lambda}$ and $\frac{x-2}{1}=\frac{y-4}{4}=\frac{z-5}{5}$ is $\frac{1}{\sqrt{3}}$, then sum of possible values of λ is: **[2023]**

(A) 16 (B) 11

(C) 12 (D) 15

195. Consider the lines L_1 : $\frac{x+1}{3}=\frac{y+2}{1}=\frac{z+1}{2}$ L_2 : $\frac{x-2}{1}=\frac{y+2}{2}=\frac{z-3}{3}$, then the unit vector perpendicular to both L_1 and L_2 is **[2023]**

(A) $\frac{-\hat{i}+7\hat{j}+5\hat{k}}{5\sqrt{3}}$ (B) $\frac{-\hat{i}-7\hat{j}+5\hat{k}}{5\sqrt{3}}$

(C) $\frac{+\hat{i}-7\hat{j}+5\hat{k}}{5\sqrt{3}}$ (D) $\frac{\hat{i}+7\hat{j}+5\hat{k}}{5\sqrt{3}}$

196. The mirror image of $P(2, 4, -1)$ in the plane $x - y + 2z - 2 = 0$ is (a, b, c), then the value of $a + b + c$ is **[2023]**

(A) 4 (B) 5

(C) 7 (D) 9

197. A vector parallel to the line of intersection of the planes $\bar{r}\cdot\left(3\hat{i}-\hat{j}+\hat{k}\right)=1$ and $\bar{r}\cdot\left(\hat{i}+4\hat{j}-2\hat{k}\right)=2$ is **[2023]**

(A) $-2\hat{i}+7\hat{j}+13\hat{k}$

(B) $2\hat{i}-7\hat{j}+13\hat{k}$

(C) $-\hat{i}+4\hat{j}+7\hat{k}$

(D) $\hat{i}-4\hat{j}+7\hat{k}$

198. Equation of the plane containing the straight line $\frac{x}{2}=\frac{y}{3}=\frac{z}{4}$ and perpendicular to the plane containing the straight lines $\frac{x}{3}=\frac{y}{4}=\frac{z}{2}$ and $\frac{x}{4}=\frac{y}{2}=\frac{z}{3}$ is **[2024, 2023]**

(A) $x + 2y - 2z = 0$

(B) $3x + 2y - 2z = 0$

(C) $x - 2y + z = 0$

(D) $5x + 2y - 4z = 0$

199. A line with positive direction cosines passes through the point $P(2, -1, 2)$ and makes equal angles with co-ordinate axes. The line meets the plane $2x + y + z = 9$ at point Q. Then the length of the line segment PQ equals **[2024, 2023]**

(A) 1 units (B) $\sqrt{2}$ units

(C) $\sqrt{3}$ units (D) 2 units

200. The length of the projection of the line segment joining the points (5, -1, 4) and (4, -1, 3) on the plane $x + y + z = 7$ is **[2024, 2023]**

(A) $\frac{\sqrt{2}}{\sqrt{3}}$ units (B) $\frac{2}{\sqrt{3}}$ units

(C) $\frac{2}{3}$ units (D) $\frac{\sqrt{2}}{3}$ units

201. If the line $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$ intersect, then the value of k is **[2025, 2024, 2023]**

(A) $\frac{3}{2}$

(B) $\frac{9}{2}$

(C) $-\frac{2}{9}$

(D) $-\frac{3}{2}$

202. The lines $\frac{6x-6}{18}=\frac{y+1}{3}=\frac{z-1}{5}$ and $\frac{3x+6}{12}=\frac{y-1}{3}=\frac{z+1}{2}$ are ... **[2025, 2023]**

(A) intersecting at point (1, -1, 2)
(B) intersecting at right angles
(C) do not intersect
(D) intersecting at point (3, 1, - 1)

203. The graphical solution set for the system of inequations $x-2y\le 2$, $5x+2y\ge 10$, $4x+5y\le 20$, $x\ge 0$, $y\ge 0$ is given by **[2023]**

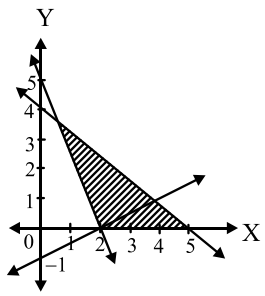


Fig. 1

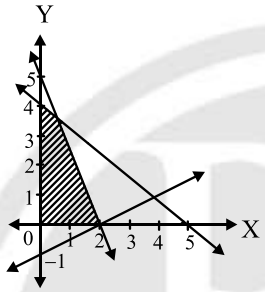


Fig. 2

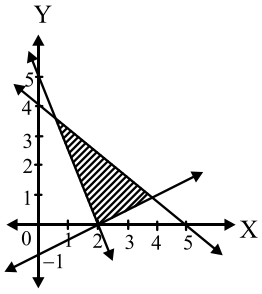


Fig. 3

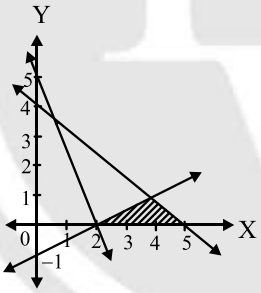
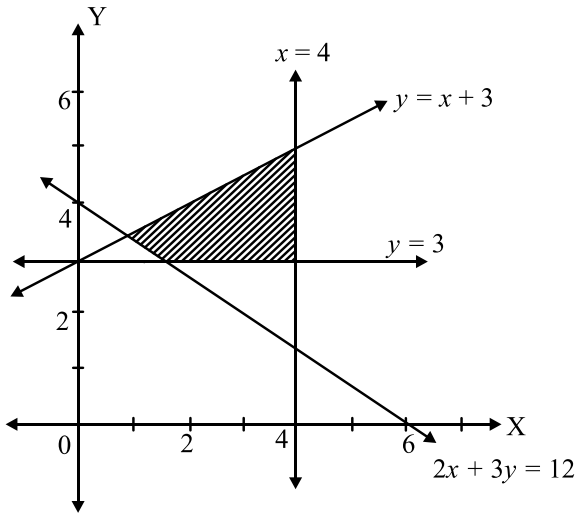


Fig. 4

(A) Fig.2
(B) Fig.4
(C) Fig.1
(D) Fig.3

204. The shaded area in the figure given below is a solution set of a system of inequations. The minimum value of objective function $3x + 5y$, subject to the linear constraints given by this system of inequations is:



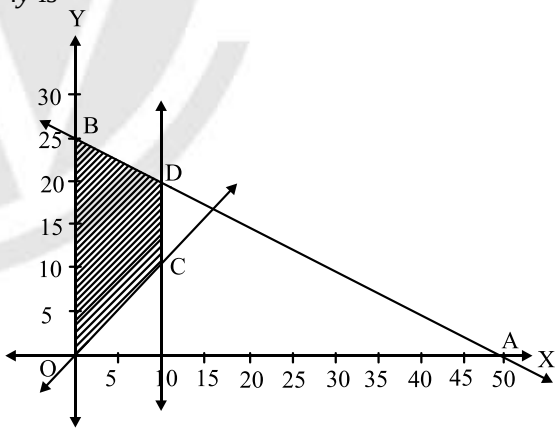
[2023]

(A) 19.5 (B) 21
(C) 15 (D) 19.8

205. The maximum value of $z = 3x + 5y$ subject to the constraints $3x + 2y \le 18$, $x \le 4$, $y \le 6$, $x, y \ge 0$, is **[2023]**

(A) 27 (B) 36
(C) 42 (D) 30

206. For a feasible region OCDBO given below, the maximum value of the objective function $z = 3x + 4y$ is



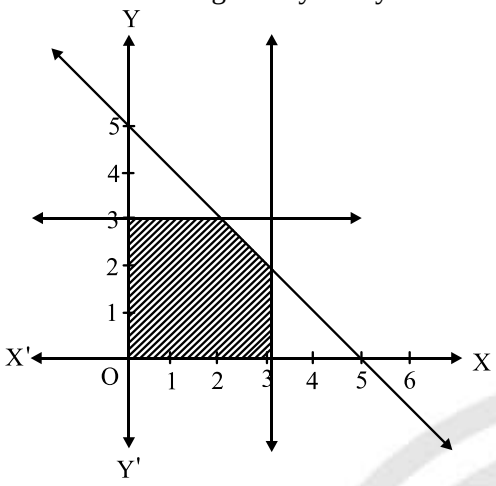
[2023]

(A) 70 (B) 100
(C) 110 (D) 130

207. The maximum value of $z = 7x + 8y$ subject to the constraints $x + y \le 20$, $y \ge 5$, $x \le 10$, $x \ge 0$, $y \ge 0$ is **[2023]**

(A) 150 (B) 160
(C) 110 (D) 180

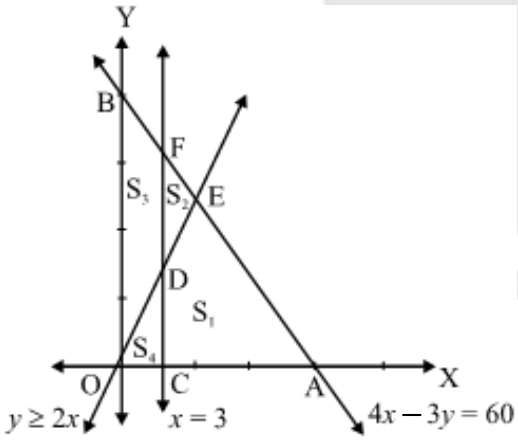
208. The shaded area in the given figure is a solution set for some system of inequations. The maximum value of the function $z = 10x + 25y$ subject to the linear constraints given by the system is



[2023]

(A) 80
(B) 100
(C) 95
(D) 105

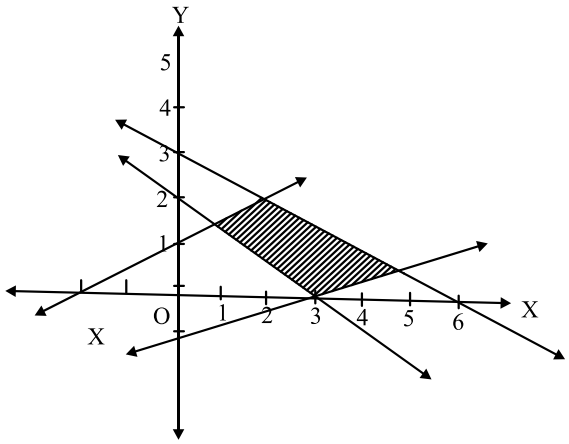
209. The solution set of the inequalities $4x + 3y \le 60$, $y \ge 2x$, $x \ge 3$, $x, y \ge 0$ is represented by region



[2023]

(A) S_2 region (B) S_1 region
(C) S_3 region (D) S_4 region

210. The shaded region in the following figure represents the solution set for a certain linear programming problem. Then linear constraints for this region are given by



[2023]

(A) $2x + 3y \ge 6, -x + 2y \ge 2, 3x + 6y \le 18, x - 3y \ge 3, x \ge 0, y \ge 0$
(B) $2x + 3y \ge 6, -x + 2y \le 2, x - 3y \le 3, x + 2y \ge 18, x \ge 0, y \ge 0$
(C) $2x + 3y \le 6, -x + 2y \ge 2, 3x + 6y \le 18, x - 3y \le 3, x \ge 0, y \ge 0$
(D) $2x + 3y \ge 6, 3x + 6y \le 18, x - 3y \le 3, -x + 2y \le 2, x \ge 0, y \ge 0$

211. If y is a function of x and $\log(x + y) = 2xy$, then the value of $y'(0)$ is **[2023]**

(A) 1
(B) –1
(C) 2
(D) 0

212. If $x^k + y^k = a^k$ (a, k > 0) and $\frac{dy}{dx} + \left(\frac{y}{x}\right)^{\frac{1}{3}} = 0$, then k has the value **[2023]**

(A) $\frac{1}{3}$
(B) $\frac{2}{3}$
(C) $\frac{1}{4}$
(D) $\frac{2}{7}$

213. If g is the inverse of f and $f'(x)=\frac{1}{1+x^3}$, then g'(x) is **[2023]**

(A) $\frac{1}{1+(g(x))^3}$ (B) $1+(g(x))^3$

(C) $\frac{g(x)}{1+(g(x))^3}$ (D) $\frac{(g(x))^3}{1+(g(x))^3}$

214. Let $S=\{t\in R/\ f(x)=|x-\pi|\left(e^{|x|}-1\right)sin|x|$ is not differentiable at $t\}$, then S is **[2023]**

(A) ϕ (an empty set)

(B) $\{0\}$

(C) $\{\pi\}$

(D) $\{0, \pi\}$

215. If y is a function of x and $\log(x+y)=2xy$, then $\frac{dy}{dx}$ at $x=0$ is **[2023]**

(A) 0 (B) –1

(C) 1 (D) 2

216. If $x = 3\tan t$ and $y = 3\sec t$, then the value of $\frac{d^2y}{dx^2}$ at $t=\frac{\pi}{4}$ is **[2023]**

(A) $\frac{-1}{6\sqrt{2}}$ (B) $\frac{1}{6\sqrt{2}}$

(C) $\frac{1}{3\sqrt{2}}$ (D) $\frac{3}{2\sqrt{2}}$

217. If $y=tan^{-1}\left(\frac{log\left(\frac{e}{x^2}\right)}{log\left(ex^2\right)}\right)+tan^{-1}\left(\frac{4+2log\,x}{1-8log\,x}\right)$, then $\frac{dy}{dx}$ is **[2023]**

(A) 0

(B) $\frac{1}{2}$

(C) $\frac{1}{4}$

(D) 1

218. If $f(x) = 3^x$; $g(x) = 4^x$, then $\frac{f'(0)-g'(0)}{1+f'(0)g'(0)}$ is **[2023]**

(A) $\frac{log\left(\frac{3}{4}\right)}{1+(log\,3)(log\,4)}$

(B) $\frac{log\left(\frac{3}{4}\right)}{1+log\,12}$

(C) $\frac{log\,12}{1+log\,12}$

(D) $\frac{log\left(\frac{3}{4}\right)}{1-log\,12}$

219. The set of all points, where the derivative of the functions $f(x)=\frac{x}{1+|x|}$ exists, is **[2023]**

(A) $(-\infty,\infty)$ (B) $[0,\infty)$

(C) $(-\infty,0)\cup(0,\infty)$ (D) $(0,\infty)$

220. If $y=\left[(x+1)(2x+1)(3x+1).....(nx+1)\right]^{\frac{3}{2}}$, then $\frac{dy}{dx}$ at $x=0$ is **[2023]**

(A) $\frac{3n(n+1)}{4}$ (B) $\frac{n(n+1)}{2}$

(C) $\frac{3n(n+1)}{2}$ (D) $\frac{n(n+1)}{4}$

221. The derivative of $f(\tan x)$ w.r.t. $g(\sec x)$ at $x=\frac{\pi}{4}$ where $f'(1) = 2$ and $g'(\sqrt{2}) = 4$ is **[2023]**

(A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{2}$

(C) 1 (D) 0

222. If $\log(x+y)=2xy$, then $\frac{dy}{dx}$ at $x=0$ is **[2023]**

(A) 1 (B) –1

(C) 2 (D) –2

223. $y = (1 + x)(1 + x^2)(1 + x^4) \ldots \ldots (1 + x^{2n})$, then the value of $\frac{dy}{dx}$ at $x = 0$ is **[2023]**

(A) 0 (B) –1
(C) 1 (D) 2

224. If $f(x) = \sin^{-1}\left(\frac{2\log x}{1+(\log x)^2}\right)$, then $f'(e)$ is **[2023]**

(A) $\frac{2}{e}$ (B) $\frac{1}{2e}$
(C) e (D) $\frac{1}{e}$

225. For $x > 1$, if $(2x)^{2y} = 4e^{2x-2y}$, then $(1 + \log_e 2x)^2 \frac{dy}{dx}$ is equal to **[2023]**

(A) $\frac{x\log_e 2x + \log_e 2}{x}$
(B) $\frac{x\log_e 2x - \log_e 2}{x}$
(C) $x\log_e 2x + \frac{\log_e 2}{x}$
(D) $x\log_e 2x - \frac{\log_e 2}{2}$

226. If $\tan y = \frac{x\sin\alpha}{1 - x\cos\alpha}$ and $\frac{dy}{dx} = \frac{m}{x^2 + 2nx + 1}$, then $m^2 + n^2$ is **[2023]**

(A) 2 (B) 3
(C) 1 (D) 4

227. Differentiation of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ w.r.t. $\cos^{-1}\sqrt{\left(\frac{1+\sqrt{1+x^2}}{2\sqrt{1+x^2}}\right)}$ is **[2023]**

(A) $\frac{1}{2}$ (B) 1
(C) 2 (D) $\frac{1}{4}$

228. $y = \frac{\sqrt[3]{1+3x}\sqrt[4]{1+4x}\sqrt[5]{1+5x}}{\sqrt[7]{1+7x}\sqrt[8]{1+8x}}$. Then $\frac{dy}{dx}$ at $x = 0$ is **[2023]**

(A) 3 (B) –1
(C) 1 (D) 2

229. Let $f : R \to R$ be a function such that $f(x) = x^3 + x^2 f'(1) + x f''(2) + 6$, $x \in R$, then $f(2)$ equals **[2023]**

(A) 30 (B) –2
(C) –4 (D) 8

230. If $x = \log_e\left(\frac{\cos\frac{y}{2} - \sin\frac{y}{2}}{\cos\frac{y}{2} + \sin\frac{y}{2}}\right)$, $\tan\frac{y}{2} = \sqrt{\frac{1-t}{1+t}}$. Then $(y_1)_{t=\frac{1}{2}}$ has the value **[2023]**

(A) $\frac{1}{2}$ (B) $-\frac{1}{2}$
(C) $\frac{1}{4}$ (D) $-\frac{1}{4}$

231. If $y = (\sin^{-1}x)^2 + (\cos^{-1}x)^2$, then $(1 - x^2)y_2 - xy_1 =$ **[2023]**

(A) 1
(B) 4
(C) –4
(D) –1

232. If $y = [(x + 1)(2x + 1)(3x + 1) \ldots\ldots (nx + 1)]^n$, then $\frac{dy}{dx}$ at $x = 0$ is **[2023]**

(A) $\frac{n(n+1)}{2}$ (B) $\frac{n^2(n+1)}{2}$
(C) $\frac{n(n+1)}{4}$ (D) $\frac{n^2(n-1)}{2}$

233. If $x = \sqrt{e^{\sin^{-1}t}}$ and $y = \sqrt{e^{\cos^{-1}t}}$, then $\frac{d^2y}{dx^2}$ is **[2023]**

(A) $\frac{-y}{x^2}$ (B) $\frac{y^2}{2x^2}$
(C) $\frac{2y}{x^2}$ (D) $\frac{-2y}{x^2}$

234. If $f'(x) = sin(log\, x)$ and $y = f\left(\frac{2x+3}{3-2x}\right)$, then $\frac{dy}{dx}$ at $x = 1$ is **[2023]**

(A) 6 sin (log 5)
(B) 5 sin (log 6)
(C) 12 sin (log 5)
(D) 5 sin (log 12)

235. If $y = \sqrt{(x - sin\, x) + \sqrt{(x - sin\, x) + \sqrt{(x - sin\, x)}....}}$, then $\frac{dy}{dx} =$ **[2023]**

(A) $\frac{1 - cos\, x}{2y - 1}$
(B) $\frac{1 + cos\, x}{2y - 1}$
(C) $\frac{1 - cos\, x}{2y + 1}$
(D) $\frac{1 - sin\, x}{2y - 1}$

236. If y is a function of x and $\log(x + y) = 2xy$, then the value of $y'(0)$ is **[2024, 2023]**

(A) 1 (B) –1
(C) 2 (D) 0

237. If $f(1)=1$, $f'(1)=3$, then the derivative of $f(f(f(x)))+(f(x))^2$ at $x = 1$ is **[2025, 20242023]**

(A) 9 (B) 12
(C) 15 (D) 33

238. If $y = \tan^{-1}[\frac{4\, sin\, 2x}{cos\, 2x - 6\, sin^2\, x}]$, then $\frac{dy}{dx}$ at $x = 0$ is **[2025, 2023]**

(A) $\frac{1}{8}$ (B) –8
(C) 8 (D) $\frac{1}{8}$

239. If slope of a tangent to the curve $xy + ax + by = 0$ at the point (1, 1) on it is 2, then $a - b$ is **[2023]**

(A) 3 (B) 1
(C) 2 (D) –1

240. The equation $x^3 + x - 1 = 0$ has **[2023]**

(A) no real root.
(B) exactly two real roots.
(C) exactly one real root.
(D) all three real roots.

241. The range of values of x for which $f(x) = x^3 + 6x^2 - 36x + 7$ is increasing in **[2023]**

(A) $(-\infty, -6) \cup (2, \infty)$
(B) $(-6, 2)$
(C) $(-\infty, -2) \cup (6, \infty)$
(D) $(-6, 2]$

242. A water tank has a shape of inverted right circular cone whose semi-vertical angle is $tan^{-1}\left(\frac{1}{2}\right)$. Water is poured into it at constant rate of 5 cubic meter/minute. The rate in meter/ minute at which level of water is rising, at the instant when depth of water in the tank is 10 m is **[2023]**

(A) $\frac{1}{5\pi}$ (B) $\frac{1}{15\pi}$
(C) $\frac{2}{\pi}$ (D) $\frac{1}{10\pi}$

243. Let $f(0) = -3$ and $f'(x) \le 5$ for all real values of x. The $f(2)$ can have possible maximum value as **[2023]**

(A) 10 (B) 5
(C) 7 (D) 13

244. The value of c for the function $f(x) = \log x$ on [1, e] if LMVT can be applied, is **[2023]**

(A) e – 2 (B) e + 1
(C) e – 1 (D) e

245. Let $f(x) = e^x - x$ and $g(x) = x^2 - x$, $\forall\, x \in R$, then the set of all $x \in R$, where the function $h(x) = (fog)(x)$ is increasing is **[2023]**

(A) $\left[0, \frac{1}{2}\right] \cup [1, \infty)$
(B) $\left[-1, -\frac{1}{2}\right] \cup \left[\frac{1}{2}, \infty\right)$
(C) $[0, \infty)$
(D) $\left[-\frac{1}{2}, 0\right] \cup [1, \infty)$

246. The displacement 'S' of a moving particle at a time t is given by $S = 5 + \frac{48}{t} + t^3$. Then its acceleration when the velocity is zero, is **[2023]**

(A) 12 (B) 20
(C) 16 (D) 24

247. If the surface area of a spherical balloon of radius 6 cm is increasing at the rate 2 cm²/sec, then the rate of increase in its volume in cm³/sec is **[2023]**

(A) 16 (B) 6
(C) 12 (D) 8

248. The value of α, so that the volume of parallelopiped formed by $\hat{i} + \alpha\hat{j} + \hat{k}$, $\hat{j} + \alpha\hat{k}$ and $\alpha\hat{i} + \hat{k}$ becomes minimum, is **[2023]**

(A) –3 (B) 3
(C) $\frac{1}{\sqrt{3}}$ (D) $-\frac{1}{\sqrt{3}}$

249. For all real x, the minimum value of $\frac{1-x+x^2}{1+x+x^2}$ is **[2023]**

(A) 0 (B) 1
(C) $\frac{1}{3}$ (D) 3

250. If the function f is given by $f(x) = x^3 - 3(a-2)x^2 + 3ax + 7$, for some $a \in R$, is increasing in (0, 1] and decreasing in [1, 5), then a root of the equation $\frac{f(x)-14}{(x-1)^2} = 0 (x \neq 1)$ is **[2023]**

(A) –7 (B) 6
(C) 7 (D) 5

251. If a and b are positive number such that $a > b$, then the minimum value of $a \sec\theta - b\tan\theta$ $(0 < \theta < \frac{\pi}{2})$ is **[2023]**

(A) $\frac{1}{\sqrt{a^2-b^2}}$ (B) $\frac{1}{\sqrt{a^2+b^2}}$
(C) $\sqrt{a^2+b^2}$ (D) $\sqrt{a^2-b^2}$

252. A rod AB, 13 feet long moves with its ends A and B on two perpendicular lines OX and OY respectively. When A is 5 feet from O, it is moving away at the rate of 3 feet/sec. At this instant, B is moving at the rate **[2023]**

(A) $\frac{5}{4}$ ft/sec upwards.

(B) $\frac{4}{5}$ ft/sec upwards.

(C) $\frac{5}{4}$ ft/sec downwards.

(D) $\frac{4}{5}$ ft/sec downwards.

253. The equation of the tangent to the curve $y = \sqrt{9-2x^2}$, at the point where the ordinate and abscissa are equal, is **[2023]**

(A) $2x + y + \sqrt{3} = 0$ (B) $2x + y + 3\sqrt{3} = 0$
(C) $2x - y - 3\sqrt{3} = 0$ (D) $2x + y - 3\sqrt{3} = 0$

254. The approximate value of sin (60° 0′ 10″) is (given that $\sqrt{3} = 1.732$, 1° = 0.0175°) **[2023]**

(A) 0.08660243
(B) 0.0008660243
(C) 0.8660243
(D) 0.008660243

255. The angle between the tangents to the curves $y = 2x^2$ and $x = 2y^2$ at (1, 1) is **[2023]**

(A) $\tan^{-1}(\frac{15}{8})$ (B) $\tan^{-1}(\frac{7}{8})$
(C) $\tan^{-1}(\frac{3}{4})$ (D) $\tan^{-1}(\frac{1}{4})$

256. If $x = -1$ and $x = 2$ are extreme points of $f(x) = \alpha \log x + \beta x^2 + x$, α and β are constants, then the value of $\alpha^2 + 2\beta$ is **[2023]**

(A) –3
(B) 3
(C) $\frac{3}{2}$
(D) 5

257. A ladder 5 meters long rests against a vertical wall. If its top slides downwards at the rate of 10 cm/s, then the angle between the ladder and the floor is decreasing at the rate of _____ rad./s when it's lower end is 4 m away from the wall. **[2023]**

(A) –0.1 (B) –0.025
(C) 0.1 (D) 0.025

258. The equation of the normal to the curve $3x^2 - y^2 = 8$, which is parallel to the line $x + 3y = 10$, is **[2023]**

(A) $x + 3y + 6 = 0$ (B) $x + 3y - 3 = 0$
(C) $x + 3y + 8 = 0$ (D) $x + 3y - 4 = 0$

259. A tank with a rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 4 meter and volume is 36 cubic meters. If building of the tank costs ₹ 100 per square meter for the base and ₹ 50 per square meter for the sides, then the cost of least expensive tank is **[2023]**

(A) ₹ 3000 (B) ₹ 3300
(C) ₹ 2400 (D) ₹ 3500

260. The slope of the normal to the curve $x = \sqrt{t}$ and $y = t - \frac{1}{\sqrt{t}}$ at $t = 4$ is **[2023]**

(A) $-\frac{17}{4}$ (B) $\frac{4}{17}$
(C) $-\frac{4}{17}$ (D) $\frac{17}{4}$

261. Values of c as per Rolle's theorem for $f(x) = \sin x + \cos x + 6$ on $[0, 2\pi]$ are

(A) $\frac{\pi}{3}, \frac{5\pi}{3}$ (B) $\frac{\pi}{6}, \frac{5\pi}{6}$
(C) $\frac{\pi}{4}, \frac{5\pi}{4}$ (D) $\frac{\pi}{4}, \frac{7\pi}{4}$

262. $A(1, -3)$, $B(4, 3)$ are two points on the curve $y = x - \frac{4}{x}$. The points on the curve, the tangents at which are parallel to the chord AB, are **[2023]**

(A) (1, 2), (–1, –2) (B) (2, 0), (–2, 0)
(C) (0, 2), (1, –2) (D) (3, 2), (–3, 1)

263. Slope of the tangent to the curve $y = 2e^x \sin\left(\frac{\pi}{4} - \frac{x}{2}\right) \cos\left(\frac{\pi}{4} - \frac{x}{2}\right)$ where $0 \le x \le 2\pi$ is minimum at $x =$ **[2023]**

(A) 0 (B) π
(C) 2π (D) 1

264. If slope of the tangent to the curve $xy + ax + by = 0$ at the point (1, 1) on it is 2, then the value of $3a + b$ is **[2023]**

(A) 3 (B) 1
(C) 2 (D) –1

265. Let the curve be represented by $x = 2(\cos t + t \sin t)$, $y = 2(\sin t - t \cos t)$. Then normal at any point 't' of the curve is at a distance of _____ units from the origin. **[2023]**

(A) 1 (B) 0
(C) 2 (D) 4

266. Let $B \equiv (0, 3)$ and $C \equiv (4, 0)$. The point A is moving on the line $y = 2x$ at the rate of 2 units/second. The area of ΔABC is increasing at the rate of **[2023]**

(A) $\frac{11}{\sqrt{5}}$ (units)²/sec (B) $\frac{11}{5}$ (units)²/sec
(C) $\frac{13}{\sqrt{5}}$ (units)²/sec (D) $\frac{13}{5}$ (units)²/sec

267. The function $f(x) = x^3 - 6x^2 + 9x + 2$ has maximum value when x is **[2023]**

(A) 1 (B) 2
(C) 3 (D) 6

268. If $y = 4x - 5$ is a tangent to the curve $y^2 = px^3 + q$ at (2, 3), then $p - q$ is **[2023]**

(A) –5 (B) 5
(C) 9 (D) –9

269. The diagonal of a square is changing at the rate of 0.5 cm/sec. Then the rate of change of area when the area is 400 cm² is equal to **[2023]**

(A) $20\sqrt{2}$ cm²/sec (B) $10\sqrt{2}$ cm²/sec
(C) $\frac{1}{10\sqrt{2}}$ cm²/sec (D) $\frac{10}{\sqrt{2}}$ cm²/sec

270. Let $P(x)$ be a polynomial of degree 2, with $P(2) = -1$, $P'(2) = 0$, $P''(2) = 2$, then $P(1.001)$ is **[2023]**

(A) 0.002 (B) –0.002
(C) 0.004 (D) –0.004

271. Let $f(x) = 5 - |x-2|$ and $g(x) = |x+1|$, $x \in R$ If $f(x)$ attains maximum value at α and $g(x)$ attains minimum value at β, then $\lim\limits_{x\to-\alpha\beta} \dfrac{(x-1)(x^2-5x+6)}{x^2-6x+8}$ is equal to **[2023]**

(A) $\dfrac{1}{2}$ (B) $\dfrac{-3}{2}$
(C) $\dfrac{-1}{2}$ (D) $\dfrac{3}{2}$

272. If x_0 is the point of local minima of $f(x) = \bar{a}\cdot(\bar{b}\times\bar{c})$ where $\bar{a} = x\hat{i} - 2\hat{j} + 3\hat{k}$, $\bar{b} = -2\hat{i} + x\hat{j} - \hat{k}$, $\bar{c} = 7\hat{i} - 2\hat{j} + x\hat{k}$, then value of $\bar{a}\cdot\bar{b}$ at $x = x_0$ is **[2024, 2023]**

(A) −3 (B) −15
(C) −12 (D) −9

273. If Rolle's theorem holds for the function $f(x) = x^3 + bx^2 + ax + 5$ on $[1, 3]$ with $c = 2 + \dfrac{1}{\sqrt{3}}$, then the values of a and b respectively are **[2024, 2023]**

(A) −11, 6 (B) 11, 6
(C) −11, −6 (D) 11, −6

274. The maximum value of the function $f(x) = 3x^3 - 18x^2 + 27x - 40$ on the set $S = \{x \in \mathbb{R} / x^2 + 30 \le 11x\}$ is **[2024, 2023]**

(A) –122 (B) –222
(C) 222 (D) 122

275. If $f(x) = x^3 + bx^2 + cx + d$ and $0 < b^2 < c$, then in $(-\infty, \infty)$ **[2024, 2023]**

(A) $f(x)$ is strictly increasing function
(B) $f(x)$ is bounded
(C) $f(x)$ has a local maxima
(D) $f(x)$ is a strictly decreasing function

276. A poster is to be printed on a rectangular sheet of paper of area 18 m². The margins at the top and bottom of 75 cm each and at the sides 50 cm each are to be left. Then the dimensions i.e. height and breadth of the sheet so that the space available for printing is maximum, are_____ respectively. **[2024, 2023]**

(A) $2\sqrt{3}$ m, $3\sqrt{3}$ m
(B) $3\sqrt{3}$ m, $2\sqrt{3}$ m
(C) 3 m,6 m
(D) 6 m,3 m

277. If the line $ax + by + c = 0$ is normal to the curve $xy = 1$, then **[2024, 2023]**

(A) a > 0, b > 0 (B) a > 0, b < 0
(C) a < 0, b ≥ 0 (D) a < 0, b < 0

278. If the curves $y^2 = 6x$ and $9x^2 + by^2 = 16$ intersect each other at right angles, then the value of b is **[2025, 2024, 2023, 2022]**

(A) 4 (B) $\dfrac{7}{2}$
(C) 6 (D) $\dfrac{9}{2}$

279. The function f(x) = sin⁴x + cos⁴x increases if **[2025, 2023]**

(A) $0 < x < \dfrac{\pi}{8}$ (B) $\dfrac{\pi}{4} < x < \dfrac{\pi}{2}$
(C) $\dfrac{3\pi}{8} < x < \dfrac{5\pi}{8}$ (D) $\dfrac{5\pi}{8} < x < \dfrac{3\pi}{4}$

280. If $f'(x) = x - \dfrac{5}{x^5}$ and $f(1) = 4$, then $f(x)$ is **[2023]**

(A) $\dfrac{x^2}{2} + \dfrac{9}{4x^4} + \dfrac{5}{4}$
(B) $\dfrac{x^2}{2} - \dfrac{5}{4x^4} + \dfrac{9}{4}$
(C) $\dfrac{x^2}{2} + \dfrac{5}{4x^4} + \dfrac{9}{4}$
(D) $\dfrac{x^2}{2} - \dfrac{9}{4x^4} + \dfrac{5}{4}$

281. If $\int \frac{\sin x}{3+4\cos^2 x}dx = A\tan^{-1}(B\cos x)+c$, (where c is a constant of integration), then the value of $A + B$ is **[2023]**

(A) $\frac{5}{2\sqrt{3}}$

(B) $\frac{-1}{2\sqrt{3}}$

(C) $\frac{-2}{\sqrt{3}}$

(D) $\frac{\sqrt{3}}{2}$

282. $\int\left(\sqrt{\tan x}+\sqrt{\cot x}\right)dx =$ **[2023]**

(A) $\sqrt{2}\sin^{-1}(\sin x-\cos x)+c$, where c is a constant of integration.

(B) $\frac{1}{\sqrt{2}}\sin^{-1}(\sin x-\cos x)+c$, where c is a constant of integration.

(C) $\sin^{-1}(\sin x-\cos x)+c$, where c is a constant of integration.

(D) $2\sin^{-1}(\sin x-\cos x)+c$, where c is a constant of integration.

283. Let $\alpha \in (0, \frac{\pi}{2})$ be fixed. If the integral $\int\frac{\tan x+\tan\alpha}{\tan x-\tan\alpha}dx = A(x)\cos 2\alpha + B(x)\sin 2\alpha + c$, (where c is a constant of integration), then functions $A(x)$ and $B(x)$ are respectively **[2023]**

(A) $x + \alpha$ and $\log|\sin(x + \alpha)|$.

(B) $x - \alpha$ and $\log|\sin(x - \alpha)|$.

(C) $x - \alpha$ and $\log|\cos(x - \alpha)|$.

(D) $x + \alpha$ and $\log|\sin(x - \alpha)|$.

284. $\int x\sqrt{\frac{2\sin\left(x^2+1\right)-\sin 2\left(x^2+1\right)}{2\sin\left(x^2+1\right)+\sin 2\left(x^2+1\right)}}dx =$ **[2023]**

(A) $\log(\sec(\frac{x^2+1}{2}))$ + c, where c is a constant of integration.

(B) $\log(\frac{x^2+1}{2})$ + c, where c is a constant of integration.

(C) $\log(\sin(\frac{x^2+1}{2}))$ + c, where c is a constant of integration.

(D) $2\log(x^2 + 1) + c$, where c is a constant of integration.

285. If $\int\frac{\cos 8x+1}{\cot 2x-\tan 2x}dx = A\cos 8x+c$, where c is an arbitrary constant, then the value of A is **[2023]**

(A) $\frac{1}{16}$ (B) $\frac{1}{8}$

(C) $\frac{-1}{8}$ (D) $\frac{-1}{16}$

286. The value of $\int(1-\cos x)\cdot\text{cosec}^2 x dx$ is **[2023]**

(A) $\frac{1}{2}\tan\frac{x}{2}+c$, where c is a constant of integration.

(B) $\tan\frac{x}{2}+c$, where c is a constant of integration.

(C) $2\cot\frac{x}{2}+c$, where c is a constant of integration.

(D) $\cot\frac{x}{2}+c$, where c is a constant of integration.

287. If $\int\cos^{3/5}x\cdot\sin^3 x dx = \frac{-1}{m}\cos^m x+\frac{1}{n}\cos^n x+c$, (where c is the constant of integration), then (m, n) = **[2023]**

(A) $\left(\frac{18}{5},\frac{8}{5}\right)$ (B) $\left(\frac{-8}{5},\frac{18}{5}\right)$

(C) $\left(\frac{8}{5},\frac{18}{5}\right)$ (D) $\left(\frac{-18}{5},\frac{-8}{5}\right)$

288. $\int\left(\frac{\tan\left(\frac{1}{x}\right)}{x}\right)^2 dx =$ **[2023]**

(A) $x-\tan x+c$, where c is a constant of integration

(B) $\frac{1}{x}-\tan\left(\frac{1}{x}\right)+c$, where c is a constant of integration.

(C) $\frac{1}{x}+\tan\left(\frac{1}{x}\right)+c$, where c is a constant of integration.

(D) $x+\tan x+c$, where c is a constant of integration.

289. If $I=\int\frac{2x-7}{\sqrt{3x-2}}dx$, then I is given by **[2023]**

(A) $\frac{106}{27}(3x-2)^{\frac{3}{2}}+c$, where c is a constant of integration.

(B) $\frac{98}{27}(3x-2)^{\frac{3}{2}}+c$, where c is a constant of integration.

(C) $\frac{4}{27}(3x-2)^{\frac{3}{2}}-\frac{34}{9}(3x-2)^{\frac{1}{2}}+c$, where c is a constant of integration.

(D) $\frac{4}{27}(3x-2)^{\frac{3}{2}}+\frac{34}{9}(3x-2)^{\frac{1}{2}}+c$, where c is a constant of integration

290. If $\int x^5e^{-4x^3}dx=\frac{1}{48}e^{-4x^3}f(x)+c$, where c is a constant of integration, then $f(x)$ is given by **[2023]**

(A) $4x^3+1$

(B) $-4x^3-1$

(C) $-2x^3-1$

(D) $-2x^3+1$

291. $\int\frac{\sin 2x\left(1-\frac{3}{2}\cos x\right)}{e^{\sin^2 x+\cos^3 x}}dx =$ **[2023]**

(A) $e^{\sin^2 x+\cos^3 x}+c$, where c is a constant of integration.

(B) $-e^{-\left(\sin^2 x+\cos^3 x\right)}+c$, where c is a constant of integration.

(C) $e^{-\left(\sin^2 x+\cos^3 x\right)^2}+c$, where c is a constant of integration.

(D) $e^{\sin^2 x+\cos x}+c$, where c is a constant of integration.

292. If $\int\frac{\cos\theta}{5+7\sin\theta-2\cos^2\theta}d\theta=A\log_e\left|f(\theta)\right|+c$ (where c is a constant of integration), then $\frac{f(\theta)}{A}$ can be **[2023]**

(A) $\frac{2\sin\theta+1}{\sin\theta+3}$ (B) $\frac{2\sin\theta+1}{5(\sin\theta+3)}$

(C) $\frac{5(\sin\theta+3)}{2\sin\theta+1}$ (D) $\frac{5(2\sin\theta+1)}{\sin\theta+3}$

293. $\int\frac{\sin x+\sin^3 x}{\cos 2x}dx=A\cos x+B\log f(x)+c$ (where c is a constant of integration). Then values of A, B and $f(x)$ are **[2023]**

(A) $A=\frac{1}{2},B=\frac{-3}{4\sqrt{2}},f(x)=\frac{\sqrt{2}\cos x-1}{\sqrt{2}\cos x+1}$

(B) $A=-\frac{1}{2},B=\frac{-3}{4\sqrt{2}},f(x)=\frac{\sqrt{2}\cos x+1}{\sqrt{2}\cos x-1}$

(C) $A=\frac{1}{2},B=\frac{-3}{4\sqrt{2}},f(x)=\frac{\sqrt{2}\cos x+1}{\sqrt{2}\cos x-1}$

(D) $A=\frac{3}{2},B=\frac{1}{2},f(x)=\frac{\sqrt{2}\cos x-1}{\sqrt{2}\cos x+1}$

294. Let $f(x)=\int\frac{x^2-3x+2}{x^4+1}dx$, then function decreases in the interval **[2023]**

(A) $(-\infty,-2)$ (B) $(-2,-1)$

(C) $(1, 2)$ (D) $(2,\infty)$

295. $\int \frac{dx}{\cot^2 x - 1} = \frac{1}{A}\log\left|\sec 2x + \tan 2x\right| - \frac{x}{B} + c$, (where c is constant of integration), then $A + B =$ **[2023]**

(A) –6 (B) 6
(C) –5 (D) 5

296. $\int \frac{\operatorname{cosec} x dx}{\cos^2\left(1 + \log\tan\frac{x}{2}\right)} =$ **[2024, 2023]**

(A) $\tan\left(1 + \log\tan\frac{x}{2}\right) + c$, where c is a constant of integration.

(B) $\frac{1}{2}\tan\left(1 + \log\tan\frac{x}{2}\right) + c$, where c is a constant of integration.

(C) $2\tan\left(1 + \log\tan\frac{x}{2}\right) + c$, where c is a constant of integration.

(D) $\frac{1}{4}\tan\left(1 + \log\tan\frac{x}{2}\right) + c$, where c is a constant of integration.

297. If $\int \frac{\log\left(t + \sqrt{1+t^2}\right)}{\sqrt{1+t^2}} dt = \frac{1}{2}\left(g(t)\right)^2 + c$ where c is a constant of integration, then $g(2)$ is equal to **[2024, 2023]**

(A) $2\log\left(2 + \sqrt{5}\right)$ (B) $\log\left(2 + \sqrt{5}\right)$
(C) $\frac{1}{\sqrt{5}}\log\left(2 + \sqrt{5}\right)$ (D) $\frac{1}{2}\log\left(2 + \sqrt{5}\right)$

298. $\int_1^2 \frac{dx}{\left(x^2 - 2x + 4\right)^{\frac{3}{2}}} = \frac{k}{k+5}$, then k has the value **[2023]**

(A) 1 (B) 2
(C) –1 (D) –2

299. Let $f : R \to R$ and $g : R \to R$ be continuous functions. Then the value of the integral $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left[f(x) + f(-x)\right]\left[g(x) - g(-x)\right] dx$ is **[2023]**

(A) π (B) 1
(C) –1 (D) 0

300. $\int_0^4 \left|2x - 5\right| dx =$ **[2023]**

(A) $\frac{13}{2}$ (B) $\frac{15}{2}$
(C) $\frac{17}{4}$ (D) $\frac{17}{2}$

301. The value of $\int_0^{\pi} \left|\sin x - \frac{2x}{\pi}\right| dx$ is **[2023]**

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$
(C) π (D) 2π

302. If $f(x) = \begin{cases} e^{\cos x}\sin x, & \text{for } |x| \le 2 \\ 2, & \text{otherwise} \end{cases}$, then $\int_{-2}^{3} f(x) dx$ is equal to **[2023]**

(A) 0 (B) 2
(C) 1 (D) 3

303. The integral $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec^{\frac{2}{3}} x \operatorname{cosec}^{\frac{4}{3}} x dx$ is equal to **[2023]**

(A) $3^{\frac{5}{6}} - 3^{\frac{2}{3}}$ (B) $3^{\frac{7}{6}} - 3^{\frac{5}{6}}$
(C) $3^{\frac{5}{3}} - 3^{\frac{1}{3}}$ (D) $3^{\frac{4}{3}} - 3^{\frac{1}{3}}$

304. If $I_n = \int_0^{\frac{\pi}{4}} \tan^n \theta d\theta$, then $I_{12} + I_{10} =$ **[2023]**

(A) $\frac{1}{8}$ (B) $\frac{1}{12}$
(C) $\frac{1}{11}$ (D) $\frac{1}{10}$

305. The area (in sq. units) bounded by the curve $y = x|x|$, X-axis and the lines $x = -1$ and $x = 1$ is **[2023]**

(A) $\frac{2}{3}$ (B) $\frac{1}{3}$

(C) 1. (D) $\frac{4}{3}$

306. The area bounded by the X-axis and the curve $y = x(x - 2)(x + 1)$ is **[2023]**

(A) $\frac{37}{12}$ sq. units (B) $\frac{27}{12}$ sq. units

(C) $\frac{37}{4}$ sq. units (D) $\frac{27}{13}$ sq. units

307. The area of the region bounded by the curves $y = e^x$, $y = \log x$ and lines $x = 1$, $x = 2$ is **[2023]**

(A) $(e - 1)^2$ sq. units

(B) $(e^2 - e + 1)$ sq. units

(C) $(e^2 - e + 1 - 2\log 2)$ sq. units

(D) $(e^2 + e - 2\log 2)$ sq. units

308. The area (in sq. units) of the smaller part of the circle $x^2 + y^2 = a^2$ cut off by the line $x = \frac{a}{\sqrt{2}}$ is **[2023]**

(A) $\frac{a^2}{4}\left|\frac{\pi}{2} - 1\right|$ (B) $a^2\left|\frac{\pi}{4} - 1\right|$

(C) $\frac{a^2}{2}\left|\frac{\pi}{2} - 1\right|$ (D) $\frac{a^2}{4}\left|\frac{\pi}{4} - 1\right|$

309. If a curve $y = a\sqrt{x} + bx$ passes through the point (1, 2) and the area bounded by the curve, line $x = 4$ and X-axis is 8 sq. units, then **[2023]**

(A) $a = 3, b = -1$

(B) $a = 3, b = 1$

(C) $a = -3, b = 1$

(D) $a = -3, b = -1$

310. The area (in sq. units) bounded by the curves $y = (x + 1)^2$, $y = (x - 1)^2$ and the line $y = \frac{1}{4}$ is **[2024, 2023]**

(A) $\frac{2}{3}$ (B) $\frac{1}{6}$

(C) $\frac{1}{3}$ (D) $\frac{1}{4}$

311. The area bounded by the curve $y = |x - 2|$, $x = 1$, $x = 3$ and X-axis is **[2025, 2023]**

(A) 3 sq. units

(B) 2 sq. units

(C) 1 sq. units

(D) 4 sq. units

312. General solution of the differential equation $\log\left(\frac{dy}{dx}\right) = ax + by$ is **[2023]**

(A) $ae^{by} + be^{ax} = c_1$, where c_1 is a constant.

(B) $ae^{-by} + be^{-ax} = c_1$, where c_1 is a constant.

(C) $ae^{-by} + be^{ax} = c_1$, where c_1 is a constant.

(D) $ae^{by} + be^{-ax} = c_1$, where c_1 is a constant.

313. The general solution of the differential equation $\frac{dy}{dx} + \left(\frac{3x^2}{1+x^3}\right)y = \frac{1}{x^3+1}$ is **[2023]**

(A) $y(1 + x^3) = x^3 + c$, where c is a constant of integration.

(B) $y(1 + x^3) = x + c$, where c is a constant of integration.

(C) $y(1 + x^3) = x^2 + c$, where c is a constant of integration.

(D) $y(1 + x^3) = 2x + c$, where c is a constant of integration.

314. The differential equation of all circles, passing through the origin and having their centres on the X-axis, is **[2023]**

(A) $y^2 = x^2 + xy\frac{dy}{dx}$

(B) $x^2 = y^2 + 2xy\frac{dy}{dx}$

(C) $y^2 = x^2 + 2xy\frac{dy}{dx}$

(D) $x^2 = y^2 - xy\frac{dy}{dx}$

315. The solution of $\frac{dx}{dy}+\frac{x}{y}=x^2$ is **[2023]**

(A) $\frac{1}{y}=cx-x\log x$, where c is a constant of integration.

(B) $\frac{1}{x}=cy-y\log y$, where c is a constant of integration.

(C) $\frac{1}{x}=cx-x\log y$, where c is a constant of integration.

(D) $\frac{1}{y}=cx-y\log x$, where c is a constant of integration.

316. The solution of the differential equation $\frac{dy}{dx}+\frac{y}{x}=\sin x$ is **[2023]**

(A) $xy + \cos x = \sin x + c$, where c is a constant of integration.

(B) $x(y + \cos x) = \sin x + c$, where c is a constant of integration.

(C) $y(x + \cos x) = \sin x + c$, where c is a constant of integration.

(D) $xy + \sin x = \cos x + c$, where c is a constant of integration.

317. At present a firm is manufacturing 1000 items. It is estimated that the rate of change of production P w.r.t. additional number of worker x is given by $\frac{dp}{dx}=100-12\sqrt{x}$.

If the firm employees 9 more workers, then the new level of production of items is **[2023]**

(A) 1684

(B) 1648

(C) 2116

(D) 1116

318. The decay rate of radio active material at any time t is proportional to its mass at that time. The mass is 27 grams when $t = 0$. After three hours it was found that 8 grams are left. Then the substance left after one more hour is **[2023]**

(A) $\frac{27}{8}$ grams

(B) $\frac{81}{4}$ grams

(C) $\frac{16}{3}$ grams

(D) $\frac{16}{9}$ grams

319. The differential equation $\cos(x + y)\, dy = dx$ has the general solution given by **[2023]**

(A) $y = \sin(x + y) + c$, where c is a constant.

(B) $y = \tan(x + y) + c$, where c is a constant

(C) $y=\tan\left(\frac{x+y}{2}\right)+c$, where c is a constant

(D) $y=\frac{1}{2}\tan(x+y)+c$, where c is a constant

320. The solution of the differential equation $e^{-x}(y+1)dy+\left(\cos^2 x-\sin 2x\right)ydx=0$ at $x = 0$, $y = 1$ is **[2023]**

(A) $(y+1)+e^x\cos^2 x=2$

(B) $y+\log y=e^x\cos^2 x$

(C) $\log(y+1)+e^x\cos^2 x=1$

(D) $y+\log y+e^x\cos^2 x=2$

321. Rate of increase of bacteria in a culture is proportional to the number of bacteria present at that instant and it is found that the number doubles in 6 hours. The number of bacteria becomes _____ times at the end of 18 hours. **[2023]**

(A) 9 (B) 6

(C) 8 (D) 3

322. The particular solution of differential equation $e^{\frac{dy}{dx}} = (x+1), y(0) = 3$ is **[2023]**

(A) $y = x\log x - x + 2$
(B) $y = (x + 1)\log(x + 1) - x + 3$
(C) $y = (x + 1)\log(x + 1) + x - 3$
(D) $y = x\log x + x - 2$

323. A right circular cone has height 9 cm and radius of base 5 cm. It is inverted and water is poured into it. If at any instant, the water level rises at the rate $\frac{\pi}{A}$ cm/sec. where A is area of the water surface at that instant, then cone is completely filled in **[2023]**

(A) 70 sec. (B) 75 sec.
(C) 72 sec. (D) 77 sec.

324. If $xdy = y(dx + ydy)$, $y(1) = 1$, $y(x) > 0$, then $y(-3)$ is **[2023]**

(A) 1 (B) 2
(C) 3 (D) 4

325. General solution of the differential equation $\cos x(1 + \cos y)\,dx - \sin y(1 + \sin x)\,dy = 0$ is **[2023]**

(A) $(1 + \cos x)(1 + \sin y) = c$, where c is a constant of integration.
(B) $1 + \sin x + \cos y = c$, where c is a constant of integration.
(C) $(1 + \sin x)(1 + \cos y) = c$, where c is a constant of integration.
(D) $1 + \sin x \cos y = c$, where c is a constant of integration.

326. If the slope of the tangent of the curve at any point is equal to $-y + e^{-x}$, then the equation of the curve passing through origin is **[2023]**

(A) $y + xe^x = 0$ (B) $ye^x + x = 0$
(C) $ye^x - x = 0$ (D) $y - xe^x = 0$

327. If a body cools from 80° C to 50° C in the room temperature of 25° C in 30 minutes, then the temperature of the body after 1 hour is **[2023]**

(A) 31.36° C (B) 32.25° C
(C) 36.36° C (D) 33.25° C

328. In a certain culture of bacteria, the rate of increase is proportional to the number present. If there are 10^4 at the end of 3 hours and $4\cdot10^4$ at the end of 5 hours, then there were _____ in the beginning. **[2024, 2023]**

(A) 10^4 (B) $\frac{10^4}{4}$
(C) 410^4 (D) $\frac{10^4}{8}$

329. A radio-active substance has a half-life of h days, then its initial decay rate is given by (where radio-active substance has initial mass m_0) **[2024, 2023]**

(A) $\frac{m_0}{h}(\log 2)$ (B) $(m_0 h)(\log 2)$
(C) $-\frac{m_0}{h}(\log 2)$ (D) $-(m_0 h)(\log 2)$

330. If $\frac{dy}{dx} = y + 3, y + 3 > 0$ and $y(0) = 2$, then $y(\log 2)$ is equal to **[2024, 2023]**

(A) 13 (B) –2
(C) 7 (D) 5

331. If $y = y(x)$ and $\left(\frac{2+\sin x}{y+1}\right)\frac{dy}{dx} = -\cos x, y(0) = 1$, then $y\left(\frac{\pi}{2}\right) =$ **[2025, 2024, 2023, 2022]**

(A) $\frac{1}{3}$
(B) $\frac{2}{3}$
(C) $-\frac{1}{3}$
(D) 1

332. A random variable X assumes values 1, 2, 3, ..., n with equal probabilities, if $\text{var}(X) = E(X)$, then n is **[2023]**

(A) 4 (B) 5
(C) 7 (D) 9

333. In a game, 3 coins are tossed. A person is paid ₹ 7/-, if he gets all heads or all tails; and he is supposed to pay ₹ 3/-, if he gets one head or two heads. The amount he can expect to win on an average per game is ₹ **[2023]**

(A) –0.5
(B) 0.5
(C) 1
(D) –1

334. A fair die is tossed twice in succession. If X denotes the number of sixes in two tosses, then the probability distribution of X is given by **[2023]**

(A)	X = x	0	1	2
	P(X = x)	$\frac{25}{36}$	$\frac{1}{36}$	$\frac{5}{18}$

(B)	X = x	0	1	2
	P(X = x)	$\frac{5}{18}$	$\frac{1}{36}$	$\frac{25}{36}$

(C)	X = x	0	1	2
	P(X = x)	$\frac{25}{36}$	$\frac{5}{18}$	$\frac{1}{36}$

(D)	X = x	0	1	2
	P(X = x)	$\frac{5}{18}$	$\frac{25}{36}$	$\frac{1}{36}$

335. Two cards are drawn successively with replacement from a well-shuffled pack of 52 cards. Then mean of number of tens is **[2023]**

(A) $\frac{1}{13}$
(B) $\frac{1}{169}$
(C) $\frac{2}{13}$
(D) $\frac{4}{169}$

336. A fair die is tossed twice in succession. If X denotes the number of fours in two tosses, then the probability distribution of X is given by **[2023]**

(A)	$X = x_i$	0	1	2
	P_i	$\frac{1}{36}$	$\frac{25}{36}$	$\frac{5}{18}$

(B)	$X = x_i$	0	1	2
	P_i	$\frac{25}{36}$	$\frac{1}{36}$	$\frac{5}{18}$

(C)	$X = x_i$	0	1	2
	P_i	$\frac{25}{36}$	$\frac{5}{18}$	$\frac{1}{36}$

(D)	$X = x_i$	0	1	2
	P_i	$\frac{5}{18}$	$\frac{1}{36}$	$\frac{25}{36}$

337. Two cards are drawn successively with replacement from a well shuffled pack of 52 cards. Then the probability distribution of number of jacks is **[2023]**

(A)	X = x	0	1	2
	P(X = x)	$\frac{144}{169}$	$\frac{24}{169}$	$\frac{1}{169}$

(B)	X = x	0	1	2
	P(X = x)	$\frac{1}{169}$	$\frac{144}{169}$	$\frac{24}{169}$

(C)	X = x	0	1	2
	P(X = x)	$\frac{24}{169}$	$\frac{1}{169}$	$\frac{144}{169}$

(D)	X = x	0	1	2
	P(X = x)	$\frac{144}{169}$	$\frac{1}{169}$	$\frac{24}{169}$

338. The discrete random variable X can take all possible integer values from 1 to k, each with a probability $\frac{1}{k}$, then its variance is **[2023]**

(A) $\frac{k^2-1}{12}$ (B) $\frac{k^2-1}{6}$

(C) $\frac{k^2+1}{12}$ (D) $\frac{k^2+1}{6}$

339. Three fair coins with faces numbered 1 and 0 are tossed simultaneously. Then variance (X) of the probability distribution of random variable X, where X is the sum of numbers on the upper most faces, is **[2023]**

(A) 0.7 (B) 0.75

(C) 0.65 (D) 0.6

340. A random variable X has the following probability distribution

X = x	0	1	2
P(X = x)	$4k-10k^2$	$5k-1$	$3k^3$

then P(X < 2) is **[2023]**

(A) $\frac{2}{9}$

(B) $\frac{5}{9}$

(C) $\frac{8}{9}$

(D) $\frac{4}{9}$

341. If a continuous random variable X has probability density function $f(x)$ given by

$$f(x)=\begin{cases} ax & ,\text{if } 0\le x<1 \\ a & ,\text{if } 1\le x<2 \\ 3a-ax & ,\text{if } 2\le x\le 3, \\ 0 & ,\text{otherwise} \end{cases}$$

then a has the value **[2023]**

(A) $\frac{1}{5}$ (B) $\frac{1}{3}$

(C) $\frac{1}{2}$ (D) 1

342. A random variable X has the probability distribution

X = x	1	2	3	4	5	6	7	8
P(X = x)	0.15	0.23	0.12	0.20	0.08	0.10	005	0.07

For the events $E = \{X \text{ is a prime number}\}$ and $F = \{x < 5\}$, $P(E \cup F)$ is **[2023]**

(A) 0.63

(B) 0.75

(C) 0.83

(D) 0.90

343. The c.d.f. $F(x)$ associated with p.d.f. $f(x)$ $f(x)=\begin{cases}12x^2(1-x) & ,\text{ if } 0<x<1 \\ 0 & ,\text{ otherwise}\end{cases}$ is **[2023]**

(A) $F(x) = 4x^3+3x^4$

(B) $F(x) = 4x^3-3x^4$

(C) $F(x) = -4x^3-3x^4$

(D) $F(x) = -4x^3+3x^4$

344. Two cards are drawn successively with replacement from well shuffled pack of 52 cards, then the probability distribution of number of queens is **[2023]**

(A)

X = x	0	1	2
P(X = x)	$\frac{144}{169}$	$\frac{24}{169}$	$\frac{1}{169}$

(B)

X = x	0	1	2
P(X = x)	$\frac{1}{169}$	$\frac{24}{169}$	$\frac{144}{169}$

(C)

X = x	0	1	2
P(X = x)	$\frac{24}{169}$	$\frac{1}{169}$	$\frac{144}{169}$

(D)

X = x	0	1	2
P(X = x)	$\frac{1}{169}$	$\frac{25}{169}$	$\frac{143}{169}$

345. If $f(x) = \begin{cases} 3\left(1-2x^2\right); & 0 < x < 1 \\ 0 \quad ; & \text{otherwise} \end{cases}$ is a probability density function of X, then $P\left(\frac{1}{4} < x < \frac{1}{3}\right)$ is **[2023]**

(A) $\frac{75}{243}$ (B) $\frac{23}{96}$

(C) $\frac{179}{864}$ (D) $\frac{52}{243}$

346. A fair die with numbers 1 to 6 on their faces is thrown. Let X denote the number of factors of the number, on the uppermost face, then the probability distribution of X is **[2023]**

(A)

X = x	1	2	3	4
P(X = x)	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{6}$

(B)

X = x	1	2	3	4
P(X = x)	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{2}$

(C)

X = x	1	2	3	4
P(X = x)	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

(D)

X = x	1	2	3	4
P(X = x)	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{6}$

347. A man takes a step forward with probability 0.4 and backwards with probability 0.6. The probability that at the end of eleven steps, he is one step away from the starting point is **[2023]**

(A) ${}^{11}C_6(0.24)^6$

(B) ${}^{11}C_6(0.24)^5$

(C) ${}^{11}C_6(0.4)^6(0.6)^5$

(D) ${}^{11}C_6(0.4)^5(0.6)^6$

348. If the sum of mean and variance of a Binomial Distribution is $\frac{15}{2}$ for 10 trials, then the variance is **[2023]**

(A) 1.5 (B) 2.5

(C) 4.5 (D) 3.5

349. An experiment succeeds twice as often as it fails. Then the probability, that in the next 6 trials there will be atleast 4 successes, is **[2023]**

(A) $\frac{1}{729}$ (B) $\frac{496}{729}$

(C) $\frac{233}{729}$ (D) $\frac{491}{729}$

350. An irregular six faced die is thrown and the probability that, in 5 throws it will give 3 even numbers is twice the probability that it will give 2 even numbers. The number of times, in 6804 sets of 5 throws, you expect to give no even number is **[2023]**

(A) 18 (B) 28

(C) 27 (D) 19

351. If the sum of the mean and the variance of a Binomial distribution for 5 trials is 1.8, then the value of p is **[2023, 2021, 2020, 20219]**

(A) 0.4 (B) 0.8

(C) 0.18 (D) 0.2

352. For an initial screening of an entrance exam, a candidate is given fifty problems to solve. If the probability that the candidate can solve any problem is $\frac{4}{5}$, then the probability, that he is unable to solve less than two problems, is **[2023]**

(A) $\frac{201}{5}\left(\frac{1}{5}\right)^{49}$

(B) $\frac{316}{25}\left(\frac{4}{5}\right)^{48}$

(C) $\frac{54}{5}\left(\frac{4}{5}\right)^{49}$

(D) $\frac{164}{25}\left(\frac{1}{5}\right)^{48}$

353. A lot of 100 bulbs contains 10 defective bulbs. Five bulbs are selected at random from the lot and are sent to retail store. Then the probability that the store will receive at most one defective bulb is

[2023]

(A) $\frac{7}{5}\left(\frac{9}{10}\right)^4$ (B) $\frac{7}{5}\left(\frac{9}{10}\right)^5$

(C) $\frac{6}{5}\left(\frac{9}{10}\right)^4$ (D) $\frac{6}{5}\left(\frac{9}{10}\right)^5$

354. Let a random variable X have a Binomial distribution with mean 8 and variance 4. If $P(x \le 2) = \frac{k}{2^{16}}$, then k is equal to

[2024, 2023, 2022]

(A) 17 (B) 121

(C) 1 (D) 137

MHTCET PYQ T(2023)

Marathi

Maths

Hints & Solutions

1. (A)

Progression of $\tan A, \tan B, \tan C$

From $\cos 2B \cos(A-C) = \cos(A+C)$, expansion gives $\tan A \tan C = \tan^2 B$. Hence the three terms are in G.P.

2. (C)

Condition involving $\cos x, \cos y$

The maximum value $3/2$ occurs for the equality case $x = y = \pi/3$; hence the required relation is $x = y$.

3. (D)

Value of $\cos 2\theta$

$\tan\theta = \tan(\alpha - \pi/4)$, so $\cos 2\theta = \cos(2\alpha - \pi/2)$

$= \sin 2\alpha$.

4. (C)

Trigonometric value

$$\cos^{248^\circ} - \sin^{212^\circ} = \frac{\cos 96^\circ + \cos 24^\circ}{2}$$

$$= \frac{1}{2}\cos 36^\circ = \frac{\sqrt{5}+1}{8}.$$

5. (A)

Equation defining an angle

After rationalising the given tangent relation, the standard half-angle identity gives $\sin 4x = x$.

6. (B)

H.P. condition on three sines

The H.P. condition gives $\sin^2\theta = 2\cos^2(\alpha/2)$; therefore $\cos 2\theta = 1 - 4\cos^2(\alpha/2)$.

7. (B)

Product-to-sum identity

The expression is $1/2\left(\cos 36^\circ - \cos 144^\circ\right) = \cos 36^\circ$

$= \sin 54^\circ$. Thus options (B) and (C) are equivalent; the stated form is (B).

8. (A)

Area of a square between parallel lines

Side $= |78-39| / \sqrt{5^2 + (-12)^2} = 39/13 = 3$. Hence area $= 3^2 = 9$.

9. (D)

Circumcentre of a triangle

The perpendicular bisectors meet at $(1,-1)$.

10. (C)

Points at unit distance from a line

Using $y = 2x - 5$ and $|3x + 4y - 5|/5 = 1$ gives $(30/11, 5/11)$ and $(20/11, -15/11)$.

11. (A)

Concurrency of three lines

The determinant condition is $(k-3)(k-2)(k+5) = 0$; hence $\sum k_i = 3 + 2 - 5 = 0$.

12. (B)

Side of an equilateral triangle

Altitude $= |2(1) - 2 - 1| / \sqrt{5} = 1/\sqrt{5}$. Since $h = \sqrt{3}s/2$, $s = 2/\sqrt{15}$.

13. (A)

Minimum intercept-triangle area

For intercepts a,b, $1/a+2/b=1$. Minimising $ab/2$ gives $a=2,b=4$, so slope = $-b/a=-2$.

14. (C)

Line parallel to a median

The median PS has slope $-2/9$. The required line is $2x+9y+7=0$, with intercepts $-7/2,-7/9$.

15. (C)

Perpendicular from origin to intercept form

For $x/a+y/b=1$, $p^{-2}=a^{-2}+b^{-2}$.

16. (C)

Angle bisector at the origin

The rays have directions π and $\pi/3$; the internal bisector has slope $-\sqrt{3}$, so $\sqrt{3}x+y=0$.

17. (D)

Centre = $(1,\frac{3}{2})$. Circle touches horizontal sides (top/bottom), so radius = distance to $y=-2$ or $y=5=7/2$.

$$(x-1)^2+\left(y-\frac{3}{2}\right)^2=\left(\frac{7}{2}\right)^2$$

$$\Rightarrow x^2+y^2-2x-3y-9=0$$

18. (B)

Circle centre on the y-axis

Equidistance from $(4,0)$ and $(0,2)$ gives centre $(0,-3)$ and $r=5$. Thus $r^2-r+1=21$.

19. (B)

Maximum distance from a circle to a line

Centre $(-1,-1)$, radius $\sqrt{5}$; centre-line distance is $2\sqrt{5}$. Maximum distance = $3\sqrt{5}$.

20. (B)

External contact of two circles

External contact reduces to $a^2b^2=c\left(a^2+b^2\right)$, i.e. $1/a^2+1/b^2=1/c$.

21. (A)

Internally tangent circle

The large circle has centre $(2,3)$ and radius 5. Moving 2 units from this centre toward $(-1,-1)$ gives $(4/5,7/5)$.

22. (D)

Circle with AB as diameter

Using the sums and products of the two root pairs gives $x^2+y^2+2ax+2py-\left(b^2+q^2\right)=0$.

23. (B)

At least one student solves

$1-(1-1/2)(1-1/3)(1-1/4)=1-1/4=3/4$.

24. (A)

Three ships arrive safely

The probabilities are $2/7,3/10,6/17$; their product is $18/595$.

25. (D)

Conditional probabilities

Since $P(A\cup B)=P(A)$, $B\subset A$. Hence $P(A'|B')=5/6$ and $P(B'|A')=1$; sum = $11/6$.

26. (D)

Independent events

$1/4+p-p/4=2p-1/4$ gives $p=P(B)=2/5$

27. (A)

Perfect-square ticket

There are ten squares from 1^2 to 10^2, so probability $10/100=1/10$.

28. **(D)**
Black card or face card
$P=(26+12-6)/52=32/52=8/13$.

29. **(A)**
Prime sum on modified dice
Nine of the 36 ordered outcomes have prime sum, so $P=9/36=1/4$.

30. **(D)**
Majority of critics in favour
Adding the three exactly-two-success cases and the all-success case gives $134/343$.

31. **(B)**
Complex quotient
Interpreting the overline over the whole quotient, $\overline{Z_1/Z_2}=(2-11i)/25$, so $-7a+b=-1$. If only Z_1 is conjugated, the printed options are inconsistent.

32. **(C)**
Argument of a complex quotient
Arguments subtract: $\arg(1+i\sqrt{3})-\arg(\sqrt{3}+i)$
$=\pi/3-\pi/6=\pi/6$.

33. **(A)**
Polynomial in a complex number
$x=5/(1-2i)=1+2i$ and direct substitution gives $x^3+x^2-x+22=7$.

34. **(C)**
Range of $|z|$ in a disk
The disk has centre $2-i$ of modulus $\sqrt{5}$ and radius 2. The extreme moduli differ by $(\sqrt{5}+2)-(\sqrt{5}-2)=4$.

35. **(D)**
Powers of i
The numerator is 1 and the denominator is i; therefore the quotient is $-i$.

36. **(B)**
Conjugate complex numbers
Equating real parts gives $x=-1/3$ and opposite imaginary parts give $y=-1/3$; hence $(x-y)/(x+y)=0$.

37. **(A)**
Modulus determines a parameter
$|z|=2/\sqrt{a^2+1}=2/\sqrt{5}$ gives $a=2$. Thus $\bar{z}=(-2-4i)/5$.

38. **(D)**
Complex equation with natural n
Solving gives $z=n(-1+i)/4$. Since $\mathrm{Im}z=10$, $n=40$ and $\mathrm{Re}z=-10$.

39. **(D)**
$=\pi/3-\pi/6=\pi/6\ 4\binom{6}{4}+4\binom{4}{1}\binom{6}{3}$
$=60+320=380$.

40. **(A)**
Greatest binomial coefficients
$\binom{10}{5}/\binom{11}{5}=252/462=6/11$.

41. **(C)**
Restricted arrangement
With B_1 fixed second, G_1,G_2 can occupy $(3,4)$ or $(4,5)$ in either order; the other two can permute.
Count = $2\cdot2\cdot2=8$.

42. **(B)**
Triangles from polygon vertices
$T_{n+1}-T_n=\binom{n}{2}=21$, so $n(n-1)=42$ and $n=7$.

43. **(C)**
Words from CALCULATE
Counting repeated consonants at both ends gives $12600=5\cdot7!/2$.

44. **(C)**

Selection ratio

$\binom{n-2}{3}/\binom{n-2}{5}=2/3$ gives $(n-5)(n-6)=30$,

so $n=11$.

45. **(B)**

Diagonals of a polygon

$n(n-3)/2=54$ gives $n=12$.

46. **(C)**

Logarithmic equation

With $x=2^k$, the left side is $k(1+1/2+1/3+1/4)$

$=25k/12$. Hence $k=1/3$.

47. **(C)**

Domain of an inverse-sine function

Require $(|x|+5)/(x^2+1)\le 1$. Thus $t^2-t-4\ge 0$

for $t=|x|$, giving $|x|\ge(1+\sqrt{17})/2$.

48. **(C)**

Composition of two fractional functions

Direct substitution gives $f(g(x))=x=g(f(x))$.

49. **(D)**

Let $t=1+\sqrt{x}\Rightarrow\sqrt{x}=t-1$

$f(t)=3+2(t-1)+(t-1)^2=t^2+2$

So $f(x)=x^2+2$

$f(f(x))=(x^2+2)^2+2=x^4+4x^2+6$

50. **(C)**

Nested composition

At $x=1$: $f(1)=2$, $g(2)=1/2$, $g(1/2)=2$, and

$f(2)=5$.

51. **(A)**

Functional equation and progression

Solving the equation together with its $x\mapsto 1/x$ form gives $f(x)=6\log_2 x$. Hence 6,12,18 are in A.P.

52. **(B)**

Range of $x^2/(x^2+1)$

The value is non-negative, equals 0 at $x=0$, and approaches but never reaches 1. Hence $0\le f(x)<1$.

53. **(A)**

Inverse of a fractional linear function

Solving $y=(2x-3)/(3x-4)$ gives

$x=(4y-3)/(3y-2)$.

54. **(D)**

Domain from $2^x+2^y=2$

A real y exists iff $2-2^x>0$, i.e. $x<1$.

55. **(A)**

Elementary limit

Using $1-\cos 2x\sim 2x^2$, $\sin 5x\sim 5x$, $\sin 3x\sim 3x$, the limit is $10/3$.

56. **(C)**

Limit of radical differences

L'Hospital's rule at $x=a$ gives $2/(3\sqrt{3})$.

57. **(C)**

Degree-mode trigonometric limit

Convert degrees to radians: the quadratic coefficient is $-(49-4)\pi^2/(2\cdot 180^2)=-\pi^2/1440$

58. **(B)**

Limit with cotangents

$x\cot 4x\to 1/4$ and $\sin^2 x\cot^{22} x\to 1/4$; ratio = 1.

59. (A)

Polynomial limit

The denominator is $3\alpha + o(\alpha)$ and numerator is $-\alpha f'(1) + o(\alpha)$. Since $f'(1) = -53$, the limit is $53/3$.

60. (C)

Limit by expansion

The numerator is $3x^2/4 + o(x^2)$ and $\tan^2(x/2) = x^2/4 + o(x^2)$; limit = 3.

61. (B)

Limit at infinity

Expansion gives $\sqrt{x^2 + \sqrt{1+x^4}}$ $= x\sqrt{2} + 1/(4\sqrt{2}x^3) + o(x^{-3})$. The limit is $1/(4\sqrt{2})$.

62. (B)

Derivative-form limit

The limit is $g'(a)f(a) - g(a)f'(a) = 2\cdot 2 - (-1)\cdot 1 = 5$.

63. (D)

Two-sided limit with $|x|$

The right-hand limit is 1 and the left-hand limit is -1; hence the limit does not exist.

64. (A)

Continuity of a piecewise function

The right limit is 8 and the left limit is $k^2/2$. Thus $k^2 = 16$; the listed positive value is $k = 4$.

65. (D)

Continuity at zero

Factor the bracket as $(1-\cos(x/2))(1-\cos(x/4))$.

The limit is $4(x^2/8)(x^2/32)/x^4 = 1/64$.

66. (D)

Continuity at $x = 3$

The left limit is $a-1$, right limit $b+1$, and value $a+b$. Solving gives $a = 1, b = -1$, so $a - b = 2$.

67. (B)

Discontinuities after substitution

$t^2 + t - 2 = 0$ at $t = 1, -2$, corresponding to $x = 2, 1/2$.

68. (C)

$$f(x) = \pi[x]\cos\left(\frac{2x-1}{2}\right)$$

At integers, [x] jumps, while $\cos\left(\frac{2x-1}{2}\right) \neq 0$, so $f(x)$ is discontinuous.

At non-integers, [x] is constant $\rightarrow$ continuous.

69. (C)

Continuity constant

Using $\log(1+u) \sim u$, the two contributions are 3 and 2; hence $k = 5$.

70. (C)

This is the law of importation/exportation in logic: $p \rightarrow (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$

So, the biconditional between them is always true, regardless of truth values. Hence it is a tautology.

71. (C)

False implication

An implication is false only for true antecedent and false consequent; this occurs for $(p, q) = (T, F)$.

72. (A)

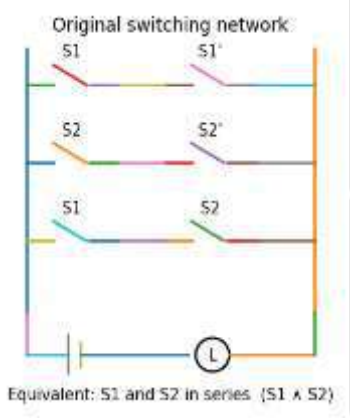
Inverse of a statement

The inverse of $p \rightarrow q$ is $\neg p \rightarrow \neg q$.

73. **(D)**
Contrapositive
The exact contrapositive is: if at least one of x, y is not odd, then xy is not odd. The intended listed choice is (D).

74. **(D)**
Relation between two statements
Statement 2 reverses hypothesis and conclusion, so it is the converse.

75. **(A)**
$Y = S_1S_2' + S_1'S_2 + S_1'S_2'$
$= S_1'(S_2 + S_2') + S_1S_2'$
$= S_1' + S_1S_2'$
Using $A + A'B = A + B$:
$= S_1' + S_2'$



76. **(D)**
Truth values
$(p \wedge q) \vee r = T$ and $(p \vee s) \leftrightarrow (q \wedge r) = F$.

77. **(A)**
Negation by De Morgan's law
$\neg[\neg s \vee (\neg r \wedge s)] = s \wedge (r \vee \neg s) = s \wedge r$.

78. **(B)**
Negation of an inverse
Simplification gives $\neg q$.

79. **(B)**
Negation of a contrapositive
The result simplifies to $(\neg p \vee q) \wedge (p \vee \neg q)$.

80. **(B)**
Tautological pattern
$(p \wedge r) \rightarrow (p \vee r)$ is a tautology for all truth values.

81. **(A)**
Negation of a biconditional
The negation is the exclusive-or: odd and not divisible by 3, or divisible by 3 and not odd.

82. **(B)**
Negation of 'if and only if'
The two statements must have different truth values.

83. **(A)**
Logical simplification
With the intended grouping, the expression reduces to $(p \wedge r) \wedge \neg q$.

84. **(D)**
Making an implication a tautology
The antecedent is $\neg p \wedge \neg q$; choosing $r \equiv \neg q$ makes the implication always true.

85. **(D)**
Equivalent circuits
Circuit A is $S_1S_2 \vee S_1S_3 = S_1(S_2 \vee S_3)$, exactly circuit C.

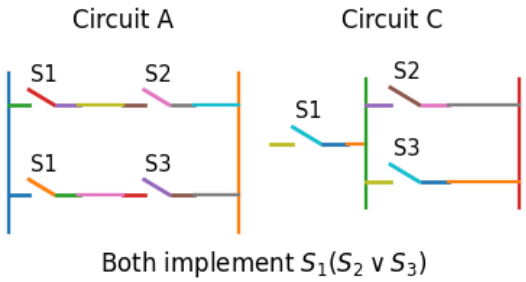


Fig: *Equivalent circuit forms*

86. **(A)**

Matrix equation $AX = B$

Solving gives $(a,b,c) = (1,2,3)$.

Therefore $2a + b + 2c = 10$.

87. **(A)**

Determinant of an adjoint

$|\text{adj}A| = |A|^2 = 16$. The determinant of the given matrix is $-4(\alpha - 5)$, so $\alpha = 1$.

88. **(D)**

Power of a complex matrix

$A^{12} = I$ and $2029 \equiv 1 (\text{mod } 12)$, so $B = A$. Since $A^{-1} = -\text{adj}A$, correct option: (D).

89. **(B)**

Cayley-Hamilton form of A^{-1}

Comparison gives $x = -1/11, y = 2/11$, hence $2x + 3y = 4/11$.

90. **(C)**

Matrix identity

Equating $A\text{adj}A$ and AA^T gives $a = b = 1$.

Therefore $2a + 3b = 5$.

91. **(D)**

Binomial expression in $\text{adj}A$

The expression is $(I - \text{adj}A)^3 = \begin{bmatrix} -1 & -3 \\ 0 & -1 \end{bmatrix}$.

Sum = -5.

92. **(B)**

Inverse of $A + B$

$A + B = \begin{bmatrix} 2 & 0 \\ 6 & -2 \end{bmatrix}$, so $(A+B)^{-1} = \begin{bmatrix} 1/2 & 0 \\ 3/2 & -1/2 \end{bmatrix}$.

93. **(B)**

Matrix trigonometric form

$A^T A^{-1} = \begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$.

94. **(B)**

Triangle sequences

By the sine rule, the sides are in A.P. Altitudes are proportional to reciprocals of the sides, hence are in H.P.

95. **(A)**

Inverse-tangent radical expression

Using $\tan(\pi/4 + u/2) = (1 + \sin u)/\cos u$ with $\cos u = x^2$, the value is $\pi/4 + 1/2\cos^{-1}x^2$.

96. **(A)**

Triangle sides from pair sums

The equations give $(a,b,c) = (4k, 3k, 5k)$. Area = $6k^2$, so $A^2/k^4 = 36$.

97. **(A)**

Roots $\tan(A/2), \tan(B/2)$

Since $A + B = \pi/2$, $(r_1 + r_2)/(1 - r_1 r_2) = 1$. With Vieta, this gives $a_1 + b_1 = c_1$.

98. **(B)**

Determinant in triangle angles

The determinant simplifies to $1 + \cos 2B = 2\cos^2 B$. Thus $B = (2n+1)\pi/2$.

99. **(D)**

Sum of three inverse tangents

Taking tangent of the sum π gives $a + b + c - abc = 0$.

100. **(B)**

Triangle identity

The expression simplifies to $a^2 + b^2 - 2ab\cos C = c^2 = 16$.

101. **(B)**

General trigonometric solution

The equation is $\tan(3x - \pi/4) = \sqrt{3}$. Hence $x = n\pi/3 + 7\pi/36$ and $p/q = 1/12$.

102. (A)

Inverse-cosine equation

The first two inverse cosines sum to $\pi/2$. Thus $\cos^{-1}\sqrt{1-q}=\pi/4$, giving $q=1/2$.

103. (C)

Sum of inverse sines

The first two angles have cosine $16/65$, so their sum plus $\sin^{-1}(16/65)$ is $\pi/2$. Adding π gives $3\pi/2$.

104. (B)

Nested inverse functions

Stepwise simplification yields $x=\sqrt{3-a^2}$; hence $x^2+a^2=3$.

105. (C)

Quadratic in $\cos\theta$

The valid root is $\cos\theta=-1/2$, so $\theta=2\pi/3,4\pi/3$

106. (A)

General solution of a trigonometric equation

The equation reduces to $\sin\theta=1/2$.

Thus $\theta=n\pi+(-1)^n\pi/6$, so $A+B=7$.

107. (B)

Circumradius from side relations

The condition gives $c^2=a^2+b^2+ab$, hence $\cos C=-1/2$ and $C=120^\circ$.

Therefore $R=c/(2\sin C)=c/\sqrt{3}$.

108. (A)

$\sin\left(\cot^{-1}x\right)$

For the principal angle, a right triangle gives $\sin\left(\cot^{-1}x\right)=1/\sqrt{1+x^2}$.

109. (C)

Equation of inverse trigonometric values

Equating $1/\sqrt{1+x^2}$ and $1/\sqrt{1+(1+x)^2}$ gives $x=-1/2$.

110. (B)

Consecutive triangle sides

Let the sides be $n,n+1,n+2$. The condition $C=2A$ gives $n=4$, hence $4,5,6$.

111. (A)

Ratio of extreme sides

By the sine rule, the ratio is $\sin45^\circ/\sin75^\circ=\sqrt{3}-1$.

112. (C)

Point dividing a triangle side

Using $BD/DC=(AB\sin BAD)/(AC\sin CAD)$ and the sine rule gives $\sin BAD/\sin CAD=1/\sqrt{6}$.

113. (C)

Inverse-tangent equation

$\tan^{-1}\left[(1-x)/(1+x)\right]=\pi/4-\tan^{-1}x$.

Hence $\tan^{-1}x=\pi/6$ and $x=1/\sqrt{3}$.

114. (D)

Area from a cosine-side condition

The condition forces $A=B=C=60^\circ$, so the triangle is equilateral with side $1/\sqrt{6}$. Area = $1/\left(8\sqrt{3}\right)$.

115. (B)

Repeated product-to-sum identity

The value is $\cos36^\circ=\sin54^\circ$.

116. (C)

Equation involving secant and cosecant

$3\sin\theta=2\cos^2\theta$ gives $\sin\theta=1/2$. General solution $n\pi+(-1)^n\pi/6$.

117. (B)

Tangent of a sum

$\tan\left(\sin^{-1}3/5\right)=3/4$. Therefore the required tangent is $\left(3/4+2/3\right)/\left(1-1/2\right)=17/6$.

118. (B)

Squares of inverse tangent and cotangent

Using $\cot^{-1}x=\pi/2-\tan^{-1}x$ gives $\tan^{-1}x=-\pi/4$ or $3\pi/4$; in either case $x=-1$.

119. (B)

Sum of cosines of triangle angles

The side ratios are $a:b:c=7:6:5$. Applying the cosine rule to all three angles gives $\cos A+\cos B+\cos C=51/35$.

120. (A)

Principal inverse-sine value

$\sin\left(3\pi/4\right)=\sqrt{2}/2$ and the principal inverse sine is $\pi/4$.

121. (A)

Repeated angle equation

The same half-angle manipulation as Q5 gives $\sin 4x=x$.

122. (B)

Solutions in $\left(0,\pi/2\right)$

$2\sin 3x\cos 2x=\sin 3x$ gives $x=\pi/6$ or $\pi/3$.

123. (B)

Repeated H.P. sine condition

The result is $\cos 2\theta=1-4\cos^2\left(\alpha/2\right)$.

124. (A)

Principal solutions

After multiplying by $\cos x$, $2\sin^2 x+\sin x-1=0$.

The valid nonsingular solutions are $\pi/6,5\pi/6$.

125. (A)

Sum of three inverse cosines

Each term is at most π; a total 3π forces $x=y=z=-1$. The power sum is -1.

126. (D)

Triangle angles in A.P.

The middle angle is $B=60^\circ$. By the sine rule, the expression becomes $2\sin\left(A+C\right)=2\sin 120^\circ=\sqrt{3}$.

127. (C)

Telescoping inverse-tangent sum

$\tan^{-1}\left[1/\left(2r^2\right)\right]=\tan^{-1}\left[1/\left(2r-1\right)\right]-\tan^{-1}\left[1/\left(2r+1\right)\right]$.

The sum gives $\tan p=50/51$.

128. (C)

Cosine-difference identity

Set $x=\cos u$ and $y/3=\cos v$. The expression becomes $9\sin^2\left(u-v\right)=9\sin^2\alpha$.

129. (A)

Triangle angles in A.P. with two sides

The middle angle is 60°. With sides 10 and 9, the listed admissible third side is $5+\sqrt{6}$.

130. (A)

Inverse-sine expression

Let $x=\sin\theta$, $0\le\theta\le\pi/6$. The inner sum is $\sin\left(\theta+\pi/4\right)$, so the tangent is $\tan\left(\pi/4\right)=1$.

131. (B)

Longest side to perimeter

The angles are $120^\circ,30^\circ,30^\circ$. The required ratio is $\sqrt{3}:\left(2+\sqrt{3}\right)$.

132. (A)

A.P. before and after inverse tangent

The tangent addition formula yields $y^2 = xz$. Together with $x + z = 2y$ and positivity, this gives $x = y = z$.

133. (B)

Triangle sine identity

$(A - B + C)/2 = \pi/2 - B$, so the expression is $2ac\cos B = a^2 + c^2 - b^2$.

134. (A)

Pair of lines and reciprocal slopes

The product of slopes is $a/2 = 1$, hence $a = 2$. Their sum is $-5/2$, so $m_1^2 + m_2^2 = 17/4$.

135. (A)

Pair of lines at 45°

The slopes are 5 and $-1/5$. Their joint equation is $5x^2 + 24xy - 5y^2 = 0$.

136. (B)

Perpendicular pair through a point

The perpendicular slopes are 1 and $-3/5$. Translation to $(3,-2)$ gives $3x^2 + 2xy - 5y^2 - 14x - 26y - 5 = 0$.

137. (B)

Sides of a right isosceles triangle

The two side slopes are 3 and $-1/3$. Through $(2,1)$ their pair is $3x^2 - 3y^2 + 8xy - 20x - 10y + 25 = 0$.

138. (C)

Perpendicular pair condition

The equation factors as $x(x\cos 2\alpha + y\sin 2\alpha) = 0$. Perpendicularity requires $\cos 2\alpha = 0$, hence $\alpha = \pi/4$.

139. (B)

Angle between a pair of lines

$\tan\theta = 2\sqrt{9/4 - \lambda}\,/\,|1 + \lambda| = 1/3$. With $\lambda \geq 0$, $\lambda = 2$.

140. (C)

Joint equation from perpendicular distances

The lines are $x + y = \sqrt{2}$ and $-x + y = \sqrt{2}$. Their product gives $x^2 - y^2 + 2\sqrt{2}y - 2 = 0$.

141. (C)

Centroid of a triangle formed by lines

$6x^2 + xy - y^2 = (3x - y)(2x + y)$. The vertices are $(0,0),(1,3),(-2,4)$, so centroid = $(-1/3, 7/3)$.

142. (B)

Projection on a plane normal

A normal to the plane is $(1,-2,1)$. The scalar projection of $(2,1,1)$ is $|1|/\sqrt{6} = 1/\sqrt{6}$.

143. (A)

Unit vector along a diagonal

The diagonal vector is $(3,-6,2)$ with magnitude 7, giving $(3\hat{i} - 6\hat{j} + 2\hat{k})/7$.

144. (B)

Vector medians

Substitution of midpoint vectors gives $\overrightarrow{AD} + \frac{2}{3}\overrightarrow{BE} + \frac{1}{3}\overrightarrow{CF} = \frac{1}{2}\overrightarrow{AC}$.

145. (C)

Vector triple products

From $(a \times b) \times c = -5a + 4b$, obtain $a \cdot c = 4, b \cdot c = 5$. Hence $a \times (b \times c) = 4b - 3c$.

146. (D)

Coplanar perpendicular vector

$p \cdot q = 0$ and $|p| = \sqrt{3}$. Therefore a vector of magnitude $5\sqrt{3}$ is $5p = 5(\hat{i} + \hat{j} + \hat{k})$.

147. (A)

Linear combination of vectors

Solving $m+2n=4$ and $-2m+3n=13$ gives $m=-2, n=3$, so $m+n=1$.

148. (C)

Volume of a tetrahedron

The scalar triple product is $22(\lambda-4)$. Thus $|22(\lambda-4)|/6=11$, giving the listed value $\lambda=7$.

149. (B)

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 4 & 2\\ 3 & -2 & 7\end{vmatrix}$$

$=\hat{i}(4\cdot 7-2\cdot(-2))-\hat{j}(1\cdot 7-2\cdot 3)+\hat{k}(1\cdot(-2)-4\cdot 3)$

$=\hat{i}(28+4)-\hat{j}(7-6)+\hat{k}(-2-12)=32\hat{i}-\hat{j}-14\hat{k}$

2. $\vec{d}$ is parallel to $\vec{a}\times\vec{b}$, so:

$\vec{d}=\lambda(32\hat{i}-\hat{j}-14\hat{k})$

3. Given $\vec{c}\cdot\vec{d}=15$, where $\vec{c}=2\hat{i}-\hat{j}+4\hat{k}$:

$(2\hat{i}-\hat{j}+4\hat{k})\cdot\lambda(32\hat{i}-\hat{j}-14\hat{k})=15$

$\lambda[2(32)+(-1)(-1)+4(-14)]=15$

$\lambda[64+1-56]=15\Rightarrow\lambda(9)=15\Rightarrow\lambda=\frac{15}{9}=\frac{5}{3}$

150. (A)

Unit vector perpendicular to two vectors

$(a+b)\times(a-b)=(-14,4,10)$; divide by $\sqrt{312}$.

151. (D)

Dot product from vector conditions

$|a\times b|=3$ and the angle condition gives $|c|=2$.

From $|c-a|=4$, $a\cdot c=-3/2$.

152. (D)

Parameter from perpendicularity

$(a+\lambda b)\cdot c=3+5\lambda=0$, so $\lambda=-3/5$.

153. (A)

Linearly dependent vectors

$c=a=(1,1,1)$ is in the span and has magnitude $\sqrt{3}$, so $\alpha=1,\beta=1$.

154. (A)

Magnitude condition in vectors

The relations lead to $\lambda^2=4$ for the listed positive value, so $\lambda=2$.

155. (A)

Centroid of a tetrahedron

Average the four coordinates to obtain $(3/4,-1/4,11/4)$.

156. (C)

Area of a triangle in space

$|(B-A)\times(C-A)|^2=4x^2-8x+12$. Setting area = $\sqrt{6}$ gives $x=-1$ or 3; the listed value is 3.

157. (D)

Special point of a triangle

The two dot products state $AD\perp BC$ and $BD\perp AC$. Thus D is the orthocentre.

158. (C)

Orthogonal coplanar unit vector

A normal to the span is $(0,-1,1)$; crossing it with $(3,2,6)$ gives $(-8,3,3)$. Unit vector = $(-8,3,3)/\sqrt{82}$.

159. (C)

Tetrahedron volume

The determinant is $7x-17$. Hence $|7x-17|/6=11/6$, giving the listed value $x=4$.

160. (C)

Scalar product with a unit vector

For the printed vectors the dot product equals $\lambda/\sqrt{(\lambda+2)^2+8}$, which cannot equal +1. The listed value $\lambda=-3$ gives -1; thus (C) is the intended answer if the sign is negative.

161. (D)

Scalar projection

$b\cdot a=3$ and $|a|=\sqrt{29}$, so the projection is $3/\sqrt{29}$.

162. (D)

Midpoint from a centroid

$M_{BC}=(3G-A)/2=(3/2,-2,-2)$.

163. (C)

Components after rotation

Rotation preserves magnitude: $1+4p^2=(p+1)^2+1$. Thus $p=1$ or $-1/3$.

164. (A)

Vector identity

$(a-b)\times(a+b)=2a\times b$. Therefore the expression is $4\left(|a\times b|^2+(a\cdot b)^2\right)=4|a|^2|b|^2=576$.

165. (B)

Solving for a vector

The equations $r\times a=b$ and $r\cdot c=3$ give $r=(5,7,9)$. Hence $|r|=\sqrt{155}$.

166. (D)

Angle between two faces

The normals are $(-1,-7,3)$ and $(1,-5,-3)$. Thus $\cos\theta=25/\left(\sqrt{59}\sqrt{35}\right)$.

167. (C)

Right angle in 3D

$(B-A)\cdot(C-A)=0$ gives $x^2-10x+21=0$. The listed value is $x=3$.

168. (B)

Cross product of unit vectors

From $|a+2c|^2=4$, $a\cdot c=-1/4$.

Therefore $|a\times c|=\sqrt{1-1/16}=\sqrt{15}/4$.

169. (C)

Double cross-product magnitude

$|a\times(a\times c)|=|a|^2|c|\sin\theta=6\sin\theta=3$. Thus $\sin\theta=1/2$ and $\sec^2\theta=4/3$.

170. (A)

Magnitude of a vector sum

$|a+b+c|^2=4+9+25+6+15+10=69$. Hence the magnitude is $\sqrt{69}$.

171. (D)

Vector equation

$b\times(c-a)=0$ gives $c=a+\lambda b$. From $c\cdot a=0$, $\lambda=-3/2$, and $c\cdot b=-1/2$.

172. (C)

Incentre in 3D

The opposite side lengths are $2,3,3$. The weighted average is $(-3/2,1/2,1)$.

173. (D)

Scalar and vector triple products

Direct application of $(u\times v)\times w$ gives the value -5.

174. (D)

Angle between two vectors

$b\cdot c=-48$ and $|c|=8\sqrt{3}$. Hence $\cos\theta=-\sqrt{3}/2.$, so $\theta=5\pi/6$.

175. (B)

Rotation to a perpendicular side

$\cos\alpha=|AB\times AD|/(|AB||AD|)=\sqrt{17}/9$.

176. (C)

Magnitude using projections

Equal projections imply $u\cdot v=u\cdot w$, while $v\cdot w=0$. Hence $|u-v+w|^2=1+4+9=14$.

177. (A)

Direction of a line of plane intersection

The cross product of the plane normals is proportional to $(1,-1,1)$. Thus $\cos\alpha=1/\sqrt{3}$ and $\sec\alpha=\sqrt{3}$.

178. (A)

Line-plane intersection

Substitution gives parameter $t=1$, hence point $(3,-1,-1)$. (A)

179. (C)

Plane through an intersection line

Using $3(2x-y-4)-(y+2z-4)=0$ gives $3x-2y-z=4$.

180. (C)

Plane parallel to the y-axis

The family parameter is $-1/3$, giving $x+4z-7=0$. Of the listed points, $(3,2,1)$ lies on it.

181. (A)

Intersection and reflection

The lines meet at $(2,-4,7)$. Reflection in the xy-plane is $(2,-4,-7)$.

182. (B)

Distance of a line-plane intersection

The intersection parameter is $1/2$, so $P=(2,1/2,4)$ and $OP=9/2$.

183. (C)

Line perpendicular to two lines

The cross product of directions is proportional to $(-4,-7,-13)$.

184. (D)

* $\vec{n}_1=-18\hat{i}$
* $\vec{n}_2=3\hat{i}-3\hat{j}-3\hat{k}$
* $\vec{A}=\vec{n}_1\times\vec{n}_2=-54(\hat{j}-\hat{k})\rightarrow$ parallel to $\hat{k}-\hat{j}$

Angle with $\vec{B}=2\hat{i}+\hat{j}-2\hat{k}$:

$$\cos\theta=\frac{(\hat{k}-\hat{j})\cdot(2\hat{i}+\hat{j}-2\hat{k})}{|\hat{k}-\hat{j}|\cdot|2\hat{i}+\hat{j}-2\hat{k}|}=\frac{-3}{\sqrt{2}\cdot 3}=-\frac{1}{\sqrt{2}}$$

$$\Rightarrow\theta=\frac{3\pi}{4}$$

185. (B)

Volume cut by a plane

The plane is $x+y+z=3$, so the intercepts are $3,3,3$. Volume = $27/6=9/2$.

186. (B)

Plane with equal acute direction angles

The normal is proportional to $(1,1,1)$. Through $(-1,1,2)$ the plane is $x+y+z-2=0$.

187. (B)

Distance from a point to a line

$d=|(P-A)\times b|/|b|=\sqrt{259/70}=\sqrt{37/10}$.

188. (A)

Perpendicular lines with parameter

The direction dot product is $77p-490$. Setting it to zero gives $p=70/11$.

189. (A)

Line parallel to two planes

The direction is the cross product $(1,-1,2)\times(3,1,1)=(-3,5,4)$.

190. (A)
Shortest distance between skew lines
$d = |(P_2 - P_1) \cdot (d_1 \times d_2)| / |d_1 \times d_2| = 8 / (3\sqrt{5})$.

191. (A)
Parallel line meeting a plane
The parameter is $-5/12$, giving $B = (1/6, 4/3, 19/12)$.

192. (B)
Intersection of vector lines
Equating components gives the point $(2,1,1)$, with position vector $2\hat{i} + \hat{j} + \hat{k}$.

193. (C)
Acute angle between two lines
The direction vectors are $(-1,0,2)$ and $(3,4,5)$.
Hence $\cos\theta = 7/(5\sqrt{10})$.

194. (A)
Shortest-distance parameter
The distance equation gives $\lambda = 5$ or 11, so their sum is 16.

195. (B)
Common perpendicular unit vector
$(3,1,2) \times (1,2,3) = (-1,-7,5)$, of magnitude $5\sqrt{3}$.

196. (D)
Mirror image in a plane
Using the reflection formula gives $(a,b,c) = (4,2,3)$, so $a + b + c = 9$.

197. (A)
Direction of plane intersection
$(3,-1,1) \times (1,4,-2) = (-2,7,13)$.

198. (C)
Plane containing a line and perpendicular to another plane
The required normal is proportional to $(1,-2,1)$, giving $x - 2y + z = 0$.

199. (C)
Equal positive direction cosines
The unit direction is $(1,1,1)/\sqrt{3}$. Substitution in the plane gives the travel parameter and length $PQ = \sqrt{3}$.

200. (A)
Projection of a segment on a plane
For $v = (-1,0,-1)$ and unit normal $(1,1,1)/\sqrt{3}$, the projected length is $\sqrt{2 - 4/3} = \sqrt{2/3}$.

201. (B)
Intersecting lines and parameter
Solving the x, z equations gives parameters $-3/2, -5$; the y equation then gives $k = 9/2$.

202. (C)
Nature of two lines
The component equations are inconsistent, so the lines do not intersect.

203. (D)
Graphical solution of inequations
The half-planes form the triangle with vertices $(2,0)$, $(50/13, 12/13)$ and $(10/17, 60/17)$, matching Fig. 3.

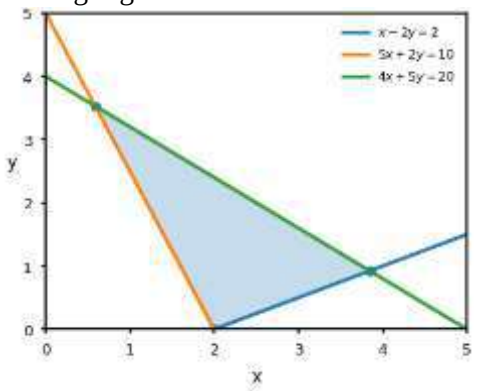


Fig: Feasible region Q 203

204. (A)

Minimum objective value from a graph

At the lower-left feasible vertex $(3/2, 3)$, $3x+5y=19.5$, the minimum.

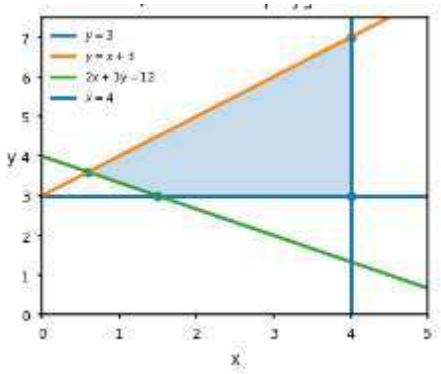


Fig: Feasible polygon for Q 204

205. (B)

Linear-programming maximum

Checking corner points gives the maximum 36 at $(2,6)$.

206. (C)

Maximum on region OCDBO

At O, C, D, B, the objective values are $0, 70, 110, 100$. Maximum = 110.

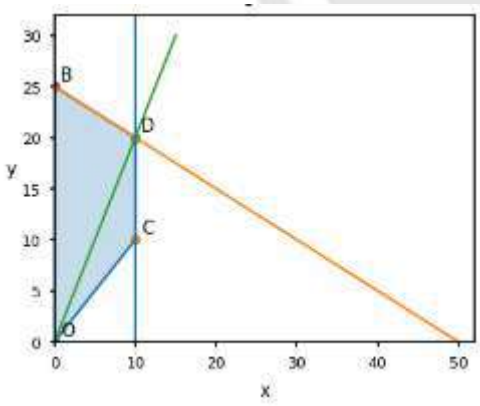


Fig: Region OCDBO

207. (B)

Constrained maximum

The feasible corners include $(0,20)$, where $7x+8y=160$, the maximum.

208. (C)

Maximum from a shaded polygon

Corner values show the maximum $10(2)+25(3)=95$ at $(2,3)$.

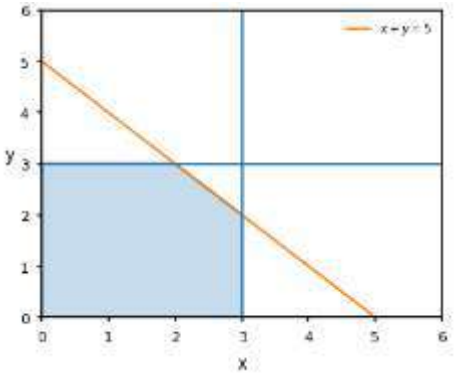


Fig: Feasible region for Q208

209. (A)

Region satisfying three inequalities

The region lies right of $x=3$, above $y=2x$, and below $4x+3y=60$; it is S_2.

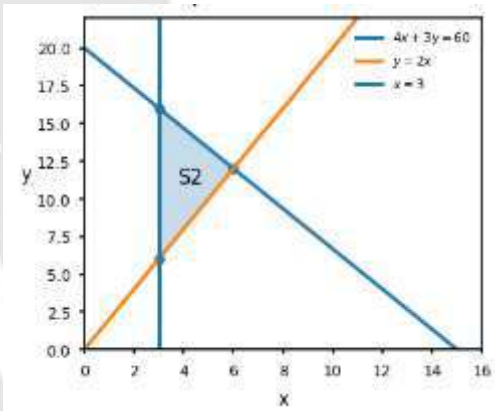


Fig: Region S_2

210. (D)

Constraints represented by a shaded region. The shaded region is above $2x+3y=6$, below $x+2y=6$, and satisfies $x-3y\le 3$, $-x+2y\le 2$.

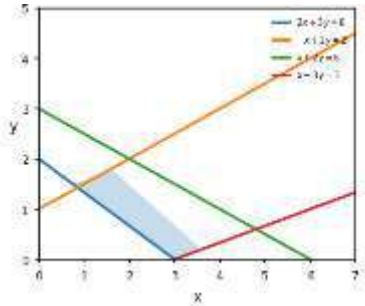


Fig: Shaded feasible region

211. (A)

Implicit differentiation

At $x=0$, the equation gives $y=1$. Differentiating yields $(1+y')/(x+y)=2y+2xy'$, hence $y'(0)=1$.

212. (B)

Parameter in a differentiated power relation

Differentiation gives $y'=-(x/y)^{k-1}$. Comparison with the given relation yields $k-1=-1/3$, so $k=2/3$.

213. (B)

Derivative of an inverse function

$g'(x)=1/f'(g(x))=1+(g(x))^3$.

214. (A)

Differentiability of an absolute-value product

The potentially troublesome points 0 and π have quadratic zero factors, so the function is differentiable there. Thus $S=$.

215. (C)

Repeated implicit derivative

As in Q211, $dy/dx=1$ at $x=0$.

216. (B)

Second derivative for parametric curves

$dy/dx=\sin t$ and $d^2y/dx^2=\cos^3 t/3$. At $t=\pi/4$, this is $1/(6\sqrt{2})$.

217. (A)

Derivative of a sum of inverse tangents

The first term is $\pi/4-\tan^{-1}(2\log x)$ and the second is $\tan^{-1}4+\tan^{-1}(2\log x)$. Their sum is constant, so $dy/dx=0$.

218. (A)

Derivative values of exponentials

$f'(0)=\log 3$ and $g'(0)=\log 4$, so the value is $\log(3/4)/[1+(\log 3)(\log 4)]$.

219. (A)

Differentiability of $x/(1+|x|)$

The two piecewise derivatives agree at $x=0$ and the function is smooth elsewhere. The derivative exists for every real x.

220. (A)

Derivative of a product power

Log differentiation gives $y'(0)=\frac{3}{2}\sum_{k=1}^{n}k$

$=3n(n+1)/4$.

221. (A)

Derivative with respect to another function

$$\frac{df(\tan x)}{dg(\sec x)}=\frac{f'(1)\sec^2 x}{g'(\sqrt{2})\sec x\tan x}=1/\sqrt{2}.$$

222. (A)

Repeated implicit derivative

The same computation gives $dy/dx=1$.

223. (C)

Derivative of a finite product

At $x=0$, only the derivative of the factor $(1+x)$ contributes, giving 1.

224. (D)

Let $t=\log x$ at $x=e, t=1$

$$f(x)=\sin^{-1}\left(\frac{2t}{1+t^2}\right)=2\tan^{-1}t \quad \text{(since } t=1 \text{ gives principa}$$

So,

$f(x)=2\tan^{-1}(\log x)$

Differentiate:

$$f'(x)=\frac{2}{1+(\log x)^2}\cdot\frac{1}{x}$$

At $x=e$:

$$f'(e)=\frac{2}{1+1}\cdot\frac{1}{e}=\frac{1}{e}$$

225. (B)

$$\frac{x\log_e(2x)-\log_e 2}{x}$$

$(2x)^{2y}=4e^{2x-2y}$

Take in

$2y\ln(2x)=2\ln 2+2x-2y$

Divide by 2 : $y\ln(2x)=\ln 2+x-y$

$y(1+\ln(2x))=x+\ln 2$

Differentiate: $y'(1+\ln(2x))+\frac{y}{x}=1$

$(1+\ln(2x))^2 y'=(1+\ln(2x))\left(1-\frac{y}{x}\right)$

Now $y=\frac{x+\ln 2}{1+\ln(2x)}$

So, $1-\frac{y}{x}=\frac{x\ln(2x)-\ln 2}{x(1+\ln(2x))}$

Thus, $(1+\ln(2x))^2 y'=\frac{x\ln(2x)-\ln 2}{x}$

226. (C)

Derivative of an inverse tangent

$dy/dx=\sin\alpha/\left(x^2-2x\cos\alpha+1\right)$.

Thus $m=\sin\alpha, n=-\cos\alpha$, and $m^2+n^2=1$.

227. (B)

Let $u=\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$

$v=\cos^{-1}\left(\frac{1+\sqrt{1+x^2}}{2\sqrt{1+x^2}}\right)$

Put $x=\tan\theta$. Then $\sqrt{1+x^2}=\sec\theta$.

For u :

$$\frac{\sec\theta-1}{\tan\theta}=\frac{\frac{1}{\cos\theta}-1}{\frac{\sin\theta}{\cos\theta}}=\frac{1-\cos\theta}{\sin\theta}=\tan\frac{\theta}{2}$$

So $u=\tan^{-1}(\tan\frac{\theta}{2})=\frac{\theta}{2}$.

For v :

$$\frac{1+\sec\theta}{2\sec\theta}=\frac{\cos\theta+1}{2}=\cos^2\frac{\theta}{2}$$

So $v=\cos^{-1}(\cos^2\frac{\theta}{2})$.

For small $\theta, v=\frac{\theta}{2}$ (since $\cos^{-1}(\cos^2\alpha)=\alpha$ for small α).

Thus $u=v$, so $\frac{du}{dv}=1$.

228. (C)

$$y=\frac{(1+3x)^{1/3}(1+4x)^{1/4}(1+5x)^{1/5}}{(1+7x)^{1/7}(1+8x)^{1/8}}$$

Take log:

$$\ln y=\frac{1}{3}\ln(1+3x)+\frac{1}{4}\ln(1+4x)+\frac{1}{5}\ln(1+5x)-\frac{1}{7}\ln(1+7x)-$$

Differentiate:

$$\frac{y'}{y}=\frac{1}{3}\cdot\frac{3}{1+3x}+\frac{1}{4}\cdot\frac{4}{1+4x}$$

$$+\frac{1}{5}\cdot\frac{5}{1+5x}-\frac{1}{7}\cdot\frac{7}{1+7x}-\frac{1}{8}\cdot\frac{8}{1+8x}$$

At $x=0$:

$\frac{y'}{y}=1+1+1-1-1-1=1$

Also $y(0)=1$, so $y'(0)=1$.

229. (B)

Let $a=f'(1), b=f''(2)$.

$f(x)=x^3+ax^2+bx+6$

$f'(x)=3x^2+2ax+b, \quad f''(x)=6x+2a$

At $x=1$:

$3+2a+b=a\Rightarrow a+b+3=0 \quad (1)$

At $x=2$:

$12+2a=b \qquad (2)$

From (1) and (2)

$a+(12+2a)+3=0\Rightarrow 3a+15=0\Rightarrow a=-5, b=2$

$f(2)=8-20+4+6=-2$

230. (B)

Given:

$$x=\ln\left(\frac{1-\tan(y/2)}{1+\tan(y/2)}\right)$$

With $\tan(y/2)=\sqrt{\frac{1-t}{1+t}}$, we get:

$$\frac{1-\tan(y/2)}{1+\tan(y/2)}=\frac{\sqrt{1+t}-\sqrt{1-t}}{\sqrt{1+t}+\sqrt{1-t}}=\tan\left(\frac{1}{2}\cos^{-1}t\right)$$

So:

$$x=\ln\left(\tan\left(\frac{1}{2}\cos^{-1}t\right)\right)$$

But also from $\tan(y/2)=\sqrt{\frac{1-t}{1+t}}$, we get:

$$y=\cos^{-1}t$$

Now:

$$\frac{dx}{dt}=\frac{1}{t\sqrt{1-t^2}},\quad \frac{dy}{dt}=-\frac{1}{\sqrt{1-t^2}}$$

Thus:

$$\frac{dy}{dx}=\frac{dy/dt}{dx/dt}=-t$$

At $t=\frac{1}{2}$:

$$\frac{dy}{dx}=-\frac{1}{2}$$

231. (B)

Second-order expression

Writing $A=\sin^{-1}x$, differentiation shows $\left(1-x^2\right)y''-xy'=4$.

232. (B)

Derivative of a powered product

Log differentiation gives

$$y'(0)=n\sum_{k=1}^{n}k=n^2(n+1)/2.$$

233. (C)

Second derivative of parametric exponentials

Since $xy=e^{\pi/4}$, $y=C/x$. Therefore $d^2y/dx^2=2y/x^2$.

234. (C)

Chain rule

At $x=1$, the inner argument is 5 and its derivative is 12. Hence $dy/dx=12\sin(\log 5)$.

235. (A)

Infinite nested radical

The equation is $y^2=x-\sin x+y$. Differentiating gives $y'=(1-\cos x)/(2y-1)$.

236. (A)

Repeated implicit derivative

Again $y'(0)=1$.

237. (D)

Derivative of repeated composition

$d\left[f\left(f\left(f(x)\right)\right)\right]/dx=3^3=27$ at $x=1$, and $d\left[f(x)^2\right]/dx=6$. Total 33.

238. (C)

Derivative of an inverse tangent at zero

The argument is $8x+o(x)$, so the derivative at $x=0$ is 8.

239. (A)

Tangent slope with parameters

The point condition gives $a+b=-1$; the slope condition gives $a=1, b=-2$. Hence $a-b=3$.

240. (C)

Number of real roots

For $F(x)=x^3+x-1$, $F'(x)=3x^2+1>0$ and F changes sign from $-\infty$ to $+\infty$. Exactly one real root.

241. (A)

Increasing intervals

$f'(x)=3(x+6)(x-2)>0$ for $x<-6$ or $x>2$.

242. (A)

Related rates in a cone

$r=h/2$ and $V=\pi h^3/12$. Thus $5=\left(\pi h^2/4\right)h'$, and at $h=10$, $h'=1/(5\pi)$.

243. (C)

Derivative bound

By the mean value theorem, $f(2)-f(0)\le 5(2)$, so $f(2)\le 7$.

244. (C)

Lagrange mean value theorem

$1/c=[\log e-\log 1]/(e-1)=1/(e-1)$,

Hence $c=e-1$.

245. (A)

$h(x)=e^{x^2-x}-(x^2-x)$

$h'(x)=(2x-1)(e^{x^2-x}-1)$

For increasing, $h'(x)\ge 0$:

* $2x-1\ge 0\Rightarrow x\ge\frac{1}{2}$

* $e^{x^2-x}-1\ge 0\Rightarrow x^2-x\ge 0\Rightarrow x\le 0$ or $x\ge 1$

Common: $x\ge 1$

OR both negative:

* $x\le\frac{1}{2}$ and $0\le x\le 1$

Common: $0\le x\le\frac{1}{2}$

So: $[0,\frac{1}{2}]\cup[1,\infty)$

246. (D)

Acceleration when velocity is zero

$v=-48/t^2+3t^2=0$ gives $t=2$.

Then $a=96/t^3+6t=24$.

247. (B)

Spherical balloon rates

$8\pi rr'=2$, so at $r=6$, $r'=1/(24\pi)$.

Then $V'=4\pi r^2r'=6$.

248. (C)

Minimum parallelepiped volume

The determinant is $\alpha^3-\alpha+1$; its stationary minimum occurs at $\alpha=1/\sqrt{3}$.

249. (C)

Minimum of a rational function

Differentiation gives critical points $x=\pm 1$; values are 3 and $1/3$. Minimum = $1/3$.

250. (C)

Monotonicity determines a parameter

The change at $x=1$ gives $a=5$. Then $f(x)-14=(x-7)(x-1)^2$, and with $x\ne 1$ the root is 7.

251. (D)

Minimum of $a\sec\theta-b\tan\theta$

The standard minimum is $\sqrt{a^2-b^2}$ for $a>b>0$.

252. (C)

Sliding rod

$x^2+y^2=169$. At $x=5, y=12$, $y'=-(x/y)x'$

$=-(5/12)3=-5/4$ ft/s, downward.

253. (D)

Tangent to $y=\sqrt{9-2x^2}$

At $x=y=\sqrt{3}$, slope = -2.

The tangent is $2x+y-3\sqrt{3}=0$.

254. (C)

Small-angle approximation

Using $\sin(A+\delta)\approx\sin A+\delta\cos A$ with $\delta=(10/3600)(0.0175)$ gives 0.8660243.

255. (A)

Angle between two tangents

Using the printed derivatives at $(1,1)$ gives slopes 4 and $1/4$, hence $\tan\theta = 15/8$. Correct option: (A).

256. (B)

Extreme points determine constants

With $\log|x|$, the equations give $\alpha = 2, \beta = -1/2$. Thus $\alpha^2 + 2\beta = 3$.

257. (D)

Sliding ladder angle

$y = 5\sin\theta$; at $x = 4$, $\cos\theta = 4/5$. Hence $-0.1 = 4\theta'$ so $\theta' = -0.025$.

258. (C)

Normal parallel to a line

The normal slope is $-y/(3x) = -1/3$, giving $y = x$ and points $(\pm 2, \pm 2)$. The listed normal is $x + 3y + 8 = 0$.

259. (B)

Least-cost open tank

Base area $xy = 9$ and cost = $900 + 400(x + y)$. Minimum $x + y = 6$ occurs at $x = y = 3$, giving ₹3300.

260. (C)

Normal slope for a parametric curve

At $t = 4$, $dx/dt = 1/4$ and $dy/dt = 17/16$. Tangent slope = $17/4$; normal slope = $-4/17$.

261. (C)

Rolle points

$f'(x) = \cos x - \sin x = 0$ gives $x = \pi/4, 5\pi/4$.

262. (B)

Tangents parallel to a chord

The chord slope is 2. Since $y' = 1 + 4/x^2$, $x = \pm 2$, giving $(2,0), (-2,0)$.

263. (B)

Minimum tangent slope

$y = e^x \cos x$, so the tangent slope is $e^x(\cos x - \sin x)$. Its minimum on $[0, 2\pi]$ occurs at $x = \pi$.

264. (B)

Second expression in the tangent parameters

From Q240, $a = 1, b = -2$, so $3a + b = 1$.

265. (C)

Distance of a normal from the origin

The tangent unit direction is $(\cos t, \sin t)$. The point vector has constant projection 2 on it, so the normal is at distance 2.

266. (A)

Rate of change of triangle area

The signed double area is $11x - 12$. Since the point speed on $y = 2x$ gives $x' = 2/\sqrt{5}$, area increases at $11/\sqrt{5} = 11\sqrt{5}/5$.

267. (A)

Local maximum

$f'(x) = 3(x-1)(x-3)$ changes from positive to negative at $x = 1$.

268. (C)

Tangent to an implicit curve

At $(2,3)$, $24 = 12p$, so $p = 2$. Then $9 = 8p + q$ gives $q = -7$, and $p - q = 9$.

269. (B)

Changing diagonal of a square

Area $A = d^2/2$, so $A' = dd'$. With $A = 400$, $d = 20\sqrt{2}$ and $d' = 1/2$, hence $A' = 10\sqrt{2}$.

270. (B)

Quadratic polynomial from derivatives

$P(x)=-1+(x-2)^2$.

Thus $P(1.001)=-0.001999\approx -0.002$.

271. (A)

* $f(x)$ max at $\alpha=2$

* $g(x)$ min at $\beta=-1$

* So $-\alpha\beta=2$

Limit:

$$\lim_{x\to 2}\frac{(x-1)(x^2-5x+6)}{x^2-6x+8}$$

$$=\lim_{x\to 2}\frac{(x-1)(x-2)(x-3)}{(x-2)(x-4)}$$

$$=\lim_{x\to 2}\frac{(x-1)(x-3)}{x-4}=\frac{(1)(-1)}{-2}=\frac{1}{2}$$

272. (B)

Local minimum of a scalar triple product

$f(x)=(x-1)\left(x^2+x-26\right)$ has a local minimum at $x_0=3$. Then $a\cdot b=-15$.

273. (D)

Rolle theorem with prescribed c

Using $f(1)=f(3)$ and $f'\left(2+1/\sqrt{3}\right)=0$ gives $a=11, b=-6$.

274. (D)

Maximum on a constrained set

The set is $[5,6]$ and f is increasing there. Thus maximum $f(6)=122$.

275. (A)

Monotonicity of a cubic

$f'(x)=3x^2+2bx+c$ has negative discriminant because $b^2<c$, and leading coefficient is positive. Thus f is strictly increasing.

276. (B)

Maximum printable poster area

With sheet dimensions h, b and $hb=18$, optimisation gives $h=3\sqrt{3}$ m and $b=2\sqrt{3}$ m.

277. (B)

Normal to $xy=1$

At $(t,1/t)$ the normal can be written with $a=t^2>0$ and $b=-1<0$.

278. (D)

Orthogonal curves

The tangent slopes satisfy $(3/y)\left(-9x/(by)\right)=-1$. With $y^2=6x$, this gives $b=9/2$.

279. (B)

Increasing interval of $\sin^4 x+\cos^4 x$

$f'(x)=-\sin 4x$. It is positive on $\pi/4<x<\pi/2$.

280. (C)

Recovering a function from its derivative

$f=x^2/2+5/\left(4x^4\right)+C$ and $f(1)=4$ gives $C=9/4$.

281. (D)

Integral with $3+4\cos^2 x$

With $u=\cos x$, the integral is $-\left(1/\left(2\sqrt{3}\right)\right)\tan^{-1}\left(2u/\sqrt{3}\right)+C$.

Thus $A+B=\sqrt{3}/2$.

282. (A)

Integral of square roots of tangent and cotangent

The integrand is $(\sin x+\cos x)/\sqrt{\sin x\cos x}$. Put $u=\sin x-\cos x$ to obtain $\sqrt{2}\sin^{-1}u+C$.

283. (B)

Parameterized trigonometric integral

The integrand is $\sin(x+\alpha)/\sin(x-\alpha)$.

Integration gives $(x-\alpha)\cos 2\alpha + \log|\sin(x-\alpha)|\sin 2\alpha + C$.

284. (A)

Let $t = x^2+1$, so $dt = 2x\,dx \to x\,dx = \frac{dt}{2}$.

The integral becomes:

$$\frac{1}{2}\int\sqrt{\frac{2\sin t - \sin 2t}{2\sin t + \sin 2t}}\,dt$$

Use $\sin 2t = 2\sin t\cos t$:

Numerator: $2\sin t - 2\sin t\cos t = 2\sin t(1-\cos t)$

Denominator: $2\sin t + 2\sin t\cos t = 2\sin t(1+\cos t)$

So the fraction:

$$\frac{1-\cos t}{1+\cos t} = \tan^2\left(\frac{t}{2}\right)$$

Square root $\to \tan\left(\frac{t}{2}\right)$

Integral:

$$\frac{1}{2}\int\tan\left(\frac{t}{2}\right)dt = \frac{1}{2}\cdot 2\ln\left|\sec\frac{t}{2}\right| + c$$

$$= \ln\left|\sec\left(\frac{x^2+1}{2}\right)\right| + c$$

285. (D)

We have $\int\frac{\cos 8x+1}{\cot 2x - \tan 2x}dx$

Step-1: Simplify denominator

$$\cot 2x - \tan 2x = \frac{\cos 2x}{\sin 2x} - \frac{\sin 2x}{\cos 2x}$$

$$= \frac{\cos^2 2x - \sin^2 2x}{\sin 2x\cos 2x}$$

$$= \frac{\cos 4x}{\frac{1}{2}\sin 4x} = 2\cot 4x$$

Step 2: Numerator $1+\cos 8x = 2\cos^2 4x$

So, integrand becomes:

$$\frac{2\cos^2 4x}{2\cot 4x} = \cos^2 4x\cdot\tan 4x = \sin 4x\cos 4x = \frac{1}{2}\sin 8x$$

Step 3: Integrate

$$\int\frac{1}{2}\sin 8x\,dx = \frac{1}{2}\cdot\left(-\frac{\cos 8x}{8}\right) = -\frac{1}{16}\cos 8x + c$$

Thus, $A = -\frac{1}{16}$.

286. (B)

Integral with cosecant squared

$(1-\cos x)\csc^2 x = 1/2\sec^2(x/2)$. Integral

$= \tan(x/2) + C$.

287. (C)

$$\int\cos^{\frac{3}{5}}x\sin^3 x\,dx.$$

Step-1: Express $\sin^3 x$

$\sin^3 x = \sin x(1-\cos^2 x)$.

Hence,

$$I = \int\cos^{\frac{3}{5}}x(1-\cos^2 x)\sin x\,dx.$$

Step 2: Substitute: Let $u = \cos x$, $du = -\sin x\,dx$.

Then

$$I = -\int u^{\frac{3}{5}}(1-u^2)\,du.$$

$$I = -\int\left(u^{\frac{3}{5}} - u^{\frac{13}{5}}\right)du.$$

Step 3: Integrate : $I = -\left(\frac{u^{\frac{8}{5}}}{\frac{8}{5}} - \frac{u^{\frac{18}{5}}}{\frac{18}{5}}\right) + C.$

Simplify:

$$I = -\frac{5}{8}u^{\frac{8}{5}} + \frac{5}{18}u^{\frac{18}{5}} + C.$$

Substitute $u = \cos x$:

$$I = -\frac{5}{8}\cos^{\frac{8}{5}}x + \frac{5}{18}\cos^{\frac{18}{5}}x + C.$$

Step 4 : Compare with the given form :

Given,

$$I = -\frac{1}{m}\cos^m x + \frac{1}{n}\cos^n x + C.$$

Comparing,

$m=\frac{8}{5},\quad n=\frac{18}{5}.$

Final Answer $\left(\frac{8}{5},\frac{18}{5}\right)$

288. (B)

Integral after $u=1/x$

$u=1/x$ gives $-\int\tan^2u\,du=u-\tan u+C$

$=1/x-\tan(1/x)+C$.

289. (C)

Algebraic radical integral

With $u=3x-2$, the result is $\frac{4}{27}u^{3/2}-\frac{34}{9}u^{1/2}+C$

.

290. (B)

Exponential integral

Put $u=x^3$. Integration by parts gives $-e^{-4x^3}\left(4x^3+1\right)/48+C$, so $f(x)=-4x^3-1$.

291. (B)

Integral recognised as a derivative

If $F=\sin^2x+\cos^3x$, the numerator is F'. Hence $\int F'e^{-F}dx=-e^{-F}+C$.

292. (D)

Partial fractions in $\sin\theta$

The integral is $\frac{1}{5}\log\left|(2\sin\theta+1)/(\sin\theta+3)\right|+C$.

Thus $f/A=5(2\sin\theta+1)/(\sin\theta+3)$.

293. (A)

Integral with $\cos2x$

Put $u=\cos x$. The result is $\frac{1}{2}\cos x-\frac{3}{4\sqrt{2}}\log\left|\left(\sqrt{2}\cos x-1\right)/\left(\sqrt{2}\cos x+1\right)\right|+C$.

294. (C)

Where an antiderivative decreases

$f'(x)=(x-1)(x-2)/\left(x^4+1\right)<0$ on $(1,2)$.

295. (B)

Integral involving $\cot^2x-1$

The integrand is $\frac{1}{2}\sec2x-\frac{1}{2}$. Hence $A=4,B=2$ and $A+B=6$.

296. (A)

Substitution in a composite trigonometric integral

Let $u=1+\log\tan(x/2)$; then $du=\csc x\,dx$ and the integral is $\tan u+C$.

297. (B)

Inverse hyperbolic substitution

Let $u=\log\left(t+\sqrt{1+t^2}\right)$; then $du=dt/\sqrt{1+t^2}$.

Thus $g(t)=u$ and $g(2)=\log\left(2+\sqrt{5}\right)$.

298. (A)

Complete square: $x^2-2x+4=(x-1)^2+3$

Let $t=x-1,dx=dt$.

Limits: $x=1\to t=0,x=2\to t=1$

$I=\int_0^1\frac{dt}{(t^2+3)^{3/2}}$

Use $t=\sqrt{3}\tan\theta,dt=\sqrt{3}\sec^2\theta d\theta$

Limits: $t=0\to\theta=0,t=1\to\tan\theta=\frac{1}{\sqrt{3}}\Rightarrow\theta=\frac{\pi}{6}$

$t^2+3=3\sec^2\theta$

$(t^2+3)^{3/2}=(3\sec^2\theta)^{3/2}=3\sqrt{3}\sec^3\theta$

So:

$I=\int_0^{\pi/6}\frac{\sqrt{3}\sec^2\theta}{3\sqrt{3}\sec^3\theta}d\theta=\frac{1}{3}\int_0^{\pi/6}\cos\theta d\theta$

$=\frac{1}{3}[\sin\theta]_0^{\pi/6}=\frac{1}{3}\cdot\frac{1}{2}=\frac{1}{6}$

Given:

$\frac{1}{6}=\frac{k}{k+5}$

Cross multiply:

$k+5=6k\Rightarrow5=5k\Rightarrow k=1$

299. (D)

Parity of an integrand

$f(x)+f(-x)$ is even and $g(x)-g(-x)$ is odd, so the product is odd. The symmetric integral is 0.

300. (D)

Absolute-value integral

Split at $x=5/2$. The two triangular areas sum to $25/4+9/4=17/2$.

301. (B)

Absolute trigonometric integral

The curves meet at 0 and $\pi/2$. Splitting there gives $\pi/2$.

302. (B)

Piecewise definite integral

On $[-2,2]$, $e^{\cos x}\sin x$ is odd, so its integral is 0.

On $[2,3]$, the integral of 2 is 2.

303. (B)

$$I=\int_{\pi/6}^{\pi/3}\sec^{2/3}x\,\mathrm{cosec}^{4/3}x\,dx$$

$$=\int_{\pi/6}^{\pi/3}\tan^{2/3}x\,\mathrm{cosec}^{2}x\,dx$$

Let

$$t=\cot x,\quad dt=-\mathrm{cosec}^{2}x\,dx$$

Then

$$I=-\int t^{-2/3}\,dt=-3t^{1/3}\Big|_{\sqrt{3}}^{1/\sqrt{3}}$$

$$=-3\left(3^{-1/6}-3^{1/6}\right)=3^{7/6}-3^{5/6}.$$

304. (C)

Recurrence for I_n

$I_n+I_{n-2}=1/(n-1)$. Hence $I_{12}+I_{10}=1/11$.

305. (A)

Area under $y=x|x|$

The two symmetric parts each have area $\int_0^1 x^2dx=1/3$. Total $2/3$.

306. (A)

Area under a cubic

Split at roots $-1,0,2$: $\int_{-1}^{0}f(x)dx-\int_{0}^{2}f(x)dx$ $=37/12$.

307. (C)

Area between e^x and $\log x$

$$\int_1^2\left(e^x-\log x\right)dx=e^2-e+1-2\log 2.$$

308. (C)

$$d=\frac{a}{\sqrt{2}}=a\cos 45^\circ\Rightarrow\theta=45^\circ$$

Central angle $=90^\circ=\dfrac{\pi}{2}$

Segment Area = Sector − Triangle

$$=\frac{\pi a^2}{4}-\frac{a^2}{2}=\frac{a^2}{2}\left(\frac{\pi}{2}-1\right)$$

309. (A)

Curve parameters from a point and area

$a+b=2$ and $\int_0^4\left(a\sqrt{x}+bx\right)dx=8$. Solving gives $a=3,b=-1$.

310. (C)

Area enclosed by two parabolas and a line

By symmetry, area

$$=2\int_0^{1/2}\left[(x-1)^2-1/4\right]dx=1/3.$$

311. (C)

Area under $|x-2|$

Two right triangles of base and height 1 give total area 1.

312. (C)

Differential equation from a logarithm

$dy/dx=e^{ax+by}$ separates as $e^{-by}dy=e^{ax}dx$.

Hence $ae^{-by}+be^{ax}=C$.

313. (B)

Given,

$$\frac{dy}{dx}+\frac{3x^2}{1+x^3}y=\frac{1}{1+x^3}\$$

$$\text{I.F.}=e^{\int\frac{3x^2}{1+x^3}dx}=1+x^3$$

$$y(1+x^3)=\int 1dx+C=x+C$$

Answer: $y(1+x^3)=x+C$

314. (C)

Differential equation of a family of circles

From $x^2+y^2-2ax=0$ and its derivative, eliminate a to get $y^2=x^2+2xyy'$.

315. (B)

Bernoulli equation for $x(y)$

For $dx/dy+x/y=x^2$, put $v=1/x$. This gives $v'-v/y=-1$ and $1/x=cy-y\log y$.

316. (B)

First-order linear equation

The integrating factor is x: $(xy)'=x\sin x$. Therefore $x(y+\cos x)=\sin x+C$.

317. (A)

Production increase

The printed rate is $dP/dx=100-12\sqrt{x}$. Integrating from 0 to 9 adds $900-216=684$, so production becomes 1684.

318. (C)

Radioactive decay

$8/27=(2/3)^3$, so the hourly factor is $2/3$. After four hours the mass is $27(2/3)^4=16/3$ g.

319. (C)

Differential equation with $x+y$

Let $z=x+y$.

Then $dz/dy=1+\cos z=2\cos^2(z/2)$,

giving $y=\tan((x+y)/2)+C$.

320. (D)

Exact differential after division

Divide by y and integrate: $y+\log y+e^x\cos^2 x=C$. Using $(0,1)$ gives $C=2$.

321. (C)

Exponential bacterial growth

The culture doubles every 6 h; in 18 h it multiplies by $2^3=8$.

322. (B)

Particular solution

Integrating $dy/dx=\log(x+1)$ gives $y=(x+1)\log(x+1)-x+C$. The initial value gives $C=3$.

323. (B)

Filling an inverted cone

With $r=5h/9$ and $dh/dt=\pi/A=1/r^2$, $t=\int_0^9(25h^2/81)dh=75$ s.

324. (C)

Differential equation with x as a function of y

$dx/dy-x/y=-y$. Thus $x/y=-y+C$; the initial condition gives $x=2y-y^2$. At $x=-3$ and $y>0$, $y=3$.

325. (C)

Exact differential product

$d[(1+\sin x)(1+\cos y)]=0$.

Hence $(1+\sin x)(1+\cos y)=C$.

326. (C)

Given, $\frac{dy}{dx}=-y+e^{-x}$

$$\frac{dy}{dx}+y=e^{-x}$$

I.F. $= e^{\int 1dx} = e^x$

$ye^x = \int 1dx + C = x + C$

Passing through $(0,0)$:

$0 = 0 + C \Rightarrow C = 0$

$ye^x = x$

or

$ye^x - x = 0$

327. (C)

Newton's law of cooling

The excess temperature factor in 30 min is $25/55 = 5/11$. After 60 min, $T - 25 = 55(5/11)^2 = 125/11$, so $T = 36.36^\circ$C.

328. (D)

Initial bacterial population

Growth from hour 3 to 5 is by factor 4, hence the hourly factor is 2. The initial number is $10^4/2^3 = 10^4/8$.

329. (C)

Initial radioactive decay rate

$M = M_0 e^{-kt}$ and $e^{-kh} = 1/2$, so $k = \log 2/h$.

Therefore $M'(0) = -m_0 \log 2/h$.

330. (C)

Linear growth equation

$y' + (-1)y = 3$ gives $y + 3 = 5e^x$. At $x = \log 2$, $y = 7$.

331. (A)

Separable differential equation

Integration gives $(y+1)(2+\sin x) = 4$. At $x = \pi/2$, $y = 1/3$.

332. (C)

Uniform discrete random variable

$(n^2 - 1)/12 = (n+1)/2$ gives $(n+1)(n-7) = 0$, so $n = 7$.

333. (A)

Expected gain in a coin game

$E = 7(2/8) - 3(6/8) = -1/2$.

334. (C)

Distribution of sixes

$X \sim \text{Bin}(2, 1/6)$,

so probabilities are $25/36, 5/18, 1/36$.

335. (C)

Mean number of tens

$X \sim \text{Bin}(2, 1/13)$, hence $E(X) = 2/13$.

336. (C)

Distribution of fours

Again $X \sim \text{Bin}(2, 1/6)$,

giving $25/36, 5/18, 1/36$.

337. (A)

Distribution of jacks

$X \sim \text{Bin}(2, 1/13)$,

giving $144/169, 24/169, 1/169$.

338. (A)

Variance of a discrete uniform variable

For X uniform on $1, \ldots, k$, $\text{Var}X = (k^2 - 1)/12$.

339. (B)

Variance of three Bernoulli trials

$X \sim \text{Bin}(3, 1/2)$, so variance

$= 3(1/2)(1/2) = 0.75$.

340. (C)

Unknown probability parameter

Normalisation gives $(k-2)(k-1)(3k-1) = 0$; validity gives $k = 1/3$.

Then $P(X < 2) = 2/9 + 2/3 = 8/9$.

341. (C)
Normalising a density
The three areas are $a/2, a, a/2$, totalling $2a = 1$. Thus $a = 1/2$.

342. (C)
Union of two events
$E \cup F = \{1,2,3,4,5,7\}$; summing the probabilities gives 0.83.

343. (B)
CDF from a density
$F(x) = \int_0^x 12t^2(1-t)\,dt = 4x^3 - 3x^4$ for $0 < x < 1$.

344. (A)
Distribution of queens
As for jacks, $X \sim \text{Bin}(2, 1/13)$, so the distribution is $144/169, 24/169, 1/169$.

345. (C)
Probability from a density
$P(1/4 < X < 1/3) = \int_{1/4}^{1/3} 3\left(1-2x^2\right)dx = 179/864$.

346. (A)
Number of factors on a die
The counts are: one factor for 1, two for $2, 3, 5$, three for 4, four for 6. Probabilities $1/6, 1/2, 1/6, 1/6$.

347. (B)
Random walk after eleven steps
One step away means 6 forward and 5 backward, or vice versa. The sum simplifies to $\binom{11}{6}(0.24)^5$.

348. (B)
Binomial mean plus variance
$10p + 10p(1-p) = 15/2$ gives $p = 1/2$. Variance $= 10(1/2)(1/2) = 2.5$.

349. (B)
At least four successes
With $p = 2/3$, $\sum_{r=4}^{6} \binom{6}{r} (2/3)^r (1/3)^{6-r}$
$= 496/729$.

350. (B)
Irregular die and expected frequency
$P(X = 3) = 2P(X = 2)$ gives $p = 2/3$ for an even result. Thus $P(0 \text{ even}) = (1/3)^5$ and $6804/243 = 28$.

351. (D)
Binomial parameter
$5p + 5p(1-p) = 1.8$ gives $p = 0.2$.

352. (C)
Fewer than two unsolved problems
For failure probability $1/5$, $P(Y < 2) = (4/5)^{50} + 50(1/5)(4/5)^{49} = \frac{54}{5}(4/5)^{49}$.

353. (A)
At most one defective bulb
Using the binomial form in the choices, $(0.9)^5 + 5(0.1)(0.9)^4 = \frac{7}{5}(9/10)^4$.

354. (D)
Binomial CDF numerator
$np = 8$ and $npq = 4$ give $p = q = 1/2, n = 16$. Thus $P(X \le 2) = [1 + 16 + 120]/2^{16} = 137/2^{16}$, so $k = 137$.

MHTCET Yearwise PYQ (2024)

Marathi

Mathematics

1. If $\tan x = \frac{3}{4}$ and $\pi < x < \frac{3\pi}{2}$, then $\cos\frac{x}{2} =$ **[2024]**

(A) $-\frac{2}{5}$ (B) $\frac{2}{5}$

(C) $\frac{1}{\sqrt{10}}$ (D) $-\frac{1}{\sqrt{10}}$

2. The value of the expression $\sqrt{3}\text{cosec}20° - \sec 20°$ is equal to **[2024]**

(A) 2 (B) $\frac{2\sin 20°}{\sin 40°}$

(C) 4 (D) $4\frac{\sin 20°}{\sin 40°}$

3. If A > B and tan A - tan B = x and cot B - cot A = y, then cot(A - B) = **[2024]**

(A) $\frac{1}{y} - \frac{1}{x}$ (B) $\frac{1}{x} - \frac{1}{y}$

(C) $\frac{1}{x} + \frac{1}{y}$ (D) $\frac{xy}{x-y}$

4. If $\alpha + \beta = \frac{\pi}{2}$ and $\beta + \gamma = \alpha$, then $\tan\alpha$ equals **[2024]**

(A) $2(\tan\beta + \tan\gamma)$ (B) $\tan\beta + \tan\gamma$

(C) $\tan\beta + 2\tan\gamma$ (D) $2\tan\beta + \tan\gamma$

5. If A, B, C are the angles of a triangle with $\tan\frac{A}{2} = \frac{1}{3}, \tan\frac{B}{2} = \frac{2}{3}$ then the value of $\tan\frac{C}{2}$ is **[2024]**

(A) $-\frac{7}{9}$ (B) $\frac{7}{9}$

(C) $\frac{9}{7}$ (D) $-\frac{9}{7}$

6. The smallest positive value of x in degrees satisfying the equation $\tan(x+100°) = \tan(x+50°)\tan(x)\tan(x-50°)$ is **[2024]**

(A) 30° (B) 15°

(C) 45° (D) 60°

7. $\cos^3\frac{\pi}{8}\cos\frac{3\pi}{8} + \sin^3\frac{\pi}{8}\sin\frac{3\pi}{8} =$ **[2024]**

(A) $\frac{1}{2\sqrt{2}}$ (B) $\frac{1}{\sqrt{2}}$

(C) $\frac{1}{2}$ (D) $\frac{\sqrt{3}}{2}$

8. The value of cos(18° - A)cos(18° + A) -cos(72° - A) cos(72° + A) is equal to **[2024, 2023]**

(A) cos54° (B) cos36°

(C) sin54° (D) sin36°

9. Let a, b, c be three non-zero real numbers such that the equation $\sqrt{3}a\ \cos x + 2\ b\ \sin x = c, x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ has two distinct real roots α and β with $\alpha + \beta = \frac{\pi}{3}$. Then the value of $\frac{b}{a}$ is **[2024]**

(A) 0.1 (B) 0.5

(C) -0.5 (D) 1

10. The maximum value of $(\cos\alpha_1)(\cos\alpha_2)\ldots(\cos\alpha_n)$ under the constraints $0 \le \alpha_1, \alpha_2, \ldots, \alpha_n \le \frac{\pi}{2}$ and $(\cot\alpha_1)(\cot\alpha_2)\ldots(\cot\alpha_n) = 1$ is **[2024]**

(A) $\frac{1}{2^{\frac{n}{2}}}$ (B) $\frac{1}{2^n}$

(C) 2^n (D) $2^{\frac{n}{2}}$

11. The value of $\cos 20° + 2\sin^2 55° - \sqrt{2}\sin 65°$ is **[2024]**

(A) 0 (B) 1

(C) -1 (D) $\frac{1}{2}$

12. If two lines $x+(a-1)y=1$ and $2x+a^2y = 1(a \in R - \{0,1\})$ are perpendicular, then the distance of their point of intersection from the origin is **[2024]**

(A) $\frac{2}{5}$ (B) $\frac{\sqrt{2}}{5}$

(C) $\frac{2}{\sqrt{5}}$ (D) $\frac{\sqrt{2}}{\sqrt{5}}$

13. The diagonals of a parallelogram ABCD are along the lines $x+3y=4$ and $6x-2y = 7$. Then ABCD must be a **[2024]**

(A) rectangle.
(B) square.
(C) rhombus.
(D) cyclic quadrilateral

14. The number of possible distinct straight lines passing through (2,3) and forming a triangle with co-ordinate axes whose area is 12 sq. units are, **[2024]**

(A) one (B) two
(C) three (D) four

15. The acute angle between the lines $x \cos 30° + y \sin 30° = 3$ and $x \cos 60° + y \sin 60° = 5$ is **[2024]**

(A) 75° (B) 30°
(C) 60° (D) 45°

16. The straight line, $2x-3y+17=0$ is perpendicular to the line passing through the points (7,17) and (15, β), then β equals **[2024]**

(A) 5
(B) $\frac{35}{3}$
(C) $-\frac{35}{3}$
(D) –5

17. If the length of the perpendicular to a line from the origin is $2\sqrt{2}$ units, which makes an angle of 135° with the X-axis, then the equation of line is **[2024]**

(A) $x + y = 4$
(B) $x - y + 4 = 0$
(C) $x - y = 4$
(D) $x + y + 4 = 0$

18. Suppose that the points (h, k), (1, 2) and (–3, 4) lie on the line l_1. If a line l_2 passing through the points (h, k) and (4, 3) is perpendicular to l_1, then $(\frac{k}{h})$ equals **[2024]**

(A) $\frac{1}{3}$ (B) 0

(C) 3 (D) $-\frac{1}{7}$

19. If $(m_i, \frac{1}{m_i})$, $m_i > 0$, i = 1,2,3,4 are four distinct points on a circle, then the product $m_1 m_2 m_3 m_4$ is equal to **[2024]**

(A) –1 (B) 1
(C) 0 (D) 2

20. The equation of the concentric circle, with the circle C_1 having equation $x^2 + y^2 – 6x – 4y – 12 = 0$ and having double area compared to the area of C_1, is **[2024]**

(A) $x^2 + y^2 – 6x – 4y = 27$
(B) $x^2 + y^2 – 6x – 4y = 13$
(C) $x^2 + y^2 – 6x – 4y = 50$
(D) $x^2 + y^2 – 6x – 4y = 37$

21. The equation of the circle which has its centre at the point (3, 4) and touches the line $5x + 12y – 11 = 0$ is **[2024]**

(A) $x^2 + y^2 – 6x – 8y + 9 = 0$
(B) $x^2 + y^2 – 6x – 8y + 25 = 0$
(C) $x^2 + y^2 – 6x – 8y – 9 = 0$
(D) $x^2 + y^2 – 6x – 8y – 25 = 0$

22. The equation of the circle, concentric with the circle $x^2 + y^2 - 6x - 4y - 12 = 0$ and touching the X-axis is **[2024]**

(A) $x^2 + y^2 - 6x - 4y + 5 = 0$
(B) $x^2 + y^2 - 6x - 4y + 17 = 0$
(C) $x^2 + y^2 - 6x - 4y + 9 = 0$
(D) $x^2 + y^2 - 6x - 4y + 4 = 0$

23. The abscissae of the two points A and B are the roots of the equation $x^2 + 2ax - b^2 = 0$ and their ordinates are roots of the equation $y^2+2py - q^2 = 0$. Then the equation of the circle with AB as diameter is given by **[2024, 2023]**

(A) $x^2 + y^2 - 2ax - 2py + (b^2 + q^2) = 0$
(B) $x^2 + y^2 - 2ax - 2py - (b^2 + q^2) = 0$
(C) $x^2 + y^2 + 2ax + 2py + (b^2 + q^2) = 0$
(D) $x^2 + y^2 + 2ax + 2py - (b^2 + q^2) = 0$

24. Let PQ and RS be tangents at the extremities of the diameter PR of a circle of radius r. If PS and RQ intersect at a point X on the circumference of the circle, then 2r equals **[2024]**

(A) $\sqrt{PQ \cdot RS}$
(B) $\frac{PQ + RS}{2}$
(C) $\frac{2 \cdot PQ \cdot RS}{PQ + RS}$
(D) $\sqrt{\frac{PQ^2 + RS^2}{2}}$

25. The parametric equations of the circle $x^2 + y^2 - ax - by = 0$ are **[2024, 2022]**

(A) $x = \frac{a}{2} + \frac{\sqrt{a^2+b^2}}{2}\cos\theta, y = \frac{b}{2} + \frac{\sqrt{a^2+b^2}}{2}\sin\theta$
(B) $x = \frac{-a}{2} + \frac{\sqrt{a^2+b^2}}{4}\sin\theta, y = \frac{-b}{2} + \frac{\sqrt{a^2+b^2}}{4}\cos\theta$
(C) $x = \frac{a}{2} + \sqrt{\frac{a^2+b^2}{2}}\sin\theta, y = \frac{b}{2} + \sqrt{\frac{a^2+b^2}{2}}\cos\theta$
(D) $x = \frac{a}{2} + \frac{\sqrt{a^2+b^2}}{4}\cos\theta, y = \frac{b}{2} + \frac{\sqrt{a^2+b^2}}{4}\sin\theta$

26. Three houses are available in a locality. Three persons apply for the houses. Each applies for one house without consulting others. The probability that all the three persons apply for the same house is **[2024]**

(A) $\frac{1}{9}$ (B) $\frac{2}{9}$
(C) $\frac{7}{9}$ (D) $\frac{8}{9}$

27. Let A and B be two events such that the probability that exactly one of them occurs is $\frac{2}{5}$ and the probability that A or B occurs is $\frac{1}{2}$, then the probability of both of them occur together is **[2024]**

(A) 0.1 (B) 0.2
(C) 0.01 (D) 0.02

28. In a class of 300 students, every student reads 5 news papers and every news paper is read by 60 students. Then the number of newspapers is **[2024]**

(A) at least 30 (B) at most 20
(C) exactly 25 (D) exactly 10

29. Four persons can hit a target correctly with probabilities $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ and $\frac{1}{5}$ respectively. If all hit at the target independently, then the probability that the target would be hit, is **[2024]**

(A) $\frac{1}{5}$ (B) $\frac{3}{5}$
(C) $\frac{2}{5}$ (D) $\frac{4}{5}$

30. There are three events A, B, C, one of which must and only one can happen. The odds are 8:3 against A, 5 : 2 against B and the odds against C is 43:17k, then value of k is **[2024]**

(A) $\frac{1}{2}$ (B) 2
(C) $\frac{1}{3}$ (D) $\frac{1}{4}$

31. A and B are independent events with P(A) $=\frac{3}{10}$, P(B) = $\frac{2}{5}$, then P(A' ∪ B) has the value **[2024]**

(A) $\frac{41}{50}$ (B) $\frac{41}{125}$

(C) $\frac{7}{25}$ (D) $\frac{7}{50}$

32. If A and B are two independent events such that P(A') = 0.75, P(A ∪ B) = 0.65 and P(B)= p, then value of p is **[2024]**

(A) $\frac{9}{14}$ (B) $\frac{7}{15}$

(C) $\frac{5}{14}$ (D) $\frac{8}{15}$

33. Let A,B and C be three events, which are pairwise independent and $\bar{E}$ denote the complement of an event E. If P(A∩B∩C) = 0 and P(C)>0, then P(($\bar{A}$∩$\bar{B}$)/C) is equal to **[2024]**

(A) P(A)+P($\bar{B}$)

(B) P($\bar{A}$)−P($\bar{B}$)

(C) P($\bar{A}$)−P(B)

(D) P($\bar{A}$)+P($\bar{B}$)

34. The probability, that a year selected at random will have 53 Mondays, is **[2024]**

(A) $\frac{1}{4}$ (B) $\frac{3}{28}$

(C) $\frac{5}{28}$ (D) $\frac{3}{4}$

35. Two friends A and B apply for a job in the same company. The probabilities of A getting selected is $\frac{2}{5}$ and that of B is $\frac{4}{7}$. Then the probability, that one of them is selected, is **[2024]**

(A) $\frac{8}{35}$ (B) $\frac{18}{35}$

(C) $\frac{26}{35}$ (D) $\frac{34}{35}$

36. If P(x,y) denotes z = x + iy x, y∈ R and i = $\sqrt{-1}$ in Argand's plane and $\left|\frac{z-1}{z+2i}\right|=1$, then the locus of P is **[2024]**

(A) parabola (B) hyperbola

(C) circle (D) straight line

37. If $z^2+z+1=0$ then $\left(z^3+\frac{1}{z^3}\right)^2+\left(z^4+\frac{1}{z^4}\right)^2=$ where $z=\omega$ = complex cube root of unity **[2024]**

(A) 4 (B) 1

(C) 5 (D) 2

38. Let $\left(-2-\frac{1}{3}i\right)^3=\frac{x+iy}{27}$, $i=\sqrt{-1}$, where x and y are real numbers, then (y - x) has the value **[2024]**

(A) -91 (B) -85

(C) 85 (D) 91

39. If Z = $\frac{-2}{1+\sqrt{3}i}$, i = $\sqrt{-1}$, then the value of arg Z is **[2024]**

(A) $\frac{2\pi}{3}$ (B) $\frac{\pi}{3}$

(C) $-\frac{\pi}{3}$ (D) $\frac{4\pi}{3}$

40. If ω = $\frac{-1+i\sqrt{3}}{2}$, where $i=\sqrt{-1}$, then the value of $\left(3+\omega+3\omega^2\right)^4$ is **[2024]**

(A) 16 (B) -16

(C) 16ω (D) 16ω²

41. If ω = $\frac{-1-i\sqrt{3}}{2}$ where $i=\sqrt{-1}$, then the value of $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$ is **[2024]**

(A) –1 (B) 0

(C) 1 (D) 3

42. If the complex number z = x + iy, where i = $\sqrt{-1}$, satisfies the condition $|z+1|=1$, then z lies on **[2024, 2021]**

(A) X-axis.
(B) circle with centre (1,0) and radius 1 unit.
(C) circle with centre (-1,0) and radius 1 unit.
(D) Y-axis.

43. Let z = x + iy be a complex number, where x and y are integers and i = $\sqrt{-1}$. Then the area of the rectangle whose vertices are the roots of the equation $z\bar{z}^3+\bar{z}z^3=350$ is **[2024]**

(A) 48 (B) 32
(C) 40 (D) 80

44. If $z_1=5-2i$ and $z_2=3+i$, where i = $\sqrt{-1}$, then $arg\left(\frac{z_1+z_2}{z_1-z_2}\right)$ is **[2024]**

(A) $tan^{-1}\left(\frac{22}{19}\right)$ (B) $tan^{-1}\left(\frac{22}{13}\right)$
(C) $tan^{-1}\left(\frac{21}{19}\right)$ (D) $tan^{-1}\left(\frac{19}{22}\right)$

45. The number of four letter words that can be formed using letters of the word BARRACK is **[2024]**

(A) 120 (B) 264
(C) 270 (D) 144

46. A committee of 11 members is to be formed from 8 males and 5 females. If m is the number of ways the committee is formed with at least 6 males and n is the number of ways the committee is formed with at least 3 females, then **[2024]**

(A) m + n = 68 (B) m = n = 78
(C) m = n = 68 (D) n=m-8

47. The number of ways in which 5 boys and 3 girls can be seated on a round table, if a particular boy B_1 and a particular girl G_1 never sit adjacent to each other, is **[2024]**

(A) 7! (B) 5×6!
(C) 6×6! (D) 5×7!

48. Five persons A,B,C,D and E are seated in a circular arangement, if each of them is given a hat of one of the three colours red, blue and green, then the number of ways, of distributing the hats such that the person seated in adjacent seats get different coloured hats, is **[2024]**

(A) 30 (B) 15
(C) 60 (D) 40

49. Consider a group of 5 boys and 7 girls. The number of different teams, consisting of 2 boys and 3 girls that can be formed from this group if there are two specific girls A and B, who refuse to be the members of the same team, is **[2024]**

(A) 350 (B) 300
(C) 200 (D) 500

50. Words of length 10 are formed by using the letters A, B, C, D, E, F, G, H, I, J. Let x be number of such words where no letter is repeated and y be number of such words where exactly two letters are repeated twice and no other letter is repeated, then the value of $\frac{y}{x}$ is **[2024]**

(A) 45 (B) 415
(C) 315 (D) 215

51. _____ numbers greater than a million can be formed with the digits 2, 3, 0, 3, 4, 2, 3. **[2024]**

(A) 60 (B) 420
(C) 360 (D) 120

52. If n(A) = 4, n(B) = 2. Then the number of subsets of the set A × B each having at least 3 elements are **[2024]**

(A) 275 (B) 510
(C) 219 (D) 256

53. There are 3 sections in a question paper and each section contains 5 questions. A candidate has to answer a total of 5 questions, choosing at least one question from each section. Then the number of ways, in which the candidate can choose the questions, is **[2024]**

(A) 750 (B) 1500
(C) 2255 (D) 2250

54. Eight chairs are numbered 1 to 8. Two women and three men wish to occupy one chair each. First the women choose chairs from amongst the chairs marked 1 to 4, and then the men select the chairs from amongst the remaining. The number of possible arrangements is **[2024]**

(A) ${}^6C_3 \times {}^4C_2$ **(B)** ${}^4P_2 \times {}^6P_3$

(C) ${}^4C_2 + {}^4P_3$ **(D)** ${}^4P_2 + {}^6P_3$

55. The domain of definition of $f(x)=\frac{log_2(x+3)}{x^2+3x+2}$ is **[2024]**

(A) R-{1, 2}

(B) (-2,∞)

(C) R-{-1,-2,-3}

(D) (-3,∞)-{-1,-2}

56. If $f(x)=log_e\left(\frac{1-x}{1+x}\right)$, $|x|<1$, then $f\left(\frac{2x}{1+x^2}\right)$ is equal to **[2024]**

(A) $2f(x^2)$ (B) $-2f(x)$

(C) $(f(x))^2$ (D) $2f(x)$

57. If $f(x)=\frac{x}{2-x}$, $g(x)=\frac{x+1}{x+2}$, then (gogof) (x) = **[2024]**

(A) $\frac{6+x}{10-2x}$ (B) $\frac{6-x}{10+2x}$

(C) $\frac{6+x}{10+2x}$ (D) $\frac{6-x}{10-2x}$

58. Let α,β be the roots of the equation $x^2 - px + r = 0$ and $\frac{\alpha}{2}, 2\beta$ be the roots of the equation $x^2 - qx + r = 0$. Then the value of r is **[2024]**

(A) $\frac{2}{9}(p-q)(2q-p)$

(B) $\frac{2}{9}(q-p)(2p-q)$

(C) $\frac{2}{9}(q-2p)(2q-p)$

(D) $\frac{2}{9}(2p-q)(2q-p)$

59. The domain of the definition of the function f(x) is given by the equation $2^x+2^y=2$ is **[2024, 2023]**

(A) $0<x\le 1$ (B) $0\le x\le 1$

(C) $-\infty<x\le 0$ (D) $-\infty<x<1$

60. If $f:[1,\infty)\to[2,\infty)$ is given by $f(x)=x+\frac{1}{x}$ then $f^{-1}(x)$ equals **[2024]**

(A) $\frac{x+\sqrt{x^2-4}}{2}$ (B) $\frac{2}{1+x^2}$

(C) $\frac{x-\sqrt{x^2-4}}{2}$ (D) $1+\sqrt{x^2-4}$

61. Let $f(x) = (x + 1)^2 - 1, x \ge -1$, then the set $\{x / f(x) = f^{-1}(x)\}$ is **[2024]**

(A) $\{0,1,-1\}$

(B) $\{0,-1\}$

(C) $\{0,-1, \frac{-3+i\sqrt{3}}{2}, \frac{-3-i\sqrt{3}}{2}$, where $i = \sqrt{-1}\}$

(D) φ

62. Let $P = \{\theta / \sin\theta - \cos\theta = \sqrt{2}\cos\theta\}$ and $Q = \{\theta / \sin\theta + \cos\theta = \sqrt{2}\sin\theta\}$ be two sets, then **[2024]**

(A) $P \subset Q$ and $Q - P \neq \varphi$ (B) $Q \not\subset P$

(C) $P \not\subset Q$ (D) $P = Q$

63. The value of $lim_{x\to 0}\frac{x}{|x|+x^2}$ is **[2024, 2023]**

(A) 1 (B) -1

(C) 0 (D) does not exist.

64. Let α (*a*) and β (*a*) be the roots of the equation $\left(\sqrt[3]{1+a}-1\right)x^2+\left(\sqrt{1+a}-1\right)x+\left(\sqrt[6]{1+a}-1\right)=0$ where a > -1 then $lim_{a\to 0^+}\alpha(a)$ and $lim_{a\to 0^+}\beta(a)$ respectively are **[2024]**

(A) 1 and $-\frac{5}{2}$ (B) -1 and $-\frac{1}{2}$

(C) 2 and $-\frac{7}{2}$ (D) 3 and $\frac{9}{2}$

65. For each x∈R, Let [x] represent greatest integer function, then $\lim_{x\to0^-}\frac{x([x]+|x|)\sin[x]}{|x|}$ is equal to **[2024]**

(A) 0 (B) 1
(C) sin 1 (D) -sin 1

66. $\lim_{x\to0}\frac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}$ has the value **[2024]**

(A) 2 (B) $\frac{1}{2}$
(C) 4 (D) 3

67. If $\lim_{x\to1}\frac{x^2-ax+b}{x-1}=7$, then a + b is equal to **[2024]**

(A) -1 (B) 1
(C) -11 (D) 11

68. If $\lim_{x\to\infty}\left(\frac{x^2+x+1}{x+1}-ax-b\right)=4$ then **[2024]**

(A) a = 1, b = 4 (B) a = 1, b = -4
(C) a = 2, b = -3 (D) a = 2, b = 3

69. $\lim_{x\to\frac{\pi}{2}}\frac{(1-\sin x)(8x^3-\pi^3)\cos x}{(\pi-2x)^4}$ **[2024]**

(A) $\frac{\pi^2}{16}$ (B) $\frac{3\pi^2}{16}$
(C) $\frac{-3\pi^2}{16}$ (D) $\frac{-\pi^2}{16}$

70. $\lim_{x\to0}\frac{9^x-4^x}{x(9^x+4^x)}$ = **[2024]**

(A) $\log(\frac{3}{2})$ (B) $\frac{1}{2}\log(\frac{3}{2})$
(C) $2\log(\frac{3}{2})$ (D) $2\log(\frac{9}{4})$

71. $\lim_{x\to2}\frac{5^x+5^{3-x}-30}{5^{3-x}-5^{\frac{x}{2}}}$ = **[2024]**

(A) $\frac{-16}{3}$ (B) $\frac{8}{3}$
(C) $\frac{-8}{3}$ (D) $\frac{16}{3}$

72. The value of $\lim_{x\to0}\left((\sin x)^{\frac{1}{x}}+\left(\frac{1}{x}\right)^{\sin x}\right)$, where x > 0 is **[2024]**

(A) 0 (B) -1
(C) 1 (D) 2

73. $\lim_{x\to0}\frac{\sin(\pi\cos^2 x)}{x^2}$ is equal to **[2024, 2022]**

(A) 1 (B) -π
(C) π (D) $\frac{\pi}{2}$

74. If f(x) is continuous over [-π, π], where f(x) is defined as f(x) = $\begin{cases}-2\sin x, & -\pi\le x\le-\frac{\pi}{2}\\ \alpha\sin x+\beta, & -\frac{\pi}{2}<x<\frac{\pi}{2}\\ \cos x, & \frac{\pi}{2}\le x\le\pi\end{cases}$

then α and β equals **[2024, 2009, 2004]**

(A) α = -1, β = 1
(B) α = 1, β = -1
(C) α = 1, β = 1
(D) α = β = 0

75. If f(x) = $\frac{1+\cos\pi x}{\pi(1-x)^2}$, for x≠1 is continuous at x=1, then f(1) is equal to **[2024]**

(A) $\frac{\pi}{2}$ (B) $\frac{2}{\pi}$
(C) $\frac{\pi^2}{4}$ (D) $\frac{4}{\pi^2}$

76. The value of k, for which the function $f(x)=\begin{cases}\left(\frac{4}{5}\right)^{\frac{\tan 4x}{\tan 5x}} & ,0<x<\frac{\pi}{2}\\ k+\frac{2}{5} & ,x=\frac{\pi}{2}\end{cases}$ is continuous at $x=\frac{\pi}{2}$, is **[2024]**

(A) $\frac{17}{20}$ (B) $\frac{3}{5}$

(C) $-\frac{2}{5}$ (D) $\frac{2}{5}$

77. Let $f(x)=\frac{1-\tan x}{4x-\pi}$, $x\neq\frac{\pi}{4}$, $x\in\left[0,\frac{1}{2}\right]$, f(x) is continuous in $\left[0,\frac{\pi}{2}\right]$, then $f(\frac{\pi}{4})$ is **[2024]**

(A) $-\frac{1}{2}$ (B) $\frac{1}{2}$

(C) 1 (D) –1

78. If the function $f(x)=\left(\frac{5x-8}{8-3x}\right)^{\frac{3}{2x-4}}$ if $x\neq 2$ and k if $x = 2$. is continuous at x=2, then k = **[2024]**

(A) e^6

(B) e^2

(C) e^{-6}

(D) e^{-2}

79. If $f(x)=\frac{x+x^2+x^3+.....+x^n-n}{x-1}$, for $x\neq1$ is continuous at x=1, then f(1) = **[2024]**

(A) $\frac{n(n+1)(4n-1)}{6}$

(B) $\frac{n(n+1)}{2}$

(C) $\frac{n(n+1)(2n+1)}{6}$

(D) $\frac{n(2n+1)}{4}$

80. Let k be a non-zero real number. If $f(x)=\begin{cases}\frac{\left(e^x-1\right)^2}{\sin\left(\frac{x}{k}\right)\log\left(1+\frac{x}{4}\right)} & ,x\neq0\\ 12 & ,x=0\end{cases}$ is a continuous function, then the value of k is **[2024]**

(A) 1

(B) 2

(C) 4

(D) 3

81. Let f(x) = x[x/2], for −10<x<10, where [t] denotes the greatest integer function. Then the number of points of discontinuity of f is equal to **[2024]**

(A) 10

(B) 9

(C) 6

(D) 8

82. If $f(x)=\left(\frac{1+\tan x}{1+\sin x}\right)^{\cos ec\,x}$ is continuous at x = 0, then f(0) is equal to **[2024]**

(A) 0

(B) 1

(C) e

(D) $\frac{1}{e}$

83. If $f(x)=\begin{cases}\frac{a}{2}\left(x-|x|\right) & ,\text{for}x<0\\ 0 & ,\text{for}x=0\\ bx^2\sin\left(\frac{1}{x}\right) & ,\text{for}x>0\end{cases}$ is continuous at x = 0, then **[2024]**

(A) a is any real value and b is any real value

(B) a is only rational value and b is any real value

(C) a is only irrational value and b is any real value

(D) a is only rational value and b is only rational value

84. Let f:[-1,3] → R be defined as $f(x)=\begin{cases}|x|+[x], & -1\le x<1\\ x+|x|, & 1\le x<2\\ x+[x], & 2\le x\le 3\end{cases}$ where [t] denotes the greatest integer function. Then f is discontinuous at **[2024]**

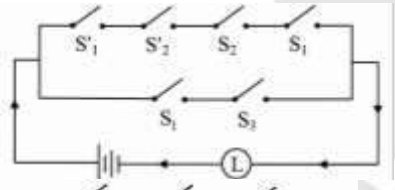
(A) only two points
(B) only three points
(C) four or more points
(D) only one point

85. If statement I : If the work is not finished on time, the contractor is in trouble. statement II : Either the work is finished on time or the contractor is in trouble. then **[2024]**

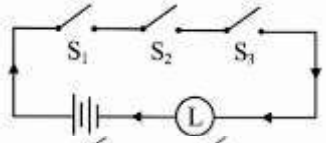
(A) statement II is negation of statement I.
(B) statement II is converse of statement I.
(C) statement II and statement I are equivalent.
(D) statement II is an inverse of statement I.

86. Consider the following statements
p : the switch S_1 is closed.
q : the switch S_2 is closed.
r : the switch S_3 is closed.
Then the switching circuit represented by the statement $(p\wedge q)\vee(\sim p\wedge(\sim q\vee p\vee r))$ is **[2024]**

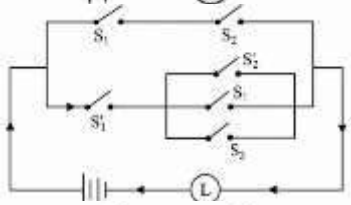
(A)



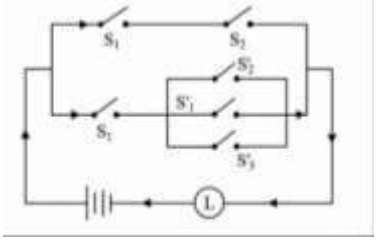
(B)



(C)



(D)



87. The converse of $[(p\wedge\sim q)]\to r$ is **[2024]**

(A) $\sim r\to(\sim p\vee q)$ (B) $r\to(\sim p\wedge\sim q)$
(C) $(\sim p\vee q)\to\sim r$ (D) $r\to(p\wedge q)$

88. Consider the statements given by following :
(A) If 3+3=7, then 4+3=8.
(B) If 5+3= 8, then earth is flat.
(C) If both (A) and (B) are true, then 5+6 = 17
Then which of the following statements is correct? **[2024]**

(A) (A) is true while (B) and (C) are false.
(B) (A) and (C) are true while (B) is false.
(C) (A) and (B) are false, while (C) is true.
(D) (A) is false but (B) and (C) are true.

89. The negation of the statement "The payment will be made if and only if the work is finished in time." Is **[2024, 2023, 2022]**

(A) The work is finished in time and the payment is not made.
(B) Either the work is finished in time and the payment is not made or the payment is made and the work is not finished in time.
(C) The payment is made and the work is not finished in time.
(D) The work is finished in time and the payment is not made or the payment is made and the work is finished in time.

90. The inverse of $p\to(q\to r)$ is logically equivalent to **[2024]**

(A) $p\to(q\to r)$ (B) $(q\to r)\to\sim p$
(C) $(p\vee q)\to r$ (D) $(q\to r)\to p$

91. Let p, q and r be the statements p : X is an equilateral triangle q : X is isosceles triangle r: $q\vee\sim p$, then the equivalent statement of r is **[2024]**

(A) If X is not an equilateral triangle, then X is not an isosceles triangle
(B) X is neither isosceles nor equilateral triangle
(C) X is isosceles but not an equilateral triangle
(D) If X is not an isosceles triangle, then X is not an equilateral triangle.

92. The contrapositive of the inverse of $p \rightarrow (p \rightarrow q)$ is **[2024]**

(A) $(\sim p \wedge q) \rightarrow p$ (B) $(\sim p \vee q) \rightarrow p$

(C) $p \rightarrow (\sim p \vee q)$ (D) $(p \vee q) \rightarrow p$

93. If $(p \wedge \sim q) \wedge (p \wedge r) \rightarrow \sim p \vee q$ is false, then the truth values of p, q and r are respectively **[2025, 2024]**

(A) T, T, T (B) F, F, F

(C) T, F, T (D) F, T, F

94. Suppose A is any 3×3 non-singular matrix and (A-3I)(A-5I)=0 where I = I_3 and O = O_3. Here O_3 represent zero matrix of order 3 and I_3 is an identity matrix of order 3. If $\alpha A + \beta A^{-1} = 4I$, then $\alpha + \beta$ is equal to **[2024]**

(A) 13 (B) 7

(C) 12 (D) 8

95. If $A + B = \begin{bmatrix} 1 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{bmatrix}$ where A is symmetric and B is skew-symmetric matrix, then the matrix $(A^{-1}B + AB^{-1})$ at $\theta = \frac{\pi}{6}$ is given by **[2024]**

(A) $\begin{bmatrix} 1 & 2\sqrt{3} \\ 2\sqrt{3} & 1 \end{bmatrix}$

(B) $\begin{bmatrix} -1 & -2\sqrt{3} \\ 2\sqrt{3} & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 0 & 2\sqrt{3} \\ 2\sqrt{3} & 0 \end{bmatrix}$

(D) $\begin{bmatrix} 0 & -2\sqrt{3} \\ 2\sqrt{3} & 0 \end{bmatrix}$

96. If $A = \begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}$ then $\left(A^2 - 5A\right)^{-1}$ is **[2024]**

(A) $\frac{-1}{4}\begin{bmatrix} -3 & 1 \\ 7 & -1 \end{bmatrix}$ (B) $\frac{1}{4}\begin{bmatrix} -3 & 1 \\ 7 & -1 \end{bmatrix}$

(C) $\frac{1}{4}\begin{bmatrix} 3 & 1 \\ 7 & 1 \end{bmatrix}$ (D) $\frac{-1}{4}\begin{bmatrix} 3 & -1 \\ 7 & -1 \end{bmatrix}$

97. Let A and B be 3x3 real matrices such that A is symmetric matrix and B is skew-symmetric matrix. Then the system of linear equations $\left(A^2B^2 - B^2A^2\right)X = O$. where X is 3x1 column matrix of unknown variables and O is a 3x1 null matrix, has **[2024]**

(A) a unique solution

(B) exactly two solutions

(C) no solution

(D) infinitely many solutions

98. Let $A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$, $x \in \mathbb{R}^+$ and $A^4 = \left[a_{ij}\right]_{2\times 2}$. If $a_{11} = 109$, then $\left(A^4\right)^{-1} =$ **[2024]**

(A) $\begin{bmatrix} 109 & 33 \\ 33 & 10 \end{bmatrix}$ (B) $\begin{bmatrix} 10 & 33 \\ 33 & 10 \end{bmatrix}$

(C) $\begin{bmatrix} 10 & 33 \\ 33 & 109 \end{bmatrix}$ (D) $\begin{bmatrix} 10 & -33 \\ -33 & 109 \end{bmatrix}$

99. If $A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$ and $A.adj\ A = AA^T$, then $5a + b =$ **[2025, 2024, 2021]**

(A) 7

(B) 9

(C) 13

(D) 5

100. The principal solutions, of the equation $\sqrt{3}\sec x + 2 = 0$, are **[2024, 2021]**

(A) $\frac{2\pi}{3}, \frac{4\pi}{3}$

(B) $\frac{4\pi}{3}, \frac{5\pi}{3}$

(C) $\frac{5\pi}{6}, \frac{7\pi}{6}$

(D) $\frac{7\pi}{6}, \frac{11\pi}{6}$

101. Let $f(\theta)=\sin\left(\tan^{-1}\left(\frac{\sin\theta}{\sqrt{\cos 2\theta}}\right)\right)$, where $-\frac{\pi}{4}<\theta<\frac{\pi}{4}$, then the value of $\frac{d}{d(\tan\theta)}(f(\theta))$ is **[2024]**

(A) -1 (B) 1

(C) $\frac{1}{\sqrt{2}}$ (D) $\sqrt{2}$

102. Let S = $\{x \in (-\pi,\pi) \mid x \neq 0,\pm\frac{\pi}{2}\}$. The sum of all distinct solutions of the equation $\sqrt{3}$ sec x + cosec x + 2(tan x-cot x) = 0 in the set S is equal to **[2024]**

(A) $-\frac{7\pi}{9}$ (B) $-\frac{2\pi}{9}$

(C) 0 (D) $\frac{5\pi}{9}$

103. If θ and α are not odd multiples of $\frac{\pi}{2}$ then tanθ = tanα implies principal solution is **[2024]**

(A) $\theta = \alpha + \frac{n\pi}{2}$, $n \in Z$

(B) $\theta = \alpha + \frac{3n\pi}{2}$, $n \in Z$

(C) $\theta = n\pi + \alpha$, $n \in Z$

(D) $\theta = \frac{n\pi}{4} + \alpha$, $n \in Z$

104. The equation $(\cos p-1)x^2 + (\cos p)x + \sin p = 0$ in the variable x, has real roots. Then p can take any value in the interval **[2024]**

(A) $(0,2\pi)$ (B) $(-\pi,0)$

(C) $(-\frac{\pi}{2},\frac{\pi}{2})$ (D) $(0,\pi)$

105. The number of all values of θ in the interval $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ satisfying the equation $(1 - \tan\theta)(1 + \tan\theta)\sec^2\theta + 2\tan^2\theta = 0$ is **[2024]**

(A) 1 (B) 0

(C) 2 (D) infinitely many.

106. If $\sin(\cot^{-1}(x+1)) = \cos(\tan^{-1}x)$ then considering positive square roots, x has the value _____. **[2024, 2022]**

(A) 0 (B) $\frac{9}{4}$

(C) $\frac{1}{2}$ (D) $-\frac{1}{2}$

107. If the angles of a triangle are in the ratio 4 : 1 : 1, then the ratio of the longest side to the perimeter is **[2024, 2023]**

(A) 1 : 6 (B) $\sqrt{3}:\left(2+\sqrt{3}\right)$

(C) $1:\left(2+\sqrt{3}\right)$ (D) 2 : 3

108. In ΔABC, with usual notations, if b = 3, c = 8, m∠A = 60°, then the circumradius of the triangle is _____ units. **[2024]**

(A) $\frac{7}{3}$ (B) $\frac{7\sqrt{2}}{3}$

(C) $\frac{7}{\sqrt{3}}$ (D) $\frac{7\sqrt{3}}{2}$

109. The Solution set of the equation $\sin^2\theta - \cos\theta = \frac{1}{4}$ in the interval $[0,2\pi]$ is **[2024]**

(A) $\{\frac{\pi}{6},\frac{5\pi}{6}\}$ (B) $\{\frac{\pi}{3},\frac{5\pi}{3}\}$

(C) $\{\frac{\pi}{3},\frac{2\pi}{3}\}$ (D) $\{\frac{2\pi}{3},\frac{4\pi}{3}\}$

110. For the triangle ABC, with usual notations, if the angles A,B,C are in A.P. and m∠A = 30°, c = 3, then the values of a and b are respectively **[2024]**

(A) $\frac{\sqrt{3}}{2},\frac{3}{2}$

(B) $\frac{3}{2},\frac{3\sqrt{3}}{2}$

(C) $\frac{3\sqrt{3}}{2},\frac{3}{2}$

(D) $\frac{3}{2},\frac{\sqrt{3}}{2}$

111. Let $2\sin^2x + 3\sin x - 2 > 0$ and $x^2 - x - 2 < 0$ (x is measured in radians). Then x lies in the interval **[2024]**

(A) $(\frac{\pi}{6}, \frac{5\pi}{6})$ (B) $(-1, \frac{5\pi}{6})$

(C) $(-1,2)$ (D) $(\frac{\pi}{6}, 2)$

112. The number of solutions, of $2^{1+|\cos x|+|\cos x|^2+\ldots} = 4$ in $(-\pi,\pi)$, is **[2024]**

(A) 2 (B) 3

(C) 4 (D) 6

113. If for certain x, $3\cos x \neq 2\sin x$, then the general solution of, $\sin^2x - \cos 2x = 2 - \sin 2x$, is **[2024]**

(A) $(2n+1)\frac{\pi}{2}, n \in \mathbb{Z}$

(B) $(2n+1)\frac{\pi}{4}, n \in \mathbb{Z}$

(C) $n\pi + (-1)^n \frac{\pi}{3}, n \in \mathbb{Z}$

(D) $\frac{n\pi}{2} + 1, n \in \mathbb{Z}$

114. If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$ and $x^2 + y^2 + z^2 + kxyz = 1$, then k is **[2024]**

(A) -1 (B) 1

(C) -2 (D) 2

115. If x, y, z are in Arithmetic Progression and $\tan^{-1}x$, $\tan^{-1}y$, $\tan^{-1}z$ are also in Arithmetic progression, where $x, z > 0$ and $xz < 1$, $y < 1$, then **[2024, 2023]**

(A) x = y = z

(B) 2x = 3y = 6z

(C) 6x = 3y = 2z

(D) 6x = 4y = 3z

116. If $\tan^{-1}(x + 2) + \tan^{-1}(x - 2) - \tan^{-1}(\frac{1}{2}) = 0$, then one value of *x* is **[2024]**

(A) –1 (B) $\frac{1}{2}$

(C) 1 (D) 2

117. If angle θ in $[0, 2\pi]$ satisfies both the equations $\cot\theta = \sqrt{3}$ and $\sqrt{3}\sec\theta + 2 = 0$, then θ is equal to **[2024]**

(A) $\frac{\pi}{6}$ (B) $\frac{7\pi}{6}$

(C) $\frac{5\pi}{6}$ (D) $\frac{11\pi}{6}$

118. The angles of a triangle are in the ratio 5:1:6, then ratio of the smallest side to the greatest side is **[2024]**

(A) $\sqrt{3}+1 : 2\sqrt{2}$ (B) $2\sqrt{2} : \sqrt{3}+1$

(C) $2\sqrt{2} : \sqrt{3}-1$ (D) $\sqrt{3}-1 : 2\sqrt{2}$

119. If the sides of a triangle *a,b,c* are in *A.P.*, then with usual notations, $a\cos^2\frac{C}{2} + c\cos^2\frac{A}{2}$ is **[2024, 2018]**

(A) $\frac{3a}{2}$ (B) $\frac{3c}{2}$

(C) $\frac{3b}{2}$ (D) $\frac{a+c}{2}$

120. If $\cos^{-1}x = \alpha (0 < x < 1)$ and $\sin^{-1}(2x\sqrt{1-x^2}) + \sec^{-1}(\frac{1}{2x^2-1}) = \frac{2\pi}{3}$, then α is **[2024]**

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{6}$

(C) $\frac{\pi}{3}$ (D) $\frac{\pi}{4}$

121. If $0 < x < 1$, then

$$\sqrt{1+x^2}\left[\left\{x\cos\left(\cot^{-1}x\right) + \sin\left(\cot^{-1}x\right)\right\}^2 - 1\right]^{\frac{1}{2}}$$ is equal to **[2024]**

(A) $x^2\sqrt{1+x^2}$

(B) x

(C) $x\sqrt{1+x^2}$

(D) $\sqrt{1+x^2}$

122. In a triangle ABC, with usual notations, $2ac\sin\left(\frac{A-B+C}{2}\right)$ is equal to **[2024, 2023]**

(A) $a^2 + b^2 - c^2$ (B) $c^2 + a^2 - b^2$
(C) $b^2 - a^2 + c^2$ (D) $a^2 - b^2 - c^2$

123. The number of values of x in the interval $(0, 5\pi)$ satisfying the equation $3\sin^2x - 7\sin x + 2 = 0$ **[2024, 2019]**

(A) 0 (B) 5
(C) 6 (D) 10

124. If $\tan^{-1}(\frac{1}{4}) + \tan^{-1}(\frac{2}{9}) = \frac{1}{2}\cos^{-1}x$, then x is **[2024]**

(A) $\frac{1}{5}$ (B) $\frac{2}{5}$
(C) $\frac{3}{5}$ (D) $\frac{4}{5}$

125. If $\cos^{-1}(\frac{12}{13}) + \sin^{-1}(\frac{3}{5}) = \sin^{-1}P$, then the value of P is **[2024, 2021]**

(A) $\frac{63}{65}$ (B) $\frac{56}{65}$
(C) $\frac{48}{65}$ (D) $\frac{36}{65}$

126. In a triangle ABC, l(AB) = $\sqrt{23}$ units, l(BC) = 3 units, l(CA) = 4 units, then $\frac{\cot A + \cot C}{\cot B}$ is **[2024]**

(A) 1 (B) 2
(C) 4 (D) 8

127. The value of $\cos^{-1}\left\{\frac{1}{\sqrt{2}}\left(\cos\frac{9\pi}{10} - \sin\frac{9\pi}{10}\right)\right\}$ is **[2025, 2024]**

(A) $\frac{7\pi}{20}$ (B) $\frac{13\pi}{20}$
(C) $\frac{17\pi}{20}$ (D) $\frac{21\pi}{20}$

128. The slopes of the lines given by $x^2 + 2hxy + 2y^2 = 0$ are in the ratio 1:2, then h is **[2024]**

(A) $\frac{1}{2}$ (B) $\frac{3}{2}$
(C) 3 (D) 1

129. The equation of pair of lines y = px and y = qx can be written as (y – px) (y - qx) = 0. Then the equation of the pair of the angle bisectors of the lines $x^2 - 4xy - 5y^2 = 0$ is **[2024]**

(A) $x^2 - 3xy + y^2 = 0$ (B) $x^2 + 4xy - y^2 = 0$
(C) $x^2 - 3xy - y^2 = 0$ (D) $x^2 + 3xy - y^2 = 0$

130. The combined equation of two lines through the origin and making an angle of 45° with the line 3x + y = 0, is **[2024]**

(A) $2x^2-3xy - 2y^2 = 0$ (B) $2x^2 + 3xy + 4y^2 = 0$
(C) $2x^2 + 3xy - 2y^2 = 0$ (D) $2x^2-3xy + 2y^2 = 0$

131. ΔOAB is formed by the lines $x^2 - 4xy + y^2 = 0$ and the line AB. The equation of line AB is 2x + 3y - 1 = 0. Then the equation of the median of the triangle drawn from the origin is **[2024]**

(A) 7x + 8y = 0 (B) 7x - 8y = 0
(C) 8x + 7y = 0 (D) 8x - 7y = 0

132. If P_1 and P_2 are perpendicular distances (in units) from point (2, -1) to the pair of lines $2x^2 - 5xy + 2y^2 = 0$, then the value of P_1P_2 is **[2024, 2021]**

(A) 2 (B) 5
(C) 10 (D) 4

133. If C is a given non-zero scalar and $\bar{A}$ and $\bar{B}$ are given non-zero vectors such that $\bar{A}$ is perpendicular to $\bar{B}$. If vector $\bar{X}$ is such that $\bar{A}\cdot\bar{X} = C$ and $\bar{A}\times\bar{X} = \bar{B}$ then $\bar{X}$ is given by **[2024]**

(A) $\frac{C\bar{A} + \bar{A}\times\bar{B}}{|\bar{A}|^2}$ (B) $\frac{C\bar{A}\times\bar{B}}{|\bar{A}|^2}$
(C) $\frac{C\bar{A} - \bar{A}\times\bar{B}}{|\bar{A}|^2}$ (D) $\frac{C\bar{A} + \bar{B}}{|\bar{A}|^2}$

134. If $\bar{a}=\hat{i}+2\hat{j}+3\hat{k}$ and $\bar{b}=\hat{i}\times(\bar{a}\times\hat{i})+\hat{j}\times(\bar{a}\times\hat{j})+\hat{k}\times(\bar{a}\times\hat{k})$ then $|\bar{b}|$ is **[2024]**

(A) $\sqrt{12}$ (B) $2\sqrt{12}$

(C) $3\sqrt{14}$ (D) $2\sqrt{14}$

135. If $A\equiv(1,-1,0)$, $B\equiv(0,1,-1)$ and $C\equiv(-1,0,1)$, then the unit vector $\bar{d}$ such that $\bar{a}$ and $\bar{d}$ are perpendiculars and $\bar{b}$, $\bar{c}$, $\bar{d}$ are coplanar is **[2024]**

(A) $+\frac{1}{\sqrt{3}}(1,1,1)$ (B) $+\frac{1}{\sqrt{3}}(-1,-1,1)$

(C) $+\frac{1}{\sqrt{6}}(1,1,-2)$ (D) $+\frac{1}{\sqrt{2}}(1,1,0)$

136. The vectors $\bar{a}$ and $\bar{b}$ are not perpendicular and $\bar{c}$ and $\bar{d}$ are two vectors satisfying $\bar{b}\times\bar{c}=\bar{b}\times\bar{d}$ and $\bar{a}\cdot\bar{d}=0$, then the vector $\bar{d}$ is equal to **[2024]**

(A) $\bar{b}+\left(\frac{\bar{b}\cdot\bar{c}}{\bar{a}\cdot\bar{b}}\right)\bar{c}$ (B) $\bar{c}-\left(\frac{\bar{a}\cdot\bar{c}}{\bar{a}\cdot\bar{b}}\right)\bar{b}$

(C) $\bar{b}-\left(\frac{\bar{b}\cdot\bar{c}}{\bar{a}\cdot\bar{b}}\right)\bar{c}$ (D) $\bar{c}+\left(\frac{\bar{a}\cdot\bar{c}}{\bar{a}\cdot\bar{b}}\right)\bar{b}$

137. If $\bar{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$, $\bar{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ and $\bar{c}=c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$ are non-zero non-coplanar vectors and m is non-zero scalar such that $\left[m\bar{a}+\bar{b}\quad m\bar{b}+\bar{c}\quad m\bar{c}+\bar{a}\right]=28\left[\bar{a}\ \bar{b}\ \bar{c}\right]$, then the value of m is equal to **[2024]**

(A) 2 (B) 3

(C) 4 (D) 7

138. Let $\bar{p}$ and $\bar{q}$ be the position vectors of P and Q respectively, with respect to O and $|\bar{p}|=p,|\bar{q}|=q$. The points R and S divide PQ internally and externally in the ratio 2 : 3 respectively. If OR and OS are perpendiculars, then **[2024]**

(A) $9p^2=4q^2$

(B) $4p^2=9q^2$

(C) $9p=4q$

(D) $4p=9q$

139. The incenter of the triangle ABC, whose vertices are A(0,2,1),B(-2,0,0) and C (–2,0,2) is **[2024, 2023]**

(A) $(\frac{3}{2},\frac{1}{2},-1)$

(B) $(\frac{3}{2},\frac{1}{2},1)$

(C) $(-\frac{3}{2},\frac{-1}{2},1)$

(D) $(\frac{-3}{2},-\frac{1}{2},-1)$

140. Suppose that $\vec{p},\vec{q}$ and $\vec{r}$ are three non-coplanar vectors in $\mathbb{R}^3$. Let the components of a vector $\vec{s}$ along $\vec{p},\vec{q}$ and $\vec{r}$ be 4, 3 and 5 respectively. If the components of this vector $\vec{s}$ along $(-\vec{p}+\vec{q}+\vec{r}),(\vec{p}-\vec{q}+\vec{r})$ and $(-\vec{p}-\vec{q}+\vec{r})$ are x, y and z respectively, then the value of $2x+y+z$ is **[2024]**

(A) 10 (B) 6

(C) 9 (D) 8

141. Let $\vec{a}=2\hat{i}+\hat{j}-2\hat{k}$ and $\vec{b}=\hat{i}+\hat{j}$ If $\vec{c}$ is a vector such that $\vec{a}\cdot\vec{c}=|\vec{c}|$, $|\vec{c}-\vec{a}|=2\sqrt{2}$ and the angle between $(\vec{a}\times\vec{b})$ and $\vec{c}$ is 60°, then the value of $\left|(\vec{a}\times\vec{b})\times\vec{c}\right|$ is **[2024, 2021]**

(A) $\frac{\sqrt{3}}{2}$ (B) $\frac{3\sqrt{3}}{2}$

(C) $\frac{5\sqrt{3}}{2}$ (D) $\frac{\sqrt{3}}{4}$

142. Let $\vec{a}=\alpha\hat{i}+3\hat{j}-\hat{k}$, $\vec{b}=3\hat{i}-\beta\hat{j}+4\hat{k}$ and $\vec{c}=\hat{i}+2\hat{j}-2\hat{k}$, where $\alpha,\beta\in R$, be three vectors. If the projection at $\vec{a}$ on $\vec{c}$ is $\frac{10}{3}$ and $\vec{b}\times\vec{c}=-6\hat{i}+10\hat{j}+7\hat{k}$, then the value of $\alpha^2+\beta^2-\alpha\beta$ is equal to **[2024]**

(A) 1 (B) 2

(C) 3 (D) 4

143. The vector $\vec{a}=\alpha\hat{i}+2\hat{j}+\beta\hat{k}$ lies in the plane of the vectors $\vec{b}=\hat{i}+\hat{j}$ and $\vec{c}=\hat{j}+\hat{k}$ and bisects the angle between $\vec{b}$ and $\vec{c}$. Then which one of the following gives possible values of α and β? **[2024]**

(A) $\alpha=1,\beta=1$ (B) $\alpha=2,\beta=2$
(C) $\alpha=1,\beta=2$ (D) $\alpha=2,\beta=1$

144. Let $\vec{a}=3\hat{i}-\alpha\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}+\alpha\hat{j}+3\hat{k}$. If the area of the parallelogram whose adjacent sides are represented by the vectors $\vec{a}$ and $\vec{b}$, is $8\sqrt{3}$ sq. units, then $\vec{a}\cdot\vec{b}$ is equal to **[2024]**

(A) 1 (B) 2
(C) 3 (D) 4

145. If $\vec{a}$ and $\vec{c}$ are unit vectors inclined at $\frac{\pi}{3}$ with each other and $\left(\vec{a}\times\left(\vec{b}\times\vec{c}\right)\right)\cdot\left(\vec{a}\times\vec{c}\right)=5$, then the value of $5[\vec{a}\ \ \vec{b}\ \ \vec{c}\,]=$ **[2024]**

(A) -10 (B) 10
(C) 50 (D) -50

146. If $|\bar{a}|=2,|\bar{b}|=3$ and $\bar{a},\bar{b}$ are mutually perpendicular vectors, then the area of the triangle whose vertices are 0, $\bar{a}+2\bar{b}$, $\bar{a}-2\bar{b}$ is **[2024]**

(A) 6 sq.units (B) 12 sq.units
(C) 24 sq.units (D) 8 sq.units

147. Let $\bar{A},\bar{B},\bar{C}$ be vectors of lengths 3 units, 4 units, 5 units respectively. let $\bar{A}$ be perpendicular to $\bar{B}+\bar{C}$, $\bar{B}$ be perpendicular to $\bar{C}+\bar{A}$ and $\bar{C}$ be perpendicular to $\bar{A}+\bar{B}$, then the length of vector $\bar{A}+\bar{B}+\bar{C}$ is **[2024, 2022]**

(A) $2\sqrt{5}$ (B) $\sqrt{30}$
(C) $\sqrt{45}$ (D) $5\sqrt{2}$

148. If $\bar{a},\bar{b},\bar{c}$ are mutually perpendicular vectors having magnitudes 1, 2, 3 respectively, then the value of $\left[\bar{a}+\bar{b}+\bar{c}\ \ \bar{b}-\bar{a}\ \bar{c}\right]$ is **[2024, 2018]**

(A) 0 (B) 6
(C) 12 (D) 18

149. $\vec{a}=\hat{i}+\hat{j}+\hat{k},\vec{b}=4\hat{i}-2\hat{j}+3\hat{k},\vec{c}=\hat{i}-2\hat{j}+\hat{k}$, then a vector of magnitude 6 units, which is parallel to the vector $2\vec{a}-\vec{b}+3\vec{c}$, is **[2024]**

(A) $2\hat{i}-4\hat{j}+4\hat{k}$ (B) $\hat{i}-\hat{j}+2\hat{k}$
(C) $4\hat{i}+4\hat{j}-2\hat{k}$ (D) $2\hat{i}+4\hat{j}+4\hat{k}$

150. If the vectors $\vec{a}=\hat{i}-\hat{j}+2\hat{k},\vec{b}=2\hat{i}+4\hat{j}+\hat{k}$ and $\vec{c}=\lambda\hat{i}+\hat{j}+\mu\hat{k}$ are mutually orthogonal, then $(\lambda,\mu)=$ **[2024]**

(A) (-3,2) (B) (2,-3)
(C) (-2,3) (D) (3,-2)

151. Let $\vec{a}=\hat{i}+\hat{j}+\hat{k},\vec{b}=\hat{i}-\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}-\hat{j}-\hat{k}$ be three vectors. A vector $\vec{v}$ in the plane of $\vec{a}$ and $\vec{b}$, whose projection on $\vec{c}$ is $\frac{1}{\sqrt{3}}$, is given by **[2024, 2022]**

(A) $\hat{i}-3\hat{j}+3\hat{k}$
(B) $-3\hat{i}-3\hat{j}-\hat{k}$
(C) $3\hat{i}-\hat{j}+3\hat{k}$
(D) $\hat{i}+3\hat{j}-3\hat{k}$

152. Let P, Q, R and S be the points on the plane with position vectors $-2\hat{i}-\hat{j},4\hat{i},3\hat{i}+3\hat{j}$ and $-3\hat{i}+2\hat{j}$ respectively. Then the quadrilateral PQRS must be a **[2024]**

(A) parallelogram, which is neither a rhombus nor a rectangle.
(B) square.
(C) rectangle, but not a square.
(D) rhombus, but not a square.

153. If the points P, Q and R are with the position vectors $\hat{i}-2\hat{j}+3\hat{k},-2\hat{i}+3\hat{j}+2\hat{k}$ and $-8\hat{i}+13\hat{j}$ respectively, then these points are **[2024]**

(A) collinear and Q lies between P and R.
(B) collinear and R lies between P and Q.
(C) collinear and P lies between Q and R.
(D) non-collinear.

154. One side and one diagonal of a parallelogram are represented by $3\hat{i}+\hat{j}-\hat{k}$ and $2\hat{i}+\hat{j}-2\hat{k}$ respectively, then the area of parallelogram in square units is **[2024]**

(A) $2\sqrt{3}$ (B) $3\sqrt{2}$

(C) $6\sqrt{2}$ (D) $4\sqrt{3}$

155. If $\vec{a},\vec{b}$ and $\vec{c}$ are unit coplanar vectors, then the scalar triple product $\left[2\vec{a}-\vec{b}\quad 2\vec{b}-\vec{c}\quad 2\vec{c}-\vec{a}\right]$ has the value **[2024, 2019]**

(A) 0 (B) 1

(C) $-\sqrt{3}$ (D) $\sqrt{3}$

156. If $\left[\bar{a}\times\bar{b}\ \bar{b}\times\bar{c}\ \bar{c}\times\bar{a}\right]=\lambda\left[\bar{a}\,\bar{b}\,\bar{c}\right]^2$, then λ is equal to **[2024]**

(A) 3 (B) 0

(C) 1 (D) 2

157. If $\bar{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$, $\bar{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$, $\bar{c}=c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$ and

$$\left[3\bar{a}+\bar{b}\quad 3\bar{b}+\bar{c}\quad 3\bar{c}+\bar{a}\right]=\lambda\begin{vmatrix}\bar{a}\cdot\hat{i} & \bar{a}\cdot\hat{j} & \bar{a}\cdot\hat{k}\\ \bar{b}\cdot\hat{i} & \bar{b}\cdot\hat{j} & \bar{b}\cdot\hat{k}\\ \bar{c}\cdot\hat{i} & \bar{c}\cdot\hat{j} & \bar{c}\cdot\hat{k}\end{vmatrix},$$

then the value of λ is **[2024]**

(A) 27 (B) 28

(C) 4 (D) 3

158. The equation of the line passing through the point (–1, 3, –2) and perpendicular to each of the lines $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and $\frac{x+2}{-3}=\frac{y-1}{2}=\frac{z+1}{5}$, is **[2024]**

(A) $\frac{x+1}{2}=\frac{y-3}{7}=\frac{z+2}{4}$

(B) $\frac{x+1}{-2}=\frac{y-3}{-7}=\frac{z+2}{4}$

(C) $\frac{x+1}{2}=\frac{y-3}{7}=\frac{z+2}{-4}$

(D) $\frac{x+1}{2}=\frac{y-3}{-7}=\frac{z+2}{4}$

159. The equation of the plane, passing through the mid point of the line segment of join of the points $P(1,2,5)$ and $Q(3,4,3)$ and perpendicular to it, is **[2024]**

(A) x+y-z+1=0

(B) x+y-z-1 = 0

(C) x+y+z+1=0

(D) x-y-z+1= 0

160. The equation of the line, through $A(1,2,3)$ and perpendicular to the vector $2\hat{i}+\hat{j}-\hat{k}$ and $\hat{i}+3\hat{j}+2\hat{k}$, is **[2024]**

(A) $\vec{r}=\left(\hat{i}+2\hat{j}+3\hat{k}\right)+\lambda\left(\hat{i}+\hat{j}+\hat{k}\right)$

(B) $\vec{r}=\left(\hat{i}+2\hat{j}+3\hat{k}\right)+\lambda\left(\hat{i}-\hat{j}-\hat{k}\right)$

(C) $\vec{r}=\left(\hat{i}+2\hat{j}+3\hat{k}\right)+\lambda\left(\hat{i}+\hat{j}-\hat{k}\right)$

(D) $\vec{r}=\left(\hat{i}+2\hat{j}+3\hat{k}\right)+\lambda\left(\hat{i}-\hat{j}+\hat{k}\right)$

161. Let P be the image of the point (3, 1, 7) with respect to the plane $x - y + z = 3$. Then the equation of the plane passing through P and containing the straight line $\frac{x}{1}=\frac{y}{2}=\frac{z}{1}$ is **[2024]**

(A) $-4y - x + 7z = 0$

(B) $x - 4y - 7z = 0$

(C) $x - 4y + 7z = 0$

(D) $x + 4y + 7z = 0$

162. The image of the line $\frac{x-1}{3}=\frac{y-3}{1}=\frac{z-4}{-5}$ in the plane $2x - y + z + 3=0$ is the line **[2024]**

(A) $\frac{x+3}{-3}=\frac{y-5}{-1}=\frac{z+2}{5}$

(B) $\frac{x-3}{3}=\frac{y+5}{1}=\frac{z-2}{-5}$

(C) $\frac{x-3}{-3}=\frac{y+5}{-1}=\frac{z-2}{5}$

(D) $\frac{x+3}{3}=\frac{y-5}{1}=\frac{z-2}{-5}$

163. The equation of the line passing through the point of intersection of the lines $3x - y = 5$ and $x + 3y = 1$ and making equal intercepts on the axes is **[2024]**

(A) $5x+5y-7 = 0$
(B) $5x-5y-7 = 0$
(C) $2x + y - 7=0$
(D) $x-y+7=0$

164. Equation of the plane containing the straight line $\frac{x}{2}=\frac{y}{3}=\frac{z}{4}$ and perpendicular to the plane containing the straight lines $\frac{x}{3}=\frac{y}{4}=\frac{z}{2}$ and $\frac{x}{4}=\frac{y}{2}=\frac{z}{3}$ is **[2024, 2023]**

(A) $x+2y-2z = 0$
(B) $3x + 2y - 2z = 0$
(C) $x-2y + z = 0$
(D) $5x + 2y - 4z = 0$

165. On which of the following lines lies the point of intersection of the line, $\frac{x-4}{2}=\frac{y-5}{2}=\frac{z-3}{1}$ and the plane $x + y + z = 2$? **[2024]**

(A) $\frac{x-1}{1}=\frac{y-3}{2}=\frac{z+4}{-5}$
(B) $\frac{x-4}{1}=\frac{y-5}{1}=\frac{z-5}{-1}$
(C) $\frac{x-2}{2}=\frac{y-3}{2}=\frac{z+3}{3}$
(D) $\frac{x+3}{3}=\frac{4-y}{3}=\frac{z+1}{-2}$

166. Let the vectors $\vec{a},\vec{b},\vec{c}$ and $\vec{d}$ be such that $\left(\vec{a}\times\vec{b}\right)\times\left(\vec{c}\times\vec{d}\right)=\vec{0}$. Let P_1 and P_2 be the planes determined by the pair of vectors $\vec{a},\vec{b}$ and $\vec{c},\vec{d}$ respectively, then the angle between P_1 and P_2 is **[2024]**

(A) 0 (B) $\frac{\pi}{4}$
(C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

167. Equation of the plane, through the points (–1, 2, –2) and (–1, 3, 2) and perpendicular to yz - plane, is **[2024]**

(A) $4y + z = 10$
(B) $4y - z + 10 = 0$
(C) $4y - z = 10$
(D) $4y + z + 10 = 0$

168. The equation of the plane through the point (2, -1, -3) and parallel to the lines $\frac{x-1}{3}=\frac{y+2}{2}=\frac{z}{-4}$ and $\frac{x}{2}=\frac{y-1}{-3}=\frac{z-2}{2}$ is **[2024]**

(A) $8x + y - 13z + 27 = 0$
(B) $2x + y + z = 0$
(C) $3x - y - z - 10 = 0$
(D) $8x + 14y + 13z + 37 = 0$

169. A variable plane passes through the fixed point (3, 2, 1) and meets X, Y and Z axes at points A, B and C respectively. A plane is drawn parallel to YZ - plane through A, a second plane is drawn parallel to ZX-plane through B, a third plane is drawn parallel to XY - plane through C. Then locus of the point of intersection of these three planes, is **[2024]**

(A) $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{11}{6}$ (B) $\frac{x}{3}+\frac{y}{2}+\frac{z}{1}=1$
(C) $\frac{3}{x}+\frac{2}{y}+\frac{1}{z}=1$ (D) $x+y+z=6$

170. The distance of the point (1,-5,9) from the plane $x - y + z = 5$ measured along the line x = y = z is _____ units. **[2024]**

(A) $3\sqrt{10}$
(B) $10\sqrt{3}$
(C) $\frac{10}{\sqrt{3}}$
(D) $\frac{20}{3}$

171. The perpendicular distance from the origin to the plane containing the two lines $\frac{x+2}{3}=\frac{y-2}{5}=\frac{z+5}{7}$ and $\frac{x-1}{1}=\frac{y-4}{4}=\frac{z+4}{7}$, is **[2024]**

(A) $\frac{11}{\sqrt{6}}$ units (B) $11\sqrt{6}$ units

(C) 11 units (D) $6\sqrt{11}$ units

172. The equation of a line passing through the point (2, -1, 1) and parallel to the line joining the points $\hat{i}+2\hat{j}+2\hat{k}$ and $-\hat{i}+4\hat{j}+\hat{k}$ is **[2024]**

(A) $\vec{r}=\left(2\hat{i}-\hat{j}+\hat{k}\right)+\lambda\left(-2\hat{i}+2\hat{j}-\hat{k}\right)$

(B) $\vec{r}=\left(2\hat{i}-\hat{j}+\hat{k}\right)+\lambda\left(2\hat{i}+6\hat{j}+3\hat{k}\right)$

(C) $\vec{r}=\left(2\hat{i}-\hat{j}+\hat{k}\right)+\lambda\left(2\hat{i}-2\hat{j}-\hat{k}\right)$

(D) $\vec{r}=\left(2\hat{i}-\hat{j}+\hat{k}\right)+\lambda\left(2\hat{i}-6\hat{j}-3\hat{k}\right)$

173. A line with positive direction cosines passes through the point P(2, -1, 2) and makes equal angles with co-ordinate axes. The line meets the plane $2x + y + z = 9$ at point Q. Then the length of the line segment PQ equals **[2024, 2023]**

(A) 1 units

(B) $\sqrt{2}$ units

(C) $\sqrt{3}$ units

(D) 2 units

174. The length of the projection of the line segment joining the points (5, -1, 4) and (4, -1, 3) on the plane $x + y + z = 7$ is **[2024, 2023]**

(A) $\sqrt{\frac{2}{3}}$ units

(B) $\frac{2}{\sqrt{3}}$ units

(C) $\frac{2}{3}$ units

(D) $\frac{\sqrt{2}}{3}$ units

175. A line having direction ratios 1, -4, 2 intersects the lines $\frac{x-7}{3}=\frac{y-1}{-1}=\frac{z+2}{1}$ and $\frac{x}{2}=\frac{y-7}{3}=\frac{z}{1}$ at the points A and B resp., then co-ordinates of points A and B are **[2024]**

(A) A(-8, 6, -7) B(-6, -2, -3)

(B) A(8, 6, 7) B(6, 2, 3)

(C) A(8, 6, 7) B(6, -2, -3)

(D) A(7, 6, 8) B(-3, -2, 6)

176. A plane makes positive intercepts of unit length on each of X and Y axis. If it passes through the point (–1, 1, 2) and makes angle θ with the X-axis, then θ is **[2024]**

(A) $\cos^{-1}(\frac{2}{3})$

(B) $\cos^{-1}(\frac{1}{3})$

(C) $\sin^{-1}(\frac{1}{3})$

(D) $\sin^{-1}(\frac{2}{3})$

177. If the line $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$ intersect, then the value of k is **[2025, 2024, 2023]**

(A) $\frac{3}{2}$

(B) $\frac{9}{2}$

(C) $\frac{-2}{9}$

(D) $-\frac{3}{2}$

178. The maximum value of $Z = x + y$, subjected to $x + y \le 10$, $5x + 3y \ge 15$, $x \le 6$, $x, y \ge 0$ **[2024]**

(A) occurs only at unique point

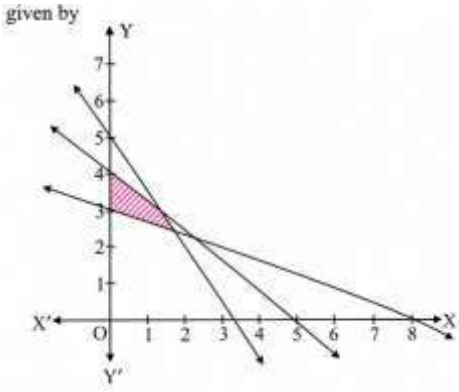
(B) occurs only at two distinct points

(C) occurs at infinitely many points

(D) does not exist

179. The function to be maximized is given by Z=3x+2y. The feasible region for this function is the shaded region given below, then the linear constraints for this region are given by **[2024]**

given by



(A) $3x+8y \le 24, 4x + 5y \le 20, 5x + 3y \ge 15, x \ge 0, y \ge 0$

(B) $3x+8y \ge 24, 4x + 5y \ge 20, 5x + 3y \le 15, x \ge 0, y \ge 0$

(C) $3x+8y \le 24, 4x+5y \ge 20, 5x + 3y \ge 15, x \ge 0, y \ge 0$

(D) $3x+8y \ge 24, 4x + 5y \le 20, 5x + 3y \le 15, x \ge 0, y \ge 0$

180. A production unit makes special type of metal chips by combining copper and brass. The standard weight of the chip must be at least 5 gms. The basic ingredients i.e. copper and brass cost ₹8 and ₹5 per gm. The durability considerations dictate that the metal chip must no contain more than 4 gms of brass and should contain minimum 2 gms of copper. Then the minimum cost of the metal chip satisfying the above conditions is **[2024]**

(A) ₹ 36

(B) ₹ 31

(C) ₹ 30

(D) ₹ 40

181. The region represented by the inequations $2x + 3y \le 18, x + y \ge 10, x \ge 0, y \ge 0$ is **[2024, 2022]**

(A) unbounded

(B) bounded region, but not a singleton set

(C) singleton set

(D) null set

182. Let K be the set of all real values of x, where the function $f(x) = \sin|x|-|x|+2(x-\pi)\cos|x|$ is not differentiable. Then the set K is **[2024]**

(A) {0} (B) an empty set

(C) $\{\pi\}$ (D) $\{0,\pi\}$

183. If $y = \sqrt{\frac{1-\sin^{-1}x}{1+\sin^{-1}x}}$, then $\left(\frac{dy}{dx}\right)$ at x = 0 is **[2024]**

(A) 1 (B) 2

(C) -2 (D) -1

184. If $f(x) = \sin^{-1}\left(\frac{2\cdot 3^x}{1+9^x}\right)$, then $f'\left(\frac{1}{2}\right)$ equals **[2024]**

(A) $\sqrt{3}\log\left(\sqrt{3}\right)$ (B) $-\sqrt{3}\log 3$

(C) $-\sqrt{3}\log\left(\sqrt{3}\right)$ (D) $\sqrt{3}\log 3$

185. If $F(x) = \left(f\left(\frac{x}{2}\right)\right)^2 + \left(g\left(\frac{x}{2}\right)\right)^2$, where f''(x) = -f(x) and g(x) = f'(x) and given by F(5)=5, then F(10) is equal to **[2024]**

(A) 5 (B) 10

(C) 15 (D) 0

186. Let f be a twice differentiable function such that f''(x)=-f(x), f'(x) = g(x) and $h(x) = [f(x)]^2 + [g(x)]^2$. If h(5) = 1, then h(10) is **[2024]**

(A) 2 (B) 4

(C) -1 (D) 1

187. If $y = \tan^{-1}\left(\frac{3+2x}{2-3x}\right) + \tan^{-1}\left(\frac{3x}{1+4x^2}\right)$, then $\frac{dy}{dx}$ is equal to **[2024]**

(A) $\frac{1}{1+16x^2}$

(B) $\frac{4}{1+16x^2}$

(C) $\frac{1}{1+4x^2}$

(D) $\frac{4}{1+4x^2}$

188. If $8f(x)+6f\left(\frac{1}{x}\right)=x+5$ and $y=x^2f(x)$, then $\frac{dy}{dx}$ at $x=-1$ is **[2024]**

(A) 14 (B) -14

(C) $\frac{1}{14}$ (D) $-\frac{1}{14}$

189. If the function $f(x)=x^3+e^{\frac{x}{2}}$ and $g(x)=f^{-1}(x)$ then the value of g'(1) is **[2024]**

(A) 1 (B) 0

(C) 2 (D) $\frac{1}{2}$

190. $\frac{d}{dx}cos^{-1}\left(\frac{x-\frac{1}{x}}{x+\frac{1}{x}}\right)=$ **[2024]**

(A) $\frac{x^2+1}{x^2-1}$

(B) $\frac{2}{1+x^2}$

(C) $\frac{-1}{1+x^2}$

(D) $\frac{-2}{1+x^2}$

191. If $f(x)=\frac{x^2-x}{x^2+2x}$ then $\frac{d}{dx}\left(f^{-1}(x)\right)$ at x = 2 is **[2024]**

(A) -3 (B) 3

(C) -1 (D) 1

192. If y = $sin^{-1}\left(\frac{3x}{2}-\frac{x^3}{2}\right)$, then $\frac{dy}{dx}$ is equal to **[2024]**

(A) $\frac{3}{2\sqrt{x^2-4}}$ (B) $\frac{3}{\sqrt{4-x^2}}$

(C) $\frac{3}{2\sqrt{1-x^2}}$ (D) $\frac{4}{\sqrt{4-x^2}}$

193. If $y=\cfrac{sin\,x}{1+\cfrac{cos\,x}{1+\cfrac{sin\,x}{1+\cfrac{cos\,x}{.....}}}}$, then $\frac{dy}{dx}$ is given by **[2024]**

(A) $\frac{y\,sin\,x+(1+y)cos\,x}{1+2y+cos\,x-sin\,x}$

(B) $\frac{y\,cos\,x+(1+y)sin\,x}{1+2y+cos\,x-sin\,x}$

(C) $\frac{y\,sin\,x-(1+y)cos\,x}{1+2y+cos\,x-sin\,x}$

(D) $\frac{y\,cos\,x-(1+y)sin\,x}{1+2y+cos\,x-sin\,x}$

194. If $x^2y^2=sin^{-1}x+cos^{-1}x$, then $\frac{dy}{dx}$ at x = 1 and y = 2 is **[2024]**

(A) $\frac{1}{2}$ (B) 2

(C) $-\frac{1}{2}$ (D) -2

195. If $y=sin^{-1}\left(\frac{log\,x^2}{1+(log\,x)^2}\right)$. , then $\left(\frac{dy}{dx}\right)_{atx=1}$ = **[2024]**

(A) 2 (B) $\frac{1}{2}$

(C) $\frac{2}{3}$ (D) –2

196. If $\left(a+\sqrt{2}b\,cos\,x\right)\left(a-\sqrt{2}b\,cos\,y\right)=a^2-b^2$, where a > b > 0, then $\frac{dx}{dy}$ at $\left(\frac{\pi}{4},\frac{\pi}{4}\right)$ is **[2024]**

(A) $\frac{a-b}{a+b}$ (B) $\frac{a+b}{a-b}$

(C) $\frac{2a+b}{2a-b}$ (D) $\frac{a-2b}{a+2b}$

197. The function y(x) represented by x = sin t, $y = ae^{t\sqrt{2}} + be^{t\sqrt{2}}$, $t \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ satisfies the equation $\left(1-x^2\right)y'' - xy' = ky$, then the value of k is **[2024]**

(A) 1 (B) 2
(C) -1 (D) 0

198. If y is a function of x and $log\left(x+y\right) = 2xy$, then the value of y'(0) is **[2024, 2023]**

(A) 1 (B) -1
(C) 2 (D) 0

199. If f(1)=1, f'(1)=3, then the derivative of $f\left(f\left(f\left(x\right)\right)\right) + \left(f\left(x\right)\right)^2$ at x=1 is **[2025, 2024, 2023]**

(A) 9 (B) 12
(C) 15 (D) 33

200. For n ∈ N if $y = ax^{n+1} + bx^{-n}$, then $x^2\frac{d^2y}{dx^2} =$ **[2025, 2024]**

(A) n(n-1)y (B) (n - 1)y
(C) n(n+1)y (D) (n + 1)y

201. The length of the longest interval, in which the function 3 sin x – 4sin³x is increasing, is **[2024]**

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{2}$
(C) $\frac{3\pi}{2}$ (D) π

202. If x_0 is the point of local minima of $f\left(x\right) = \bar{a}\cdot\left(\bar{b}\times\bar{c}\right)$ where $\bar{a} = x\hat{i} - 2\hat{j} + 3\hat{k}$, $\bar{b} = -2\hat{i} + x\hat{j} - \hat{k}, \bar{c} = 7\hat{i} - 2\hat{j} + x\hat{k}$, then value of $\bar{a}\cdot\bar{b}$ at $x = x_0$ is **[2024, 2023]**

(A) -3 (B) -15
(C) -12 (D) -9

203. If Rolle's theorem holds for the function $f\left(x\right) = x^3 + bx^2 + ax + 5$ on [1, 3] with $c = 2 + \frac{1}{\sqrt{3}}$, then the values of a and b respectively are **[2024, 2023]**

(A) -11, 6 (B) 11, 6
(C) -11, -6 (D) 11, -6

204. The normal to the curve, y(x-2)(x-3) = x + 6 at the point, where the curve intersects the Y-axis, passes through the point **[2024]**

(A) $\left(-\frac{1}{2}, \frac{-1}{2}\right)$ (B) $\left(\frac{1}{2}, \frac{1}{2}\right)$
(C) $\left(\frac{1}{2}, -\frac{1}{3}\right)$ (D) $\left(\frac{1}{2}, \frac{1}{3}\right)$

205. The maximum value of the function f(x) = 3x³ - 18x² + 27x - 40 on the set S = {x ∈ ℝ / x² + 30 ≤ 11x} is **[2024, 2023]**

(A) −122 (B) −222
(C) 222 (D) 122

206. A triangular park is enclosed on two sides by a fence and on the third side a straight river bank. The two sides having fence are of same length x. The maximum area (in sq. units) enclosed by the park is **[2024]**

(A) $\frac{3}{2}x^2$
(B) $\sqrt{\frac{x^3}{8}}$
(C) $\frac{1}{2}x^2$
(D) πx²

207. If f(x) = x³ + bx² + cx + d and 0 < b² < c, then in (–∞, ∞) **[2024, 2023]**

(A) f(x) is strictly increasing function
(B) f(x) is bounded
(C) f(x) has a local maxima
(D) f(x) is a strictly decreasing function

208. The minimum value of the function $f(x) = 2x^3-15x^2+36x-48$ on the set $A = \{x \mid x^2 + 20 \le 9x\}$ is **[2024]**

(A) –16 (B) –7
(C) 16 (D) 7

209. The sum of intercepts on coordinate axes made by tangent to the curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$ is **[2024]**

(A) a (B) 2a
(C) $2\sqrt{a}$ (D) $\sqrt{2}a$

210. If sum of two numbers is 3, then the maximum value of the product of first number and square of the second number is **[2024]**

(A) 6 (B) 4
(C) 5 (D) 3

211. Let C be a curve given by $y(x) = 1 + \sqrt{4x-3}$, $x > \frac{3}{4}$. If P is a point on C, such that the tangent at P has slope $\frac{2}{3}$, then a point through which the normal at P passes, is **[2024]**

(A) (1, 7)
(B) (3, –4)
(C) (4, -3)
(D) (2, 3)

212. The maximum value of $\frac{\log x}{x}$ is **[2024, 2017]**

(A) e (B) 2e
(C) $\frac{1}{e}$ (D) $\frac{2}{e}$

213. If Mean value theorem holds for the function $f(x) = (x-1)(x-2)(x-3), x \in [0,4]$ then the values of c as per the theorem are **[2024]**

(A) $2 \pm \frac{4}{\sqrt{3}}$ (B) $2 \pm \frac{2}{\sqrt{3}}$
(C) $2 \pm \sqrt{2}$ (D) $2 \pm \sqrt{3}$

214. A poster is to be printed on a rectangular sheet of paper of area 18 m². The margins at the top and bottom of 75 cm each and at the sides 50 cm each are to be left. Then the dimensions i.e. height and breadth of the sheet so that the space available for printing is maximum, are …….. respectively. **[2024, 2023]**

(A) $2\sqrt{3}$ m, $3\sqrt{3}$ m
(B) $3\sqrt{3}$ m, $2\sqrt{3}$ m
(C) 3 m, 6 m
(D) 6 m, 3 m

215. The approximate value of $(3.978)^{3/2}$ is **[2024]**

(A) 7.096
(B) 8.096
(C) 7.934
(D) 8.934

216. The equation of the tangent to the curve $x = a\cos^3\theta$, $y = a\sin^3\theta$ at $\theta = \frac{\pi}{4}$ is **[2024]**

(A) $x+y=\frac{a}{\sqrt{2}}$ (B) $x+y=\frac{a}{2}$
(C) $x+y=\frac{a}{2\sqrt{2}}$ (D) $x+y=\frac{a}{8}$

217. The curve $x^4 - 2xy^2 + y^2 + 3x - 3y = 0$ cuts the X-axis at (0,0) at an angle of **[2024]**

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$
(C) 0 (D) $\frac{\pi}{6}$

218. A point moves along the arc of parabola $y = 2x^2$. Its abscissa increases uniformly at the rate of 2 units/sec. At the instant, the point is passing through (1, 2), its distance from origin is increasing at the rate of **[2024]**

(A) $\frac{36}{\sqrt{5}}$ units/sec. (B) $\frac{18}{\sqrt{5}}$ units/sec.
(C) $\frac{36}{5}$ units/sec. (D) $\frac{18}{5}$ units / sec.

219. The equation of the normal to the curve $y = x\log x$, which is parallel to the line $2x - 2y + 3 = 0$, is **[2024]**

(A) $x + y = 3e^{-2}$
(B) $x - y = 3e^{-2}$
(C) $x - y = 3e^{2}$
(D) $x + y = 3e^{2}$

220. The distance 's' in meters covered by a body in t seconds is given by $s = 3t^2 - 8t + 5$. The body will stop after **[2024]**

(A) 1 sec (B) $\frac{3}{4}$ sec
(C) $\frac{4}{3}$ sec (D) 4 sec

221. Let $f(x) = \frac{x}{\sqrt{a^2+x^2}} - \frac{d-x}{\sqrt{b^2+(d-x)^2}}$, $x \in R$ where a, b, d are non-zero real constants. Then **[2024]**

(A) f' is not a continuous function of x.
(B) f is neither increasing nor decreasing function of x.
(C) f is an increasing function of x.
(D) f is a decreasing function of x.

222. Twenty meters of wire is available for fencing off a flower-bed in the form of a circular sector. Then the maximum area (in sq.m) of the flower-bed is **[2024]**

(A) 30 (B) 12.5
(C) 25 (D) 10

223. If θ denotes the acute angle between the curves $y = 10 - x^2$ and $y = 2 + x^2$, at a point of the intersection, then $|\tan\theta|$ is equal to **[2024]**

(A) $\frac{8}{15}$ (B) $\frac{8}{17}$
(C) $\frac{4}{9}$ (D) $\frac{7}{17}$

224. Let the function $g : (-\infty, \infty) \to (-\frac{\pi}{2}, \frac{\pi}{2})$ be given by $g(u) = 2\tan^{-1}(e^u) - \frac{\pi}{2}$. Then g is **[2024]**

(A) even and is strictly increasing in $(0, \infty)$.
(B) odd and is strictly decreasing in $(-\infty, \infty)$.
(C) odd and is strictly increasing in $(-\infty, \infty)$.
(D) neither even nor odd, but is strictly increasing in $(-\infty, \infty)$.

225. The abscissa of the point on the curve $y = a(e^{x/a} + e^{-x/a})$ where the tangent is parallel to the X-axis is **[2024]**

(A) 0
(B) a
(C) 2a
(D) -2a

226. If the curves $y^2 = 6x$ and $9x^2 + by^2 = 16$ intersect each other at right angles, then the value of b is **[2025, 2024, 2023, 2022]**

(A) 4 (B) $\frac{7}{2}$
(C) 6 (D) $\frac{9}{2}$

227. The equation of motion of the particle is $s = at^2 + bt + c$. If the displacement after 1 second is 20 m, velocity after 2 seconds is 30 m/seconds and the acceleration is 10 m/seconds^2, then **[2025, 2024]**

(A) $a + c = 2b$
(B) $a + c = b$
(C) $a - c = b$
(D) $a + c = 3b$

228. An open tank with a square bottom is to contain 4000 cubic cm. of liquid. The dimensions of the tank so that the surface area of the tank is minimum, is **[2025, 2024]**

(A) side = 20 cm, height = 10 cm
(B) side = 10 cm, height = 20 cm
(C) side = 10 cm, height = 40 cm
(D) side = 20 cm, height = 05 cm

229. $\int \frac{dx}{3-2\cos 2x} = \frac{\tan^{-1}\left(f(x)\right)}{\sqrt{5}} + c$, (where c is a constant of integration), then f(π/4) has the value **[2024]**

(A) $-\sqrt{5}$ (B) $\sqrt{5}$

(C) $\frac{2}{\sqrt{5}}$ (D) $\frac{1}{\sqrt{5}}$

230. If $\int \frac{x+1}{\sqrt{2x-1}}dx = f(x)\sqrt{2x-1} + c$, (where c is a constant of integration), then f(x) is equal to **[2024, 2022]**

(A) $\frac{1}{3}(x+1)$ (B) $\frac{1}{3}(x+4)$

(C) $\frac{2}{3}(x+2)$ (D) $\frac{2}{3}(x-4)$

231. $\int \frac{2x^2-1}{(x^2+4)(x^2-3)}dx =$ **[2024]**

(A) $\frac{9}{14}\tan^{-1}\left(\frac{x}{2}\right) + \frac{5}{14\sqrt{3}}\log\left(\frac{x-\sqrt{3}}{x+\sqrt{3}}\right) + c$, (where c is constant of integration)

(B) $\frac{9}{7}\tan^{-1}\left(\frac{x}{2}\right) + \frac{5}{7\sqrt{3}}\log\left(\frac{x-\sqrt{3}}{x+\sqrt{3}}\right) + c$, (where c is constant of integration)

(C) $\frac{9}{7}\tan^{-1}\left(\frac{x}{2}\right) - \frac{5}{7\sqrt{3}}\log\left(\frac{x-\sqrt{3}}{x+\sqrt{3}}\right) + c$, (where c is constant of integration)

(D) $\frac{9}{14}\tan^{-1}\left(\frac{x}{2}\right) + \frac{5}{7}\log\left(\frac{x-\sqrt{3}}{x+\sqrt{3}}\right) + c$, (where c is constant of integration)

232. If $\int (2x+4)\sqrt{x-1}dx = a(x-1)^{5/2} + b(x-1)^{3/2} + c$ where c is a constant of integration, then the value of (2a + b) is **[2024]**

(A) $\frac{20}{5}$ (B) $\frac{28}{5}$

(C) $\frac{48}{5}$ (D) $\frac{16}{5}$

233. $\int \frac{\text{cosec}x dx}{\cos^2\left(1+\log\tan\frac{x}{2}\right)} =$ **[2024, 2023]**

(A) $\tan\left(1+\log\tan\frac{x}{2}\right) + c$, where c is a constant of integration.

(B) $\frac{1}{2}\tan\left(1+\log\tan\frac{x}{2}\right) + c$, where c is a constant of integration.

(C) $2\tan\left(1+\log\tan\frac{x}{2}\right) + c$, where c is a constant of integration.

(D) $\frac{1}{4}\tan\left(1+\log\tan\frac{x}{2}\right) + c$, where c is a constant of integration.

234. If f($\frac{x-4}{x-2}$) = 2x+1, x $\in$ R-{1,-2}, then $\int f(x)dx$ is equal to **[2024]**

(A) 5x-4log(x-1)+c, where c is constant of integration.

(B) x-4log(x-1)+c, where c is constant of integration.

(C) 5x+4log(x-1)+c, where c is constant of integration.

(D) 5x+log(x-1)+c, where c is constant of integration.

235. The value of I = $\int \frac{x^2}{(a+bx)^2}dx$ is **[2024]**

(A) $\frac{1}{b^3}\left[a+bx+2a\log|a+bx| - \frac{a^2}{a+bx}\right] + c$ (where c is the constant of integration)

(B) $\frac{1}{b^3}\left[a+bx-2a\log|a+bx| + \frac{a^2}{a+bx}\right] + c$ (where c is the constant of integration)

(C) $\frac{1}{b^3}\left[a+bx-2a\log|a+bx| - \frac{a^2}{a+bx}\right] + c$ (where c is the constant of integration)

(D) $\frac{1}{b^3}\left[a+bx+2a\log|a+bx| + \frac{a^2}{a+bx}\right] + c$ (where c is the constant of integration)

236. If $I = \int e^{\sin\theta}\left(\log\sin\theta + \operatorname{cosec}^2\theta\right)\cos\theta\, d\theta$, then I is equal to **[2024]**

(A) $e^{\sin\theta}\left(\log\sin\theta + \operatorname{cosec}^2\theta\right) + c$,
(where c is the constant of integration)

(B) $e^{\sin\theta}\left(\log\sin\theta + \operatorname{cosec}\theta\right) + c$,
(where c is the constant of integration)

(C) $e^{\sin\theta}\left(\log\sin\theta - \operatorname{cosec}\theta\right) + c$,
(where c is a constant of integration)

(D) $e^{\sin\theta}\left(\log\sin\theta - \operatorname{cosec}^2\theta\right) + c$,
(where c is a constant of integration)

237. If $\int \frac{4e^x - 25}{2e^x - 5}dx = Ax + B\log\left(2e^x - 5\right) + c$ (where c is a constant of integration) then **[2024, 2016]**

(A) A = 5, B = 3
(B) A = 5, B = -3
(C) A = -5, B = 3
(D) A = -5, B = -3

238. If $\int f(x)dx = \psi(x)$, then $\int x^5 f\left(x^3\right)dx$ is equal to **[2024]**

(A) $\frac{1}{3}x^3\psi\left(x^3\right) - 3\int x^2\psi\left(x^3\right)dx + c$,
(where c is a constant of integration)

(B) $\frac{1}{3}\left(x^3\psi\left(x^3\right) - \int x^3\psi\left(x^3\right)dx\right) + c$,
(where c is a constant of integration)

(C) $\frac{1}{3}x^3\psi\left(x^3\right) - \int x^2\psi\left(x^3\right)dx + c$,
(where c is a constant of integration)

(D) $\frac{1}{3}\left(x^3\psi\left(x^3\right) - \int x^2\psi\left(x^3\right)dx\right) + c$,
(where c is a constant of integration)

239. If $\int \frac{dx}{\sqrt[3]{\sin^{11}x\cos x}} = -\left(\frac{3}{8}f(x) + \frac{3}{2}g(x)\right) + c$ then **[2024]**

(A) $f(x) = \tan^{\frac{-8}{3}}x, g(x) = \tan^{\frac{-2}{3}}x$,
(where c is a constant of integration)

(B) $f(x) = \tan^{\frac{8}{3}}x, g(x) = \tan^{\frac{-2}{3}}x$,
(where c is a constant of integration)

(C) $f(x) = \tan^{\frac{-8}{3}}x, g(x) = \tan^{\frac{2}{3}}x$,
(where c is a constant of integration)

(D) $f(x) = \tan^{\frac{8}{3}}x, g(x) = \tan^{\frac{2}{3}}x$,
(where c is a constant of integration)

240. $\int \frac{x^4 + x^2 + 1}{x^2 - x + 1}dx$ is equal to **[2024]**

(A) $\frac{x^3}{3} - \frac{x^2}{2} + x + c$,
(where c is a constant of integration)

(B) $\frac{x^3}{3} + \frac{x^2}{2} + x + c$,
(where c is a constant of integration)

(C) $\frac{x^3}{3} - \frac{x^2}{2} - x + c$,
(where c is a constant of integration)

(D) $\frac{x^3}{3} + \frac{x^2}{2} - x + c$,
(where c is a constant of integration)

241. The value of $\int \frac{x+1}{x\left(1+xe^x\right)^2}dx$ is equal to **[2024]**

(A) $log\left(\frac{xe^x}{1+xe^x}\right)+\frac{x}{1+xe^x}+c$,
where c is a constant of integration

(B) $log\left(\frac{xe^x}{1+xe^x}\right)+\frac{e^x}{1+xe^x}+c$,
where c is a constant of integration

(C) $log\left(\frac{xe^x}{1+xe^x}\right)+\frac{1}{1+xe^x}+c$,
where c is a constant of integration

(D) $log\left(\frac{xe^x}{1+xe^x}\right)-\frac{x}{1+xe^x}+c$,
where c is a constant of integration

242. $\int \frac{1+sin(log\,x)}{1+cos(log\,x)}dx =$ **[2024]**

(A) $x^2tan\left(\frac{log\,x}{2}\right)+c$,
where c is a constant of integration.

(B) $xtan\left(log\left(\frac{x}{2}\right)\right)+c$,
where c is a constant of integration.

(C) $x^3log\left(\frac{tan\,x}{2}\right)+c$,
where c is a constant of integration.

(D) $x\cdot tan\left(\frac{log\,x}{2}\right)+c$,
where c is a constant of integration.

243. $\int \frac{dx}{\sqrt{e^x-1}}=2tan^{-1}\left(f\left(x\right)\right)+c$ where x > 0 and c is a constant of integration, then f(x) is **[2024]**

(A) e^x-1

(B) $\sqrt{e^x-1}$

(C) e^x+1

(D) $\sqrt{e^x+1}$

244. $\int \frac{xdx}{(x-1)^2(x+2)} =$ **[2024]**

(A) $\frac{2}{9}log\left(x-1\right)+\frac{1}{3}\times\frac{1}{x-1}+\frac{2}{9}log\left(x+2\right)+c$,
where c is a constant of integration

(B) $\frac{2}{9}log\left(x-1\right)-\frac{1}{3}\times\frac{1}{(x-1)}+\frac{2}{9}log\left(x+2\right)+c$,
where c is a constant of integration

(C) $\frac{2}{9}log\left(x-1\right)+\frac{1}{3}\times\frac{1}{x-1}-\frac{2}{9}log\left(x+2\right)+c$,
where c is a constant of integration

(D) $\frac{2}{9}log\left(x-1\right)-\frac{1}{3}\times\frac{1}{x-1}-\frac{2}{9}log\left(x+2\right)+c$,
where c is a constant of integration

245. If
$\int(7x-2)\sqrt{3x+2}dx=A(3x+2)^{5/2}+B(3x+2)^{3/2}+c$
(where c is a constant of integration), then the values of A and B are respectively **[2024]**

(A) $\frac{14}{45},\frac{40}{27}$

(B) $\frac{14}{15},\frac{-40}{9}$

(C) $\frac{14}{15},\frac{40}{9}$

(D) $\frac{14}{45},\frac{-40}{27}$

246. The value of $\int \frac{cos^3x}{sin^2x+sin\,x}dx$ is **[2024]**

(A) log (sin x) - sin x + c, where c is a constant of integration.

(B) log (sin x) - cos x + c, where c is a constant of integration.

(C) log (sin x) + sin x + c, where c is a constant of integration.

(D) log (cos x) - cos x + c, where c is a constant of integration.

247. If f(x)=1+x;g(x)=log x, then $\int g\left(f\left(x\right)\right)dx$ is equal to **[2024]**

(A) (1+x) log(1+x)-x+c,
(where c is a constant of integration)

(B) (1+x)logx-x+c,
(where c is a constant of integration)

(C) x log(1+x)+c,
(where c is a constant of integration)

(D) (1+x) log(1+x)+x+c,
(where c is a constant of integration)

248. $\int\frac{2x+5}{\sqrt{7-6x-x^2}}dx=A\sqrt{7-6x-x^2}+B\sin^{-1}\left(\frac{x+3}{4}\right)+c$ (where c is a constant of integration) then the value of A + B is **[2024]**

(A) -3 (B) 1
(C) -1 (D) 3

249. The value of $\int\frac{\sec x\tan x}{9-16\tan^2 x}dx$ is equal to **[2024]**

(A) $\frac{1}{24}\log\left(\frac{5+4\sec x}{5-4\sec x}\right)+c$, (where c is a constant of integration)

(B) $\frac{1}{40}\log\left(\frac{5+4\sec x}{5-4\sec x}\right)+c$, (where c is a constant of integration)

(C) $\frac{1}{24}\log\left(\frac{5-4\sec x}{5+4\sec x}\right)+c$, (where c is a constant of integration)

(D) $\frac{1}{40}\log\left(\frac{5-4\sec x}{5+4\sec x}\right)+c$, (where c is a constant of integration)

250. If $f\left(x\right)=\frac{x}{x+1},x\neq-1$ and (fof) $\left(x\right)=F\left(x\right)$, then $\int F\left(x\right)dx$ is **[2024]**

(A) $\frac{x}{2}+\frac{1}{2}\log\left(2x+1\right)+c$, where c is a constant of integration.

(B) $\frac{x}{2}-\frac{1}{4}\log\left(2x+1\right)+c$, where c is a constant of integration.

(C) $\frac{x}{2}-\frac{1}{2}\log\left(2x+1\right)+c$, where c is a constant of integration.

(D) $\frac{x}{2}+\frac{1}{4}\log\left(2x+1\right)+c$, where c is a constant of integration.

251. If $f\left(x\right)=\frac{\sin^2\pi x}{1+\pi^x}$, then $\int\left(f\left(x\right)+f\left(-x\right)\right)dx$ is equal to **[2024]**

(A) $\frac{x}{2}-\frac{\sin\pi x}{2\pi}+c$,
(where c is a constant of integration)

(B) $\frac{1}{2}x-\frac{\sin 2\pi x}{4\pi}+c$,
(where c is a constant of integration)

(C) $\frac{x}{2}-\frac{\cos\pi x}{2\pi}+c$,
(where c is a constant of integration)

(D) $\frac{1}{1+\pi^x}+\frac{\cos^2\pi x}{2\pi}+c$,
(where c is a constant of integration)

252. If $\int\frac{dx}{\cos^3 x\sqrt{2\sin 2x}}=\left(\tan x\right)^A+C\left(\tan x\right)^B+k$ where k is a constant of integration, then A + B + C equals **[2024]**

(A) $\frac{27}{10}$

(B) $\frac{16}{5}$

(C) $\frac{27}{5}$

(D) $\frac{21}{5}$

253. If $\int \frac{log\left(t+\sqrt{1+t^2}\right)}{\sqrt{1+t^2}}dt = \frac{1}{2}\left(g\left(t\right)\right)^2 + c$ where c is a constant of integration, then $g(2)$ is equal to **[2024, 2023]**

(A) $2log\left(2+\sqrt{5}\right)$

(B) $log\left(2+\sqrt{5}\right)$

(C) $\frac{1}{\sqrt{5}}log\left(2+\sqrt{5}\right)$

(D) $\frac{1}{2}log\left(2+\sqrt{5}\right)$

254. $\int \frac{x^3-7x+6}{x^2+3x}dx =$ **[2024]**

(A) $\frac{x^2}{2}+3x-log\,x+c,$ where c is a constant of integration.

(B) $\frac{x^2}{2}+3x+2log\,x+c,$ where c is a constant of integration.

(C) $\frac{x^2}{2}-3x+2log\,x+c,$ where c is a constant of integration.

(D) $\frac{x^2}{2}-3x-log\,x+c,$ where c is a constant of integration.

255. $\int \sec^{\frac{2}{3}} x \cdot \text{cosec}^{\frac{4}{3}} x dx =$ **[2025, 2024]**

(A) $3tan^{-\frac{1}{3}}x+c$, where c is the constant of integration

(B) $-3tan^{-\frac{-1}{3}}x+c$, where c is the constant of integration

(C) $-3cot^{-\frac{1}{3}}x+c$, where c is the constant of integration

(D) $-\frac{3}{4}tan^{\frac{-4}{3}}x+c$, where c is the constant of integration

256. $\int e^x \frac{(x-1)}{(x+1)^3}dx =$ **[2025, 2024]**

(A) $e^x(x+1)^2+c$, where c is the constant of integration

(B) $e^x(x+1)^3+c$, where c is the constant of integration

(C) $\frac{e^x}{(x+1)^2}+c$, where c is the constant of integration

(D) $\frac{e^x}{(x+1)^3}+c$, where c is the constant of integration

257. Let f and g be continuous functions on [0, a] such that f(x) = f(a-x) and g(x)+g(a-x)=4, then $\int_0^a f(x)g(x)dx$ is equal to **[2024]**

(A) $4\int_0^a f(x)dx$

(B) $\int_0^a f(x)dx$

(C) $2\int_0^a f(x)dx$

(D) $-3\int_0^a f(x)dx$

258. If $\int_0^{\frac{\pi}{3}} \frac{\tan\theta}{\sqrt{2k\,\sec\theta}}d\theta = 1-\frac{1}{\sqrt{2}}$, (k > 0) then the value of k is **[2024]**

(A) 2 (B) 1

(C) $\frac{1}{2}$ (D) 4

259. The value of the integral $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(x^2+log\frac{\pi-x}{\pi+x}\right)cos\,x dx$ is equal to **[2024]**

(A) 0 (B) $\frac{\pi^2}{2}-4$

(C) $\frac{\pi^2}{2}$ (D) $\frac{\pi^2}{2}+4$

260. The integral $\int_{-\frac{1}{2}}^{\frac{1}{2}}\left([x]+log_e\left(\frac{1+x}{1-x}\right)\right)dx$, where [x] represent greatest integer function, equals **[2024]**

(A) $-\frac{1}{2}$ (B) $log_e\left(\frac{1}{2}\right)$

(C) $\frac{1}{2}$ (D) $2log_e\left(\frac{1}{2}\right)$

261. The value of $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}}\frac{1}{sin\,2x\left(tan^5x+cot^5x\right)}dx$ is **[2024]**

(A) $\frac{1}{5}\left(\frac{\pi}{4}-tan^{-1}\left(\frac{1}{3\sqrt{3}}\right)\right)$

(B) $\frac{1}{2}\left(\frac{\pi}{4}-tan^{-1}\left(\frac{1}{9\sqrt{3}}\right)\right)$

(C) $\frac{1}{10}\left(\frac{\pi}{4}-tan^{-1}\left(\frac{1}{9\sqrt{3}}\right)\right)$

(D) $\frac{1}{10}\left(\frac{\pi}{4}-tan^{-1}\left(\frac{1}{3\sqrt{3}}\right)\right)$

262. $\int_0^{\frac{\pi}{4}}log\left(\frac{sin\,x+cos\,x}{cos\,x}\right)dx=$ **[2024]**

(A) $\frac{\pi}{2}log\,2$

(B) $\frac{\pi}{4}log\,2$

(C) $\frac{\pi}{6}log\,2$

(D) $\frac{\pi}{8}log\,2$

263. $\int_0^a\frac{x-a}{x+a}dx$ **[2024]**

(A) a - 2a log 2

(B) a + 2a log 2

(C) a - a log 2

(D) a + a log 2

264. $\int_{0.2}^{3.5}[x]dx=$ (where [x] = greatest integer not greater than x) **[2024]**

(A) 4 (B) 4.2

(C) 4.5 (D) 4.4

265. $\int_{log\frac{1}{2}}^{log\,2}sin\left(\frac{e^x-1}{e^x+1}\right)dx=$ **[2024]**

(A) -2 log 2 (B) 0

(C) $\frac{1}{2}$ (D) 2 log 2

266. $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}sin^{-4}x\,dx=$ **[2025, 2024]**

(A) $\frac{8}{3}$ (B) $-\frac{8}{3}$

(C) $\frac{2}{3}$ (D) $-\frac{2}{3}$

267. The area of the region bounded by hyperbola $x^2 - y^2 = 9$ and its latus rectum is **[2024]**

(A) $9\left[\sqrt{2}-log\left(\sqrt{2}+1\right)\right]$ sq. units

(B) $4\left[\sqrt{2}-log\left(\sqrt{2}+1\right)\right]$ sq. units

(C) $3\left[\sqrt{2}-log\left(\sqrt{2}+1\right)\right]$ sq. units

(D) $18\left[\sqrt{2}-log\left(\sqrt{2}+1\right)\right]$ sq. units

268. The area (in sq. units) bounded by the curves $y = \sqrt{x}$, 2y - x + 3 = 0, X-axis and lying in the first quadrant is **[2024, 2022]**

(A) 36 (B) 18

(C) $\frac{27}{4}$ (D) 9

269. The area (in sq. units) bounded by the curves $y = (x + 1)^2$, $y = (x - 1)^2$ and the line $y = \frac{1}{4}$ is **[2024, 2023]**

(A) $\frac{2}{3}$ (B) $\frac{1}{6}$

(C) $\frac{1}{3}$ (D) $\frac{1}{4}$

270. The area (in sq. units), in the first quadrant bounded by the curve $y = x^2 + 2$ and the lines $y = x + 1$, $x = 0$ and $x = 2$, is **[2024]**

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$

(C) $\frac{5}{3}$ (D) $\frac{8}{3}$

271. The area (in sq. units) of the region $\{(x, y)/x \geq 0, x + y \leq 3, x^2 \leq 4y$ and $y \leq 1 + \sqrt{x}\ \}$ is **[2024]**

(A) $\frac{9}{2}$ (B) $\frac{3}{2}$

(C) $\frac{7}{2}$ (D) $\frac{5}{2}$

272. The area (in sq. units) of the region bounded by $y-x=2$ and $x^2 = y$ is equal to **[2024]**

(A) $\frac{2}{3}$ (B) $\frac{4}{3}$

(C) $\frac{9}{2}$ (D) $\frac{16}{3}$

273. The area (in sq. units) bounded between the parabolas $x^2 = \frac{y}{4}$ and $x^2 = 9y$ and the line $y = 2$ is **[2024]**

(A) $20\sqrt{2}$ (B) $\frac{10\sqrt{2}}{3}$

(C) $\frac{20\sqrt{2}}{3}$ (D) $10\sqrt{2}$

274. The solution of the differential equation $\frac{dy}{dx} = (x-y)^2$ when $y(1) = 1$ is **[2024]**

(A) $log\left|\frac{2-y}{2-x}\right| = 2(y-1)$

(B) $-log\left|\frac{1+x-y}{1-x+y}\right| = x+y-2$

(C) $log\left|\frac{2-x}{2-y}\right| = x-y$

(D) $-log\left|\frac{1-x+y}{1+x-y}\right| = 2(x-1)$

275. If $y = asinx + bcosx$ (where a and b are constants), then $y^2 + \left(\frac{dy}{dx}\right)^2$ is **[2024]**

(A) a function of x. (B) a function of x and y.

(C) a function of y. (D) a constant.

276. If order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^5 + 4\left(\frac{\left(\frac{d^2y}{dx^2}\right)^5}{\frac{d^3y}{dx^3}}\right) + \frac{d^3y}{dx^3} = \sin x$, are m and n respectively, then the value of $(m^2 + n^2)$ is equal to **[2024]**

(A) 29 (B) 13

(C) 5 (D) 8

277. Let $y = y(x)$ be the solution of the differential equation $sin\,x\frac{dy}{dx} + y\,cos\,x = 4x, x \in (0, \pi)$. If $y\left(\frac{\pi}{2}\right) = 0$, then $y\left(\frac{\pi}{6}\right)$ is equal to **[2024]**

(A) $\frac{-4}{9}\pi^2$

(B) $\frac{4}{9\sqrt{3}}\pi^2$

(C) $\frac{-8}{9\sqrt{3}}\pi^2$

(D) $\frac{-8}{9}\pi^2$

278. Given that the slope of the tangent to a curve $y = y(x)$ at any point (x, y) is $\frac{2y}{x^2}$. If the curve passes through the centre of the circle $x^2 + y^2 - 2x - 2y = 0$, then its equation is **[2024]**

(A) $x\log|y| = x - 1$

(B) $x\log|y| = -2(x-1)$

(C) $x\log|y| = 2(x-1)$

(D) $x^2\log|y| = -2(x-1)$

279. The decay rate of radium is proportional to the amount present at any time t. If initially 60 gms was present and half life period of radium is 1600 years, then the amount of radium present after 3200 years is **[2024]**

(A) 20 grams (B) 15 grams
(C) 12 grams (D) 10 grams

280. The particular solution of differential equation $(1+y^2)(1+\log x)dx + x\,dy = 0$ at x=1, y=1 is **[2024]**

(A) $\log x - \frac{1}{2}(\log x)^2 - \tan^{-1} y = -\frac{\pi}{4}$

(B) $\log x + \frac{1}{2}(\log x)^2 + \tan^{-1} y = \frac{\pi}{4}$

(C) $\log x - \frac{1}{2}(\log x)^2 + \tan^{-1} y = \frac{\pi}{4}$

(D) $\log x + \frac{1}{2}(\log x)^2 - \tan^{-1} y = \frac{\pi}{4}$

281. The curve satisfying the differential equation $ydx-(x+3y^2)dy=0$ and passing through the point (1,1) also passes through the point **[2024]**

(A) $(\frac{1}{4},\frac{1}{2})$ (B) $(\frac{1}{4},-\frac{1}{2})$
(C) $(\frac{1}{3},-\frac{1}{3})$ (D) $(-\frac{1}{3},\frac{1}{3})$

282. The general solution of the differential equation $\frac{dy}{dx} = \frac{y+\sqrt{x^2-y^2}}{x}$ is **[2024]**

(A) $\sin^{-1}y = \log x + c$, where c is a constant of integration.

(B) $\frac{y}{x} = \sin^{-1} x + c$, where c is a constant of integration.

(C) $\frac{y}{x} = \sqrt{x^2-y^2} + c$, where c is a constant of integration.

(D) $\sin^{-1}\left(\frac{y}{x}\right) = \log x + c$, where c is a constant of integration.

283. In a certain culture of bacteria, the rate of increase is proportional to the number present. If there are 10^4 at the end of 3 hours and $4\cdot10^4$ at the end of 5 hours, then there were _____ in the beginning. **[2024, 2023]**

(A) 10^4 (B) $\frac{10^4}{4}$
(C) 410^4 (D) $\frac{10^4}{8}$

284. Integrating factor of the differential equation $\frac{dy}{dx} + y = \frac{1+y}{x}$ is **[2024]**

(A) $\frac{x}{e^x}$ (B) xe^x
(C) e^x (D) $\frac{e^x}{x}$

285. If $\cos x\frac{dy}{dx} - y\sin x = 6x, 0 < x < \frac{\pi}{2}$, then general solution of the differential equation is **[2024]**

(A) $y = \cos x + 3x^2 + c$, where c is a constant of integration.
(B) $y + \cos x = 3x^2 + c$, where c is a constant of integration.
(C) $y = 3x^2\cos x + c\cos x$, where c is a constant of integration.
(D) $y\cdot\cos x = 3x^2 + c$, where c is a constant of integration.

286. The general solution of $\frac{dy}{dx} = \frac{x+y+1}{x+y-1}$ is **[2024, 2009]**

(A) $y = x + \log(x+y) + c$, where c is a constant of integration.
(B) $y = x - \log(x+y) + c$, where c is a constant of integration.
(C) $y = x - \log(2x+y) + c$, where c is a constant of integration.
(D) $y = x^2 + \log(x+y) + c$, where c is a constant of integration.

287. A radio-active substance has a half-life of h days, then its initial decay rate is given by (where radio-active substance has initial mass m_0) **[2024, 2023]**

(A) $\frac{m_0}{h}(log\,2)$

(B) $(m_0 h)(log\,2)$

(C) $\frac{-m_0}{h}(log\,2)$

(D) $-(m_0 h)(log\,2)$

288. The general solution of the differential equation $e^{y-x}\frac{dy}{dx} = y\left(\frac{sin\,x + cos\,x}{1 + y\,log\,y}\right)$ is **[2024]**

(A) $e^y log\,y = e^x sin\,x + c$, where c is a constant of integration.

(B) $e^y = e^x sin\,x + c$, where c is a constant of integration.

(C) log y = e^x sin x + c, where c is a constant of integration.

(D) y log y = e^x sin x + c, where c is a constant of integration.

289. Let y= y(x) be the solution of the differential equation $x\frac{dy}{dx} + y = x\,log\,x, (x > 1)$. If 2(y(2)) = log 4 – 1 then the value of y(e) is **[2024]**

(A) $\frac{e^2}{4}$ (B) $-\frac{e^2}{2}$

(C) $-\frac{e}{2}$ (D) $\frac{e}{4}$

290. If y(x) is the solution of the differential equation $(x+2)\frac{dy}{dx} = x^2 + 4x - 9, x \neq -2$ and $y(0) = 0$, then $y(-4)$ is equal to **[2024]**

(A) 0

(B) 1

(C) –1

(D) 2

291. The differential equation, having general solution as $Ax^2 + By^2 = 1$, where A and B are arbitrary constants, is **[2024]**

(A) $xy\frac{d^2y}{dx^2} - x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$

(B) $xy\frac{d^2y}{dx^2} - x\left(\frac{dy}{dx}\right)^2 + y\frac{dy}{dx} = 0$

(C) $xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 + y\frac{dy}{dx} = 0$

(D) $xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$

292. The general solution of the differential equation $\frac{dy}{dx} = \frac{3e^{2x} + 3e^{4x}}{e^x + e^{-x}}$ is **[2024]**

(A) $y = e^{-3x} + c$, where c is a constant of integration.

(B) $y = e^x + c$, where c is a constant of integration.

(C) $y = e^{3x} + c$, where c is a constant of integration.

(D) $y = e^{-x} + c$, where c is a constant of integration.

293. After t seconds, the acceleration of a particle, which starts from rest and moves in a straight line is $\left(8 - \frac{t}{5}\right)$ cm/s², then velocity of the particle at the instant, when the acceleration is zero, is **[2024]**

(A) 160 cm/s

(B) 80 cm/s

(C) 320 cm/s

(D) 480 cm/s

294. The general solution of the differential equation $\frac{dy}{dx}+sin\left(\frac{x+y}{2}\right)=sin\left(\frac{x-y}{2}\right)$ is **[2025, 2024]**

(A) $log\,tan\left(\frac{y}{2}\right)=c-2sin\frac{x}{2}$, where c is the constant of integration

(B) $log\,tan\left(\frac{y}{4}\right)=c-2sin\left(\frac{x}{2}\right)$, where c is the constant of integration

(C) $log\left[tan\left(\frac{y}{2}+\frac{\pi}{4}\right)\right]=c-2sin\,x$, where c is the constant of integration

(D) $log\left[tan\left(\frac{y}{4}+\frac{\pi}{4}\right)\right]=c-2sin\frac{x}{2}$, where c is the constant of integration

295. If $\frac{dy}{dx}=y+3, y+3>0$ and y(0) = 2, then y(log 2) is equal to **[2024, 2023]**

(A) 13 (B) –2

(C) 7 (D) 5

296. If y= y(x) and $\left(\frac{2+\sin x}{y+1}\right)\frac{dy}{dx}=-\cos x, y(0)=1,$ then $y\left(\frac{\pi}{2}\right)=$ **[2025, 2024, 2023, 2022]**

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$

(C) $-\frac{1}{3}$ (D) 1

297. If $x\frac{dy}{dx}=y(log\,y-log\,x+1)$, then the solution of the equation is **[2025, 2024]**

(A) $log\frac{x}{y}=cy$, where c is the constant of integration

(B) $log\frac{y}{x}=cy$, where c is the constant of integration

(C) $log\frac{x}{y}=cx$, where c is the constant of integration

(D) $log\frac{y}{x}=cx$, where c is the constant of integration

298. The assets of a person reduced in his business such that the rate of reduction is proportional to the square root of the existing assets. If the assets were initially ₹ 10 lakhs and due to loss they reduce to ₹ 10000 after 3 years, then the number of years required for the person to be bankrupt will be **[2025, 2024]**

(A) $\frac{20}{3}$ years (B) $\frac{10}{3}$ years

(C) $\frac{10}{9}$ years (D) $\frac{20}{9}$ years

299. A wet substance in the open air loses its moisture at a rate proportional to the moisture content. If a sheet, hung in the open air, loses half its moisture during the first hour, then 90% of the moisture will be lost in _____ hours. **[2025, 2024]**

(A) 2 $\log_2 10$ (B) $\frac{4log10}{log2}$

(C) $\log_2 10$ (D) $\frac{3log10}{log2}$

300. A person throws an unbiased die. If the number shown is even, he gains an amount equal to the number shown. If the number is odd, he loses an amount equal to the number shown. Then his expectation is ₹. **[2024]**

(A) 1

(B) 1.5

(C) 2

(D) 0.5

301. For the probability distribution

X:	-2	-1	0	1	2	3
p(x):	0.1	0.2	0.2	0.3	0.15	0.05

Then the Var(*X*) is (Given: $(0.25)^2 = 0.0625, (0.35)^2 = 0.1225, (0.45)^2 = 0.2025$) **[2024]**

(A) 0.8275

(B) 1.1225

(C) 1.8275

(D) 2.0725

302. Two cards are drawn successively with replacement from a well shuffled pack of 52 cards. Then mean of number of kings is **[2024, 2022]**

(A) $\frac{1}{13}$

(B) $\frac{1}{169}$

(C) $\frac{2}{13}$

(D) $\frac{4}{169}$

303. If P(X = 2) = 0.3, P(X = 3) = 0.4, P(X = 4) = 0.3, then the variance of random variable X is **[2024]**

(A) 1.6

(B) 3.6

(C) 6.6

(D) 0.6

304. A bag contains 4 red and 3 black balls. One ball is drawn and then replaced in the bag and the process is repeated. Let X denote the number of times black ball is drawn in 3 draws. Assuming that at each draw each ball is equally likely to be selected, then probability distribution of X is given by **[2024, 2022]**

(A)

x	0	1	2	3
$P(x)$	$\left(\frac{4}{7}\right)^3$	$\frac{9}{7}\cdot\left(\frac{4}{7}\right)^2$	$\frac{12}{7}\cdot\left(\frac{3}{7}\right)^2$	$\left(\frac{3}{7}\right)^3$

(B)

x	0	1	2	3
$P(x)$	$\left(\frac{3}{7}\right)^3$	$\frac{12}{7}\cdot\left(\frac{3}{7}\right)^2$	$\frac{9}{7}\cdot\left(\frac{4}{7}\right)^2$	$\left(\frac{4}{7}\right)^3$

(C)

x	0	1	2	3
$P(x)$	$\left(\frac{3}{7}\right)^3$	$\frac{9}{7}\cdot\left(\frac{4}{7}\right)^2$	$\frac{12}{7}\cdot\left(\frac{3}{7}\right)^2$	$\left(\frac{4}{7}\right)^3$

(D)

x	0	1	2	3
$P(x)$	$\left(\frac{4}{7}\right)^3$	$\frac{12}{7}\cdot\left(\frac{4}{7}\right)^2$	$\frac{9}{7}\cdot\left(\frac{3}{7}\right)^2$	$\left(\frac{3}{7}\right)^3$

305. A service station manager sells gas at an average of ₹ 100 per hour on a rainy day, ₹ 150 per hour on a dubious day, ₹ 250 per hour on a fair day and ₹ 300 on a clear sky. If weather bureau statistics show the probabilities of weather as follows, then his mathematical expectation is **[2024]**

Weather	Clear	Fair	Dubious	Rainy
Probability	0.50	0.30	0.15	0.05

(A) 257.5 (B) 252.5

(C) 250 (D) 247.5

306. A random variable X assumes values 1, 2, 3, ..., n with equal probabilities. If var(X) : E(X) = 4 : 1, then n is equal to **[2024]**

(A) 20 (B) 15

(C) 25 (D) 10

307. If three fair coins are tossed, then variance of number of heads obtained, is **[2024]**

(A) 0.25 (B) 3

(C) 0.75 (D) 1.5

308. The cumulative distribution function of a discrete random variable X is given by **[2024]**

$X = x$	–1	0	1	2
$F(X = x)$	0.3	0.7	0.8	1

Then $E(X^2)$ =

(A) 0.2 (B) 1.2

(C) 0.8 (D) 2.5

309. Let $X \sim B(6, \frac{1}{2})$, then $P[|x-4| \le 2]$ is **[2024]**

(A) $\frac{115}{128}$

(B) $\frac{63}{64}$

(C) $\frac{57}{64}$

(D) $\frac{7}{64}$

310. Let a random variable X have a Binomial distribution with mean 8 and variance 4. If $P(x \leq 2) = \frac{k}{2^{16}}$, then k is equal to

[2024, 2023, 2022]

(A) 17
(B) 121
(C) 1
(D) 137

311. A multiple choice examination has 5 questions. Each question has three alternative answers of which exactly one is correct. The probability, that a student will get 4 or more correct answers just by guessing, is **[2024]**

(A) $\frac{10}{3^5}$ (B) $\frac{17}{3^5}$
(C) $\frac{13}{3^5}$ (D) $\frac{11}{3^5}$

312. In a Binomial distribution consisting of 5 independent trials, probabilities of exactly 1 and 2 successes are 0.4096 and 0.2048 respectively, then the probability, of getting exactly 4 successes, is **[2024]**

(A) $\frac{80}{243}$ (B) $\frac{40}{243}$
(C) $\frac{32}{625}$ (D) $\frac{4}{625}$

313. The probability that a person who undergoes a bypass surgery will recover is 0.6. the probability that of the six patients who undergo similar operations, half of them will recover is …..**[2024]**

(A) 0.2762 (B) 0.1852
(C) 0.2074 (D) 0.7235

314. Four fair dice are thrown independently 27 times. Then the expected number of times, at least two dice show up a three or a five is **[2024]**

(A) 11 (B) 12
(C) 9 (D) 10

MHTCET PYQ M (2024)

Marathi

Hints & Solutions

1. (D)

Given $\tan x = \frac{3}{4}$ and $\pi < x < \frac{3\pi}{2}$, find $\cos\frac{x}{2}$.

Since x is in the third quadrant, $\cos x = -\frac{4}{5}$.

Hence

$$\cos\frac{x}{2} = -\sqrt{\frac{1+\cos x}{2}} = -\frac{1}{\sqrt{10}}.$$

2. (C)

Evaluate $\sqrt{3}\csc 20^\circ - \sec 20^\circ$.

$$\frac{\sqrt{3}\cos 20^\circ - \sin 20^\circ}{\sin 20^\circ \cos 20^\circ} = \frac{2\sin 40^\circ}{\frac{1}{2}\sin 40^\circ} = 4.$$

3. (C)

If $\tan A - \tan B = x$ and $\cot B - \cot A = y$, find $\cot(A-B)$.

Using $x = \frac{\sin(A-B)}{\cos A \cos B}$ and $y = \frac{\sin(A-B)}{\sin A \sin B}$,

$$\cot(A-B) = \frac{1}{x} + \frac{1}{y}.$$

4. (C)

If $\alpha + \beta = \frac{\pi}{2}$ and $\beta + \gamma = \alpha$, find $\tan\alpha$.

Here $2\beta + \gamma = \frac{\pi}{2}$, so $\cot(2\beta) = \tan\gamma$. Therefore

$$\tan\alpha = \cot\beta = \tan\beta + 2\tan\gamma.$$

5. (B)

In triangle ABC, $\tan\frac{A}{2} = \frac{1}{3}$ and $\tan\frac{B}{2} = \frac{2}{3}$. Find $\tan\frac{C}{2}$.

Since $\frac{A}{2} + \frac{B}{2} + \frac{C}{2} = \frac{\pi}{2}$,

$$\tan\frac{C}{2} = \frac{1 - \frac{1}{3}\cdot\frac{2}{3}}{\frac{1}{3} + \frac{2}{3}} = \frac{7}{9}.$$

6. (A)

Find the smallest positive x satisfying $\tan(x+100^\circ) = \tan(x+50^\circ)\tan x \tan(x-50^\circ)$.

Testing the admissible acute choices, $x = 30^\circ$ satisfies the identity exactly and is the smallest.

7. (A)

Use:

$$\cos^3\theta = \frac{\cos 3\theta + 3\cos\theta}{4}, \quad \sin^3\theta = \frac{3\sin\theta - \sin 3\theta}{4}$$

With $\theta = \frac{\pi}{8}$:

Expression =

$$\frac{1}{4}\left[\cos^2\frac{3\pi}{8} - \sin^2\frac{3\pi}{8} + 3\left(\cos\frac{\pi}{8}\cos\frac{3\pi}{8} + \sin\frac{\pi}{8}\sin\frac{3\pi}{8}\right)\right]$$

$$= \frac{1}{4}\left[\cos\frac{3\pi}{4} + 3\cos\left(-\frac{\pi}{4}\right)\right] = \frac{1}{4}\left[-\frac{1}{\sqrt{2}} + \frac{3}{\sqrt{2}}\right] = \frac{1}{2\sqrt{2}}$$

8. (B, C)

Evaluate

$$\cos(18^\circ - A)\cos(18^\circ + A) - \cos(72^\circ - A)\cos(72^\circ + A)$$

Using $\cos(u-v)\cos(u+v) = \frac{1}{2}(\cos 2u + \cos 2v)$,

$$\frac{1}{2}(\cos 36^\circ - \cos 144^\circ) = \cos 36^\circ = \sin 54^\circ$$

Answer: Options (B) and (C) are equivalent; the value is $\cos 36^\circ$.

9. (B)

For $\sqrt{3}a\cos x + 2b\sin x = c$, two roots have sum $\frac{\pi}{3}$.

Find $\frac{b}{a}$.

Writing the left side as $R\cos(x-\phi)$ gives root sum $2\phi = \frac{\pi}{3}$. Thus

$$\tan\phi = \frac{2b}{\sqrt{3}a} = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}, \frac{b}{a} = \frac{1}{2}.$$

10. (A)

Let $x_i = \cot\alpha_i$

Given,

$x_1 x_2 \cdots x_n = 1$

Using AM $\geq$ GM,

$$\frac{x_i + \frac{1}{x_i}}{2} \geq 1$$

$$\Rightarrow \frac{\cot\alpha_i + \tan\alpha_i}{2} \geq 1\$$

But,

$$\cot\alpha + \tan\alpha = \frac{1}{\sin\alpha\cos\alpha}$$

$$\Rightarrow \cos\alpha_i \leq \frac{1}{\sqrt{2}}.$$

Hence,

$$\prod_{i=1}^{n} \cos\alpha_i \leq \left(\frac{1}{\sqrt{2}}\right)^n = \frac{1}{2^{n/2}}.$$

Equality when

$$\alpha_1 = \alpha_2 = \cdots = \alpha_n = \frac{\pi}{4}.$$

Answer: $\frac{1}{2^{n/2}}$

11. (B)

Evaluate $\cos 20^\circ + 2\sin^{255^\circ} - \sqrt{2}\sin 65^\circ$.

Using $2\sin^{255^\circ} = 1 + \sin 20^\circ$ and $\sqrt{2}\sin 65^\circ = \sin 20^\circ + \cos 20^\circ$, the value is 1.

12. (D)

The lines $x + (a-1)y = 1$ and $2x + a^2 y = 1$ are perpendicular. Find the distance of their intersection from the origin.

The product of slopes gives $a^2(a-1) = -2$, whose real solution is $a = -1$. The lines become $x - 2y = 1$ and $2x + y = 1$, meeting at $(3/5, -1/5)$. Hence

$$d = \sqrt{\frac{9}{25} + \frac{1}{25}} = \frac{\sqrt{10}}{5} = \sqrt{\frac{2}{5}}.$$

13. (C)

The diagonals of a parallelogram lie along $x + 3y = 4$ and $6x - 2y = 7$. Classify it.

Their slopes are $-\frac{1}{3}$ and 3, whose product is -1. Perpendicular diagonals imply a rhombus.

14. (C)

Count lines through $(2,3)$ forming a triangle of area 12 with the coordinate axes.

For intercept form $\frac{x}{a} + \frac{y}{b} = 1$, $ab = 24$ and $\frac{2}{a} + \frac{3}{b} = 1$. Eliminating b gives a quadratic with three admissible signed-intercept configurations.

15. (B)

Find the acute angle between $x\cos 30^\circ + y\sin 30^\circ = 3$ and $x\cos 60^\circ + y\sin 60^\circ = 5$.

The angle between their normals, and hence between the lines, is $|60^\circ - 30^\circ| = 30^\circ$.

16. (A)

A line through $(7,17)$ and $(15,\beta)$ is perpendicular to $2x - 3y + 17 = 0$. Find β.

The given line has slope $\frac{2}{3}$, so the perpendicular slope is $-\frac{3}{2}$:

$$\frac{\beta-17}{8}=-\frac{3}{2}\Rightarrow\beta=5.$$

17. (B)

A line is at perpendicular distance $2\sqrt{2}$ from the origin and its normal makes angle 135° with the x-axis. Find its equation.

Normal form gives

$$x\cos135^\circ+y\sin135^\circ=2\sqrt{2}\Rightarrow x-y+4=0.$$

18. (A)

Determine k/h from the stated perpendicular-line conditions.

The line through $(1,2)$ and $(-3,4)$ has slope $-\frac{1}{2}$, so its perpendicular through $(4,3)$ has slope 2. Their intersection is $(3,1)$.

19. (B)

Four points $(m_i,1/m_i)$ lie on a circle. Find $m_1m_2m_3m_4$.

Substitution in a general circle produces a reciprocal quartic whose constant and leading coefficients are equal.

Thus the product of roots is 1.

20. (D)

Find the concentric circle with double the area of $x^2+y^2-6x-4y-12=0$.

The centre is $(3,2)$ and $r^2=25$. Double area gives $R^2=50$:

$$x^2+y^2-6x-4y=37.$$

21. (A)

Find the circle centred at $(3,4)$ tangent to $5x+12y-11=0$.

The radius is $\frac{|15+48-11|}{13}=4$. Hence

$$x^2+y^2-6x-8y+9=0.$$

22. (C)

Find the circle concentric with $x^2+y^2-6x-4y-12=0$ and tangent to the x-axis.

Its centre is $(3,2)$, so the radius is 2:

$$x^2+y^2-6x-4y+9=0.$$

23. (D)

The abscissae and ordinates of diameter endpoints are roots of two quadratics. Find the circle.

Using sums and products of roots, the midpoint is $(-a,-p)$ and $\overrightarrow{OA}\cdot\overrightarrow{OB}=-\left(b^2+q^2\right)$. Thus

$$x^2+y^2+2ax+2py-\left(b^2+q^2\right)=0.$$

24. (A)

Circle with diameter PR, Radius r, so PR = 2r

Tangents at P and R : PQ ⊥PR and RS⊥PR

So PQ ‖ RS both perpendicular to PR.

PS and RQ meet at X on the circle.

$$\frac{PQ}{RS}=\frac{PX}{RX}$$

And since X lies on the circle with PR as diameter, $\angle PXR=90^\circ$.

From geometry, we get:

$$PQ\cdot RS=(2r)^2=4r^2$$

So:

$$2r=\sqrt{PQ\cdot RS}$$

Answer: (A) $\sqrt{PQ\cdot RS}$

25. (A)

Give parametric equations of $x^2+y^2-ax-by=0$

.

The centre is $\left(\frac{a}{2},\frac{b}{2}\right)$ and radius $\frac{1}{2}\sqrt{a^2+b^2}$:

$x=\frac{a}{2}+\frac{\sqrt{a^2+b^2}}{2}\cos\theta, y=\frac{b}{2}+\frac{\sqrt{a^2+b^2}}{2}\sin\theta.$

26. (A)

Three people independently choose one of three houses. Find the probability all choose the same house.

$P=3\left(\frac{1}{3}\right)^3=\frac{1}{9}.$

27. (A)

If exactly one of A,B has probability $\frac{2}{5}$ and $P(A\cup B)=\frac{1}{2}$, find $P(A\cap B)$.

$P(A\cap B)=P(A\cup B)-P(\text{exactly one})$

$=\frac{1}{2}-\frac{2}{5}=\frac{1}{10}.$

28. (C)

In a class of 300, each student reads 5 newspapers and each newspaper is read by 60 students. Find the number of newspapers.

Double counting incidences gives $300\cdot 5=60N$, so $N=25$.

29. (D)

Four independent persons hit with probabilities $\frac{1}{2},\frac{1}{3},\frac{1}{4},\frac{1}{5}$. Find the chance the target is hit.

$1-\left(\frac{1}{2}\right)\left(\frac{2}{3}\right)\left(\frac{3}{4}\right)\left(\frac{4}{5}\right)=1-\frac{1}{5}=\frac{4}{5}.$

30. (B)

Exactly one of A,B,C occurs. Given odds against them, find k.

$P(A)=\frac{3}{11},P(B)=\frac{2}{7},P(C)=1-\frac{3}{11}-\frac{2}{7}=\frac{34}{77}.$

Since $P(C)=\frac{17k}{43+17k}$, $k=2$.

31. (A)

Independent events have $P(A)=\frac{3}{10}$ and $P(B)=\frac{2}{5}$. Find $P(A'\cup B)$.

$P(A'\cup B)=1-P(A\cap B')=1-\frac{3}{10}\cdot\frac{3}{5}=\frac{41}{50}.$

32. (D)

Independent A,B satisfy $P(A')=.75$, $P(A\cup B)=.65$. Find $P(B)$.

With $P(A)=.25$,

$.65=.25+p-.25p\Rightarrow p=\frac{8}{15}.$

33. (C)

Pairwise-independent A,B,C satisfy $P(A\cap B\cap C)=0$ and $P(C)>0$. Find $P(A'\cap B'|\,C)$.

Pairwise independence gives $P(A\cap B|\,C)=0$ and $P(A|\,C)=P(A)$, $P(B|\,C)=P(B)$. Hence

$P(A'\cap B'|\,C)=1-P(A)-P(B)=P(A')-P(B).$

34. (C)

Find the probability that a random year has 53 Mondays.

A common year contributes $\frac{1}{7}$ and a leap year $\frac{2}{7}$;

in the 28-year cycle,

$P=\frac{21}{28}\cdot\frac{1}{7}+\frac{7}{28}\cdot\frac{2}{7}=\frac{5}{28}.$

35. (B)

Let $P(A)=\frac{2}{5}, P(B)=\frac{4}{7}$

Assume selections are independent.

Probability that exactly one is selected:

$P(A\cap\bar{B})+P(\bar{A}\cap B)=P(A)P(\bar{B})+P(\bar{A})P(B)$

$=\frac{2}{5}\cdot\frac{3}{7}+\frac{3}{5}\cdot\frac{4}{7}=\frac{6}{35}+\frac{12}{35}=\frac{18}{35}$

36. (D)

Find the locus $\left|\frac{z-1}{z+2i}\right|=1$.

It means distance from $(1,0)$ equals distance from $(0,-2)$, the perpendicular bisector of the joining segment.

37. (C)

For a cube root of unity $z=\omega$, evaluate the stated expression.

Using $\omega^3=1$ and $1+\omega+\omega^2=0$, the two squared terms reduce to 1 and 4.

38. (D)

If $\left(-2-\frac{i}{3}\right)^3=\frac{x+iy}{27}$, find $y-x$.

$(-6-i)^3=-198-107i,$

so $x=-198$, $y=-107$, and $y-x=91$.

39. (A)

Find $\arg\left(\frac{-2}{1+i\sqrt{3}}\right)$.

$\frac{-2}{1+i\sqrt{3}}=\frac{-1+i\sqrt{3}}{2}=\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}.$

40. (C)

For $\omega=\frac{-1+i\sqrt{3}}{2}$, evaluate $\left(3+\omega+3\omega^2\right)^4$.

Since $1+\omega+\omega^2=0$, $3+\omega+3\omega^2=2\omega^2$. Thus

$\left(2\omega^2\right)^4=16\omega^8=16\omega.$

41. (B)

Evaluate the determinant involving $1,\omega,\omega^2$.

The rows are linearly dependent because

42. (C)

Describe the locus $|z+1|=1$.

It is a circle of radius 1 centred at $(-1,0)$.

43. (A)

Integer-coordinate roots of the stated complex equation form a rectangle. Find its area.

Separating real and imaginary parts gives $x^2=16$ and $y^2=9$, so the vertices are $(\pm4,\pm3)$. Area $=8\cdot6=48$.

44. (A)

For $z_1=5-2i$, $z_2=3+i$, find $\arg\frac{z_1+z_2}{z_1-z_2}$.

$\frac{8-i}{2-3i}=\frac{19+22i}{13},$

so the argument is $\tan^{-1}\frac{22}{19}$.

45. (C)

Count four-letter words from BARRACK.

Casewise counting by multiplicity gives

$$\underset{\text{all distinct}}{\binom{5}{4}4!}+\underset{\text{one repeated pair}}{2\binom{4}{2}\frac{4!}{2!}}+\underset{\text{both pairs}}{\frac{4!}{2!2!}}=120+144+6=270.$$

46. (B)

Form an 11-member committee from 8 men and 5 women. Compare the two specified counts.

At least six men permits $(6,5),(7,4),(8,3)$, giving $28+40+10=78$. At least three women gives the same cases.

47. (B)

Seat 5 boys and 3 girls around a table so a specified boy and girl are not adjacent.

Total $7!$; adjacent count $2\cdot 6!$. Hence

$7!-2\cdot 6!=5\cdot 6!$.

48. (A)

Colour a 5-cycle with three colours so adjacent persons receive different hats.

The chromatic polynomial of a cycle is $(q-1)^5-(q-1)$ for odd 5. At $q=3$, it is $2^5-2=30$.

49. (B)

Choose 2 boys and 3 girls, excluding teams containing both specified girls.

$$\binom{5}{2}\left[\binom{7}{3}-\binom{5}{1}\right]=10(35-5)=300.$$

50. (C)

Compare 10-letter arrangements with no repetition and with exactly two letters repeated twice.

$$x=10!,\ y=\binom{10}{2}\binom{8}{2}\frac{10!}{2!2!},$$

so $y/x=45\cdot 28/4=315$.

51. (C)

Count numbers greater than one million using digits $2,3,0,3,4,2,3$.

All seven-digit arrangements except those beginning with 0:

$$\frac{7!}{2!3!}-\frac{6!}{2!3!}=420-60=360.$$

52. (C)

If $|A|=4$, $|B|=2$, count subsets of $A\times B$ having at least 3 elements.

$|A\times B|=8$, hence

$$2^8-\binom{8}{0}-\binom{8}{1}-\binom{8}{2}=256-1-8-28=219.$$

53. (D)

Choose 5 questions from three sections of five, at least one from each.

Possible section counts are $(3,1,1)$ and $(2,2,1)$:

$$3\binom{5}{3}\binom{5}{1}^2+3\binom{5}{2}^2\binom{5}{1}=750+1500=2250.$$

54. (B)

Two women choose distinct chairs among 1-4, then three men choose remaining chairs.

Women: ${}^4P_2$; six chairs remain for men: ${}^6P_3$.

55. (D)

Find the domain of $\dfrac{\log_2(x+3)}{x^2+3x+2}$.

Require $x>-3$ and $x\neq -2,-1$. $(-3,\infty)\setminus\{-2,-1\}$.

56. (D)

If $f(x)=\log\dfrac{1-x}{1+x}$, evaluate $f\left(\dfrac{2x}{1+x^2}\right)$.

$$\frac{1-\dfrac{2x}{1+x^2}}{1+\dfrac{2x}{1+x^2}}=\left(\frac{1-x}{1+x}\right)^2,$$

so the value is $2f(x)$.

57. (D)

For $f(x)=\dfrac{x}{2-x}$ and $g(x)=\dfrac{x+1}{x+2}$, find $(g\circ g\circ f)(x)$.

Direct composition yields

$$(g\circ g\circ f)(x)=\frac{6-x}{10-2x}.$$

58. (D)

Relate roots of two quadratics and find r.

Using sums and products for roots α,β and $\alpha/2, 2\beta$ gives

$$r=\frac{2}{9}(2p-q)(2q-p).$$

59. (D)

Find the domain defined by $2^x + 2^y = 2$.

For real y, $2 - 2^x > 0$, so $2^x < 2$ and $x < 1$.

60. (A)

Invert $f(x) = x + \frac{1}{x}$ for $x \geq 1$.

Solving $x = t + 1/t$ gives

$t = \frac{x + \sqrt{x^2 - 4}}{2}$.

61. (B)

For $f(x) = (x+1)^2 - 1$, $x \geq -1$,

solve $f(x) = f^{-1}(x)$.

A function and its inverse meet on $y = x$.

Thus $f(x) = x$ gives $x(x+1) = 0$.

62. (D)

Compare the solution sets of $\sin\theta - \cos\theta = \sqrt{2}\cos\theta$ and $\sin\theta + \cos\theta = \sqrt{2}\sin\theta$.

Each reduces to $\tan\theta = 1 + \sqrt{2}$, so the sets coincide.

63. (D)

Evaluate $\lim_{x\to 0} \frac{x}{|x| + x^2}$.

The right limit is 1 and the left limit is -1.

64. (B)

Let $t = (1+a)^{1/6} - 1$.

Then

$(1+a)^{1/3} - 1 = t^2 + 2t, \quad (1+a)^{1/2} - 1 = t^3 + 3t^2 + 3t.$

Equation becomes

$(t^2 + 2t)x^2 + (t^3 + 3t^2 + 3t)x + t = 0.$

Divide by t and let $t \to 0$:

$2x^2 + 3x + 1 = 0.$

$(2x+1)(x+1) = 0$

$x = -\frac{1}{2}, -1.$

Hence, $\lim_{a\to 0} \alpha(a) = -1, \quad \lim_{a\to 0} \beta(a) = -\frac{1}{2}$.

65. (D)

Evaluate the stated greatest-integer limit as $x \to 0^-$

.

For $-1 < x < 0$, $[x] = -1$ and $|x| = -x$. The expression tends to $-\sin 1$.

66. (A)

Evaluate $\lim_{x\to 0} \frac{(1 - \cos 2x)(3 + \cos x)}{x \tan 4x}$.

Using $1 - \cos 2x \sim 2x^2$ and $\tan 4x \sim 4x$, the limit is 2.

67. (C)

If $\lim_{x\to 1} \frac{x^2 - ax + b}{x - 1} = 7$, find $a + b$.

Finiteness gives $1 - a + b = 0$; the derivative value is $2 - a = 7$. Thus $a = -5$, $b = -6$, and $a + b = -11$.

68. (B)

If $\lim_{x\to\infty} \left(\frac{x^2 + x + 1}{x + 1} - ax - b \right) = 4$, find a, b.

Polynomial division gives $\frac{x^2 + x + 1}{x + 1} = x + \frac{1}{x+1}$.

Hence $a = 1$, $b = -4$.

69. (C)

Evaluate the limit at $x = \frac{\pi}{2}$.

Put $h = x - \frac{\pi}{2}$. Then $1 - \sin x \sim \frac{h^2}{2}$, $8x^3 - \pi^3 \to 0$ to first required order, and the quotient simplifies to

$-\frac{3\pi^2}{16}$.

70. (A)

Evaluate $\lim_{x\to 0}\dfrac{9^x-4^x}{x\left(9^x+4^x\right)}$.

Using $a^x=1+x\ln a+o(x)$,

$\dfrac{\ln 9-\ln 4}{2}=\ln\dfrac{3}{2}$.

71. (C)

Given, $L=\lim\limits_{x\to 2}\dfrac{5^x+5^{3-x}-30}{5^{3-x}-5^{x/2}}$

At $x=2$, it is $0/0$, so apply L'Hôpital's Rule.

Differentiate numerator:

$\dfrac{d}{dx}(5^x+5^{3-x}-30)=\ln 5(5^x-5^{3-x})$

Differentiate denominator:

$\dfrac{d}{dx}(5^{3-x}-5^{x/2})=-\ln 5\left(5^{3-x}+\dfrac{1}{2}5^{x/2}\right)$

Now put $x=2$:

Numerator:

$\ln 5(25-5)=20\ln 5$

Denominator:

$-\ln 5\left(5+\dfrac{5}{2}\right)=-\dfrac{15}{2}\ln 5$

Hence,

$L=\dfrac{20}{\frac{-15}{2}}=\dfrac{-40}{15}=\dfrac{-8}{3}$.

Answer: $\dfrac{-8}{3}$

72. (C)

Evaluate $\lim_{x\to 0^+}\left((\sin x)^{1/x}+x^{-\sin x}\right)$.

The first term tends to 0 and $x^{-\sin x}=e^{-\sin x\ln x}\to 1$.

73. (C)

Evaluate $\lim_{x\to 0}\dfrac{\sin\left(\pi\cos^2 x\right)}{x^2}$.

$\pi\cos^2 x=\pi-\pi x^2+o\left(x^2\right)$, so the numerator is $\sin\left(\pi x^2\right)+o\left(x^2\right)\sim\pi x^2$.

74. (A)

Make the piecewise function continuous on $[-\pi,\pi]$.

Continuity at $-\dfrac{\pi}{2}$ gives $-2=-\alpha+\beta$; at $\dfrac{\pi}{2}$ gives $0=\alpha+\beta$. Hence $\alpha=-1$, $\beta=1$.

75. (A)

The printed function is $f(x)=\dfrac{1+\cos x}{\pi(1-x)^2}$ for $x\neq 1$.

Determine $f(1)$ for continuity.

As printed, the numerator tends to $1+\cos 1\neq 0$, so the limit diverges and no finite $f(1)$ can make it continuous. If the intended numerator was $1+\cos(\pi x)$, the limit would be $\dfrac{\pi}{2}$.

76. (B)

Find k for continuity at $x=\dfrac{\pi}{2}$.

The limiting expression reduces by small-angle expansion to $\dfrac{3}{5}$, so $k=\dfrac{3}{5}$.

77. (A)

Define the removable value of $\dfrac{1-\tan x}{4x-\pi}$ at $x=\dfrac{\pi}{4}$.

By L'Hopital's rule,

$f\left(\dfrac{\pi}{4}\right)=\dfrac{-\sec^2(\pi/4)}{4}=-\dfrac{1}{2}$.

78. (A)

Choose k to make the exponential-type function continuous at $x=2$.

Writing the base as $1+6(x-2)+o(x-2)$ and the exponent as $(x-2)^{-1}$ gives the limit e^6.

79. **(B)**

Define the continuous value at $x=1$ of $\frac{x+x^2+x^3+\cdots+x^n-n}{x-1}$.

L'Hopital's rule gives

$1+2+\cdots+n=\frac{n(n+1)}{2}$.

80. **(D)**

Find k so the piecewise function is continuous at 0.

Using $e^x-1\sim x$ and $\sin u\sim u$, the nonzero branch tends to $12/k$. Equating to 4 gives $k=3$.

81. **(D)**

Count discontinuities of $f(x)=x\lfloor x/2\rfloor$ on $(-10,10)$.

Jumps occur at the nonzero even integers $-8,-6,-4,-2,2,4,6,8$; at 0 multiplication by x removes the jump.

82. **(B)**

Find the continuous value at $x=0$ of the stated trigonometric expression.

After simplification the expression tends to 1 as $x\to 0$.

83. **(A)**

For continuity at $x = 0$

$\lim_{x\to 0} f(x)=\lim_{x\to 0^+} f(x)=f(0)=0$

Left-hand limit ($x < 0$)

For $x < 0$, $|x|=-x$ so: $x-|x|=x-(-x)=2x$

Thus:

$f(x)=\frac{a}{2}(2x)=ax$

$\lim_{x\to 0} ax=0$

This hold for any real a right hand limit ($x > 0$) :

$f(x)=bx^2\sin\left(\frac{1}{x}\right)$

Since $-1\le\sin(1/x)\le 1$:

$|bx^2\sin(1/x)|\le|b|x^2\to 0$

So:

$\lim_{x\to 0} f(x)=0$ This holds for any real b

Thus, both a and b can be any real numbers.

Answer: (A) a is any real value and b is any real value

84. **(B)**

Count discontinuities of the greatest-integer piecewise function on $[-1,3]$.

Checking integer junctions and the piece changes gives discontinuities at three points.

85. **(C)**

Compare the two logical statements about finishing work and contractor trouble.

"If not finished, then trouble" is $\neg p\to q\equiv p\vee q$, exactly Statement II.

86. **(C)**

Identify the switching circuit for $(p\wedge q)\vee[\neg p\wedge(\neg q\vee r)]$.

Series represents conjunction and parallel represents disjunction. The correct network has a top p - q series branch and a lower p' branch followed by parallel q' and r.

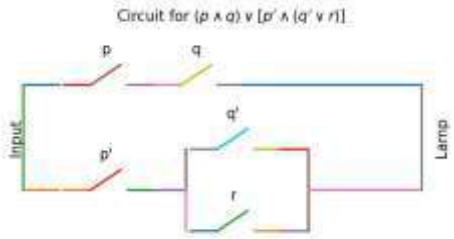


Fig: Correct switching network for Q86

87. **()**

Given:

$(p \wedge \neg q) \to r$

The converse of $P \to Q$ is

$Q \to P.$

Hence,

$r \to (p \wedge \neg q).$

Among the given options, this exact form is not present Answer.

$r \to (p \wedge \neg q)$

Note: The printed options contain a typing error.

88. (B)

Determine truth values of the three compound statements.

A false antecedent makes (A) true; (B) has true antecedent and false consequent, so it is false; therefore (C) has false antecedent and is true.

89. (B)

Negate "Payment is made iff work is finished in time."

The negation of $p \leftrightarrow q$ is exclusive-or:

$(q \wedge \neg p) \vee (p \wedge \neg q).$

90. (D)

Find an equivalent form of the inverse of $p \to (q \to r)$.

The inverse is $\neg p \to \neg(q \to r)$, whose contrapositive is

$(q \to r) \to p.$

91. (D)

Interpret $r = q \vee \neg p$ for equilateral and isosceles triangles.

$q \vee \neg p \equiv p \to q \equiv \neg q \to \neg p$.

92. (B)

We have:

$p \to (p \to q)$

Step 1: Find the inverse: Inverse of A to B is $A \to B$.

Here $A = p, B = (p \to q)$.

So inverse is:

$\neg p \to \neg(p \to q)$

Now, $\neg(p \to q) \equiv p \wedge \neg q$.

So inverse becomes:

$\neg p \to (p \wedge \neg q)$

Step 2: Find the contrapositive of the inverse , contrapositive of X to Y is $\neg Y \to \neg X$

Here $X = \neg p, Y = (p \wedge \neg q)$.

So contrapositive is:

$\neg(p \wedge \neg q) \to \neg(\neg p)$

Simplify: $\neg(p \wedge \neg q) \equiv \neg p \vee q \neg(\neg p) \equiv p$

Thus, we get:

$(\neg p \vee q) \to p$

93. (C)

If the implication is false, find truth values of p, q, r.

An implication is false only when its antecedent is true and consequent false. This yields $p = T$, $q = F$, $r = T$.

94. (D)

A nonsingular matrix satisfies $(A-3I)(A-5I) = 0$ and $\alpha A + \beta A^{-1} = 4I$. Find $\alpha + \beta$.

From $A^2 - 8A + 15I = 0$, $A^{-1} = \frac{1}{15}(8I - A)$.

Matching coefficients gives $\alpha = \frac{\beta}{15}$ and $\frac{8\beta}{15} = 4$, so $\alpha + \beta = 8$.

95. (D)

Decompose the matrix into symmetric A and skew-symmetric B, then evaluate $A^{-1}B + AB^{-1}$ at $\theta = \frac{\pi}{6}$.

Using $A=\left(M+M^T\right)/2$ and $B=\left(M-M^T\right)/2$ gives the displayed matrix in option (D).

96. (B)

For $A=\begin{bmatrix}2 & 1\\ 7 & 4\end{bmatrix}$, evaluate $\left(2A-5A^{-1}\right)^{-1}$.

Direct inversion and simplification gives the matrix shown in option (B).

97. (D)

Determine the number of solutions of $\left(A^2-B^2-B^2+A^2\right)X=O$ for symmetric A and skew-symmetric B.

The coefficient matrix is skew-symmetric of odd order, hence singular. The homogeneous system therefore has infinitely many solutions.

98. (D)

Given $A=\begin{bmatrix}x & 1\\ 1 & 0\end{bmatrix}$ and an entry of A^4, find $\left(A^4\right)^{-1}$.

The entry condition gives the required x; direct multiplication and inversion yields $\begin{bmatrix}10 & -33\\ -33 & 109\end{bmatrix}$.

99. (D)

If $A=\begin{bmatrix}5a & -b\\ 3 & 2\end{bmatrix}$ and $A\text{adj}A=AA^T$, find $5a+b$.

Since $A\text{adj}A=(\det A)I$, comparison of off-diagonal entries and diagonals gives $5a+b=5$.

100. (C)

Find the principal solutions of $\sqrt{3}\sec x+2=0$.

$\cos x=-\dfrac{\sqrt{3}}{2}$, so in $[0,2\pi)$,

$x=\dfrac{5\pi}{6},\dfrac{7\pi}{6}$.

101. (B)

For the given inverse-trigonometric function $f(\theta)$, find $\dfrac{d}{d(\tan\theta)}f(\theta)$.

On $-\dfrac{\pi}{4}<\theta<\dfrac{\pi}{4}$, the principal-value restrictions simplify the expression to $f(\theta)=\tan\theta$. Therefore $\dfrac{df}{d(\tan\theta)}=1$.

102. (C)

Find the sum of all solutions in $(-\pi,\pi)$ of $\sqrt{3}\sec x+\csc x+2(\tan x-\cot x)=0$.

After multiplying by $\sin x\cos x$ and reducing, the roots occur in symmetric pairs whose sum is 0.

103. (C)

Give the general solution of $\tan\theta=\tan\alpha$.

Tangent has period π:

$\theta=n\pi+\alpha, n\in\mathbb{Z}$.

104. (D)

Determine the interval of p for which $(\cos p-1)x^2+(\cos p)x+\sin p=0$ has real roots.

The discriminant condition simplifies to $\sin p>0$, while the quadratic coefficient is nonzero. Thus $0<p<\pi$.

105. (C)

Count solutions in $\left(-\dfrac{\pi}{2},\dfrac{\pi}{2}\right)$ of $(1-\tan\theta)(1+\tan\theta)\sec^2\theta+2\tan^2\theta=0$.

Put $t=\tan\theta$. Then

$\left(1-t^2\right)\left(1+t^2\right)+2t^2=1+2t^2-t^4=0$.

There is one positive value of t^2, giving two opposite t values.

106. (D)

Solve $\sin\left(\cot^{-1}(x+1)\right)=\cos\left(\tan^{-1}x\right)$ using positive square roots.

$$\frac{1}{\sqrt{1+(x+1)^2}}=\frac{1}{\sqrt{1+x^2}}\Rightarrow(x+1)^2=x^2,$$

so $x=-\frac{1}{2}$.

107. (B)

A triangle has angle ratio $4:1:1$. Find longest side : perimeter.

The angles are $120^\circ,30^\circ,30^\circ$. By the sine rule, sides are proportional to $\sqrt{3}:1:1$, so

ratio $=\sqrt{3}:\left(2+\sqrt{3}\right)$.

108. (C)

In ΔABC, $b=3$, $c=8$, $A=60^\circ$. Find the circumradius.

$a^2=b^2+c^2-2bc\cos A=49\Rightarrow a=7,$

then $a=2R\sin A$ gives $R=\frac{7}{\sqrt{3}}$.

109. (B)

Solve $\sin^2\theta-\cos\theta=\frac{1}{4}$ on $[0,2\pi]$.

With $u=\cos\theta$,

$1-u^2-u=\frac{1}{4}\Rightarrow 4u^2+4u-3=0.$

The admissible root is $u=1/2$, so

$\theta=\frac{\pi}{3},\frac{5\pi}{3}$.

110. (B)

If angles A,B,C are in A.P., $A=30^\circ$, and $c=3$, find a,b.

Since $B=60^\circ$ and $C=90^\circ$, the side ratio is $1:\sqrt{3}:2$. Thus

$a=\frac{3}{2},b=\frac{3\sqrt{3}}{2}$.

111. (D)

Solve the simultaneous inequalities $2\sin^2x+3\sin x-2>0$ and $x^2-x-2<0$.

The first gives $\sin x>\frac{1}{2}$; the second gives $-1<x<2$. Their intersection is $\left(\frac{\pi}{6},2\right)$.

112. (C)

Count solutions of $2^{1+|\cos x|+|\cos x|^2+\cdots}=4$ in $(-\pi,\pi)$.

The geometric exponent equals $\frac{1}{1-|\cos x|}=2$,

hence $|\cos x|=\frac{1}{2}$. There are four roots in the interval.

113. (A)

Solve $\sin^2x-\cos2x=2-\sin2x$, given $3\cos x\neq 2\sin x$.

The equation factorises as

$\cos x(3\cos x-2\sin x)=0.$

The condition excludes the second factor, hence

$x=\frac{(2n+1)\pi}{2}$.

114. (D)

If $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=\pi$ and $x^2+y^2+z^2+kxyz=1$, find k.

The cosine identity for three angles summing to π gives

$x^2+y^2+z^2+2xyz=1.$

115. (A)

If x,y,z and $\tan^{-1}x,\tan^{-1}y,\tan^{-1}z$ are both in A.P. under the stated restrictions, determine the relation.

The tangent addition formula and $x+z=2y$ force $(x-y)^2=0$, hence $x=y=z$.

116. (C)

Solve $\tan^{-1}(x+2)+\tan^{-1}(x-2)-\tan^{-1}\frac{1}{2}=0$.

Combining tangents gives

$$\frac{2x}{5-x^2}=\frac{1}{2}\Rightarrow x^2+4x-5=0.$$

The principal-value check admits $x=1$ among the choices.

117. (B)

In $[0,2\pi]$, solve $\cot\theta=\sqrt{3}$ and $\sqrt{3}\sec\theta+2=0$.

The first permits $\theta=\frac{\pi}{6},\frac{7\pi}{6}$; the second requires $\cos\theta=-\frac{\sqrt{3}}{2}$.

118. (D)

Angles of a triangle are in ratio $5:1:6$. Find smallest side : greatest side.

The angles are $75^\circ,15^\circ,90^\circ$. Thus

$$\sin15^\circ:1=(\sqrt{3}-1):2\sqrt{2}.$$

119. (C)

If sides a,b,c are in A.P., evaluate $a\cos^2\frac{C}{2}+c\cos^2\frac{A}{2}$.

Using half-angle formulae and $a+c=2b$, simplification gives $\frac{3b}{2}$.

120. (B)

Under the stated inverse-trigonometric equation, find $\alpha=\cos^{-1}x$.

Write $x=\cos\alpha$. Then $2x\sqrt{1-x^2}=\sin2\alpha$ and the principal branches reduce the equation to $4\alpha=\frac{2\pi}{3}$.

121. (C)

Simplify the expression involving $\cot^{-1}x$ for $0<x<1$.

With $\sin(\cot^{-1}x)=\frac{1}{\sqrt{1+x^2}}$ and $\cos(\cot^{-1}x)=\frac{x}{\sqrt{1+x^2}}$, the expression reduces to $x\sqrt{1+x^2}$.

122. (B)

In a triangle, evaluate $2ac\sin\frac{A-B+C}{2}$.

Since $A+B+C=\pi$, the angle is $\frac{\pi}{2}-B$, so the expression is $2ac\cos B$. By the cosine rule, $2ac\cos B=a^2+c^2-b^2$.

123. (C)

Equation: $3\sin^2x-7\sin x+2=0$

Let $t=\sin x$:

$$3t^2-7t+2=0\quad\Rightarrow\quad t=\frac{7\pm5}{6}$$

$t=2$ or $t=\frac{1}{3}$

Only $t=\frac{1}{3}$ is valid.

Now count solutions of $\sin x=\frac{1}{3}$ in $(0,5\pi)$:

In $(0,2\pi)$: 2 solutions

In $(2\pi,4\pi)$: 2 solutions

In $(4\pi,5\pi)$: 2 solutions (since $5\pi-4\pi=\pi$, which is a full half-period, so 2 solutions)

Total $=2+2+2=6$

124. (C)

If $\tan^{-1}\frac{1}{4}+\tan^{-1}\frac{2}{9}=\frac{1}{2}\cos^{-1}x$, find x.

The tangent of the sum is

$$\frac{\frac{1}{4}+\frac{2}{9}}{1-\frac{1}{18}}=\frac{1}{2},$$

so the sum is $\tan^{-1}\frac{1}{2}$. Hence

$$x=\cos\left(2\tan^{-1}\frac{1}{2}\right)=\frac{3}{5}.$$

125. (B)

If $\cos^{-1}\frac{12}{13}+\sin^{-1}\frac{3}{5}=\sin^{-1}P$, find P.

Taking sine of the sum,

$$P=\frac{5}{13}\cdot\frac{4}{5}+\frac{12}{13}\cdot\frac{3}{5}=\frac{56}{65}.$$

126. (B)

For sides $AB=\sqrt{23}$, $BC=3$, $CA=4$, evaluate $\dfrac{\cot A+\cot C}{\cot B}$.

Using $\cot A+\cot C=\dfrac{b}{2\Delta}(a+c)$ in equivalent cosine-rule form and substituting the sides gives 2.

127. (C)

Evaluate $\cos^{-1}\left[\frac{1}{\sqrt{2}}\left(\cos\frac{9\pi}{10}-\sin\frac{9\pi}{10}\right)\right]$.

The argument is

$$-\frac{\cos18^\circ+\sin18^\circ}{\sqrt{2}}=-\cos27^\circ=\cos153^\circ.$$

Therefore the principal value is $153^\circ=17\pi/20$.

128. (B)

Slopes of $x^2+2hxy+2y^2=0$ are in ratio $1:2$. Find h.

For slopes m_1,m_2, $m_1m_2=\frac{1}{2}$ and $m_1:m_2=1:2$, so $m_1=\frac{1}{2}$, $m_2=1$. Since $m_1+m_2=-h$ up to the common sign convention, $|h|=\frac{3}{2}$.

129. (D)

Find the pair of angle bisectors of $x^2-4xy-5y^2=0$.

The original lines have slopes -1 and $\frac{1}{5}$. Applying the angle-bisector formula gives

$$x^2+3xy-y^2=0.$$

130. (C)

Find the combined equation of lines through the origin making 45° with $3x+y=0$.

Rotating the slope -3 by $\pm45^\circ$ and combining the two resulting lines gives

$$2x^2+3xy-2y^2=0.$$

131. (B)

Find the median from the origin in the triangle cut by $x^2-4xy+y^2=0$ and $2x+3y-1=0$.

The two intersection points are obtained on the pair of lines; their midpoint lies on

$$7x-8y=0.$$

132. (D)

Let P_1,P_2 be distances from $(2,-1)$ to the pair $2x^2-5xy+2y^2=0$. Find P_1P_2.

The pair factors as $(2x-y)(x-2y)=0$. Thus

$$P_1P_2=\frac{|5|}{\sqrt{5}}\cdot\frac{|4|}{\sqrt{5}}=4.$$

133. (C)

Given $\mathbf{A}\perp\mathbf{B}$, $\mathbf{A}\cdot\mathbf{X}=C$, and $\mathbf{A}\times\mathbf{X}=\mathbf{B}$, find X.

Using $\mathbf{A}\times(\mathbf{A}\times\mathbf{X})=\mathbf{A}(\mathbf{A}\cdot\mathbf{X})-|\mathbf{A}|^2\mathbf{X}$,

$$\mathbf{X}=\frac{C\mathbf{A}-\mathbf{A}\times\mathbf{B}}{|\mathbf{A}|^2}.$$

134. (D)

For $\mathbf{a}=\mathbf{i}+2\mathbf{j}+3\mathbf{k}$, evaluate the magnitude of the stated sum of vector triple products.

Each term removes one component, and the sum equals $2\mathbf{a}$. Hence

$|\mathbf{b}|=2|\mathbf{a}|=2\sqrt{14}.$

135. (C)

Find a unit vector $\mathbf{d}$ perpendicular to $\mathbf{a}=(1,-1,0)$ and coplanar with $\mathbf{b}=(0,1,-1)$, $\mathbf{c}=(-1,0,1)$.

Solving $\mathbf{a}\cdot\mathbf{d}=0$ and $[\mathbf{b},\mathbf{c},\mathbf{d}]=0$ gives direction $(1,1,-2)$. Thus

$$\mathbf{d}=\pm\frac{(1,1,-2)}{\sqrt{6}}.$$

136. (B)

Given $\mathbf{b}\times\mathbf{c}=\mathbf{b}\times\mathbf{d}$ and $\mathbf{a}\cdot\mathbf{d}=0$, find $\mathbf{d}$.

The first condition gives $\mathbf{d}=\mathbf{c}+\lambda\mathbf{b}$. Dotting with $\mathbf{a}$ yields $\lambda=-\dfrac{\mathbf{a}\cdot\mathbf{c}}{\mathbf{a}\cdot\mathbf{b}}$.

137. (B)

Use the scalar triple-product condition to find m.

Multilinearity gives

$[m\mathbf{a}+\mathbf{b},\,m\mathbf{b}+\mathbf{c},\,m\mathbf{c}+\mathbf{a}]=(m^3+1)[\mathbf{abc}].$

Thus $m^3+1=28$, so $m=3$.

138. (A)

Points divide PQ internally and externally in ratio $2:3$; $OR\perp OS$. Relate $p=|\mathbf{p}|$ and $q=|\mathbf{q}|$.

Using section formulae and setting the dot product to zero gives $9p^2=4q^2$.

139. (C)

Find the incenter of the triangle with vertices $A(0,2,1)$, $B(-2,0,0)$, $C(-2,0,2)$.

The opposite side lengths are $2,3,3$, so the weighted-coordinate formula gives

$$I=\frac{2A+3B+3C}{8}=\left(-\frac{3}{2},\frac{1}{2},1\right).$$

140. (C)

Transform vector components between the two stated bases and find $2x+y+z$.

Equating coefficients of $\mathbf{p},\mathbf{q},\mathbf{r}$ gives a 3×3 linear system; its solution yields

$2x+y+z=9.$

141. (B)

Under the given vector conditions, evaluate $|(\mathbf{a}\times\mathbf{b})\times\mathbf{c}|$.

$|\mathbf{a}\times\mathbf{b}|=3$, and its angle with $\mathbf{c}$ is 60°. The remaining conditions give $|\mathbf{c}|=1$, hence

$3\cdot1\cdot\sin60^\circ=\dfrac{3\sqrt{3}}{2}.$

142. (C)

Find $\alpha^2+\beta^2-\alpha\beta$ from the projection and cross-product data.

The projection condition gives $\alpha=1$; comparing $\mathbf{b}\times\mathbf{c}$ gives $\beta=-1$. Thus

$1+1-(-1)=3.$

143. (A)

A vector lies in the plane of $\mathbf{b}=\mathbf{i}+\mathbf{j}$ and $\mathbf{c}=\mathbf{j}+\mathbf{k}$ and bisects their angle. Find possible α,β.

Since $|\mathbf{b}|=|\mathbf{c}|$, the internal bisector is parallel to $\mathbf{b}+\mathbf{c}=\mathbf{i}+2\mathbf{j}+\mathbf{k}$.

144. (B)

The parallelogram formed by $\mathbf{a}=(3,-\alpha,1)$ and $\mathbf{b}=(1,\alpha,3)$ has area $8\sqrt{3}$. Find $\mathbf{a}\cdot\mathbf{b}$.

Using $|\mathbf{a}\times\mathbf{b}|^2=|\mathbf{a}|^2|\mathbf{b}|^2-(\mathbf{a}\cdot\mathbf{b})^2$ gives the admissible dot product 2.

145. (D)

Unit vectors $\mathbf{a},\mathbf{c}$ make angle $\pi/3$ and satisfy $(\mathbf{a}\times(\mathbf{b}\times\mathbf{c}))\cdot(\mathbf{a}\times\mathbf{c})=5$. Find $5[\mathbf{abc}]$.

Vector triple-product expansion gives the left side $-(\mathbf{a}\cdot\mathbf{c})[\mathbf{abc}]=-rac12[\mathbf{abc}]$. Hence $[\mathbf{abc}]=-10$.

146. (B)

For perpendicular vectors $|\mathbf{a}|=2$, $|\mathbf{b}|=3$, find the area of the triangle with vertices 0, $\mathbf{a}+2\mathbf{b}$, $\mathbf{a}-2\mathbf{b}$.

$$\text{Area}=\frac{1}{2}|(\mathbf{a}+2\mathbf{b})\times(\mathbf{a}-2\mathbf{b})|=2|\mathbf{a}\times\mathbf{b}|=12.$$

147. (D)

Given the stated perpendicularity relations among vectors of lengths $3,4,5$, find $|\mathbf{A}+\mathbf{B}+\mathbf{C}|$.

Adding the three perpendicularity equations gives

$$2(\mathbf{A}\cdot\mathbf{B}+\mathbf{B}\cdot\mathbf{C}+\mathbf{C}\cdot\mathbf{A})=0.$$

Therefore

$$|\mathbf{A}+\mathbf{B}+\mathbf{C}|^2=3^2+4^2+5^2=50.$$

148. (C)

Mutually perpendicular vectors have magnitudes $1,2,3$. Evaluate the scalar triple product in the question.

By multilinearity and orthogonality, the expression is $2|\mathbf{a}||\mathbf{b}||\mathbf{c}|=12$.

149. (A)

Find a vector of magnitude 6 parallel to $2\mathbf{a}-\mathbf{b}+3\mathbf{c}$.

The given combination equals a scalar multiple of $(1,-2,2)$, whose magnitude is 3. Scaling to magnitude 6 gives

$2\mathbf{i}-4\mathbf{j}+4\mathbf{k}$.

150. (A)

If the three given vectors are mutually orthogonal, find (λ,μ).

From $\mathbf{a}\cdot\mathbf{c}=0$ and $\mathbf{b}\cdot\mathbf{c}=0$,

$\lambda-1+2\mu=0, 2\lambda+4+\mu=0.$

Thus $(\lambda,\mu)=(-3,2)$.

151. (C)

Find the vector in the plane of $\mathbf{a},\mathbf{b}$ whose scalar projection on $\mathbf{c}$ is $1/\sqrt{3}$.

Checking the coplanarity and projection conditions gives

$3\mathbf{i}-\mathbf{j}+3\mathbf{k}$.

152. (A)

Classify the quadrilateral from its four position vectors.

Opposite-side vectors are equal, so it is a parallelogram. Adjacent sides are neither equal nor perpendicular.

153. (A)

Position vectors:

$\overrightarrow{OP}=\mathbf{i}-2\mathbf{j}+3\mathbf{k}$

$\overrightarrow{OQ}=-2\mathbf{i}+3\mathbf{j}+2\mathbf{k}$

$\overrightarrow{OR}=-8\mathbf{i}+13\mathbf{j}+0\mathbf{k}$

Find vectors:

$$\overrightarrow{PQ}=\overrightarrow{OQ}-\overrightarrow{OP}=(-2-1)\mathbf{i}+(3+2)\mathbf{j}+(2-3)\mathbf{k}=-3\mathbf{i}+5\mathbf{j}-\mathbf{k}$$

$$\overrightarrow{QR}=\overrightarrow{OR}-\overrightarrow{OQ}=(-8+2)\mathbf{i}+(13-3)\mathbf{j}+(0-2)\mathbf{k}=-6\mathbf{i}+10\mathbf{j}$$

Clearly:

$\overrightarrow{QR}=2\cdot\overrightarrow{PQ}$

So, they are collinear, and since $\overrightarrow{QR}=2\overrightarrow{PQ}$, the order is

P $\rightarrow$ Q $\rightarrow$ R

Thus Q lies between P and R.

154. (B)

One side and one diagonal of a parallelogram are given. Find its area.

If side is $\mathbf{a}$ and diagonal $\mathbf{a}+\mathbf{b}$, then area is $|\mathbf{a}\times(\mathbf{a}+\mathbf{b})|$. Substitution gives

$3\sqrt{2}$.

155. (A)

Unit vectors $\mathbf{a},\mathbf{b},\mathbf{c}$ are coplanar. Evaluate the stated scalar triple product.

Every linear combination remains in the same plane, so the scalar triple product is 0.

156. (C)

If the stated nested cross-product identity equals $\lambda[\mathbf{abc}]^2$, find λ.

Expanding by $\mathbf{u}\times(\mathbf{v}\times\mathbf{w})=\mathbf{v}(\mathbf{u}\cdot\mathbf{w})-\mathbf{w}(\mathbf{u}\cdot\mathbf{v})$ leaves exactly one squared scalar triple product.

157. (B)

Compare the given scalar triple product with a determinant of scalar components. Find λ.

The determinant on the right is $[\mathbf{abc}]^2$, and expansion of the left produces 28 times this value.

158. (D)

Find the line through $(-1,3,-2)$ perpendicular to both given lines.

The direction is the cross product of $(1,2,3)$ and $(-3,2,5)$, proportional to $(2,-7,4)$. Hence

$$\frac{x+1}{2}=\frac{y-3}{-7}=\frac{z+2}{4}.$$

159. (B)

Find the perpendicular-bisector plane of $P(1,2,5)$ and $Q(3,4,3)$.

The midpoint is $(2,3,4)$ and a normal is $Q-P=(2,2,-2)$. Thus

$$x+y-z-1=0.$$

160. (D)

Find the line through $(1,2,3)$ perpendicular to both given vectors.

Their cross product is proportional to $(1,-1,1)$, so

$$\mathbf{r}=(\mathbf{i}+2\mathbf{j}+3\mathbf{k})+\lambda(\mathbf{i}-\mathbf{j}+\mathbf{k}).$$

161. (C)

Reflect $(3,1,7)$ in $x-y+z=3$, then find the plane through the image and the given line.

The reflected point is found by the standard plane-reflection formula. The plane containing it and direction $(1,2,1)$ is

$$x-4y+7z=0.$$

162. (D)

Find the image of the given line in $2x-y+z+3=0$.

Reflecting a point and the direction vector gives

$$\frac{x+3}{3}=\frac{y-5}{1}=\frac{z-2}{-5}.$$

163. (A)

Find the line through the intersection of $3x-y=5$ and $x+3y=1$ making equal intercepts.

The intersection is $\left(\frac{8}{5},-\frac{1}{5}\right)$. A line with equal intercepts has form $x+y=c$, so $c=\frac{7}{5}$.

164. (C)

Find the plane containing $\frac{x}{2}=\frac{y}{3}=\frac{z}{4}$ and perpendicular to the plane containing the other two lines.

The required normal is perpendicular to $(2,3,4)$ and parallel to the normal of the second plane. Calculation gives $x-2y+z=0$.

165. (A)

Determine which listed line contains the intersection of the given line and $x+y+z=2$.

Parameter substitution gives the intersection point; only option (A)'s line passes through it.

166. (A)

If $(\mathbf{a}\times\mathbf{b})\times(\mathbf{c}\times\mathbf{d})=0$, find the angle between the two planes.

Their normal vectors $\mathbf{a}\times\mathbf{b}$ and $\mathbf{c}\times\mathbf{d}$ are parallel, so the planes are parallel.

167. (C)

Find the plane through $(-1,2,-2)$ and $(-1,3,2)$ perpendicular to the yz-plane.

Its normal has zero x-component and is perpendicular to $(0,1,4)$, giving

$4y-z=10.$

168. (D)

Find the plane through $(2,-1,-3)$ parallel to the two given lines.

The cross product of their directions is proportional to $(8,14,13)$. Thus

$8x+14y+13z+37=0.$

169. (C)

A variable plane through $(3,2,1)$ has intercepts a,b,c. Find the locus of the intersection of the three coordinate-parallel planes through those intercepts.

The intersection is (a,b,c). Since the original plane is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ and contains $(3,2,1)$,

$\frac{3}{x}+\frac{2}{y}+\frac{1}{z}=1.$

170. (B)

Find the distance from $(1,-5,9)$ to $x-y+z=5$ measured along a line parallel to $(1,1,1)$.

Let $Q=P+t(1,1,1)$. Substitution gives $15+t=5$, so $t=-10$ and $PQ=|t|\sqrt{3}=10\sqrt{3}.$

171. (A)

Find the perpendicular distance from the origin to the plane containing the two given lines.

The lines intersect and their direction cross product gives the plane. Reducing its equation and applying the point-plane formula yields $\frac{11}{\sqrt{6}}$.

172. (A)

Find the line through $(2,-1,1)$ parallel to the line joining the two given position vectors.

The joining direction is $(-2,2,-1)$. Hence

$\mathbf{r}=(2\mathbf{i}-\mathbf{j}+\mathbf{k})+\lambda(-2\mathbf{i}+2\mathbf{j}-\mathbf{k}).$

173. (C)

A line through $P(2,-1,2)$ has equal positive direction cosines and meets $2x+y+z=9$. Find PQ.

Its unit direction is $(1,1,1)/\sqrt{3}$. Setting $Q=P+s(1,1,1)/\sqrt{3}$ in the plane gives $s=\sqrt{3}$.

174. (A)

Find the length of the projection of the segment joining $(5,-1,4)$ and $(4,-1,3)$ on $x+y+z=7$.

The segment vector is $(-1,0,-1)$, with squared length 2. Removing its normal component along $(1,1,1)$ gives projected squared length $\frac{2}{3}$.

175. (A)

A line with direction ratios $(1,-4,2)$ meets two given lines at A,B. Find the points.

Solving the two line parameters with $\overrightarrow{AB}\parallel(1,-4,2)$ gives

$A=(-8,6,-7),B=(-6,-2,-3).$

176. (D)

A plane has unit positive intercepts on the x- and y-axes and passes through $(-1,1,2)$. Find its angle with the x-axis.

The plane is $x+y+cz=1$, and the point gives $c=\frac{1}{2}$. Its normal is proportional to $(2,2,1)$. The angle θ between line and plane satisfies

$$\sin\theta=\frac{|2|}{3}=\frac{2}{3}.$$

177. (B)

Two symmetric-form lines intersect. Find k.

Equating their coordinates through the two parameters gives $k=\frac{9}{2}$.

178. (C)

Maximise $Z=x+y$ subject to the stated linear constraints.

The constraint $x+y\le 10$ is itself the objective boundary, and a nonzero segment of $x+y=10$ is feasible. Thus the maximum occurs at infinitely many points.

179. (D)

Step 1: Find the equations of the lines

From the intercepts:s

1. Line passing through $(0,3)$ and $(8,0)$:

$$\frac{x}{8}+\frac{y}{3}=1$$

$$3x+8y=24$$

2. Line passing through (0, 4) and (5, 0)

$$\frac{x}{5}+\frac{y}{4}=1$$

$$4x+5y=20$$

3. Line passing through (0 ,5) and (3, 0):

$$\frac{x}{3}+\frac{y}{5}=1$$

$$5x+3y=15$$

Step 2: Determine the inequality signs

Take appoint inside the shaded region, say (0.5, 3.2)

For $3x+8y=24$

$$3(0.5)+8(3.2)=1.5+25.6=27.1>24$$

So,

$$3x+8y\ge 24$$

For $4x+5y=20$

$$4(0.5)+5(3.2)=2+16=18<20$$

So,

$$4x+5y\le 20$$

For $5x+3y=15$

$$5(0.5)+3(3.2)=2.5+9.6=12.1<15$$

So,

$$5x+3y\le 15$$

Also, the region is in the first quadrant:

$x\ge 0,\quad y\ge 0.$

Final Answer:

$$\begin{aligned}3x+8y&\ge 24,\\ 4x+5y&\le 20,\\ 5x+3y&\le 15,\\ x\ge 0,&\,y\ge 0.\end{aligned}$$

180. (B)

Minimise the cost of a chip containing copper x g and brass y g.

Constraints are $x+y\ge 5$, $x\ge 2$, $0\le y\le 40\le y\le 4$. Since brass is cheaper, take $x=2.$, $y=3$:

$$C=8(2)+5(3)=31.$$

181. (D)

Classify the region $2x+3y\le 18$, $x+y\ge 10$, $x,y\ge 0$.

For $x+y\ge 10$, the minimum possible $2x+3y$ is 20, contradicting $2x+3y\le 18$.

182. (B)

Find where $f(x)=\sin|x|-|x|+2(x-\pi)\cos|x|$ is not differentiable.

At $x=0$, both one-sided values ual -2π and both derivatives equal 2. Elsewhere the function is smooth.

183. (D)

For $y=\sqrt{\frac{1-\sin^{-1}x}{1+\sin^{-1}x}}$, find $y'(0)$.

Logarithmic differentiation gives

$$\frac{y'}{y}=-\frac{\left(\sin^{-1}x\right)'}{1-\left(\sin^{-1}x\right)^2},$$

so at $x=0$, $y=1$ and $y'=-1$.

184. (C)

If $f(x)=\sin^{-1}\dfrac{2\cdot 3^x}{1+9^x}$, find $f'\left(\dfrac{1}{2}\right)$.

For $x>0$, $f(x)=\pi-2\tan^{-1}\left(3^x\right)$.

Therefore

$$f'\left(\frac{1}{2}\right)=-\frac{2\sqrt{3}\ln 3}{1+3}=-\sqrt{3}\ln\sqrt{3}.$$

185. (A)

$F(x)=f^2(x/2)+g^2(x/2)$, where $f''=-f$ and $g=f'$. Given $F(5)=5$, find $F(10)$.

$$\frac{d}{dt}\left(f^2+g^2\right)=2ff'+2gg'=2fg-2gf=0.$$

Thus F is constant.

186. (D)

If $h(x)=f^2(x)+\left[f'(x)\right]^2$, $f''=-f$, and $h(5)=1$, find $h(10)$.

As above, $h'(x)=0$.

187. (B)

Differentiate the sum of two inverse-tangent functions.

Direct differentiation and simplification gives

$$\frac{dy}{dx}=\frac{4}{1+16x^2}.$$

188. (D)

If $8f(x)+6f(1/x)=x+5$ and $y=x^2f(x)$, find $y'(-1)$.

Solving the equation together with its $x\mapsto 1/x$ form,

$$f(x)=\frac{4x+5-3/x}{14}, y=\frac{4x^3+5x^2-3x}{14}.$$

Hence $y'(-1)=-\dfrac{1}{14}$.

189. (C)

If $f(x)=x^3+e^{x/2}$ and $g=f^{-1}$, find $g'(1)$.

$f(0)=1$, so $g(1)=0$. Thus

$$g'(1)=\frac{1}{f'(0)}=\frac{1}{1/2}=2.$$

190. (D)

Differentiate $\cos^{-1}\left(\dfrac{x-1/x}{x+1/x}\right)$.

The argument is $\dfrac{x^2-1}{x^2+1}$. Differentiation gives

$$-\frac{2}{1+x^2}.$$

191. (B)

For $f(x)=\dfrac{x^2-x}{x^2+2x}$, find $\left(f^{-1}\right)'(2)$.

For $x\neq 0$, $f(x)=\dfrac{x-1}{x+2}$. Solving $f(t)=2$ gives $t=-5$, and $f'(-5)=\dfrac{1}{3}$.

192. (B)

If $y=\sin^{-1}\left(\dfrac{3x-x^3}{2}\right)$, find dy/dx on the intended principal interval.

Put $x=2\sin u$; the argument becomes $\sin 3u$. Hence

$$\frac{dy}{dx}=\frac{3}{\sqrt{4-x^2}}.$$

193. (A)

Differentiate the infinite continued fraction.

The repeating tail gives $y=\dfrac{\sin x(1+y)}{1+y+\cos x}$.

Implicit differentiation yields

$$y'=\frac{y\sin x+(1+y)\cos x}{1+2y+\cos x-\sin x}.$$

194. (D)

If $x^2y^2 = \sin^{-1}x + \cos^{-1}x$, find dy/dx at $(1,2)$.

The right side is $\frac{\pi}{2}$, so differentiation gives $2xy^2 + 2x^2yy' = 0$. Thus $y' = -\frac{y}{x} = -2$.

195. (A)

Differentiate $y = \sin^{-1}\frac{\log x^2}{1+(\log x)^2}$ at $x = 1$.

At $x = 1$, the inner value is 0 and its derivative is 2. Hence $y'(1) = 2$.

196. (B)

Implicitly find dx/dy at $(\pi/4, \pi/4)$ from the given equation.

Differentiating and substituting $\sin\frac{\pi}{4} = \cos\frac{\pi}{4} = 1/\sqrt{2}$ gives $\frac{dx}{dy} = \frac{a+b}{a-b}$.

197. (B)

The parametric function has $x = \sin t$ and $y = ae^{\sqrt{2}t} + be^{-\sqrt{2}t}$. Find k in $(1-x^2)y'' - xy' = ky$.

Because $dx/dt = \cos t$,

$(1-x^2)y'' - xy' = \frac{d^2y}{dt^2} = 2y$.

198. (A)

If $\log(x+y) = 2xy$, find $y'(0)$.

At $x = 0$, $y = 1$. Differentiating,

$\frac{1+y'}{x+y} = 2y + 2xy'$,

and substituting gives $1 + y' = 2$.

199. (D)

Given $f(1) = 1$, $f'(1) = 3$, differentiate $f(f(f(x))) + [f(x)]^2$ at $x = 1$.

$3 \cdot 3 \cdot 3 + 2(1)(3) = 27 + 6 = 33$.

200. (C)

If $y = ax^{n+1} + bx^{-n}$, evaluate x^2y''.

Each power satisfies the same Euler relation:

$x^2y'' = n(n+1)y$.

201. (A)

Find the longest interval on which $3\sin x - 4\sin^3 x$ is increasing.

Since $3\sin x - 4\sin^3 x = \sin 3x$, its derivative is $3\cos 3x$. Each maximal interval with $\cos 3x > 0$ has length $\frac{\pi}{3}$.

202. (B)

Let x_0 be a local-minimum point of $f(x) = \mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$. Find $\mathbf{a} \cdot \mathbf{b}$ at $x = x_0$.

The scalar triple product is

$f(x) = (x-1)(x^2 + x - 26)$,

so $f'(x) = 0$ at $x = \pm 3$ and $x = 3$ is the local minimum. Then

$\mathbf{a} \cdot \mathbf{b} = -4x - 3 = -15$.

203. (D)

Rolle's theorem holds for $f(x) = x^3 + bx^2 + ax + 5$ on $[1,3]$ with $c = 2 + 1/\sqrt{3}$. Find a, b.

Solving $f(1) = f(3)$ and $f'(c) = 0$ gives

$a = 11, b = -6$.

204. (B)

Find a point on the normal to $y(x-2)(x-3) = x + 6$ at its y-axis intersection.

At $x = 0$, $y = 1$. Implicit differentiation gives $y' = 1$, so the normal is $y - 1 = -x$. The point $\left(\frac{1}{2}, \frac{1}{2}\right)$ lies on it.

205. (D)

Maximise $3x^3 - 18x^2 + 27x - 40$ on $x^2 + 30 \le 11x$.

The domain is $[5,6]$ and $f'(x)=9(x-1)(x-3)>0$ there. Hence $f(6)=122$.

206. (C)

A triangular park has two fenced sides of equal fixed length x and a river-bank third side. Find its maximum area.

For included angle θ,

$$A=\frac{1}{2}x^2\sin\theta\le\frac{1}{2}x^2,$$

with equality at $\theta=90^\circ$.

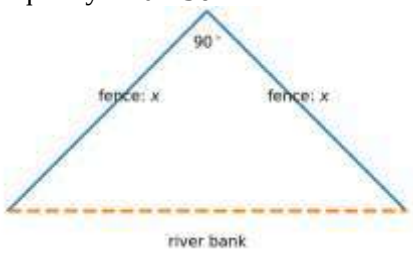


Fig: Isosceles triangular park with the river as the unfenced side

207. (A)

If $f(x)=x^3+bx^2+cx+d$ and $0<b^2<c$, determine its monotonicity.

$f'(x)=3x^2+2bx+c.$

Its discriminant $4b^2-12c<0$ and leading coefficient is positive, so $f'(x)>0$ for all x.

208. (A)

Minimise $2x^3-15x^2+36x-48$ on $x^2+20\le 9x$.

The domain is $[4,5]$ and $f'(x)=6(x-2)(x-3)>0$ there. Hence $f(4)=-16.$

209. (A)

Find the sum of coordinate-axis intercepts of the tangent to $\sqrt{x}+\sqrt{y}=\sqrt{a}$.

At (x_1,y_1), the tangent has intercepts $\sqrt{ax_1}$ and $\sqrt{ay_1}$. Their sum is

$$\sqrt{a}\left(\sqrt{x_1}+\sqrt{y_1}\right)=a.$$

210. (B)

Two numbers sum to 3. Maximise the first times the square of the second.

Let the first be x. Then $P=x(3-x)^2$ and

$P'=(3-x)(3-3x).$

The maximum occurs at $x=1$, giving $P=4$.

211. (A)

On $y=1+\sqrt{4x-3}$, the tangent slope is $2/3$. Find a point on the normal.

$y'=2/\sqrt{4x-3}=2/3$ gives $P=(3,4)$. The normal is

$$y-4=-\frac{3}{2}(x-3),$$

which passes through $(1,7)$.

212. (C)

Find the maximum of $\frac{\log x}{x}$ for $x>0$.

$$f'(x)=\frac{1-\log x}{x^2}=0\Rightarrow x=e,$$

and the maximum is $1/e$.

213. (B)

Apply the mean-value theorem to $f(x)=(x-1)(x-2)(x-3)$ on $[0,4]$.

The secant slope is 3. Thus

$$3c^2-12c+11=3\Rightarrow c=2\pm\frac{2}{\sqrt{3}}.$$

214. (B)

A rectangular sheet has area $18\,\text{m}^2$, top/bottom margins 0.75 m and side margins 0.50 m. Maximise printable area.

Let height H and breadth $B=18/H$. Printable area is $(H-1.5)(B-1)$.

Differentiation gives $H=3\sqrt{3}$ m and $B=2\sqrt{3}$ m.

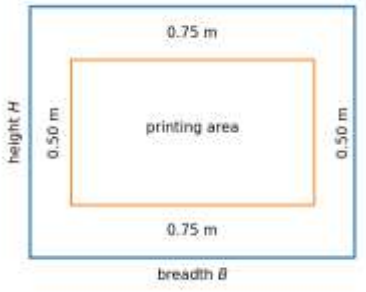


Fig: Sheet and printable rectangle with the stated margins

215. (C)

Approximate $(3.978)^{3/2}$.

For $f(x)=x^{3/2}$ near $x=4$,

$f(3.978)\approx 8+3(-0.022)=7.934.$

216. (A)

Find the tangent to $x=a\cos^3\theta$, $y=a\sin^3\theta$ at $\theta=\pi/4$.

The tangent to the astroid is

$x\sec\theta+y\csc\theta=a.$

At $\theta=\pi/4$, $x+y=a/\sqrt{2}$.

217. (A)

Find the angle at which $x^4-2xy^2+y^2+3x-3y=0$ cuts the x-axis at the origin.

The lowest-degree terms give the tangent $3x-3y=0$, or $y=x$.

218. (B)

A point moves on $y=2x^2$ with $dx/dt=2$. Find dr/dt at $(1,2)$, where r is distance from the origin.

$dy/dt=4x\,dx/dt=8$. From $r^2=x^2+y^2$,

$$\frac{dr}{dt}=\frac{x\dot{x}+y\dot{y}}{r}=\frac{18}{\sqrt{5}}.$$

219. (B)

Find the normal to $y=x\log x$ parallel to $2x-2y+3=0$.

Normal slope 1 means tangent slope -1:

$\log x+1=-1\Rightarrow x=e^{-2}, y=-2e^{-2}.$

Thus $x-y=3e^{-2}$.

220. (C)

If $s=3t^2-8t+5$, when does the body stop?

Velocity $v=ds/dt=6t-8$, so $v=0$ at $t=\frac{4}{3}$ s.

221. (C)

Given, $f(x)=\dfrac{x}{\sqrt{a^2+x^2}}-\dfrac{d-x}{\sqrt{b^2+(d-x)^2}}$ \$

where $a,b,d\neq 0$.

Differentiate each term

Use the standard result:

$$\frac{d}{dx}\left(\frac{x}{\sqrt{a^2+x^2}}\right)=\frac{a^2}{(a^2+x^2)^{3/2}}.$$

For the second term, let $u=d-x$.

Then,

$$\frac{d}{dx}\left(\frac{d-x}{\sqrt{b^2+(d-x)^2}}\right)=-\frac{b^2}{(b^2+(d-x)^2)^{3/2}}.$$

Since this term is subtracted,

$$f'(x)=\frac{a^2}{(a^2+x^2)^{3/2}}+\frac{b^2}{(b^2+(d-x)^2)^{3/2}}.$$

Both terms are strictly positive for all *x*.

Hence,

$f'(x)>0$ for all *x*.

Therefore,

$f(x)$ is an increasing function.

222. (C)

Twenty metres of wire forms the perimeter of a circular sector. Find its maximum area.

If radius is r and angle θ, then $2r+r\theta=20$.

Therefore

$$A=\frac{1}{2}r^2\theta=10r-r^2\leq 25.$$

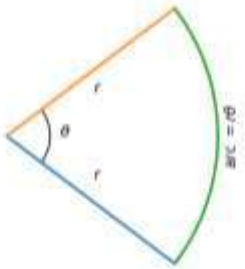


Fig: Circular sector with two radii and an arc

223. (A)

Find the acute angle between $y=10-x^2$ and $y=2+x^2$ at an intersection.

The curves meet at $x=\pm 2$. Their slopes are $-2x$ and $2x$, so

$$\tan\theta=\left|\frac{m_2-m_1}{1+m_1m_2}\right|=\frac{8}{15}.$$

224. (C)

Determine parity and monotonicity of $g(u)=2\tan^{-1}\left(e^u\right)-\pi/2$.

Using $\tan^{-1}t+\tan^{-1}(1/t)=\pi/2$, $g(-u)=-g(u)$. Also

$$g'(u)=\frac{2e^u}{1+e^{2u}}>0.$$

225. (A)

Find the abscissa where the tangent to $y=a\left(e^{x/a}+e^{-x/a}\right)$ is horizontal.

$$y'=e^{x/a}-e^{-x/a}=0\Rightarrow x=0.$$

226. (D)

Curves $y^2=6x$ and $9x^2+by^2=16$ intersect orthogonally. Find b.

Their slopes satisfy

$$\frac{3}{y}\left(-\frac{9x}{by}\right)=-1.$$

Using $x=y^2/6$ gives $b=9/2$.

227. (B)

For $s=at^2+bt+c$, use the given displacement, velocity and acceleration data.

$2a=10$, so $a=5$; $4a+b=30$, so $b=10$; $a+b+c=20$, so $c=5$. Hence $a+c=b$.

228. (A)

An open square-bottom tank holds $4000\,\text{cm}^3$. Minimise its surface area.

Let side x and height $h=4000/x^2$. Then

$$S=x^2+4xh=x^2+\frac{16000}{x}.$$

$S'=0$ gives $x=20$ cm and $h=10$ cm.

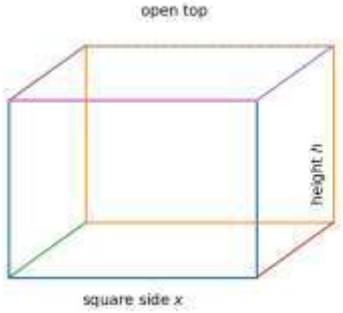


Fig: Open tank with square base

229. (B)

If $\int\frac{dx}{3-2\cos 2x}=\frac{1}{\sqrt{5}}\tan^{-1}\left(f(x)\right)+c$, find $f(\pi/4)$.

With $t=\tan x$,

$$\int\frac{dx}{3-2\cos 2x}=\frac{1}{\sqrt{5}}\tan^{-1}\left(\sqrt{5}\tan x\right)+c.$$

Thus $f(\pi/4)=\sqrt{5}$.

230. (B)

If $\int\frac{x+1}{\sqrt{2x-1}}dx=f(x)\sqrt{2x-1}+c$, find $f(x)$.

Let $u=\sqrt{2x-1}$. Integration gives

$$\frac{x+4}{3}\sqrt{2x-1}+c.$$

231. (A)

Evaluate $\int \frac{2x^2-1}{\left(x^2+4\right)\left(x^2-3\right)}dx$.

$$\frac{2x^2-1}{\left(x^2+4\right)\left(x^2-3\right)}=\frac{9}{7\left(x^2+4\right)}+\frac{5}{7\left(x^2-3\right)}.$$

Hence

$$\frac{9}{14}\tan^{-1}\frac{x}{2}+\frac{5}{14\sqrt{3}}\log\left|\frac{x-\sqrt{3}}{x+\sqrt{3}}\right|+c.$$

232. (B)

If $\int(2x+4)\sqrt{x-1}\,dx=a(x-1)^{5/2}+b(x-1)^{3/2}+c$, find $2a+b$.

Put $u=x-1$:

$$\int(2u+6)u^{1/2}du=\frac{4}{5}u^{5/2}+4u^{3/2}+c.$$

Thus $2a+b=28/5$.

233. (A)

Evaluate the integral involving $\csc x$ and $\cos^2\left(1+\log\tan\left(x/2\right)\right)$.

Let $u=1+\log\tan\left(x/2\right)$. Since $du=\csc x\,dx$,

$$\int\sec^2 u\,du=\tan u+c.$$

234. (A)

If $f\left(\frac{x-4}{x-2}\right)=2x+1$, find $\int f\left(x\right)dx$.

Solving $t=\frac{x-4}{x-2}$ for x gives $f\left(t\right)=5-\frac{4}{t-1}$.

Thus

$$\int f\left(x\right)dx=5x-4\log\left|x-1\right|+c.$$

235. (C)

Evaluate $I=\int\frac{x^2}{\left(a+bx\right)^2}dx$.

With $u=a+bx$,

$$I=\frac{1}{b^3}\left[u-2a\log\left|u\right|-\frac{a^2}{u}\right]+c.$$

236. (C)

Evaluate $\int e^{\sin\theta}\left(\log\sin\theta+\csc^2\theta\right)\cos\theta\,d\theta$.

Let $u=\sin\theta$. Since

$$\frac{d}{du}\left[e^u\left(\log u-\frac{1}{u}\right)\right]=e^u\left(\log u+\frac{1}{u^2}\right),$$

the integral is $e^{\sin\theta}\left(\log\sin\theta-\csc\theta\right)+c$.

237. (B)

If $\int\frac{4e^x-25}{2e^x-5}dx=Ax+B\log\left(2e^x-5\right)+c$, find A,B.

Differentiating the proposed form and equating coefficients gives

$A=5,B=-3.$

238. (C)

Given $\int f\left(x\right)dx=\psi\left(x\right)$, evaluate $\int x^5 f\left(x^3\right)dx$ by parts.

Since $d\psi\left(x^3\right)=3x^2 f\left(x^3\right)dx$,

$$\int x^5 f\left(x^3\right)dx=\frac{1}{3}x^3\psi\left(x^3\right)-\int x^2\psi\left(x^3\right)dx+c.$$

239. (A)

Identify f,g in the antiderivative of $1/\sqrt[3]{\sin^{11}x\cos x}$.

With $t=\tan x$,

$$\int\left(t^{-11/3}+t^{-5/3}\right)dt=-\frac{3}{8}t^{-8/3}-\frac{3}{2}t^{-2/3}+c.$$

240. (B)

Evaluate $\int\frac{x^4+x^2+1}{x^2-x+1}dx$.

Since

$x^4+x^2+1=\left(x^2-x+1\right)\left(x^2+x+1\right)$,

the integral is

$\frac{x^3}{3}+\frac{x^2}{2}+x+c.$

241. (C)

Evaluate $\int\frac{x+1}{x\left(1+xe^x\right)^2}dx$.

Differentiation verifies

$\log\left(\frac{xe^x}{1+xe^x}\right)+\frac{1}{1+xe^x}+c.$

242. (D)

Evaluate $\int\frac{1+\sin\left(\log x\right)}{1+\cos\left(\log x\right)}dx$.

Let $t=\tan\left(\frac{1}{2}\log x\right)$. The integrand becomes $\frac{1}{2}\left(1+t\right)^2$, which is the derivative of $x\tan\left(\frac{\log x}{2}\right)$.

243. (B)

If $\int\frac{dx}{\sqrt{e^x-1}}=2\tan^{-1}\left(f\left(x\right)\right)+c$, find $f\left(x\right)$.

Put $u=\sqrt{e^x-1}$. Then the integral is $2\tan^{-1}u+c$.

244. (D)

Evaluate $\int\frac{x\,dx}{\left(x-1\right)^2\left(x+2\right)}$.

Partial fractions give

$\frac{x}{\left(x-1\right)^2\left(x+2\right)}=\frac{2}{9\left(x-1\right)}+\frac{1}{3\left(x-1\right)^2}-\frac{2}{9\left(x+2\right)}.$

Thus the answer is option (D).

245. (D)

If

$\int\left(7x-2\right)\sqrt{3x+2}\,dx=A\left(3x+2\right)^{5/2}+B\left(3x+2\right)^{3/2}+c$

, find A,B.

With $u=3x+2$,

$A=\frac{14}{45},B=-\frac{40}{27}.$

246. (A)

Evaluate $\int\frac{\cos^3x}{\sin^2x+\sin x}dx$.

Let $u=\sin x$:

$\frac{1-u^2}{u\left(u+1\right)}=\frac{1}{u}-1.$

Hence the integral is $\log\left|\sin x\right|-\sin x+c$.

247. (A)

If $f\left(x\right)=1+x$ and $g\left(x\right)=\log x$, evaluate $\int g\left(f\left(x\right)\right)dx$.

$\int\log\left(1+x\right)dx=\left(1+x\right)\log\left(1+x\right)-x+c.$

248. (A)

Determine $A+B$ in the stated antiderivative of $\frac{2x+5}{\sqrt{7-6x-x^2}}$.

Since $7-6x-x^2=16-\left(x+3\right)^2$ and $2x+5=2\left(x+3\right)-1$,

$\int=-2\sqrt{7-6x-x^2}-\sin^{-1}\frac{x+3}{4}+c.$

Thus $A+B=-3$.

249. (B)

Evaluate $\int\frac{\sec x\tan x}{9-16\tan^2x}dx$.

Let $u=\sec x$. Then the denominator is $25-16u^2$:

$\int\frac{du}{25-16u^2}=\frac{1}{40}\log\left|\frac{5+4u}{5-4u}\right|+c.$

250. (B)

If $f(x)=\frac{x}{x+1}$ and $F=f\circ f$, find $\int F(x)dx$.

$F(x)=\frac{x}{2x+1}$,

so

$\int F(x)dx=\frac{x}{2}-\frac{1}{4}\log|2x+1|+c.$

251. (B)

If $f(x)=\frac{\sin^2(\pi x)}{1+\pi^x}$, evaluate $\int\left[f(x)+f(-x)\right]dx$

.

$f(x)+f(-x)=\sin^2(\pi x)$. Hence

$\int=\frac{x}{2}-\frac{\sin 2\pi x}{4\pi}+c.$

252. (B)

If the integral has form $(\tan x)^A+C(\tan x)^B+k$, find $A+B+C$.

With $t=\tan x$, the integrand becomes

$\frac{1}{2}\left(t^{-1/2}+t^{3/2}\right)dt,$

so the antiderivative is $t^{1/2}+\frac{1}{5}t^{5/2}$. Therefore

$A+B+C=\frac{1}{2}+\frac{5}{2}+\frac{1}{5}=\frac{16}{5}.$

253. (B)

Determine $g(2)$ from the given integral involving $\log\left(t+\sqrt{1+t^2}\right)$.

Let $u=\log\left(t+\sqrt{1+t^2}\right)$. Then $du=dt/\sqrt{1+t^2}$

and the integral is $u^2/2$. Thus

$g(2)=\log\left(2+\sqrt{5}\right).$

254. (C)

Evaluate $\int\frac{x^3-7x+6}{x^2+3x}dx$.

Polynomial division and partial fractions give

$\frac{x^3-7x+6}{x^2+3x}=x-3+\frac{2}{x}.$

Hence

$\frac{x^2}{2}-3x+2\log|x|+c.$

255. (B)

Evaluate $\int\sec^{2/3}x\csc^{4/3}x\,dx$.

With $t=\tan x$, the integrand becomes $t^{-4/3}dt$, so $-3\tan^{-1/3}x+c.$

256. (C)

Evaluate $\int e^x\frac{x-1}{(x+1)^3}dx$.

$\frac{d}{dx}\left(\frac{e^x}{(x+1)^2}\right)=e^x\frac{x-1}{(x+1)^3}.$

257. (C)

If $f(x)=f(a-x)$ and $g(x)+g(a-x)=4$, evaluate $\int_0^a f(x)g(x)dx$.

Replacing x by $a-x$ and adding,

$2I=\int_0^a 4f(x)dx,$

so $I=2\int_0^a f(x)dx$.

258. (A)

Given:

$\int_0^{\pi/3}\frac{\tan\theta}{\sqrt{2k\sec\theta}}d\theta=1-\frac{1}{\sqrt{2}}$

Step 1 : Simplify integrand

$\frac{\tan\theta}{\sqrt{2k\sec\theta}}=\frac{\sin\theta/\cos\theta}{\sqrt{2k/\cos\theta}}$

$=\frac{\sin\theta}{\cos\theta}\cdot\frac{\sqrt{\cos\theta}}{\sqrt{2k}}=\frac{\sin\theta}{\sqrt{2k}\sqrt{\cos\theta}}$

So:

$$I=\frac{1}{\sqrt{2k}}\int_0^{\pi/3}\frac{\sin\theta}{\sqrt{\cos\theta}}d\theta$$

Step 2 : Substitute Let $t=\cos\theta, dt=-\sin\theta d\theta$

Limits: $\theta=0\to t=1, \theta=\pi/3\to t=1/2$.

$$\int_0^{\pi/3}\frac{\sin\theta}{\sqrt{\cos\theta}}d\theta=\int_1^{1/2}\frac{-dt}{\sqrt{t}}=\int_{1/2}^1 t^{-1/2}dt$$

$$=\left[2\sqrt{t}\right]_{1/2}^1=2\left(1-\frac{1}{\sqrt{2}}\right) \text{ ** Step3: Equate **}$$

$$\frac{1}{\sqrt{2k}}\cdot 2\left(1-\frac{1}{\sqrt{2}}\right)=1-\frac{1}{\sqrt{2}}$$

Cancel $1-1/\sqrt{2}\neq 0$:

$$\frac{2}{\sqrt{2k}}=1 \quad\Rightarrow\quad \sqrt{2k}=2 \quad\Rightarrow\quad 2k=4 \quad\Rightarrow\quad k=2$$

259. (B)

Evaluate the symmetric definite integral containing $x^2+\log\frac{\pi-x}{\pi+x}$.

The logarithmic term is odd after multiplication by $\cos x$, so its integral vanishes. Thus

$$2\int_0^{\pi/2}x^2\cos x\,dx=\frac{\pi^2}{2}-4.$$

260. (A)

Evaluate the greatest-integer/logarithm integral on $[-1/2,1/2]$.

The logarithm is odd. Also $[x]=-1$ on $[-1/2,0)$ and 0 on $[0,1/2)$, giving $-\frac{1}{2}$.

261. (C)

Evaluate $\int_{\pi/6}^{\pi/4}\frac{dx}{\sin 2x\left(\tan^5 x+\cot^5 x\right)}$..

Let $t=\tan x$.

The integrand becomes $\frac{t^4}{2\left(1+t^{10}\right)}dt$. Therefore

$$\frac{1}{10}\left[\frac{\pi}{4}-\tan^{-1}\left(\frac{1}{9\sqrt{3}}\right)\right].$$

262. (D)

Evaluate $\int_0^{\pi/4}\log\frac{\sin x+\cos x}{\cos x}dx$.

The integrand is $\log(1+\tan x)$. Pairing x with $\pi/4-x$ gives $\frac{\pi}{8}\log 2$.

263. (A)

Evaluate $\int_0^a\frac{x-a}{x+a}dx$.

$$\frac{x-a}{x+a}=1-\frac{2a}{x+a},$$

so the value is

$$a-2a\log 2.$$

264. (C)

Evaluate $\int_{0.2}^{3.5}[x]dx$.

$$0(0.8)+1(1)+2(1)+3(0.5)=4.5.$$

265. (B)

Evaluate the definite integral of $\sin^{-1}\frac{e^x-1}{e^x+1}$ over symmetric logarithmic limits.

The fraction equals $\tanh(x/2)$, so the integrand is odd. The integral is 0.

266. (B)

Evaluate $\int_{-\pi/4}^{\pi/4}\sin^{-4}x\,dx$.

The printed integrand has a non-integrable singularity at $x=0$, so the improper integral diverges to $+\infty$. Formal substitution of an antiderivative across the singularity would incorrectly give $-8/3$.

267. (A)

Find the area bounded by $x^2-y^2=9$ and its latus rectum.

The latus rectum is $x=3\sqrt{2}$, with endpoints $y=\pm 3$. Thus

$$A=2\int_0^3\left(3\sqrt{2}-\sqrt{y^2+9}\right)dy=9\left[\sqrt{2}-\log\left(1+\sqrt{2}\right)\right].$$

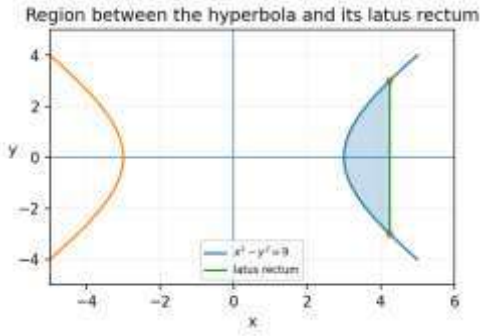


Fig: Hyperbola and its right latus rectum

268. (D)

Find the first-quadrant area bounded by $y=\sqrt{x}$, $2y-x+3=0$, and the x-axis.

The curves meet at $(9,3)$, and the line meets the axis at $(3,0)$. Hence

$$A=\int_0^9\sqrt{x}\,dx-\int_3^9\frac{x-3}{2}dx=18-9=9.$$

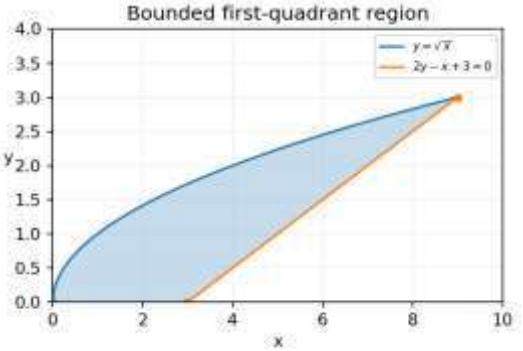


Fig: Region bounded by the square-root curve, line and x-axis

269. (C)

Find the area bounded by $y=(x+1)^2$, $y=(x-1)^2$, and $y=1/4$.

By symmetry,

$$A=2\int_0^{1/2}\left[(1-x)^2-\frac{1}{4}\right]dx=\frac{1}{3}.$$

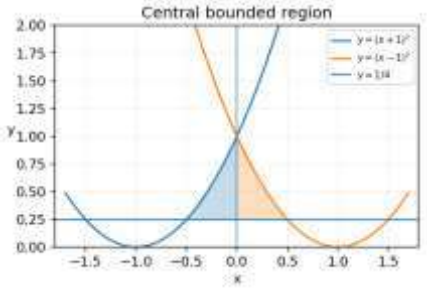


Fig: Central region between two parabolas and y=1/4

270. (D)

Find the area between $y=x^2+2$ and $y=x+1$ for $0\le x\le 2$.

$$A=\int_0^2\left(x^2-x+1\right)dx=\frac{8}{3}.$$

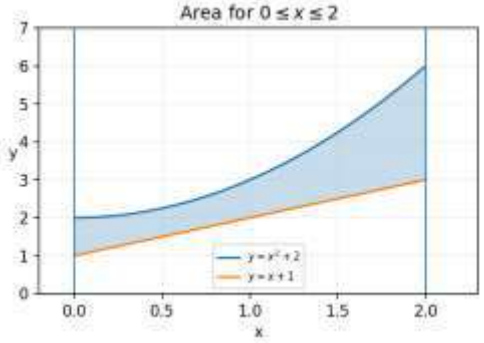


Fig: Area between the parabola and line from x=0 to x=2

271. (D)

Find the area of the region $x\ge 0$, $x+y\le 3$, $x^2\le 4y$, $y\le 1+\sqrt{x}$.

The upper boundary changes at $x=1$, and the lower parabola meets the line at $x=2$:

$$A=\int_0^1\left(1+\sqrt{x}-\frac{x^2}{4}\right)dx+\int_1^2\left(3-x-\frac{x^2}{4}\right)dx=\frac{5}{2}.$$

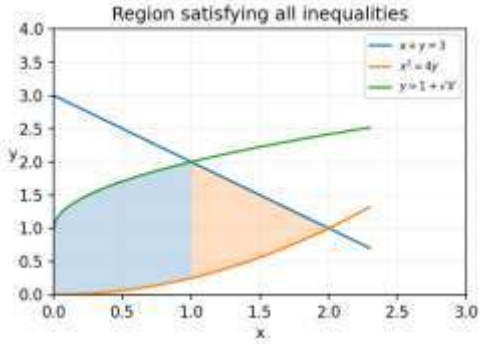


Fig: Region satisfying the four inequalities

272. (C)

Find the area bounded by $y-x=2$ and $y=x^2$.

Intersections are $x=-1,2$. Therefore

$$A=\int_{-1}^{2}\left(x+2-x^2\right)dx=\frac{9}{2}.$$

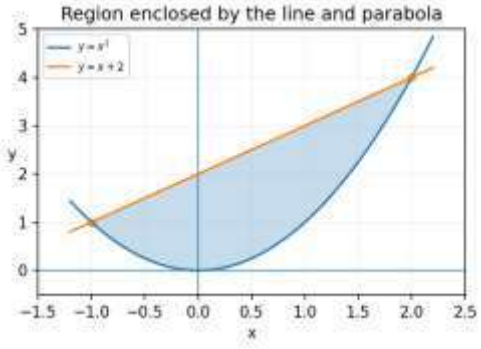


Fig: Region enclosed by y=x+2 and y=x^2

273. (C)

Find the area between $x^2=y/4$, $x^2=9y$, and $y=2$.

For each y, the total horizontal width is

$$2\left(3\sqrt{y}-\frac{1}{2}\sqrt{y}\right)=5\sqrt{y}.$$

Thus

$$A=\int_0^2 5\sqrt{y}\,dy=\frac{20\sqrt{2}}{3}.$$

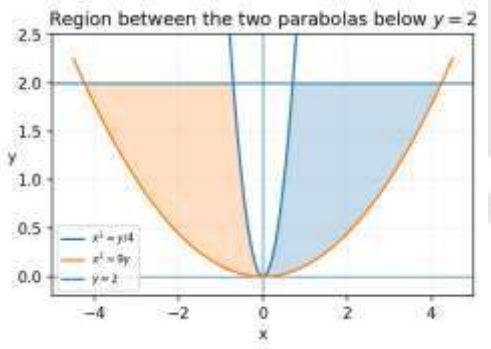


Fig: Region between the two parabolas below y=2

274. (D)

Solve $dy/dx=(x-y)^2$ with $y(1)=1$.

Let $v=x-y$. Then $dv/dx=1-v^2$ and $v(1)=0$:

$$\log\left|\frac{1+v}{1-v}\right|=2(x-1).$$

Substituting $v=x-y$ gives option (D).

275. (D)

If $y=a\sin x+b\cos x$, classify $y^2+(dy/dx)^2$.

$$y^2+(y')^2=(a\sin x+b\cos x)^2+(a\cos x-b\sin x)^2=a^2+b^2.$$

276. (B)

Find order and degree of the printed differential equation.

After clearing the denominator containing y, the highest derivative is y and its highest power is 2. Thus $m=3$, $n=2$ and

$$m^2+n^2=13.$$

277. (D)

Solve $\sin x\,y'+y\cos x=4x$, $y(\pi/2)=0$, and find $y(\pi/6)$.

$$\frac{d}{dx}(y\sin x)=4x\Rightarrow y\sin x=2x^2-\frac{\pi^2}{2}.$$

Hence

$$y\left(\frac{\pi}{6}\right)=-\frac{8\pi^2}{9}.$$

278. (C)

A curve has $dy/dx=2y/x^2$ and passes through the centre of $x^2+y^2-2x-2y=0$. Find its equation.

The centre is $(1,1)$. Integrating, $\log|y|=-\frac{2}{x}+C$, and the point gives $C=2$. Thus $x\log|y|=2(x-1)$.

279. (B)

Radium initially weighs 60 g and has half-life 1600 years. Find the amount after 3200 years.

Two half-lives elapse:

$$60\left(\frac{1}{2}\right)^2=15\text{ g}.$$

280. (B)

Solve $(1+y^2)(1+\log x)\,dx+x\,dy=0$ through $(1,1)$.

$$\frac{dy}{1+y^2} = -\frac{1+\log x}{x}dx.$$

Therefore

$$\log x + \frac{1}{2}(\log x)^2 + \tan^{-1} y = \frac{\pi}{4}.$$

281. (D)

Solve $y\,dx - (x + 3y^2)dy = 0$ through $(1,1)$ and identify another point.

Treating x as a function of y,

$$\frac{dx}{dy} - \frac{x}{y} = 3y \Rightarrow x = 3y^2 - 2y.$$

At $y = 1/3$, $x = -1/3$.

282. (D)

Solve $dy/dx = \left(y + \sqrt{x^2 - y^2}\right)/x$.

Put $v = y/x$. Then

$$x\frac{dv}{dx} = \sqrt{1 - v^2},$$

so

$$\sin^{-1}\left(\frac{y}{x}\right) = \log x + c.$$

283. (D)

Bacteria number is 10^4 at 3 h and $4 \cdot 10^4$ at 5 h. Find the initial number.

$N = N_0 e^{kt}$ and $e^{2k} = 4$, so $e^k = 2$. Therefore

$$N_0 = \frac{10^4}{2^3} = \frac{10^4}{8}.$$

284. (D)

Find the integrating factor of $y' + y = (1 + y)/x$.

Standard form is

$$y' + \left(1 - \frac{1}{x}\right)y = \frac{1}{x}.$$

Thus

$$\text{I.F.} = e^{x - \log x} = \frac{e^x}{x}.$$

285. (D)

Solve $\cos x\, y' - y \sin x = 6x$.

The left side is $d(y\cos x)/dx$. Hence

$$y\cos x = 3x^2 + c.$$

286. (A)

Solve $dy/dx = (x + y + 1)/(x + y - 1)$.

Let $v = x + y$. Then

$$\frac{dx}{dv} = \frac{v - 1}{2v}.$$

Integration and rearrangement give

$$y = x + \log(x + y) + c.$$

287 (C)

For radioactive decay,

$$m = m_0 e^{-kt} \quad Half - life\,\text{h}$$

$$\frac{m_0}{2} = m_0 e^{-kh}$$

$$k = \frac{\ln 2}{h}$$

Decay rate:

$$\frac{dm}{dt} = -km$$

Initial decay rate ($t = 0$, $m = m_0$):

$$\left.\frac{dm}{dt}\right|_{t=0} = -\frac{m_0 \ln 2}{h}$$

288. (A)

Solve the given differential equation involving e^{y-x}.

It separates as

$$e^y \frac{1 + y\log y}{y}dy = e^x(\sin x + \cos x)dx.$$

Both sides are exact derivatives, giving

$$e^y \log y = e^x \sin x + c.$$

289. (D)

Solve $xy' + y = x\log x$ with the stated condition, then find $y(e)$.

$$\frac{d}{dx}(xy) = x\log x,$$

so $y=\frac{x}{2}\log x-\frac{x}{4}+C/x$. The condition gives $C=0$, hence

$y(e)=\frac{e}{4}$.

290. (A)

Solve $(x+2)y'=x^2+4x-9$, $y(0)=0$, and find $y(-4)$.

$y'=x+2-\frac{13}{x+2}$.

After integration and use of the condition, substitution gives $y(-4)=0$.

291. (D)

Eliminate arbitrary constants from $Ax^2+By^2=1$.

Differentiation and elimination give

$xyy''+x(y')^2-yy'=0$.

292. (C)

Solve $dy/dx=\frac{3e^{2x}+3e^{4x}}{e^x+e^{-x}}$.

The right side simplifies to $3e^{3x}$. Therefore

$y=e^{3x}+c$.

293. (A)

A particle starts from rest with acceleration $a=8-t/5$. Find its velocity when acceleration becomes zero.

$a=0$ at $t=40$. Thus

$v=\int_0^{40}\left(8-\frac{t}{5}\right)dt=160$ cm/s.

294. (B)

Solve $y'+\sin\frac{x+y}{2}=\sin\frac{x-y}{2}$.

The sine difference gives

$y'=-2\cos\frac{x}{2}\sin\frac{y}{2}$.

Separating and integrating,

$\log\tan\frac{y}{4}=c-2\sin\frac{x}{2}$.

295. (C)

Solve $y'=y+3$, $y(0)=2$, and find $y(\log 2)$.

$y+3=5e^x$. Hence

$y(\log 2)=10-3=7$.

296. (A)

Solve $\frac{2+\sin x}{y+1}y'=-\cos x$, $y(0)=1$, and find $y(\pi/2)$.

$\log(y+1)=-\log(2+\sin x)+C$,

so $(y+1)(2+\sin x)=4$. At $x=\pi/2$, $y=1/3$.

297. (D)

Solve $xy'=y(\log y-\log x+1)$.

Put $v=y/x$. Then $xv'=v\log v$, so

$\log\left(\frac{y}{x}\right)=cx$.

298. (B)

Assets decrease at a rate proportional to the square root of assets. Initially Rs. 10 lakh; after 3 years Rs. 10000. Find bankruptcy time.

$\sqrt{A}$ decreases linearly. Since $\sqrt{0.1/10}=0.1$, after 3 years 90% of the linear interval has elapsed:

$T=\frac{3}{0.9}=\frac{10}{3}$ years.

299. (C)

Moisture halves in one hour. When is 90% lost?

The remaining fraction is 2^{-t}. Set $2^{-t}=0.1$:

$t=\log_2 10$.

300. (D)

An unbiased die gives gain on even faces and equal loss on odd faces. Find expectation.

$E=\frac{-1+2-3+4-5+6}{6}=\frac{1}{2}$.

301. (C)

$=(-2)(0.1)+(-1)(0.2)+0(0.2)+1(0.3)$
$+2(0.15)+3(0.05)=0.35$

$E(X^2)=4(0.1)+1(0.2)+0+1(0.3)$
$+4(0.15)+9(0.05)=1.95$

$\text{Var}(X)=E(X^2)-[E(X)]^2$

$=1.95-(0.35)^2=1.95-0.1225=1.8275$

302. (C)

Two cards are drawn with replacement. Find the mean number of kings.

This is binomial with $n=2$, $p=4/52=1/13$.

Hence

$$E(X)=np=\frac{2}{13}.$$

303. (D)

For $P(X=2)=0.3$, $P(X=3)=0.4$,

$P(X=4)=0.3$, find variance.

$E(X)=3$ and

$$\text{Var}(X)=0.3(2-3)^2+0.3(4-3)^2=0.6.$$

304. (A)

A black ball has probability $3/7$ on each of three replacement draws. Give the distribution of the number X of black balls.

$$P(X=r)=\binom{3}{r}\left(\frac{3}{7}\right)^r\left(\frac{4}{7}\right)^{3-r},\ r=0,1,2,3.$$

This is the distribution displayed in option (A).

305. (B)

Find expected hourly sales from the weather probabilities.

$E=300(0.50)+250(0.30)+150(0.15)+100(0.05)$

$=252.5.$

306. (C)

X is uniform on $1,2,\ldots,n$ and $\text{Var}(X):E(X)=4:1$. Find n.

$$E(X)=\frac{n+1}{2},\ \text{Var}(X)=\frac{n^2-1}{12}.$$

Thus $(n-1)/6=4$, so $n=25$.

307. (C)

Three fair coins are tossed. Find variance of the number of heads.

$X\sim B(3,1/2)$, so

$$\text{Var}(X)=npq=\frac{3}{4}.$$

308. (B)

A *CDF* is given at 1, 0, 1, 2. Find $E(X^2)$.

The point probabilities are 0.3, 0.4, 0.1, 0.2.

Therefore,

$$E(X^2)=0.3(1)+0.4(0)+0.1(1)+0.2(4)=1.2$$

309. (C)

$|X-4|\le 2\Rightarrow 2\le X\le 6$

So,

$P(|X-4|\le 2)=1-P(X=0)-P(X=1).$

Now,

$$P(X=0)=\binom{6}{0}\left(\frac{1}{2}\right)^6=\frac{1}{64},$$

$$P(X=1)=\binom{6}{1}\left(\frac{1}{2}\right)^6=\frac{6}{64}.$$

Hence,

$$P=1-\frac{1}{64}-\frac{6}{64}=\frac{57}{64}.$$

310. (D)

A binomial variable has mean 8 and variance 4. If $P(X\le 2)=k/2^{16}$, find k.

$np=8$, $npq=4$ give $p=q=1/2$, $n=16$. Hence

$$k=\binom{16}{0}+\binom{16}{1}+\binom{16}{2}=1+16+120=137.$$

311. (D)

Five three-choice questions are guessed. Find the probability of at least four correct.

With $p=1/3$,

$$P(X\ge 4)=\binom{5}{4}\left(\frac{1}{3}\right)^4\frac{2}{3}+\left(\frac{1}{3}\right)^5=\frac{11}{3^5}.$$

312. (D)

In $B(5,p)$, $P(X=1)=0.4096$ and

$P(X=2)=0.2048$.

Find $P(X=4)$.

$\frac{P(X=2)}{P(X=1)}=\frac{2p}{q}=\frac{1}{2}$,

so $p=1/5$, $q=4/5$. Thus

$P(X=4)=5\left(\frac{1}{5}\right)^4\frac{4}{5}=\frac{4}{625}$.

313. (A)

Six patients independently recover with probability 0.6. Find the probability exactly half recover.

$P(X=3)=\binom{6}{3}(0.6)^3(0.4)^3=0.27648\approx 0.2765.$

314. (A)

Four fair dice are thrown in each of 27 trials. Find the expected number of trials in which at least two dice show 3 or 5.

For each die, $p=2/6=1/3$. Thus

$P(X\geq 2)=1-\left(\frac{2}{3}\right)^4-4\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^3=\frac{11}{27}$.

Expected count $=27\cdot 11/27=11$.

MHTCET Yearwise PYQ (2025)

Marathi

Mathematics

Select and write **correct** answer of the following questions:

1. If $\tan A=\frac{1}{\sqrt{x\left(x^2+x+1\right)}}, \tan B=\frac{\sqrt{x}}{\sqrt{x^2+x+1}}$ and $\tan C=\sqrt{x^{-1}+x^{-2}+x^{-3}}$ then

(A) $A+B=C$

(B) $A+B=2C$

(C) $A+B=3C$

(D) $A+B=4C$

2. The value of $\sqrt{3}\cot 20^\circ-4\cos 20^\circ$ is equal to

(A) 1 (B) –1

(C) 0 (D) $\frac{1}{2}$

3. If $3\sin 2\theta=2\sin 3\theta$ and $0<\theta<\pi$, then the value of $\sin\theta$ is equal to **[2025]**

(A) $\frac{\sqrt{17}}{4}$ (B) $\frac{5\sqrt{2}}{4}$

(C) $\frac{3\sqrt{2}}{4}$ (D) $\frac{\sqrt{15}}{4}$

4. $3\tan^6 10^\circ-27\tan^4 10^\circ+33\tan^2 10^\circ=$ **[2025]**

(A) 0

(B) 1

(C) 2

(D) 3

5. If $\sin A=n\sin(A+2B)$, then $\tan(A+B)=$ **[2025]**

(A) $\frac{1+n}{2-n}\cdot\tan B$ (B) $\frac{1-n}{1+n}\cdot\tan B$

(C) $\frac{1-n}{2+n}\cdot\tan B$ (D) $\frac{1+n}{1-n}\cdot\tan B$

6. If $\sin A+\sin B=x$ and $\cos A+\cos B=y$, then $\sin(A+B)=$ **[2025]**

(A) $\frac{2xy}{x^2+y^2}$

(B) $\frac{xy}{x^2+y^2}$

(C) $\frac{2xy}{y^2-x^2}$

(D) $\frac{xy}{y^2-x^2}$

7. If sec x + tan x = 2, $0<x<\frac{\pi}{2}$ then $\sin\frac{x}{4}=$ **[2025]**

(A) $\frac{1}{\sqrt{10+3\sqrt{10}}}$

(B) $\frac{1}{\sqrt{2\left(10+3\sqrt{10}\right)}}$

(C) $\frac{1}{\sqrt{10-3\sqrt{10}}}$

(D) $\frac{1}{2\sqrt{10-3\sqrt{10}}}$

8. If $\tan(\pi\cos\theta)=\cot(\pi\sin\theta)$, then $\sin(\frac{\pi}{4}+\theta)=$ **[2025]**

(A) $\frac{1}{2}$ (B) $\frac{1}{\sqrt{2}}$

(C) $\frac{1}{4}$ (D) $\frac{1}{2\sqrt{2}}$

9. If $\sin(\alpha+\beta)=1$, $\sin(\alpha-\beta)=\frac{1}{2}$, $\alpha,\beta\in\left[0,\frac{\pi}{2}\right]$, then $\tan(\alpha+2\beta)\cdot\tan(2\alpha+\beta)=$ **[2025]**

(A) 1
(B) -1
(C) 0
(D) 4

10. If $3\sin\alpha=5\sin\beta$, then $\tan\left(\frac{\alpha+\beta}{2}\right)\div\tan\left(\frac{\alpha-\beta}{2}\right)=$ **[2025]**

(A) 1
(B) 2
(C) 3
(D) 4

11. The value of $\tan20^\circ\tan80^\circ\cot50^\circ=$

(A) $\sqrt{3}$ (B) $\frac{1}{\sqrt{3}}$
(C) $\frac{1}{2\sqrt{3}}$ (D) $2\sqrt{3}$

12. If $\sin\theta=\frac{1}{2}\left(x+\frac{1}{x}\right)$, then $\sin3\theta+\frac{1}{2}\left(x^3+\frac{1}{x^3}\right)=$

(A) 0
(B) 1
(C) $\frac{1}{4}$
(D) 2

13. In a triangle PQR with usual notations, $\angle R=\frac{\pi}{2}$. If $\tan\frac{P}{2}$ and $\tan\frac{Q}{2}$ are the roots of the equation $ax^2+bx+c=0(a\neq0)$, then

(A) a + b = c
(B) $b+c=a$
(C) $a+c=b$
(D) $b=c$

14. The point of intersection of the diagonals of the rectangle whose sides are contained in the line $x=8, x=10, y=11$ and $y=12$ is

(A) $\left(\frac{9}{2},23\right)$ (B) $\left(9,\frac{23}{2}\right)$
(C) $\left(7,\frac{21}{2}\right)$ (D) $\left(\frac{7}{2},21\right)$

15. If the equation of the median through vertex $A(3,k)$ of ΔABC with vertices $B(2,1)$ and $C(-4,5)$ is $x+4y=p$, then $k=$....., where p and k are constants

(A) 1 (B) 2
(C) -2 (D) 3

16. If the line $2x+y=k$ passes though the point which divides the line segment joining the points $(1,1)$ and $(2,4)$ internally in the ratio $3:2$ then, $(k+1):(k-1)=$

(A) $\frac{5}{7}$ (B) $\frac{7}{5}$
(C) $\frac{8}{5}$ (D) $\frac{6}{5}$

17. The line *MN* whose equation is $x-y-2=0$ cuts the X -axis at *M* and co-ordinates of *N* are $(4,2)$. The line *MN* is rotated about *M* through 45° in anticlockwise direction. The equation of the line *MN* in the new position is

(A) $y=-\sqrt{2}$ (B) $y=2$
(C) $x=-2$ (D) $x=2$

18. The acute angle between the diagonals of a parallelogram whose vertices are $A(2,-1)$, $B(0,2), C(2,3)$ and $D(4,0)$ is

(A) $\cot^{-1}2$ (B) $\cot^{-1}\left(\frac{1}{3}\right)$
(C) $\tan^{-1}2$ (D) $\tan^{-1}\left(\frac{2}{3}\right)$

19. The lines x + 2ay + a = 0, x + 3by + b = 0, x + 4cy + c = 0 are concurrent then a, b, c are in *[2025]*

(A Harmonic progression
(B) Geometric progression
(C) Arithmetic progression
(D) Arithmetico geometric progression

20. The distance of the point (1, 2) from the line x + y = 0 measured parallel to the line 3x – y = 2 is **[2025]**

(A) $\frac{3\sqrt{2}}{8}$ units

(B) $\frac{3\sqrt{10}}{4}$ units

(C) 10 units

(D) $5\sqrt{5}$ units

21. The acute angle between the line 4x-2y + 13 = 0 and the line which makes equal intercepts with the co-ordinate axes is **[2025]**

(A) $\tan^{-1}(2)$
(B) $\tan^{-1}(3)$
(C) $\tan^{-1}\left(\frac{1}{2}\right)$
(D) $\tan^{-1}\left(\frac{1}{3}\right)$

22. A straight line through the origin O meets the line 3y = 10 - 4x and 8x + 6y + 5 = 0 at the points A and B respectively. Then O divides the segment AB in the ratio *[2025]*

(A) 4 : 1
(B) 2 : 3
(C) 1 : 5
(D) 1 : 3

23. From the following options, the nearest line to the origin is .... *[2025]*

(A) $3x - 4y + 4 = 0$
(B) $2x - 3y = 5$
(C) $4x - 3y + 12 = 0$
(D) $5x - 2y = 3$

24. The equation of the circle passing through the point $(1,1)$ and having two diameters along the pair of lines $x^2 - y^2 - 2x + 4y - 3 = 0$ is

(A) $(x+2)^2 + (y-2)^2 = 4$
(B) $(x-3)^2 + (y-1)^2 = 4$
(C) $(x-1)^2 + (y-2)^2 = 1$
(D) $(x+1)^2 + (y+2)^2 = 1$

25. The number of integral values of k for which $x^2 + y^2 + kx + (1-k)y + 5 = 0$ represents a circle whose radius cannot exceeds 5, are **[2025]**

(A) 16 (B) 15
(C) 14 (D) 12

26. The minimum distance and maximum distance of the point $P(2,-7)$ from the circle $x^2 + y^2 - 14x - 10y - 151 = 0$ are respectively ____ units

(A) 2,28
(B) 5,25
(C) 6,24
(D) 3,27

27. Let the circle with centre at origin pass through the vertices of an equilateral triangle ABC. If A = (2,4), then the length of the median through A is *[2025]*

(A) $2\sqrt{5}$ units
(B) $3\sqrt{5}$ units
(C) $4\sqrt{5}$ units
(D) $6\sqrt{5}$ units

28. The least distance of the point A(10, 7) from the circle $x^2 + y^2 - 4x - 2y - 20 = 0$ is length of seg AM. If MM' is the diameter of the circle, then the lengths of AM and AM' are respectively _____, _____ units **[2025]**

(A) 5, 10 (B) 4, 15
(C) 5, 15 (D) 2, 10

29. The equations of the tangents to the circle $x^2 + y^2 = 36$ which are perpendicular to the line $5x + y = 2$, are **[2025]**

(A) $x + 5y \pm 6\sqrt{26} = 0$

(B) $x - 5y \pm 6\sqrt{26} = 0$

(C) $5x - y \pm 6\sqrt{26} = 0$

(D) $5x + y \pm 6\sqrt{26} = 0$

30. A pair of tangents are drawn to the circle $x^2 + y^2 + 6x - 4y - 12 = 0$ from a point $P(-4,-5)$, then the area enclosed between these tangents and the area of the circle is

(A) $25\left(\frac{4+\pi}{4}\right)$ sq. units

(B) $25\left(\frac{4+\pi}{2}\right)$ sq. units

(C) $25\left(\frac{4-\pi}{2}\right)$ sq. units

(D) $25\left(\frac{4-\pi}{4}\right)$ sq. units

31. The locus of point of intersection of the tangents to the circle $x^2 + y^2 = 16$, such that the angle between them is 60°, is

(A) $x^2 + y^2 = 4$

(B) $x^2 + y^2 = 64$

(C) $x^2 + y^2 = 32$

(D) $x^2 + y^2 = 48$

32. The area of the triangle formed by the lines joining the vertex of the parabola $x^2 = 20y$ to the end of its latus rectum is

(A) 100 sq. units

(B) 20 sq. units

(C) 40 sq. units

(D) 50 sq. units

33. The equation of the directrix of the parabola $y^2 + 4y + 4x + 2 = 0$ is

(A) $x = -1$

(B) $x = 1$

(C) $x = \frac{-3}{2}$

(D) $x = \frac{3}{2}$

34. The angle between the tangents drawn from the point $(1,4)$ to the parabola $y^2 = 4x$, is **[2025]**

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{2}$

(C) $\frac{\pi}{3}$ (D) $\frac{\pi}{4}$

35. The line $y = mx + 3$ is tangent to the parabola $y^2 = 4x$, if the value of m is **[2025]**

(A) 3

(B) $\frac{1}{3}$

(C) 4

(D) $\frac{1}{4}$

36. The equations of two ellipses are $\frac{x^2}{4} + \frac{y^2}{2} = 1$ and $\frac{x^2}{36} + \frac{y^2}{b^2} = 1$. If the product of their eccentricities is $\frac{\sqrt{2}}{3}$, then the product of the length of the major axis and minor axis of the second ellipse is …. **[2025]**

(A) $12\sqrt{5}$ (B) 720

(C) $6\sqrt{20}$ (D) $48\sqrt{5}$

37. The tangent to the ellipse $9x^2+16y^2=288$ making equal intercepts on the co-ordinate axes intersects the X -axis and the Y -axis in the points A and B respectively. Then $A(\Delta OAB)=$ (where O is origin) **[2025]**

(A) $\frac{25}{2}$ sq. units

(B) 25 sq. units

(C) $\frac{25\sqrt{5}}{2}$ sq. units

(D) $25\sqrt{5}$ sq. units

38. The eccentric angle of the point $P(-6,2)$ of the ellipse $\frac{x^2}{48}+\frac{y^2}{16}=1$ is **[2025]**

(A) 30° (B) 135°

(C) 150° (D) 120°

39. The foci of the conic $25x^2+16y^2-150x=175$ are **[2025]**

(A) $(0,\pm 3)$

(B) $(3,\pm 3)$

(C) $(0,\pm 5)$

(D) $(5,\pm 5)$

40. The foci of a hyperbola coincide with the foci of the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$. The equation of the hyperbola with eccentricity 2 is **[2025]**

(A) $\frac{x^2}{12}-\frac{y^2}{4}=1$

(B) $\frac{x^2}{4}-\frac{y^2}{12}=1$

(C) $\frac{x^2}{12}-\frac{y^2}{16}=1$

(D) $\frac{x^2}{16}-\frac{y^2}{12}=1$

41. If the tangent at the point $(2\sec\theta,3\tan\theta)$ to the hyperbola $\frac{x^2}{4}-\frac{y^2}{9}=1$ is parallel to $3x-y+4=0$, then the value of θ is **[2025]**

(A) 45° (B) 60°

(C) 30° (D) 90°

42. The X and Y intercepts of the tangent to the hyperbola $\frac{x^2}{20}-\frac{y^2}{5}=1$ which is perpendicular to the line $4x+3y=7$, are respectively **[2025]**

(A) $\frac{-10}{3},\frac{-5}{3}$

(B) $\frac{10}{3},\frac{-5}{2}$

(C) $\frac{10}{3},\frac{5}{2}$

(D) $\frac{10}{3},\frac{5}{3}$

43. If two numbers p and q are chosen randomly from the set {1, 2, 3, 4}, one by one, with replacement, then the probability of getting $p^2 \geq 4q$ is *[2025]*

(A) $\frac{1}{4}$ (B) $\frac{7}{16}$

(C) $\frac{1}{2}$ (D) $\frac{9}{16}$

44. The probability that in a random arrangement of the letters of the word 'UNIVERSITY', the two 'I' s do not come together is *[2025]*

(A) $\frac{1}{5}$

(B) $\frac{1}{10}$

(C) $\frac{4}{5}$

(D) $\frac{3}{10}$

45. Three urns respectively contain 2 white and 3 black, 3 white and 2 black and 1 white and 4 black balls. If one ball is drawn from each urn, then the probability that the selection contains 1 black and 2 white balls is **[2025]**

(A) $\frac{13}{125}$

(B) $\frac{37}{125}$

(C) $\frac{28}{125}$

(D) $\frac{33}{125}$

46. A box contains 9 tickets numbered 1 to 9 both inclusive. If 3 tickets are drawn from the box one at a time, then the probability that they are alternatively either {odd, even, odd} or {even, odd, even} is *[2025]*

(A) $\frac{5}{17}$

(B) $\frac{4}{17}$

(C) $\frac{5}{16}$

(D) $\frac{5}{18}$

47. Let A and B are independent events with $P(B)=\frac{2}{5}, P(A\cup B)=\frac{11}{20}$, then $P(A'/B)$ is root of the equation **[2025]**

(A) $4x^2-7x+3=0$

(B) $4x^2+7x+3=0$

(C) $4x^2-3x-7=0$

(D) $6x^2-5x+1=0$

48. A family has 3 children. The probability that all the three children are girls, given that at least one of them is a girl is **[2025]**

(A) $\frac{7}{8}$ (B) $\frac{1}{8}$

(C) $\frac{1}{7}$ (D) $\frac{2}{7}$

49. If A, B, C are mutually exclusive and exhaustive events of a sample space S such that $P(B)=\frac{3}{2}P(A)$ and $P(C)=\frac{1}{2}P(B)$, then $P(A)=$ **[2025]**

(A) $\frac{4}{13}$ (B) $\frac{6}{13}$

(C) $\frac{8}{13}$ (D) $\frac{3}{13}$

50. If A and B are independent events such that $P(A\cap B')=\frac{3}{25}$ and $P(A'\cap B)=\frac{8}{25}$, then $P(A)=$

(A) $\frac{3}{8}$ (B) 4

(C) $\frac{1}{5}$ (D) $\frac{2}{5}$

51. For $k=1,2,3$ the box B_k contains k red balls and $(k+1)$ white balls. Let $P(B_1)=\frac{1}{2}, P(B_2)=\frac{1}{3}$ and $P(B_3)=\frac{1}{6}$. A box is selected at random and a ball is drawn from it. If a red ball is drawn from it, then the probability that it comes from box B_2 is

(A) $\frac{35}{78}$ (B) $\frac{14}{39}$

(C) $\frac{10}{13}$ (D) $\frac{12}{13}$

52. A doctor assumes that patient has one of three diseases d1, d2 or d3. Before any test he assumes an equal probability for each disease. He carries out a test that will be positive with probability 0.7 if the patient has disease $d1, 0.5$ if the patient has disease d2 and 0.8 if the patient has disease d3. Given that the outcome of the test was positive then probability that patient has disease d2 is

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$

(C) $\frac{1}{5}$ (D) $\frac{1}{7}$

53. In a single toss of a fair die, the odds against the event that number 4 or 5 turns up is

(A) $2:1$ (B) $1:3$

(C) $2:3$ (D) $1:1$

54. Let z be the complex number with $Im(z)=10$ and satisfying $\frac{2z-n}{2z+n}=2i-1$, where $i=\sqrt{-1}$, for some natural number ' n ' then

(A) $n=20$ and $Re(z)=10$

(B) $n=20$ and $Re(z)=-10$

(C) $n=40$ and $Re(z)=10$

(D) $n=40$ and $Re(z)=-10$

55. If $\frac{z-1}{2z+1}$ is an imaginary number and if it represents a circle then its radius is

(A) $\frac{9}{16}$ units

(B) $\frac{3}{4}$ units

(C) $\frac{1}{4}$ units

(D) $\frac{1}{2}$ units

56. The equation $|z+1-i|=|z-1+i|$ represents a (where z is a complex number)

(A) Straight line passing through the origin and the first and third quadrant.

(B) Straight line passing through the origin and the second and fourth quadrant.

(C) Straight line passing through the point (1, –1) and having slope –1.

(D) Straight line passing through the point (2, 1) and having slope $\frac{1}{2}$.

57. A particle P starts from the point $Z_0=1+2i$ where $i=\sqrt{-1}$. It moves first horizontally away from the origin by 5 units and then vertically upwards parallel to positive Y -axis by 3 units to reach a point Z_1. From Z_1 the particle moves $\sqrt{2}$ units in the direction of vector $\hat{i}+\hat{j}$ and then it moves through an angle $\frac{\pi}{2}$ in anticlockwise direction on a circle with centre at origin to reach at point Z_2, then $Z_2=$

(A) $6+7i$ (B) $-7+6i$

(C) $-6+7i$ (D) $7-6i$

58. Let z be the complex number such that $|z|+z=3+i$ where $i=\sqrt{-1}$, then $|z|=$

(A) $\frac{\sqrt{34}}{3}$

(B) $\frac{5}{3}$

(C) $\frac{\sqrt{41}}{4}$

(D) $\frac{5}{4}$

59. The modulus of the square root of the conjugate of $-7+24\sqrt{-1}$ is

(A) 3 (B) 4

(C) 16 (D) 5

60. If $z = x + iy$ is a complex number, then the equation $\left|\frac{z+i}{z-i}\right| = \sqrt{3}$ represents the

(A) circle with centre $(2,0)$ and radius $\sqrt{3}$

(B) circle with centre $(0,2)$ and radius $\sqrt{3}$

(C) circle with centre $(0,0)$ and radius $\sqrt{3}$

(D) circle with centre $(0,-2)$ and radius $\sqrt{3}$

61. The area of the triangle whose vertices are i, ω and ω^2 is (Where ω is a complex cube root of unity other than $1, i$ is an imaginary number) ____ sq.units

(A) $\frac{3\sqrt{3}}{4}$ (B) $\frac{\sqrt{3}}{2}$

(C) $\frac{3\sqrt{3}}{2}$ (D) $\frac{\sqrt{3}}{4}$

62. Total number of 3-digit numbers, whose g.c.d with 36 is 2 , is **[2025]**

(A) 140 (B) 150

(C) 165 (D) 170

63. If four digit numbers are formed by using the digits $1,2,3,4,5,6,7$ without repetition, then out of these numbers, the numbers exactly divisible by 25 are **[2025]**

(A) 20 (B) 40

(C) 50 (D) 51

64. A family consisting of a mother, father and their 8 children (4 boys and 4 girls) are to be seated at a round table in a party. How many ways can this be done if the mother and father sit together and the males and females alternate? **[2025]**

(A) 567 (B) 765

(C) 657 (D) 576

65. A regular polygon has 20 sides. The number of triangles that can be drawn by using the vertices but not using the sides are **[2025]**

(A) 1140 (B) 800

(C) 340 (D) 20

66. 4 red balls and 5 green balls are selected from n balls. If the sum of both the selections is greater than ${}^{n+1}C_4$ then the value of n is equal to

(A) $n > 8$

(B) $n < 8$

(C) $n > 10$

(D) $n > 12$

67. The greatest possible number of points of intersection of 8 distinct straight lines and 4 distinct circles is **[2025]**

(A) 28 (B) 104

(C) ${}^{12}C_2$ (D) ${}^{4}C_2$

68. If ${}^nC_0 + \frac{1}{2}{}^nC_1 + \frac{1}{3}{}^nC_2 + \ldots\ldots\ldots\ldots + \frac{1}{n}{}^nC_{n-1} + \frac{1}{n+1}{}^nC_n = \frac{1023}{10}$ then $n =$ **[2025]**

(A) 7 (B) 8

(C) 9 (D) 10

69. There are 11 points in a plane of which 5 points are collinear. Then the total number of distinct quadrilaterals with vertices at these points is **[2025]**

(A) 265 (B) 330

(C) 250 (D) 325

70. The number of ways in which a team of 11 players can be formed out of 25 players, if 6 out of them are always to be included and 5 of them are always to be excluded, is **[2025]**

(A) 2002 (B) ${}^{20}C_{11}$

(C) ${}^{20}C_6$ (D) ${}^{14}C_6$

71. 21 friends were invited for a party. Two round tables can accommodate 12 and 9 friends each. The number of ways of the seating arrangements of friends is _____ **[2025]**

(A) $11!\times 8!$

(B) $12!\times 9\ !$

(C) $\frac{35}{9}\times 19\ !$

(D) $\frac{20!}{12!8!}\times 11!\times 9\ !$

72 If ${}^{n+4}C_{n+1}-{}^{n+3}C_n=15(n+2)$, then $n=$ [2025]

(A) 15 (B) 23

(C) 21 (D) 27

73. The domain of the function $f(x)={}^{7-x}P_{x-1}$ is **[2025]**

(A) R

(B) $x\in\mathbb{R}-\{1\}$

(C) $\{1,2,3,4\}$

(D) $\{1,2,3,4,5,6\}$

74. The function defined by $f(x)=\frac{2x+3}{3x+4}, x\neq -\frac{4}{3}$ is

(A) only one one

(B) only onto

(C) onto for $y\neq\frac{2}{3}$ and one-one

(D) neither one-one nor onto

75. If $f(x)=2x^3+mx^2-13x+n$ and 2,3 are the roots of the equation $f(x)=0$ then the value of $4m+5n$ is **[2025]**

(A) 30

(B) 100

(C) 130

(D) 150

76. If $f(x)=\cos(\log x)$ then $f(x^2)\cdot f(y^2)-\frac{1}{2}\left[f\left(\frac{x^2}{y^2}\right)+f(x^2y^2)\right]$ has the value **[2025]**

(A) –2 (B) –1

(C) $\frac{1}{2}$ (D) 0

77. If $p^3=q^4=r^6=t^7=s^2$, then $\log_t(pqrs)=\ldots$ ... **[2025]**

(A) $\frac{168}{5}$ (B) 28

(C) $\frac{31}{4}$ (D) $\frac{35}{4}$

78. If $\sqrt{\log_3 x^{16}}+9\log_{27}\sqrt[3]{\frac{3}{x}}=5$, then $x=\ldots$ **[2025]**

(A) 81 (B) $\frac{1}{405}$

(C) 27 (D) 405

79. For a real number $x,[x]$ denotes the greatest integer less than or equal to x. Then the value of $\left[\frac{1}{2}\right]+\left[\frac{1}{2}+\frac{1}{100}\right]+\left[\frac{1}{2}+\frac{2}{100}\right]+\left[\frac{1}{2}+\frac{3}{100}\right]+\ldots\ldots\ldots\ldots+\left[\frac{1}{2}+\frac{99}{100}\right]=$ **[2025]**

(A) 49 (B) 100

(C) 0 (D) 50

80. If $f(x)=3x+10,\ g(x)=x^2-1,$ then $(fog)^{-1}(x)=$

(A) $\left(\frac{x-7}{3}\right)$ (B) $\left(\frac{x-7}{3}\right)^{\frac{1}{2}}$

(C) $\left(\frac{x-7}{3}\right)^{\frac{1}{3}}$ (D) $\left(\frac{3}{x-7}\right)^{\frac{3}{2}}$

81. If $f(x)=\log\left(\frac{1+x}{1-x}\right)$ and $g(x)=\frac{3x+x^3}{1+3x^2}$, then $(\text{fog})(x)=$

(A) $2f(x)$ (B) $3f(x)$

(C) $4f(x)$ (D) $-f(x)$

82. Let $f:\mathbb{R}-\{2\}\to\mathbb{R}-\{1\}$ defined by $f(x)=\frac{x-3}{x-2}$ and $g:\mathbb{R}\to\mathbb{R}$ defined by $g(x)=3x-2$, then sum of all values of x for which $f^{-1}(x)+g^{-1}(x)=\frac{19}{6}$ is

(A) $\frac{5}{2}$ (B) $\frac{7}{2}$

(C) $\frac{9}{2}$ (D) $\frac{11}{2}$

83. If $[x]^2-5[x]+6=0$, where $[\cdot]$ denotes the greatest integer function, then

(A) $x\in(2,4)$ (B) $x\in[2,4]$

(C) $x\in[2,4)$ (D) $x\in(2,4]$

84. $f(x)=\begin{cases}3-x, & -1\leqslant x<0\\ 1+\frac{5x}{3}, & -3\leqslant x\leqslant 2\end{cases}$ and $g(x)=\begin{cases}-x, & -2\le x\le 3\\ x, & 0\le x\le 1\end{cases}$ then range of (*fog*) (*x*) is **[2025]**

(A) $\left[1,\frac{8}{3}\right]$ (B) $\left[-4,\frac{8}{3}\right]$

(C) $\left[-4,\frac{13}{3}\right]$ (D) $\left[\frac{8}{3},\frac{10}{3}\right]$

85. Define $f(x)=\begin{cases}b-ax & ,\text{ if } x<2\\ 3 & ,\text{ if } x=2\\ a+2bx & ,\text{ if } x>2\end{cases}$ and if $\lim\limits_{x\to2}f(x)$ exists, then $\frac{a}{b}=$ **[2025]**

(A) 1

(B) –1

(C) $\frac{2}{3}$

(D) $\frac{3}{2}$

86. $\lim\limits_{x\to5}\sqrt{\frac{2-2\cos(x^2-12x+35)}{(x-5)}}=$..... **[2025]**

(A) $\frac{2}{-5}$ (B) –2

(C) $\frac{-1}{2}$ (D) –5

87. $\lim\limits_{x\to3}\frac{(84-x)^{\frac{1}{4}}-3}{x-3}$ is **[2025]**

(A) $\frac{-1}{108}$ (B) $\frac{-1}{84}$

(C) $\frac{-1}{27}$ (D) $\frac{-1}{4}$

88. $\lim\limits_{x\to0}\frac{(7^x-1)^4}{\tan\left(\frac{x}{k}\right).\log\left(1+\frac{x^2}{3}\right).\sin 4x}=3(\log 7)^3$, then $k=$ **[2025]**

(A) $4(\log 7)^{-1}$

(B) $\frac{1}{4}(\log 7)^{-1}$

(C) $4\log\left(\frac{1}{7}\right)$

(D) $\frac{1}{4}\log 7$

89. $\lim\limits_{x\to\infty}\dfrac{e^{x^4}-1}{e^{x^4}+1}=$

(A) 1 (B) e

(C) $\dfrac{1}{e}$ (D) not defined

90. $\lim\limits_{x\to\infty}\dfrac{\left(2x+1\right)^{50}+\left(2x+2\right)^{50}+\left(2x+3\right)^{50}+\cdots+\left(2x+100\right)^{50}}{\left(2x\right)^{50}+\left(10\right)^{50}}=$ **[2025]**

(A) 50
(B) 100
(C) 25
(D) 200

91. $\lim\limits_{x\to\infty}\left(\dfrac{x+8}{x+1}\right)^{x+5}=\ldots$ **[2025]**

(A) e^4 (B) e^5

(C) e^{11} (D) e^7

92. $\lim\limits_{n\to\infty}\left[\dfrac{1}{1-n^4}+\dfrac{8}{1-n^4}+\ldots\ldots\ldots\ldots+\dfrac{n^3}{1-n^4}\right]=$ **[2025]**

(A) $\dfrac{1}{4}$ (B) $\dfrac{1}{2}$

(C) $-\dfrac{1}{2}$ (D) $-\dfrac{1}{4}$

93. If $f\left(x\right)=\dfrac{\sin\left(\pi\cos^2x\right)}{3x^2}, x\neq 0$ is continuous at $x=0$ then $f\left(0\right)=$ **[2025]**

(A) 0
(B) $\dfrac{\pi}{3}$
(C) $\dfrac{-\pi}{3}$
(D) $\dfrac{3}{\pi}$

94. If the function

$$f\left(x\right)=\begin{cases}\dfrac{\cos ax-\cos bx}{\cos cx-\cos bx} & ,\ \text{if } x\neq 0\\ -1 & ,\ \text{if } x=0\end{cases}$$ is

continuous at $x=0$, then a^2, b^2, c^2 are in

(A) Geometric progression
(B) Arithmetic progression
(C) Harmonic progression
(D) Arithmetico-Geometric progression

95 If $f\left(x\right)=\begin{cases}\dfrac{8^x-4^x-2^x+1}{x^2}, & \text{if } x>0\\ e^x\sin x+x+\lambda\log 4, & \text{if } x\leqslant 0\end{cases}$ is

continuous at $x=0$ then the value of $1000e^{\lambda}=$ **[2025]**

(A) 1000
(B) 3000
(C) 2000
(D) 4000

96. Let $f:\mathbb{R}\to\mathbb{R}$ is differentiable function having $f\left(3\right)=3, f'\left(3\right)=\dfrac{1}{27}$ and

$$g\left(x\right)=\begin{cases}\int_3^{f(x)}\dfrac{3t^2}{x-3}\,dt, & \text{if } x\neq 3\\ K, & \text{if } x=3\end{cases}$$

is continuous at $x=3$, then $K=$ **[2025]**

(A) 1 (B) 3

(C) $\dfrac{1}{3}$ (D) 9

97. If $f\left(x\right)=\begin{cases}\dfrac{9^x-2\cdot 3^x+1}{\log\left(1+3x\right)\cdot\tan 2x} & ,\ \text{if } x\neq 0\\ a(\log b)^c & ,\ \text{if } x=0\end{cases}$

continuous at $x=0$, then $a+b+c=$ **[2025]**

(A) $\dfrac{31}{6}$ (B) $\dfrac{1}{6}$

(C) $\dfrac{5}{6}$ (D) $\dfrac{3}{20}$

98. The function $f(x)=2x-|x-x^2|$ is

(A) continuous at $x=1$

(B) discontinuous at $x=1$

(C) not defined at $x=1$

(D) discontinuous at $x=0$

99. If the function

$$f(x)=\begin{cases} x+a\sqrt{2}\sin x & \text{if } 0\le x\le \frac{\pi}{4} \\ 2x\cot x+b & \text{if } \frac{\pi}{4}<x\le\frac{\pi}{2} \\ a\cos 2x-b\sin x & \text{if } \frac{\pi}{2}<x\le\pi \end{cases}$$

is continuous in $[0,\pi]$ then $a-b=$

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{12}$

(C) $\frac{5\pi}{12}$ (D) $\frac{7\pi}{12}$

100. If $(p\wedge\sim q)\wedge(p\wedge r)\rightarrow\sim p\vee q$ is false, then the truth values of p,q and r are respectively

(A) T,T,T (B) F,F,F

(C) T,F,T (D) F,T,F

101. If a statement q has truth value False and $(p\wedge q)\leftrightarrow r$ has truth value True then which of the following has truth value true?

(A) $p\wedge q$

(B) $p\vee r$

(C) $p\wedge r$

(D) $(p\wedge r)\rightarrow(p\vee r)$

102. If the statement pattern $(p\wedge q)\rightarrow(r\vee\sim s)$ is false, then the truth values of p,q,r and s are respectively

(A) T, F, T, F (B) T,T,T,F

(C) T,T,F,F (D) T,T,F,T

103. If the truth value of the statement pattern $[p\wedge\sim r]\rightarrow\sim r\wedge q$ is False, then which of the following has truth value False?

(A) $(p\vee r)\rightarrow\sim r$

(B) $(r\vee q)\rightarrow\sim p$

(C) $\sim(p\vee q)\rightarrow\sim r$

(D) $\sim(r\vee q)\rightarrow\sim p$

104. p : If 7 is an odd number then 7 is divisible by 2.
q: If 7 is prime number then 7 is an odd number.
If V_1 and V_2 are respective truth values of contrapositive of p and q then $(V_1, V_2)\equiv$

(A) (T, T) (B) (T, F)
(C) (F, T) (D) (F, F)

105. The contrapositive of the statement $\sim p\vee(q\wedge\sim r)$ is

(A) $p\rightarrow q\wedge r$ (B) $(q\wedge r)\rightarrow p$

(C) $\sim q\vee\sim r\rightarrow p$ (D) $(r\vee\sim q)\rightarrow\sim p$

106. The equivalent statement of "If three vertices of a triangle are represented by cube roots of unity, then the triangle is an equilateral triangle" is **[2025]**

(A) Three vertices of a triangle are represented by cube roots of unity and the triangle is not an equilateral triangle.

(B) If a triangle is an equilateral triangle then the three vertices of a triangle are represented by cube roots of unity.

(C) If three vertices of triangle are not represented by cube roots of unity then the triangle is not an equilateral triangle.

(D) If a triangle is not an equilateral triangle then the three vertices of the triangle can not be represented by cube roots of unity.

107. The last column in the truth table of the statement pattern $[p\rightarrow(q\wedge\sim p)]\vee[(p\vee\sim q)\wedge p]$ is

(A) TTTF

(B) TFFF

(C) TTTT

(D) FFTT [1]

108. The logical statement
$[\sim(\sim p \vee q) \vee (p \wedge r) \wedge (\sim q \wedge r)]$
is equivalent to
(A) $(p \wedge r) \wedge \sim q$
(B) $(\sim p \wedge \sim q) \wedge r$
(C) $\sim p \vee r$
(D) $(p \wedge \sim q) \vee r$

109. The statement pattern $[(p \to q) \wedge \sim q] \to r$ is a tautology when r is equivalent to [2025, 2023]
(A) $p \wedge \sim q$
(B) $q \vee p$
(C) $p \wedge q$
(D) $\sim q$

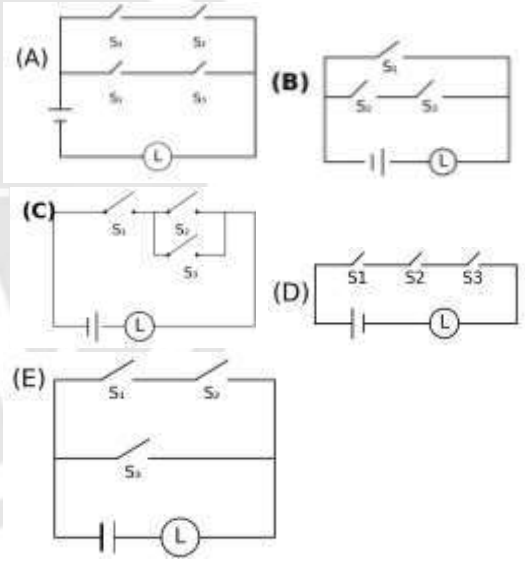
110. The negation of $(p \wedge \sim q) \to (p \vee \sim q)$ is **[2025]**
(A) a tautology (B) a contingency
(C) a contradiction (D) equivalent to $p \wedge q$

111. Which of the following statement is a tautology? **[2025]**
(A) $(\sim q \wedge p) \wedge (p \wedge \sim p)$
(B) $(p \wedge q) \wedge (\sim p \wedge q)$
(C) $[(\sim q) \wedge p] \wedge q$
(D) $(\sim q \wedge p) \vee (p \vee \sim p)$

112. If the statements p, q and r are true, false and true statements respectively, then the truth value of the statement pattern $[\sim q \wedge (p \vee \sim q) \wedge \sim r] \vee p$ and the truth value of its dual statement respectively are **[2025]**
(A) T, T
(B) F, T
(C) T, F
(D) F, F

113. The negation of the statement "The triangle is an equilateral or isosceles triangle and the triangle is not isosceles and it is right angled" is
(A) The triangle is not an equilateral or not an isosceles triangle or it is not an isosceles or it is not right angled
(B) The triangle is not an equilateral triangle or not isosceles triangle and it is isosceles or it is not right angled
(C) If the triangle is an equilateral triangle or an isosceles triangle then it is an isosceles triangle or not right angled
(D) If the triangle is an equilateral triangle or an isosceles triangle then it is not isosceles triangle and it is not right angled.

114. Which of the following are pairs of equivalent circuits **[2025, 2024, 2023]**

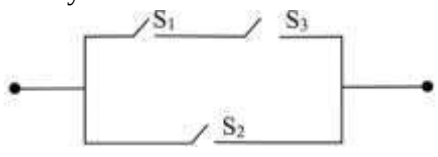


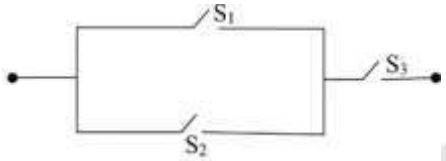
(A) (A) and (B)
(B) (B) and (D)
(C) (C) and (E)
(D) (A) and (C)

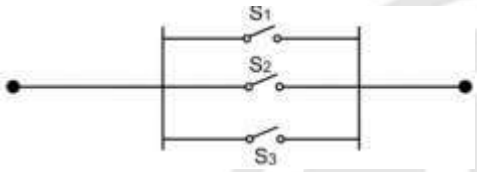
115. The correct simplified circuit diagram for the logical statement

$\left[\{q\wedge(\sim q\vee r)\}\wedge\{\sim p\vee(p\wedge\sim r)\}\right]\vee(p\wedge r)$

Where p,q,r represents switches S_1, S_2, S_3 respectively. **[2025]**

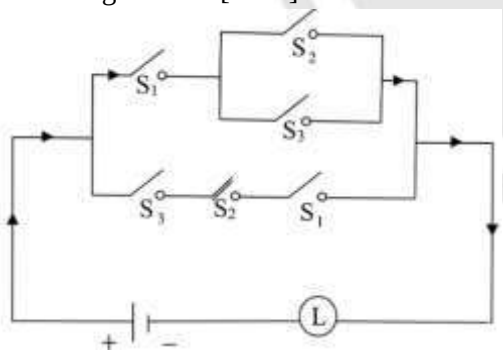
(A) 

(B) 

(C) 

(D) 

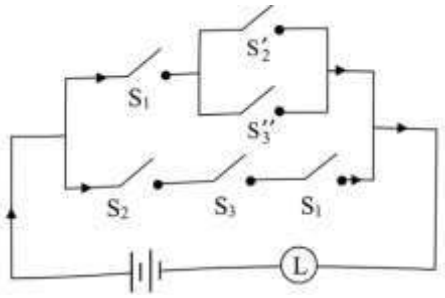
116. If p : switch S_1 is closed, q : switch S_2 is closed, r : switch S_3 closed, then the symbolic form of the following switching circuit is equivalent to Switching Circuit: [2025]



(A) $p\wedge(q\vee r)$ (B) $q\vee r$

(C) p (D) $(\sim q\wedge\sim r)$

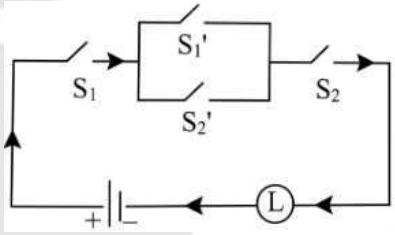
117. Number of switches in alternative equivalent simple circuit for the circuit is (are) **[2025]**



(A) 0 (B) 1

(C) 2 (D) 3

118. If p : switch S_1 is closed, q : switch S_2 is closed then correct interpretation from the following circuit is



(A) The lamp is always on

(B) The lamp is always off

(C) Symbolic form is $p\vee(\sim p\wedge\sim q)\vee q$

(D) is equivalent to $p\vee q$

119. Consider the three statements

$p:\forall n\in\mathbb{N},10n-3$ is a prime number, when n is not divisible by 3

$q:\frac{2}{\sqrt{3}},\frac{-2}{\sqrt{3}},\frac{-1}{\sqrt{3}}$ are the direction cosines of a directed line.

$r:\sin x$ is an increasing function in the interval $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$ Then which of the following statement pattern has truth value true? **[2025]**

(A) $(p\wedge q)\leftrightarrow r$

(B) $(p\rightarrow q)\rightarrow\sim r$

(C) $(\sim p\vee q)\wedge r$

(D) $(\sim p\wedge\sim q)\leftrightarrow\sim r$

120. Which of the following statements has the truth value T?

A: cube roots of unity are in Geometric progression and their sum is 1

B: $4+7>10$ iff $2+8<10$

C: $\exists x \in \mathbb{N}$ such that $x^2-3x+2=0$ and $\exists n \in \mathbb{N}$ such that n is an odd number

D: $3+i$ is a complex number or $\sqrt{2}+\sqrt{3}=\sqrt{5}$

(A) only A (B) $B, C \& D$

(C) both A and C (D) both C and D

121. If p, q, r, s are statements, where,

$p: A^2-B^2=(A-B)(A+B); A, B$ are matrices, $AB \neq BA$

$q: 5 \leq 5$

$r: {}^8C_1+{}^8C_2+{}^8C_3+\ldots\ldots+{}^8C_8=256$

s: Maximum value of ${}^8C_r$ is 70 then the statement from the following having truth value true is .... **[2025]**

(A) $(p \wedge \sim r) \vee (\sim q \wedge \sim s)$

(B) $(p \vee \sim q) \leftrightarrow (\sim r \rightarrow s)$

(C) $(p \leftrightarrow q) \wedge (\sim p \vee \sim q)$

(D) $(s \vee \sim p) \leftrightarrow (\sim p \wedge \sim r)$

122. The third element in the second row of adjoint of a matrix $A=\left[a_{ij}\right]_{3\times 3}$ (where $a_{ij}=2i+j$) is **[2025]**

(A) 2 (B) –2

(C) 4 (D) –4

123. If $A=\begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$ and $A \cdot \text{adj } A = AA^T$, then $5a+b=$

(A) 7 (B) 9

(C) 13 (D) 5

124. If $A=\begin{bmatrix} 1 & -2 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$ then $A(I+adjA)=$

(A) $\begin{bmatrix} 9 & -2 & 2 \\ 0 & 10 & -3 \\ 3 & -2 & 11 \end{bmatrix}$ (B) $\begin{bmatrix} 8 & -2 & 2 \\ 0 & 9 & -3 \\ 3 & -2 & 10 \end{bmatrix}$

(C) $\begin{bmatrix} 9 & -2 & 2 \\ 0 & 10 & -3 \\ 3 & -2 & 12 \end{bmatrix}$ (D) $\begin{bmatrix} 3 & 2 & -2 \\ 0 & 10 & 3 \\ -3 & 2 & 12 \end{bmatrix}$

125. If $A=\begin{bmatrix} 1 & \cot\frac{\theta}{2} \\ -\cot\frac{\theta}{2} & 1 \end{bmatrix}$ then $A^{-1}=$ **[2025]**

(A) $cosec^2\frac{\theta}{2}\,A^T$ (B) $\frac{-\sin^2\theta}{2}A^T$

(C) $\left(\frac{1+\cos\theta}{2}\right)A^T$ (D) $\left(\frac{1-\cos\theta}{2}\right)A^T$

126. If $A=\begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$, then $A^TA^{-1}=$

(A) $\begin{bmatrix} \cos 2x & -\sin 2x \\ -\sin 2x & \cos 2x \end{bmatrix}$

(B) $\begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$

(C) $\begin{bmatrix} -\cos 2x & \sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$

(D) $\begin{bmatrix} -\cos 2x & \sin 2x \\ -\sin 2x & \cos 2x \end{bmatrix}$

127. If matrix $A=\frac{1}{11}\begin{bmatrix}-1 & 7 & -24\\ 2 & a & 4\\ 2 & -3 & 15\end{bmatrix}$ and $A^{-1}=\begin{bmatrix}3 & 3 & 4\\ 2 & -3 & 4\\ b & -1 & c\end{bmatrix}$, then the values of a,b,c respectively are _____

(A) $3,1,0$ (B) $\frac{-6}{11},0,\frac{1}{11}$

(C) $-3,0,1$ (D) $\frac{-3}{11},0,\frac{1}{11}$

128. If A and B are non-singular matrices of order 2 such that $(AB)^{-1}=\frac{1}{6}\begin{bmatrix}-7 & -3\\ 2 & 3\end{bmatrix}$ and $A^{-1}=\frac{1}{3}\begin{bmatrix}4 & 3\\ -1 & 0\end{bmatrix}$ then $B^{-1}=$ **[2025]**

(A) $\frac{1}{2}\begin{bmatrix}2 & 3\\ 1 & -1\end{bmatrix}$

(B) $\frac{1}{2}\begin{bmatrix}3 & 1\\ 2 & 4\end{bmatrix}$

(C) $\frac{1}{2}\begin{bmatrix}-1 & 3\\ 1 & 2\end{bmatrix}$

(D) $\frac{1}{6}\begin{bmatrix}1 & 1\\ 2 & 3\end{bmatrix}$

129. If A is a matrix of order 2 and I is the identity matrix of order 2 such that $A^2-4\,\mathrm{A}+3I=0$ then $(A+3I)^{-1}=$

(A) $\frac{A}{24}-\frac{7}{24}I$ (B) $\frac{A}{21}-\frac{7}{21}I$

(C) $\frac{7I}{24}-\frac{1}{24}\,\mathrm{A}$ (D) $A-3I$

130. Let A be a non-singular matrix of order n and $|A|=k$, then $(adjA)^{-1}$ is

(A) $\frac{A}{k}$ (B) $k^{n-1}(adj\ \mathrm{A})$

(C) $k^{n-2}\,\mathrm{A}$ (D) kA

131. If $A=\begin{bmatrix}1 & -1 & 1\\ 0 & 2 & -3\\ 2 & 1 & 0\end{bmatrix}, B=adjA$ and $C=5A$, then $\frac{|\mathrm{adj\,B}|}{|C|}=$ **[2025, 2022]**

(A) 2 (B) 4

(C) 1 (D) 5

132. If $\begin{bmatrix}1 & 3 & 3\\ 1 & 4 & 4\\ 1 & 3 & 4\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}12\\ 15\\ 13\end{bmatrix}$, then the value of $x^2+y^2+z^2=$ **[2025, 2020]**

(A) 6 (B) 12

(C) 3 (D) 14

133. The possible values of $\theta\in(0,\pi)$ such that $\sin\theta+\sin(4\theta)+\sin(7\theta)=0$ are

(A) $\frac{\pi}{4},\frac{5\pi}{12},\frac{\pi}{2},\frac{2\pi}{3},\frac{3\pi}{4},\frac{8\pi}{9}$

(B) $\frac{2\pi}{9},\frac{\pi}{4},\frac{\pi}{2},\frac{2\pi}{3},\frac{3\pi}{4},\frac{35\pi}{36}$

(C) $\frac{2\pi}{9},\frac{\pi}{4},\frac{\pi}{2},\frac{2\pi}{3},\frac{3\pi}{4},\frac{8\pi}{10}$

(D) $\frac{2\pi}{9},\frac{\pi}{4},\frac{4\pi}{9},\frac{\pi}{2},\frac{3\pi}{4},\frac{8\pi}{9}$

134. The number of solutions of $16^{\sin^2 x}+16^{\cos^2 x}=10$ in $0\leqslant x\leqslant 2\pi$ are

(A) 8 (B) 10

(C) 6 (D) 4

135. The principal solution of $(5+3\sin\theta)(2\cos\theta+1)=0$ are

(A) $\frac{-\pi}{3},\frac{2\pi}{3}$ (B) $\frac{2\pi}{3},\frac{5\pi}{3}$

(C) $\frac{2\pi}{3},\frac{4\pi}{3}$ (D) $\frac{2\pi}{3},\frac{7\pi}{3}$

136. The ratios of sides in a triangle ABC are $5:12:13$ and its area is 270. Then sides of the triangle are

(A) $5,12,13$ (B) $10,24,26$

(C) $15,36,39$ (D) $20,48,52$

137. In a triangle ABC with usual notations, if a,b,c are in arithmetic progression, then, $\tan\frac{A}{2}\cdot\tan\frac{C}{2}=$

(A) 3 (B) $\frac{1}{13}$

(C) –3 (D) $\frac{1}{3}$

138. With usual notations, in a triangle ABC, if θ is any real number, then $a\cos(B-\theta)+b\cos(A+\theta)$ is

(A) $a\cos\theta$

(B) $b\cos\theta$

(C) $\cos\theta$

(D) $c\cos\theta$

139. In a triangle ABC with usual notations if $a=13$, $b=14, c=15$ Then $\sin A=$

(A) $\frac{4}{5}$ (B) $\frac{3}{5}$

(C) $\frac{1}{2}$ (D) $\frac{4}{7}$

140. With usual notation, in triangle ABC, $m\angle A=30^\circ$ then the value of $\left(1+\frac{a}{c}+\frac{b}{c}\right)\left(1+\frac{c}{b}-\frac{a}{b}\right)$ is equal to **[2025]**

(A) $\frac{2+\sqrt{3}}{2}$

(B) $2+\sqrt{3}$

(C) $\frac{1+\sqrt{3}}{2}$

(D) $1+\sqrt{3}$

141. In ΔABC, with usual notations, $a\cos B=b\cos A, a\cos C\neq c\cos A$ then $A(\Delta ABC)$ ____ sq. units. **[2025]**

(A) $\frac{c}{2}\sqrt{4a^2-b^2}$ (B) $\frac{c}{4}\sqrt{4a^2-c^2}$

(C) $\frac{b}{2}\sqrt{4b^2-c^2}$ (D) $\frac{b}{4}\sqrt{4b^2-c^2}$

142. In a triangle ABC, with usual notations if $a=4$, $b=8, \angle C=60^\circ$, then the value of $\angle B$ and the ratio $\cos A:\cos C$ respectively are**, [2025]**

(A) $\frac{\pi}{4}, 1:\sqrt{3}$ (B) $\frac{\pi}{2}, \sqrt{3}:1$

(C) $\frac{\pi}{2}, 2:\sqrt{3}$ (D) $\frac{\pi}{6}, \sqrt{3}:2$

143. In a triangle ABC, with usual notations, $\tan\left(\frac{A}{2}\right)=\frac{5}{6}, \tan\left(\frac{C}{2}\right)=\frac{2}{5}$, then **[2025]**

(A) a,c,b are in A.P.

(B) b,a,c are in A.P.

(C) a,b,c are in A.P.

(D) a,b,c are in G.P.

144. In ΔABC, with usual notations, if $a^4+b^4+c^4-2a^2c^2-2c^2\ b^2=0$, then $\angle C=\ldots.$ **[2025]**

(A) 135° (B) 120°

(C) 150° (D) 125°

145. In a triangle ABC with usual notations if, $\cot\frac{A}{2}=\frac{b+c}{a}$, then the triangle ABC is **[2025]**

(A) an isosceles triangle.

(B) an equilateral triangle.

(C) a right angled triangle.

(D) an obtuse angled triangle.

146. In ΔABC, with usual notations, if $\cos\frac{B}{2}=\sqrt{\frac{c+a}{2a}}$ then $a^2=$ **[2025]**

(A) b^2-c^2 (B) $b+c$

(C) b^2+c^2 (D) $b-c$

147. In a triangle ABC with usual notations, $\cot\frac{A}{2}+\cot\frac{B}{2}+\cot\frac{C}{2}=$ **[2025]**

(A) $\frac{s^2}{\Delta}$, where Δ is the area of the triangle ABC

(B) $\frac{s}{\Delta}$, where Δ is the area of the triangle ABC

(C) $\frac{\Delta}{s}$, where Δ is the area of the triangle ABC

(D) Δ, where Δ is the area of the triangle ABC

148. With usual notations, in ΔABC, the lengths of two sides are 10 cm and 9 cm respectively. If angles A,B,C are in A.P. then perimeter of ΔABC is

(A) $24+2\sqrt{6}$ cm

(B) $24+\sqrt{6}$ cm

(C) $24-2\sqrt{6}$ cm

(D) $22-\sqrt{6}$ cm

149. In a triangle *ABC*, with usual notations, if $a=5,\ b=4,\cos(A-B)=\frac{31}{32}$, then $c=$ **[2025]**

(A) 6 (B) 7

(C) 5 (D) 2

150. In a triangle *ABC* with usual notations if $b\sin C(b\cos C+c\cos B)=42$, then area of triangle $ABC=$ **[2025]**

(A) 42 sq. units (B) 21 sq. units

(C) 24 sq. units (D) 12 sq. units

151. In a triangle with one of the angles 120°, the lengths of the sides form an A.P. If length of the greatest side is 7 m, then the area of the triangle is **[2025]**

(A) $\frac{15\sqrt{3}}{4}$ m^2 (B) $\frac{15\sqrt{3}}{2}$ m^2

(C) $\frac{15}{2}$ m^2 (D) $\frac{15}{4}$ m^2

152. In a triangle *ABC*, with usual notations, if $a=5$, $b=7\sin A=\frac{3}{4}$, then total number of triangles possible are **[2025]**

(A) 1 (B) 0

(C) 2 (D) 5

153. In a triangle *ABC*, with usual notations, $(a+b+c)(a+b-c)=3ab$, then $\angle C=$ **[2025]**

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$ (D) $\frac{\pi}{6}$

154. If $4\sin^{-1}x+\cos^{-1}x=\pi$ then $x=$ **[2025]**

(A) $\frac{\sqrt{3}}{2}$ (B) 0

(C) $\frac{1}{2}$ (D) $\frac{1}{\sqrt{2}}$

155. If $2\tan^{-1}(\cos x)=\tan^{-1}(2\,\mathrm{cosec}\,x)$, then the value of x is **[2025, 2021]**

(A) $-\frac{\pi}{4}$ (B) 0

(C) $\frac{\pi}{8}$ (D) $\frac{\pi}{4}$

156. If $\sin^{-1}\frac{1}{3}+\sin^{-1}\frac{3}{5}+\sin^{-1}x=\frac{\pi}{2}$ then $x=$ **[2025]**

(A) $\frac{8\sqrt{2}+3}{5}$ (B) $\frac{8\sqrt{2}-3}{5}$

(C) $\frac{8\sqrt{2}+3}{15}$ (D) $\frac{8\sqrt{2}-3}{15}$

157. The value of $2\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{3}{8}$ is **[2025]**

(A) $\tan^{-1}\left(\frac{42}{24}\right)$ (B) $2\tan^{-1}\left(\frac{42}{24}\right)$

(C) $\tan^{-1}\left(\frac{24}{41}\right)$ (D) $\tan^{-1}\left(\frac{41}{12}\right)$

158. If $\sin^{-1}x+\sin^{-1}y=\frac{\pi}{3}$ and $\cot^{-1}\left(\frac{1}{x}\right)-\cot^{-1}\left(\frac{1}{y}\right)=0$ then $2x^2+y^2-xy=$ .. **[2025]**

(A) $\frac{1}{4}$ (B) 1

(C) $\frac{1}{2}$ (D) 0

159. The number of solutions of $\tan^{-1}\left(x+\frac{2}{x}\right)-\tan^{-1}\left(\frac{4}{x}\right)-\tan^{-1}\left(x-\frac{2}{x}\right)=0$ are **[2025]**

(A) 1 (B) 2

(C) 3 (D) 0

160. If $\left(\cos^{-1}x\right)^2-\left(\sin^{-1}x\right)^2>0$, then **[2025]**

(A) $x<\frac{1}{2}$

(B) $-1<x<\sqrt{2}$

(C) $0\leqslant x<\frac{1}{\sqrt{2}}$

(D) $-1\leqslant x<\frac{1}{\sqrt{2}}$

161. If $\tan^{-1}(x+1)+\tan^{-1}x+\tan^{-1}(x-1)=\tan^{-1}3$, then for $x<0$ the value of $500x^4+270x^2+997=$ **[2025]**

(A) 6716 (B) 1767

(C) 1768 (D) 6717

162. The number of positive integral solutions of $\tan^{-1}x+\cos^{-1}\left(\frac{y}{\sqrt{1+y^2}}\right)=\sin^{-1}\left(\frac{3}{\sqrt{10}}\right)$ are **[2025]**

(A) 1 (B) 2

(C) 3 (D) 4

163. If $\cot^{-1}\left(\sqrt{\cos\alpha}\right)-\tan^{-1}\left(\sqrt{\cos\alpha}\right)=x$, then the value of $\sin x$ is **[2025]**

(A) $\cot^2\frac{\alpha}{2}$ (B) $\cot\frac{\alpha}{2}$

(C) $\tan\frac{\alpha}{2}$ (D) $\tan^2\frac{\alpha}{2}$

164. $z=\frac{3+2i\sin\theta}{1-2i\sin\theta},\left(i=\sqrt{-1}\right)$ will be purely imaginary if $\theta=$ **[2025]**

(A) $2n\pi\pm\frac{\pi}{8}$, where $n\in\mathbb{Z}$

(B) $n\pi+\frac{\pi}{8}$, where $n\in\mathbb{Z}$

(C) $n\pi\pm\frac{\pi}{3}$, where $n\in\mathbb{Z}$

(D) $n\pi$, where $n\in\mathbb{Z}$

165. In a triangle ABC, the sides a,b,c are such that they are the roots of the equation $x^3-11x^2+38x-40=0$ Then $\frac{\cos A}{a}+\frac{\cos B}{b}+\frac{\cos C}{c}=$ **[2025]**

(A) $\frac{3}{4}$

(B) 1

(C) $\frac{9}{16}$

(D) $\frac{1}{16}$

166. The sum to infinite terms of the series $\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{2}{9}\right)+\ldots\ldots+\tan^{-1}\left(\frac{2^{n-1}}{1+2^{2n-1}}\right)+\ldots\ldots$ is **[2025]**

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$

(C) $\frac{\pi}{6}$ (D) $\frac{\pi}{3}$

167. $\cot^{-1}\left(2\cdot1^2\right)+\cot^{-1}\left(2\cdot2^2\right)+\cot^{-1}\left(2\cdot3^2\right)+\ldots\infty=$ **[2025]**

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$

(C) $\frac{\pi}{4}$ (D) $\frac{\pi}{8}$

168. The value of $\cos^{-1}\left\{\frac{1}{\sqrt{2}}\left(\cos\frac{9\pi}{10}-\sin\frac{9\pi}{10}\right)\right\}$ is **[2025, 2024]**

(A) $\frac{7\pi}{20}$ (B) $\frac{13\pi}{20}$

(C) $\frac{17\pi}{20}$ (D) $\frac{21\pi}{20}$

169. The joint equation of the bisectors of the angles between the lines $x=5$ and $y=3$ is **[2025]**

(A) $x^2-y^2-10x+6y+16=0$

(B) $x^2+y^2-10x-6y-16=0$

(C) $x^2+y^2+10x+6y-16=0$

(D) $x^2+y^2-5x-2y-7=0$

170. The joint equation of two lines passing through $(-2,3)$ and parallel to the bisectors of the angle between the co-ordinate axes is **[2025]**

(A) $x^2-y^2+4x+6y-4=0$

(B) $x^2+y^2+4x+6y-5=0$

(C) $x^2-y^2+4x+6y-5=0$

(D) $x^2+y^2+4x+6y+4=0$

171. If m_1 and m_2 are the slopes of the lines represented by $ax^2+2\,hxy+by^2=0$ satisfying the condition $16\,h^2=25ab$, then .... **[2025]**

(A) $m_1=m_2^2$

(B) $m_1=4m_2$

(C) $|m_1-m_2|=2$

(D) $m_1m_2=1$

172. The perpendicular distance between the lines given by $(x-2y+1)^2+k(x-2y+1)=0$ is $\sqrt{5}$, then $k=$ **[2025]**

(A) 5 (B) 2

(C) 4 (D) 6

173. The joint equation of the bisector of the angle between the lines $2x^2+11xy+3y^2=0$ is **[2025]**

(A) $11x^2+2xy-11y^2=0$

(B) $x^2+2xy-y^2=0$

(C) $3x^2-11xy+2y^2=0$

(D) $11x^2-2xy-11y^2=0$

174. If the equation $kxy+10x+8y+16=0$ represents a pair of lines, then **[2025]**

(A) $k=5$ only

(B) $k=0$ only

(C) $k=0$ or $k=5$

(D) the value of k does not exist

175. If the pair of straight lines $xy-x+y-1=0$ and the line $x+ky-3=0$ are concurrent, then the value of k is equal to **[2025]**

(A) 4

(B) 3

(C) –1

(D) 2

176. The distance between the lines represented by $16x^2+9y^2+48x-24xy-36y+35=0$ is ____ units **[2025]**

(A) $\frac{2}{5}$ (B) $\frac{35}{2}$

(C) $\frac{5}{2}$ (D) $\frac{7}{5}$

177. The circumradius of the triangle formed by the lines $xy+2x+2y+4=0$ and $x+y+2=0$ is **[2025]**

(A) 2 units (B) 1 unit

(C) $\sqrt{2}$ units (D) $\sqrt{3}$ units

178. The number of integral values of p for which the vectors $(p+1)\hat{i}-3\hat{j}+p\hat{k}, p\hat{i}+(p+1)\hat{j}-3\hat{k}$ and $-3\hat{i}+p\hat{j}+(p+1)\hat{k}$ are linearly dependent vectors, are **[2025]**

(A) 0 (B) 1

(C) 2 (D) 3

179. The area of the rectangle having vertices P,Q,R,S with position vectors $-\hat{i}+\hat{j}+\hat{k}, \hat{i}+\hat{j}+\hat{k}, \hat{i}-\hat{j}+\hat{k}, -\hat{i}-\hat{j}+\hat{k}$ respectively is **[2025]**

(A) 1 square unit

(B) 2 square units

(C) 3 square units

(D) 4 square units

180. Let $\overline{OA}=\bar{a}, \overline{OB}=\bar{b}$ and if the vector along the angle bisector of $\angle AOB$ is given by $x\frac{\bar{a}}{|\bar{a}|}+y\frac{\bar{b}}{|\bar{b}|}$ then **[2025]**

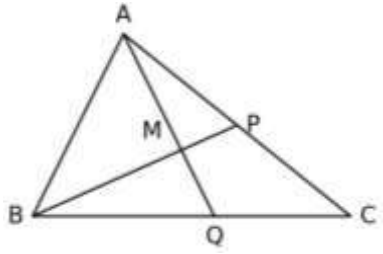
(A) $x-y=0$

(B) $x+y=0$

(C) $x=2y$

(D) $y=2x$

181.



In the above figure, P divides AC in the ratio $3:4$ and Q divides BC in the ratio $4:3$. Then M divides AQ in the ratio **[2025]**

(A) $15:14$

(B) $29:13$

(C) $21:16$

(D) $28:9$

182. If $\bar{a},\bar{b},\bar{c}$ are three vectors such that $|\bar{a}|=3$, $|\bar{b}|=5, |\bar{c}|=7$ then $|\bar{a}-\bar{b}|^2+|\bar{b}-\bar{c}|^2+|\bar{c}-\bar{a}|^2$ does not exceed

(A) 83 (B) 166

(C) 249 (D) 105

183. In a triangle ABC with usual notations if $|\overline{BC}|=8, |\overline{CA}|=7, |\overline{AB}|=10$ then the projection of $\overline{AB}$ on $\overline{AC}$ is **[2025]**

(A) $\frac{14}{85}$ units (B) $\frac{1}{85}$ units

(C) $\frac{85}{14}$ units (D) $\frac{7}{85}$ units

184. If the projection of $\bar{a}$ on $\bar{b}+\bar{c}$ is twice the projection of $\bar{b}+\bar{c}$ on $\bar{a}$ also if $|\bar{b}|=2\sqrt{2}, |\bar{c}|=4$ and the angle between $\bar{b}$ and $\bar{c}$ is $\frac{\pi}{4}$ then $|\bar{a}|=$

(A) $2\sqrt{10}$ (B) $3\sqrt{10}$

(C) $4\sqrt{10}$ (D) $5\sqrt{10}$

185. If $\bar{a}$ and $\bar{b}$ are unit vectors such that $|\bar{a}+\bar{b}|=\sqrt{3}$ then the angle between $\bar{a}$ and $\bar{b}$ is **[2025]**

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$

(C) $\frac{\pi}{4}$ (D) $\frac{\pi}{2}$

186. $\bar{a},\bar{b},\bar{c}$ are nonzero vectors such that $\bar{a}$ is perpendicular to $\bar{b}$ and $\bar{c}, |\bar{a}|=1, |\bar{b}|=2, |\bar{c}|=1$ and $\bar{b}\cdot\bar{c}=1$. There is nonzero vector $\bar{d}$ coplanar with $\bar{a}+\bar{b}$ and $2\bar{b}-\bar{c}$. If $\bar{d}\cdot\bar{a}=1$, then $|\bar{d}|^2=$

(A) $13y^2+14y+5$ (B) $y^2+14y+5$

(C) $y^2-14y-5$ (D) $y^2-14y+5$

187. If $\bar{a},\bar{b},\bar{c}$ are three-unit vectors such that $|\bar{a}+\bar{b}|^2+|\bar{a}+\bar{c}|^2=8$, then $|\bar{a}+3\bar{b}|^2+|\bar{a}+3\bar{c}|^2=$ **[2025]**

(A) 26 (B) 32

(C) 22 (D) 36

188. If the vectors $\bar{a}=c(\log_7 x)\hat{i}+2\hat{j}+3\hat{k}$ and $\bar{b}=(\log_7 x)\hat{i}+3c(\log_7 x)\hat{j}-4\hat{k}$ make obtuse angle for any $x>0$, then c belongs to **[2025]**

(A) $\left(0,\frac{3}{4}\right)$ (B) $\left(\frac{-3}{4},0\right)$

(C) $\left(\frac{-4}{3},0\right)$ (D) $\left(0,\frac{4}{3}\right)$

189. Let $\bar{a},\bar{b}$, and $\bar{c}$ be unit vectors. Suppose that $\bar{a}\cdot\bar{b}=\bar{a}\cdot\bar{c}=0$ and if the angle between $\bar{b}$ and $\bar{c}$ is $\frac{\pi}{6}$, then $\bar{a}$ is **[2025]**

(A) $\pm(\bar{b}\times\bar{c})$ (B) $\pm\frac{1}{2}(\bar{b}\times\bar{c})$

(C) $\pm2(\bar{b}\times\bar{c})$ (D) $\pm4(\bar{b}\times\bar{c})$

190. The length of the altitude through the point D of tetrahedron where the vertices of the tetrahedron are $A(2,3,1), B(4,1,-2), C(6,3,7),$, $D(-5,-4,8)$ is **[2025]**

(A) 5.5 units (B) 22 units

(C) 33 units (D) 11 units

191. The area of a parallelogram whose diagonals are the vectors $2\bar{a}-\bar{b}$ and $4\bar{a}-5\bar{b}$, where $\bar{a}$ and $\bar{b}$ are unit vectors forming an angle of 45° is **[2025]**

(A) $3\sqrt{2}$ sq. units (B) $\frac{3}{\sqrt{2}}$ sq. units

(C) $\sqrt{2}$ sq. units (D) $\frac{\sqrt{2}}{3}$ sq. units

192. If $\bar{p}=2\hat{i}+\hat{k}, \bar{q}=\hat{i}+\hat{j}+\hat{k}, \bar{r}=4\hat{i}-3\hat{j}+7\hat{k}$ and a vector $\bar{m}$ is such that $\bar{m}\times\bar{q}=\bar{r}\times\bar{q}, \bar{m}\cdot\bar{p}=0$. then $\bar{m}=\ldots.$,

(A) $\hat{i}-8\hat{j}-2\hat{k}$ (B) $-10\hat{i}+3\hat{j}+7\hat{k}$

(C) $-\hat{i}-8\hat{j}+2\hat{k}$ (D) $2\hat{i}+4\hat{j}+\hat{k}$

193. If the area of parallelogram, whose diagonals are $\hat{i}-\hat{j}+2\hat{k}$ and $2\hat{i}+3\hat{j}+\alpha\hat{k}$ is $\frac{\sqrt{93}}{2}$ sq. units, then $\alpha=$

(A) $-4,2$ (B) $-3,-2$

(C) 2,1 (D) 4,2

194. If $\bar{a},\bar{b},\bar{c}$ are three vectors such that $|\bar{a}|=\sqrt{31}, 4|\bar{b}|=|\bar{c}|=2$ and $2(\bar{a}\times\bar{b})=3(\bar{c}\times\bar{a})$ and if the angle between $\bar{b}$ and $\bar{c}$ is $\frac{2\pi}{3}$ then $\left|\frac{\bar{a}\times\bar{c}}{\bar{a}\cdot\bar{b}}\right|^2=$

(A) 1 (B) 2

(C) 3 (D) 11

195. If $\bar{a}=2\hat{i}+3\hat{j}+4\hat{k}, \bar{b}=\hat{i}-2\hat{j}-2\hat{k}, \bar{c}=-\hat{i}+4\hat{j}+3\hat{k}$ and if $\bar{d}$ is vector perpendicular to both $\bar{b}$ and $\bar{c}, \bar{a}\cdot\bar{d}=18$, then $\left|\bar{a}\times\bar{d}\right|^2=$ **[2025]**

(A) 640 (B) 680
(C) 720 (D) 740

196. The position vectors of the points A, B, C are $\hat{i}+2\hat{j}-\hat{k}, \hat{i}+\hat{j}+\hat{k}, 2\hat{i}+3\hat{j}+2\hat{k}$ respectively. If A is chosen as the origin, then the cross product of position vectors of B and C are **[2025]**

(A) $-5\hat{i}+2\hat{j}+\hat{k}$ (B) $-\hat{i}+0\hat{j}-\hat{k}$
(C) $\hat{i}-\hat{k}$ (D) $5\hat{i}-2\hat{j}-\hat{k}$

197. The altitude through vertex A of ΔABC with position vectors of points A, B, C as $\bar{a}, \bar{b}, \bar{c}$ respectively is **[2025]**

(A) $\dfrac{\left|\bar{b}\times\bar{c}\right|}{\left|\bar{c}-\bar{b}\right|}$

(B) $\dfrac{\left|\bar{a}\times\bar{b}+\bar{b}\times\bar{c}+\bar{c}\times\bar{a}\right|}{\left|\bar{c}-\bar{b}\right|}$

(C) $\dfrac{\left|\bar{a}\times\bar{b}+\bar{b}\times\bar{c}+\bar{c}\times\bar{a}\right|}{\left|\bar{c}\times\bar{b}\right|}$

(D) $\dfrac{\left|\bar{b}\times\bar{c}\right|}{\left|\bar{a}\right|}$

198. The unit vectors perpendicular to the plane determined by the points $A(1,-1,2), B(2,0,-1)$, $C(0,2,1)$ is **[2025]**

(A) $\pm\left(\dfrac{3\hat{i}+\hat{j}+\hat{k}}{\sqrt{11}}\right)$ (B) $\pm\left(\dfrac{-\hat{i}+2\hat{j}+\hat{k}}{\sqrt{6}}\right)$
(C) $\pm\left(\dfrac{2\hat{i}+\hat{j}+\hat{k}}{\sqrt{6}}\right)$ (D) $\pm\left(\dfrac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}\right)$

199. $ABCD$ is a quadrilateral with $\overline{AB}=\bar{a}, \overline{AD}=\bar{b}$ and $\overline{AC}=2\bar{a}+3\bar{b}$. If its area is α times the area of the parallelogram with AB, AD as adjacent sides, then the value of α is

(A) $\dfrac{1}{2}$

(B) $\dfrac{5}{2}$

(C) $\dfrac{3}{2}$

(D) 2

200. If $\bar{a}=\dfrac{1}{\sqrt{10}}\left(3\hat{i}+\hat{k}\right)$ and $\bar{b}=\dfrac{1}{7}\left(2\hat{i}+3\hat{j}-6\hat{k}\right)$, then the value of $\left(2\bar{a}-\bar{b}\right)\cdot\left(\left(\bar{a}\times\bar{b}\right)\times\left(\bar{a}+2\bar{b}\right)\right)=$ **[2025, 2024, 2023, 2020]**

(A) 3 (B) –3
(C) 5 (D) –5

201. If $[2\bar{p}-3\bar{r}\ \bar{q}\ \bar{s}]+[3\bar{p}+2\bar{q}\ \bar{r}\ \bar{s}] = m[\bar{p}\ \bar{r}\ \bar{s}] + n[\bar{q}\ \bar{r}\ \bar{s}]+t[\bar{p}\ \bar{q}\ \bar{s}]$, then the values of m, n, t respectively are …. **[2025]**

(A) $2, 3, 3$
(B) $3, 4, 5$
(C) $1, 2, 3$
(D) $3, 5, 2$

202. The volume of the tetrahedron whose co-terminus edges are $\bar{a}, \bar{b}, \bar{c}$ is 12 cubic units. If the scalar projection of $\bar{a}$ on $\bar{b}\times\bar{c}$ is 4, then $\left|\bar{b}\times\bar{c}\right|=$ **[2025]**

(A) 18

(B) $\dfrac{1}{18}$

(C) 16

(D) $\dfrac{1}{16}$

203. If the points $A(2-x,2,2), B(2,2-y,2)$, $C(2,2,2-z)$ and $D(1,1,1)$ are coplanar, then the locus of point $P(x,y,z)$ is

(A) $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=1$

(B) $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=0$

(C) $\frac{1}{1+x}+\frac{1}{1+y}+\frac{1}{1+z}=1$

(D) $\frac{1}{x}+\frac{1}{2y}+\frac{1}{3z}=0$

204. Let $\bar{a}$ and $\bar{b}$ be two vectors such that $|\bar{a}|=1, |\bar{b}|=4, \bar{a}\cdot\bar{b}=2$. If $\bar{c}=(2\bar{a}\times\bar{b})-3\bar{b}$, then the angle between $\bar{b}$ and $\bar{c}$ is **[2025]**

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{6}$

(C) $\frac{3\pi}{4}$ (D) $\frac{5\pi}{6}$

205. If $\bar{a},\bar{b},\bar{c},\bar{d}$ are unit vectors such that $\bar{a}\cdot\bar{b}=\frac{1}{2}, \bar{c}\cdot\bar{d}=\frac{1}{2}$ and the angle between $\bar{a}\times\bar{b}$ and $\bar{c}\times\bar{d}$ is $\frac{\pi}{6}$, then the value of $\left|\left[\bar{a}\bar{b}\bar{d}\right]\bar{c}-\left[\bar{a}\bar{b}\bar{c}\right]\bar{d}\right|=$ **[2025]**

(A) $\frac{3}{2}$ (B) $\frac{3}{4}$

(C) $\frac{3}{8}$ (D) 2

206. If $\bar{a}=4\hat{i}+3\hat{j}+\hat{k}, \bar{b}=\hat{i}-2\hat{j}+2\hat{k}$ then $\bar{a}\times(\bar{a}\times(\bar{a}\times(\bar{a}\times\bar{b})))=$ **[2025]**

(A) $676\bar{a}$ (B) $676\bar{b}$

(C) $625\bar{a}$ (D) $625\bar{b}$

207. Three vectors $\hat{i}-\hat{k}, \lambda\hat{i}+\hat{j}+(1-\lambda)\hat{k}$ and $\mu\hat{i}+\lambda\hat{j}+(1+\lambda-\mu)\hat{k}$ represents coterminous edges of a parallelopiped, then the volume of the parallelepiped depends on. **[2025]**

(A) only λ

(B) only μ

(C) both λ and μ

(D) neither λ nor μ

208. The lines $\bar{r}=\bar{a}+\lambda(\bar{b}\times\bar{c})$ and $\bar{r}=\bar{c}+\lambda(\bar{a}\times\bar{b})$ will intersect if **[2025]**

(A) $\bar{a}\times\bar{b}=\bar{b}\times\bar{c}$ (B) $\bar{a}\cdot\bar{b}=\bar{b}\cdot\bar{c}$

(C) $\bar{a}\cdot\bar{c}=|\bar{b}|^2$ (D) $\bar{a}\times\bar{b}=\bar{c}\times\bar{a}$

209. Let $\bar{a},\bar{b},\bar{c},\bar{d}$ are vectors such that $\bar{a}\times\bar{b}=2\hat{i}+3\hat{j}-\hat{k}$ and $\bar{c}\times\bar{d}=3\hat{i}+2\hat{j}+\lambda\hat{k}$ and if $\begin{vmatrix}\bar{a}\cdot\bar{c} & \bar{b}\cdot\bar{c}\\ \bar{a}\cdot\bar{d} & \bar{b}\cdot\bar{d}\end{vmatrix}=0$, then $\lambda=$

(A) 6 (B) –6

(C) 12 (D) –12

210. Let $\bar{a}=\hat{i}+\hat{j}+\hat{k}, \bar{b}$ and $\bar{c}=\hat{j}-\hat{k}$ be three vectors such that $\bar{a}\times\bar{b}=\bar{c}$ and $\bar{a}\cdot\bar{c}=1$. If the length of projection vector of the vector $\bar{b}$ on the vector $\bar{a}\times\bar{c}$ is l, then the value of $3l^2$ is **[2025]**

(A) 1 (B) 2

(C) 4 (D) 6

211. Let $\bar{a}=\hat{i}+\hat{j}, \bar{b}=2\hat{i}-\hat{k}, \bar{c}=3\hat{i}-\hat{j}+\hat{k}$, then vector $\bar{p}$ satisfying $\bar{p}\cdot\bar{a}=0$ and $\bar{p}\times\bar{b}=\bar{c}\times\bar{b}$ is **[2025]**

(A) $\hat{i}-\hat{j}+\hat{k}$ (B) $\hat{i}-2\hat{j}+\hat{k}$

(C) $-\hat{i}+\hat{j}+\hat{k}$ (D) $\hat{i}-\hat{j}+2\hat{k}$

212. If $\bar{b}$ and $\bar{c}$ are unit vectors and $|\bar{a}|=7$, $\bar{a}\times(\bar{b}\times\bar{c})+\bar{b}\times(\bar{c}\times\bar{a})=\frac{1}{2}\bar{a}$, then angle between the vectors $\bar{a}$ and $\bar{c}$ and angle between the vectors $\bar{b}$ and $\bar{c}$ are respectively **[2025]**

(A) $90^\circ, 60^\circ$

(B) $30^\circ, 60^\circ$

(C) $90^\circ, 120^\circ$

(D) $45^\circ, 90^\circ$

213. If $\bar{a}, \bar{b}, \bar{c}$ are three coplanar vectors such that $|\bar{a}|=1, |\bar{b}|=2, \bar{b}\cdot\bar{c}=8$ and the angle between $\bar{b}$ and $\bar{c}$ is 45° then the value of $|\bar{a}\times(\bar{b}\times\bar{c})|$ is **[2025]**

(A) 8

(B) $\sqrt{2}$

(C) 2

(D) 5

214. If $\bar{a}=\lambda x\hat{i}+y\hat{j}+4z\hat{k}, \bar{b}=y\hat{i}+x\hat{j}+3y\hat{k}$, $\bar{c}=-z\hat{i}-2z\hat{j}-(\lambda+1)\hat{k}x$ are the sides of the triangle ABC, where x, y, z are not all zero, such that $\bar{a}+\bar{b}-\bar{c}=\bar{0}$, then value of λ is **[2025]**

(A) 0 (B) 1

(C) 2 (D) 3

215. The vectors $\bar{p}=\hat{i}+a\hat{j}+a^2\hat{k}, \bar{q}=\hat{i}+b\hat{j}+b^2\hat{k}$ and $\bar{r}=\hat{i}+c\hat{j}+c^2\hat{k}$ are non-coplanar and $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix}=0$ then the value of (abc) is **[2025]**

(A) 0 (B) –1

(C) 1 (D) 2

216. The angle between the lines whose direction cosines are $\frac{-\sqrt{3}}{4}, \frac{1}{4}, \frac{-\sqrt{3}}{2}$ and $\frac{-\sqrt{3}}{4}, \frac{1}{4}, \frac{\sqrt{3}}{2}$ is **[2025]**

(A) 90° (B) 120°

(C) 45° (D) 30°

217. If θ is the angle between the lines whose direction cosines are given by $6mn-2nl+5l\ m=0$ and $3l+m+5n=0$, then $\sin\theta=$ **[2025]**

(A) $\frac{\sqrt{35}}{6}$ (B) $\frac{1}{6}$

(C) $\frac{\sqrt{37}}{6}$ (D) $\frac{5}{6}$

218. The maximum value and minimum value of the volume of the parallelopiped having coterminous edges $\hat{i}+x\hat{j}+\hat{k}, \hat{j}+x\hat{k}$ and $x\hat{i}+\hat{k}$ are respectively **[2025]**

(A) $\frac{1}{3\sqrt{3}}+1, \frac{-1}{3\sqrt{3}}+1$

(B) $\frac{2}{3\sqrt{3}}+1, \frac{-2}{3\sqrt{3}}+1$

(C) $\frac{1}{\sqrt{3}}+1, \frac{-1}{\sqrt{3}}+1$

(D) $\frac{2}{\sqrt{3}}+1, \frac{-2}{\sqrt{3}}+1$

219. Two adjacent sides of a parallelogram ABCD are given by $\overline{AB}=2\hat{i}+10\hat{j}+11\hat{k}$ and $\overline{AD}=-\hat{i}+2\hat{j}+2\hat{k}$. The side AD is rotated by an acute angle α in the plane of parallelogram so that AD becomes AD'. If AD' makes a right angle with side AB, then the cosine of the angle α is given by **[2025, 2023]**

(A) $\frac{8}{9}$ (B) $\frac{\sqrt{17}}{9}$

(C) $\frac{1}{9}$ (D) $\frac{4\sqrt{5}}{9}$

220. $\bar{u},\bar{v},\bar{w}$ are three vectors such that $|\bar{u}|=1$, $|\bar{v}|=2, |\bar{w}|=3$. If the projection of $\bar{v}$ along $\bar{u}$ is equal to projection of $\bar{w}$ along $\bar{u}$ and $\bar{v},\bar{w}$ are perpendicular to each other, then $|\bar{u}-\bar{v}+\bar{w}|=$

[2025, 2023]

(A) 4 (B) $\sqrt{7}$

(C) $\sqrt{14}$ (D) 2

221. If the angle θ between the line $\frac{x+1}{1}=\frac{y-1}{2}=\frac{z-2}{2}$ and the plane $2x-y+\sqrt{\lambda}z+4=0$ is such that $\sin\theta=\frac{1}{3}$, then the value of λ is:

(A) $\frac{-3}{5}$ (B) $\frac{-5}{3}$

(C) $\frac{5}{3}$ (D) $\frac{3}{5}$

222. The cartesian equation of the plane passing through the point $A(7,8,6)$ and parallel to the XY plane is:

(A) $z=1$ (B) $y=8$

(C) $x=7$ (D) $z=6$

223. If the line $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$ intersect, then the value of k is:

(A) $\frac{3}{2}$

(B) $\frac{9}{2}$

(C) $-\frac{2}{9}$

(D) $-\frac{3}{2}$

224. The acute angle between the lines $x=-2+2t, y=3-4t, z=-4+t$ and $x=-2-t, y=3+2t, z=-4+3t$ is:

(A) $\cos^{-1}\left(\frac{1}{\sqrt{6}}\right)$

(B) $\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$

(C) $\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$

(D) $\cos^{-1}\left(\frac{2}{\sqrt{6}}\right)$

225. The equation of the line passing through the point of intersection of $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-4}{5}=\frac{y-1}{2}=z$ and also through the point (2, 1, –2) is:

(A) $\vec{r}=\left(-\hat{i}-\hat{j}-\hat{k}\right)+\lambda\left(\hat{i}+2\hat{j}+\hat{k}\right)$

(B) $\vec{r}=\left(-\hat{i}-\hat{j}+\hat{k}\right)+\lambda\left(2\hat{i}+2\hat{j}+\hat{k}\right)$

(C) $\frac{x+1}{3}=\frac{y+1}{2}=\frac{z+1}{-1}$

(D) $\frac{x-1}{3}=\frac{y-1}{2}=\frac{z+1}{1}$

226. The angle between the lines $x=y, z=0$ and $y=0, z=0$ is

(A) 30° (B) 45°

(C) 60° (D) 90°

227. A triangle ABC is formed by $A(1,-1,0)$, $B(3,5,3), C(-11,-5,6)$. The equation of internal angle bisector of angle A is.

(A) $\frac{(1-x)}{2}=\frac{y-(-1)}{2}=\frac{z}{3}$

(B) $\frac{x+1}{2}=\frac{y-1}{2}=\frac{z}{3}$

(C) $\frac{x+2}{1}=\frac{y-2}{1}=\frac{z}{3}$

(D) $\frac{x-2}{1}=\frac{y+3}{2}=\frac{z}{3}$

228. A line L is passing through points $A(1,3,2)$ and $B(2,2,1)$. If mirror image of point $P(1,1,-1)$ in the line L is (x,y,z) then $x+y+z=$

(A) $\frac{10}{3}$ (B) $\frac{13}{3}$

(C) $\frac{14}{3}$ (D) $\frac{23}{3}$

229. The lines $\bar{r}=\left(\hat{i}+\hat{j}-\hat{k}\right)+\lambda\left(3\hat{i}-\hat{j}\right)$ and $\bar{r}=\left(4\hat{i}-\hat{k}\right)+\mu\left(2\hat{i}+3\hat{k}\right)$ are

(A) intersecting but not perpendicular
(B) perpendicular
(C) parallel
(D) skew lines

230. The distance of the point $(-3,2,3)$ from the line passing through $(4,6,-2)$ and having direction ratios $-1,2,3$ is …… units.

(A) $2\sqrt{17}$ (B) $4\sqrt{17}$

(C) $2\sqrt{19}$ (D) $4\sqrt{19}$

231. If the shortest distance between the lines $\bar{r}_1=\alpha\hat{i}+2\hat{j}+2\hat{k}+\lambda\left(\hat{i}-2\hat{j}+2\hat{k}\right),\lambda\in\mathbb{R},\alpha>0$ and $\bar{r}_2=-4\hat{i}-\hat{k}+\mu\left(3\hat{i}-2\hat{j}-2\hat{k}\right),\mu\in R$, is 9, then the value of α is:

(A) 4 (B) 6

(C) 8 (D) 3

232. The distance of the point $(2,4,0)$ from the point of intersection of the lines $\frac{x+6}{3}=\frac{y}{2}=\frac{z+1}{1}$ and $\frac{x-7}{4}=\frac{y-9}{3}=\frac{z-4}{2}$ is:

(A) 3 units (B) $3\sqrt{3}$ units

(C) 2 units (D) $2\sqrt{3}$ units

233. The line L is passing through $(1,2,3)$. The distance of any point on the line L from the line $\bar{r}=(3\lambda-1)\hat{i}+(-2\lambda+3)\hat{j}+(4+\lambda)\hat{k}$ is constant. Then the line L does not pass through the point

(A) $(4,0,4)$ (B) $(-2,4,2)$

(C) $(7,-2,5)$ (D) $(-5,6,2)$

234. The lines $\frac{6x-6}{18}=\frac{y+1}{3}=\frac{z-1}{5}$ and $\frac{3x+6}{12}=\frac{y-1}{3}=\frac{z+1}{2}$ are …

(A) intersecting at point $(1,-1,2)$
(B) intersecting at right angles
(C) do not intersect
(D) intersecting at point $(3,1,-1)$

235. Let M and N be foots of the perpendiculars drawn from the point $P(a,a,a)$ on the lines $x-y=0,z=1$ and $x+y=0,z=-1$ respectively and if $\angle MPN=90^\circ$ then $a^2=$

(A) 1 (B) 4

(C) 6 (D) 9

236. A plane passes through $(2,1,2)$ and $(1,2,1)$ and parallel to the line $2x=3y$ and $z=1$, then the plane also passes through the point

(A) $(-6,2,0)$ (B) $(6,-2,0)$

(C) $(-2,0,1)$ (D) $(2,0,1)$

237. The equation of the plane passing through the line of intersection of the planes $x+y+z=1$ and $3x+4y+5z=2$ and perpendicular to the XY-plane is:

(A) $2x+y-3=0$
(B) $x-2y+3=0$
(C) $x-3y-2=0$
(D) $2x-y+6=0$

238. The direction cosines of the plane $x-y+2z=5$ and $3x+y+z=6$ are:

(A) $\frac{-3}{5\sqrt{2}},\frac{5}{5\sqrt{2}},\frac{4}{5\sqrt{2}}$

(B) $\frac{3}{5\sqrt{2}},\frac{-5}{5\sqrt{2}},\frac{4}{5\sqrt{2}}$

(C) $\frac{3}{5\sqrt{2}},\frac{5}{5\sqrt{2}},\frac{4}{5\sqrt{2}}$

(D) $\frac{3}{5\sqrt{2}},\frac{5}{5\sqrt{2}},\frac{-4}{5\sqrt{2}}$

239. The equation of the plane passing through the point of intersection of the planes $2x-y+z-3=0$ and $4x-3y+5z+9=0$ and parallel to the line $\frac{x+1}{2}=\frac{y+3}{4}=\frac{z-3}{5}$ is $\alpha x+\beta y+\gamma z+d=0$ Then $\alpha+\beta+\gamma+d=$

(A) 48 (B) -48

(C) 84 (D) 45

240. The co-ordinates of the point where the line joining the points $(2,-3,1)$ and $(3,-4,-5)$ and intersects the plane $2x+y+z=7$ are

(A) $(-1,2,7)$ (B) $(1,2,3)$

(C) $(2,1,2)$ (D) $(1,-2,7)$

241. Let the plane passing through point $(2,1,-1)$ containing line joining the points $(1,3,2)$ and ($1,2,1$) makes intercepts p,q,r on co-ordinate axes, then $p+q+r=$

(A) 0 (B) 3

(C) 2 (D) -2

242. In 3-dimensional space, the equation $x^2-8x+12=0$ represents ....

(A) two straight lines

(B) a pair of straight lines passing through the origin

(C) 2 planes parallel to YZ -plane

(D) 2 planes parallel to XZ -plane

243. The Cartesian equation of the plane $\bar{r}=\left(2\hat{i}-3\hat{j}\right)+\lambda\left(\hat{i}+2\hat{j}-\hat{k}\right)+\mu\left(2\hat{i}+3\hat{j}+\hat{k}\right)$ is:

(A) $5x-4y+z=22$

(B) $5x-3y+z=19$

(C) $5x-3y-z=19$

(D) $5x-4y-z=22$

244. The line $\frac{x-1}{2}=\frac{y+2}{-1}=\frac{z}{1}$ intersects the XY plane and the YZ plane at points A and B respectively. The equation of line through the points A and B is:

(A) $\left[\bar{r}-\left(\hat{i}-2\hat{j}+0\hat{k}\right)\right]\times\left(-\hat{i}+\frac{1}{2}\hat{j}-\frac{1}{2}\hat{k}\right)=\bar{0}$

(B) $\left[\bar{r}+\left(\hat{i}-2\hat{j}+0\hat{k}\right)\right]\times\left(-\hat{i}+\frac{1}{2}\hat{j}+\frac{1}{2}\hat{k}\right)=\bar{0}$

(C) $\bar{r}=\left(-\hat{i}-2\hat{j}+0\hat{k}\right)+\lambda\left(-\hat{i}+\frac{1}{2}\hat{j}-\frac{1}{2}\hat{k}\right)$

(D) $\bar{r}=\left(\hat{i}+2\hat{j}\right)+\lambda\left(-\hat{i}+\frac{1}{2}\hat{j}-\frac{1}{2}\hat{k}\right)$

245. The direction cosines of a normal to the plane passing through $(4,2,3),(-1,4,2)$ and $(3,2,1)$ are .....

(A) $\frac{-2}{\sqrt{101}},\frac{3}{\sqrt{101}},\frac{8}{\sqrt{101}}$

(B) $\frac{-3}{\sqrt{49}},\frac{2}{\sqrt{49}},\frac{6}{\sqrt{49}}$

(C) $\frac{-4}{\sqrt{101}},\frac{-9}{\sqrt{101}},\frac{2}{\sqrt{101}}$

(D) $\frac{4}{22},\frac{-12}{22},\frac{18}{22}$

246. If the angle between the planes $x-2y+3z-5=0$ and $x+\alpha y+2z+7=0$ is $\cos^{-1}\left(\frac{1}{14}\right)$ then the difference between the values of α is:

(A) $\frac{12}{11}$ (B) $\frac{62}{55}$

(C) $\frac{31}{11}$ (D) $\frac{8}{5}$

247. The equation of plane passing through $(1,0,0)$ and $(0,1,0)$ and making an angle 45° with the plane $x+y-3=0$ is

(A) $x+y\pm\sqrt{2}z-1=0$

(B) $3x+y\pm\sqrt{3}z-3=0$

(C) $x+y\pm\sqrt{3}z-1=0$

(D) $2x+2y\pm\sqrt{3}z-2=0$

248. The angle between the lines $x-3y-4=0, 4y-z+5=0$ and $x+3y-11=0, 2y-z+6=0$ is

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{6}$ (D) $\frac{\pi}{3}$

249. The distance between the line $\bar{r}=3\hat{i}-2\hat{j}+\hat{k}+\lambda\left(\hat{i}+\hat{j}+\hat{k}\right)$ and the plane $\bar{r}\cdot\left(2\hat{i}+\hat{j}+\hat{k}\right)=4$ is:

(A) $\frac{1}{\sqrt{6}}$ units (B) $\frac{3}{\sqrt{6}}$ units

(C) $\frac{2}{\sqrt{6}}$ units (D) $\frac{5}{\sqrt{6}}$ units

250. The distance of the point $P(3,8,2)$ from the line $\frac{x-1}{2}=\frac{y-3}{4}=\frac{z-2}{3}$ measured parallel to the plane $3x+2y-2z+15=0$ is:

(A) 7 units (B) 6 units

(C) 8 units (D) 10 units

251. If the lines $\frac{x-1}{2}=\frac{y+1}{k}=\frac{z}{2}$ and $\frac{x+1}{5}=\frac{y+1}{2}=\frac{z}{k}$ are coplanar, then the equation of the plane containing these lines are

(A) $x\pm y+z=0$ (B) $y\pm z+1=0$

(C) $2x\pm y=0$ (D) $x\pm z+1=0$

252. The distance of the plane $\bar{r}=\left(\hat{i}-\hat{j}\right)+\lambda\left(\hat{i}+\hat{j}+\hat{k}\right)+\mu\left(\hat{i}-2\hat{j}+3\hat{k}\right)$ from the origin is:

(A) $\frac{7}{\sqrt{38}}$ units

(B) $\frac{1}{\sqrt{38}}$ units

(C) $\frac{5}{\sqrt{38}}$ units

(D) $\frac{2}{\sqrt{38}}$ units

253. The distance of the point $P(3,4,4)$ from the point of intersection of the line joining the points $Q(3,-4,-5), R(2,-3,1)$ and the plane $2x+y+z=7$ is: **[2025]**

(A) 7 units (B) 9 units

(C) 11 units (D) 6 units

254. If the lines $x=ay-1=z-2$ and $x=3y-2=bz-2(ab\neq 0)$ are coplanar, then [2025]

(A) $a=1, b=\frac{1}{2}$ (B) $a=2, b=2$

(C) $a=\frac{1}{2}, b=\frac{1}{2}$ (D) $b=1, a\in\mathbb{R}-\{0\}$

255. The distance of the point $(5,3,-1)$ from the plane passing through points $(2,1,0),(3,-2,4)$ and $(1,-3,3)$ is: **[2025]**

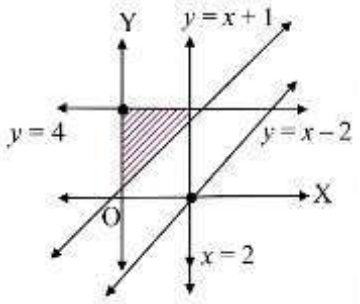
(A) $\frac{2}{\sqrt{3}}$ units

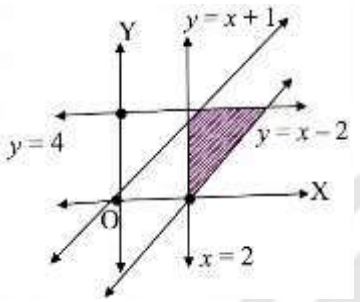
(B) $\frac{4}{\sqrt{3}}$ units

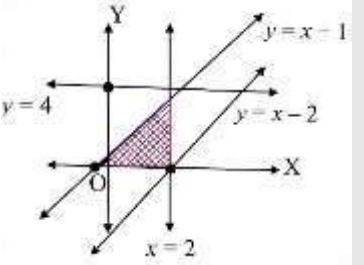
(C) $\sqrt{3}$ units

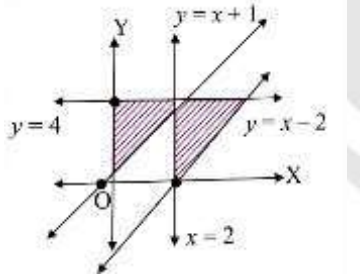
(D) $\frac{1}{\sqrt{3}}$ units

256. The feasible region for the constraints $x-2\leqslant y, x\geqslant y-1, x\geqslant 2, y\leqslant 4, x, y\geqslant 0$, is ____

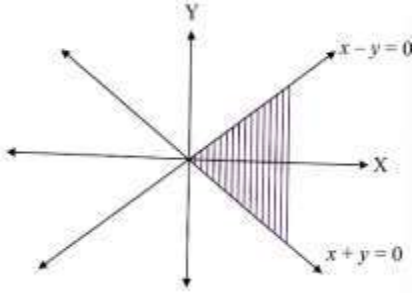
(A) 

(B) 

(C) 

(D) 

257. The shaded region in the following figure represents a solution set of:



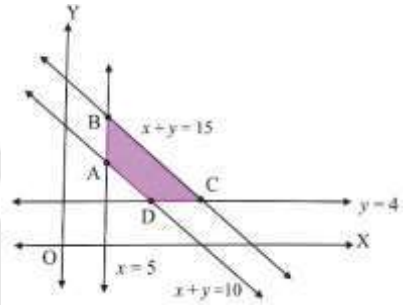
(A) $x-y\geq 0, x+y\geq 0$

(B) $x-y\leq 0, x+y\geq 0$

(C) $x-y\geq 0, x+y\leq 0$

(D) $x-y\leq 0, x+y\leq 0$

258. A scholarship amount is given by $z=550x+300y$ and is to be distributed among x boys and y girls. From the graph given below the maximum amount of scholarship is ____.



(A) 7250

(B) 9250

(C) 4250

(D) 5750

259. The solution set of the constraints $|x-y|\leq 1, x, y\geq 0$ is:

(A) a finite set

(B) an unbounded set

(C) a convex polygon

(D) such that feasible region does not exist

260. In L.P.P., the maximum value of objective function $Z=6x+3y$ subject to constraints $x+y\leq 5, x+2y\geq 4, 4x+y\leq 12, x, y\geq 0$ is:

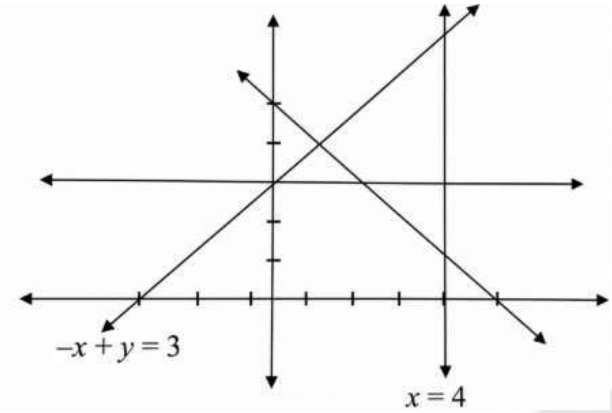
(A) $\frac{132}{7}$

(B) 22

(C) 15

(D) $\frac{122}{7}$.

261. The feasible region represented by the given constraints $2x+3y\geq 12, -x+y\leq 3, x\leq 4, y\geq 3$ is denoted by:

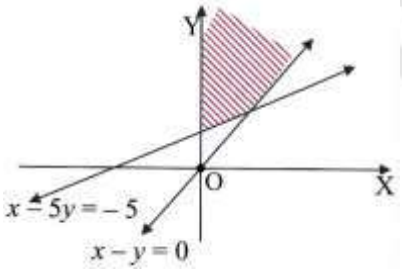


(A) S_1 (B) S_2
(C) S_3 (D) S_4

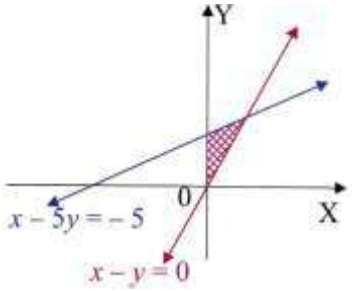
262. If the difference between the maximum and minimum values of the objective function $z=7x-8y$ subject to the constraints $x+y\leqslant 20, y\geqslant 5, x, y\geqslant 0$ is $5k+200$, then the value of k is
(A) 3 (B) 4
(C) 5 (D) 6

263. The feasible region for the constraints $x-y\geq 0, x-5y\leq -5, x\geq 0, y\geq 0$ is shown by the figure:

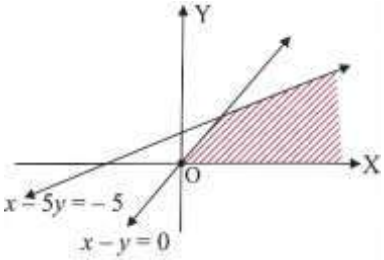
(A)



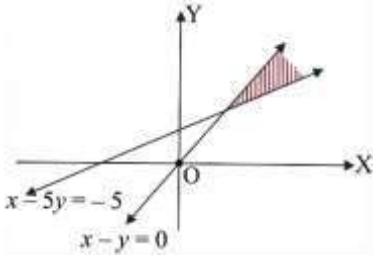
(B)



(C)



(D)



264. The L.P.P., minimize $z=30x+20y, x+y\leq 8$, $x+2y\geq 4, 6x+4y\geq 12, x\geqslant 0, y\geqslant 0$ has
(A) a unique solution
(B) infinitely many solutions
(C) minimum value at $(4,0)$
(D) minimum value at $(8,0)$

265. The solution for minimizing the function $z=x+y$ under an L.P.P. with constraints $x+y\geq 2, x+2y\leq 8, y\leq 3, x, y\geq 0$ is
(A) at the point $(0,3)$
(B) at the point $(8,0)$
(C) at infinite number of points but bounded set
(D) at unbounded set

266. If $f(1)=1, f'(1)=3$, then the derivative of $f(f(f(x)))+(f(x))^2$ at $x=1$ is
(A) 9
(B) 12
(C) 15
(D) 33

267. If $f(x)=\frac{\sin^2 x}{1+\cot x}+\frac{\cos^2 x}{1+\tan x}$, then the value of $f'\left(\frac{\pi}{6}\right)$ is equal to

(A) 0 (B) $\frac{1}{2}$

(C) $-\frac{1}{2}$ (D) $\frac{\sqrt{3}}{2}$

268. The derivative of $\tan^{-1}\left(\sqrt{1+x^2}-1\right)$ is:

(A) $\frac{x}{\sqrt{1+x^2}\left(x^2-2\sqrt{x+1}+1\right)}$

(B) $\frac{x}{\sqrt{1+x^2}\left(x^2-2\sqrt{1+x^2}+3\right)}$

(C) $\frac{x}{\sqrt{1+x^2}\left(x^2-2\sqrt{x^2+1}+2\right)}$

(D) $\frac{x}{\sqrt{1+x^2}\left(x^2+2\sqrt{1+x^2}-3\right)}$

269. If $y=\log_e x^3+3\sin^{-1}x+kx^2$ and $y'\left(\frac{1}{2}\right)=2\sqrt{3}$, then $k=$

(A) 6 (B) -6

(C) $2\sqrt{3}$ (D) 1

270. If $y=\tan^{-1}\left(\frac{1}{1+x+x^2}\right)+\tan^{-1}\left(\frac{1}{x^2+3x+3}\right)+\tan^{-1}\left(\frac{1}{x^2+5x+7}\right)$ then the value of $y'(0)$ is

[2025]

(A) $\frac{9}{10}$ (B) $\frac{1}{10}$

(C) $-\frac{9}{10}$ (D) $-\frac{1}{10}$

271. If $y=\tan^{-1}\left[\frac{4\sin 2x}{\cos 2x-6\sin^2 x}\right]$, then $\frac{dy}{dx}$ at $x=0$ is

(A) $\frac{1}{8}$ (B) -8

(C) 8 (D) $-\frac{1}{8}$

272. If $y=\tan^{-1}\left(\frac{4x}{1+5x^2}\right)+\cot^{-1}\left(\frac{3-2x}{2+3x}\right)$, then $\frac{dy}{dx}$ is equal to

(A) $\frac{5}{1+25x^2}$ (B) $\frac{1}{1+25x^2}$

(C) $\frac{1}{1+5x^2}$ (D) $\frac{5}{1+5x^2}$

273. If $f(\theta)=\cos\theta_1\cdot\cos\theta_2\cdot\cos\theta_3\ \ldots\ \cos\theta_n$, then $\tan\theta_1+\tan\theta_2+\tan\theta_3+\ldots\ldots+\tan\theta_n=$

(A) $\frac{-f'(\theta)}{f(\theta)}$

(B) $\frac{f'(\theta)}{f(\theta)}$

(C) $\frac{-f''(\theta)}{f'(\theta)}$

(D) $\frac{f''(\theta)}{f'(\theta)}$

274. Derivative of $x^{\left(x^x\right)}$ is:

(A) $x^{\left(x^x\right)}\left(x^x+1+\log x\right)$

(B) $x^{\left(x^x\right)}\left(x^x+\log x\right)$

(C) $x^{\left(x^x\right)}\left(x^x+x^{x-1}\log x(1+\log x)\right)$

(D) $x^{\left(x^x\right)}\left(x^{x-1}+x^x\log x(1+\log x)\right)$

275. If $x^y + y^x = a^b$, then $\frac{dy}{dx}$ at $x=1, y=2$ is

(A) $-2(1+\log 2)$ (B) $2(1+\log 2)$

(C) $2+\log 2$ (D) $1+\log 2$

276. If $\sqrt{y-\sqrt{y-\sqrt{y-.....\infty}}} = \sqrt{x+\sqrt{x+\sqrt{x+.....\infty}}}$ then $\frac{dy}{dx} =$

(A) $\frac{y+x+1}{y-x+1}$ (B) $\frac{y-x-1}{y-x+1}$

(C) $\frac{y-x+1}{y-x-1}$ (D) 1

277. If $x\cdot\log_e(\log_e x) - x^2 + y^2 = 4 (y>0)$, then $\frac{dy}{dx}$ at $x=e$ is

(A) $\frac{e}{\sqrt{4+e^2}}$ (B) $\frac{2e-1}{2\sqrt{4+e^2}}$

(C) $\frac{1+2e}{\sqrt{4+e^2}}$ (D) $\frac{1+2e}{2\sqrt{4+e^2}}$

278. If $u = \frac{\tan^{-1}x}{\tan^{-1}x+1}$ and $v = \tan^{-1}(\tan^{-1}x)$ then $\frac{du}{dv} =$ .....

(A) 1 (B) $\frac{1+(\tan^{-1}x)^2}{(1+\tan^{-1}x)^2}$

(C) $\frac{\tan^{-1}x}{(1+\tan^{-1}x)^2}$ (D) $\frac{1}{(1+\tan^{-1}x)^2}$

279. If $x = a\cos^3\theta, y = a\sin^3\theta$ Then $\sqrt{1+\left(\frac{dy}{dx}\right)^2} =$

(A) $\tan^2\theta$ (B) $\sec^2\theta$

(C) $\sec\theta$ (D) $\tan\theta$

280. The derivative of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ w.r.t. $\tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$ at $x=0$ is:

(A) $\frac{1}{8}$ (B) $\frac{1}{4}$

(C) $\frac{1}{2}$ (D) 1

281. If $x = \tan^{-1}\left\{\frac{\sqrt{1+t^2}-1}{t}\right\}, y = \cos^{-1}\left\{\frac{1-t^2}{1+t^2}\right\}$, then $\frac{dy}{dx}$ is equal to

(A) 2 (B) $\frac{1}{2}$

(C) 4 (D) $\frac{1}{4}$

282. For $n \in \mathbb{N}$ if $y = ax^{n+1} + bx^{-n}$, then $x^2\frac{d^2y}{dx^2} =$

(A) $n(n-1)y$

(B) $(n-1)y$

(C) $n(n+1)y$

(D) $(n+1)y$

283. If $(a+bx)e^{\frac{y}{x}} = x$, then $x^3\frac{d^2y}{dx^2}$ is equal to

(A) $\left(y\frac{dy}{dx} - x\right)^2$

(B) $\left(x\frac{dy}{dx} - y\right)^2$

(C) $\left(x\frac{dy}{dx} + y\right)^2$

(D) $\left(y\frac{dy}{dx} + x\right)^2$

284. For $N \in \mathbb{N}, \frac{d^n}{dx^n}(\log x) =$

(A) $\frac{(n-1)!}{x^n}$

(B) $\frac{n!}{x^n}$

(C) $\frac{(n-2)!}{x^n}$

(D) $(-1)^{n-1}\frac{(n-1)!}{x^n}$

285. If $f(x)=3x^3+2x^2f'(1)+xf''(2)+f'''(3)$ then $f(x)=\ldots\ldots$

(A) $\frac{1}{7}\left(3x^3-90x^2+72x+18\right)$

(B) $\frac{1}{7}\left(21x^3-90x^2+72x+126\right)$

(C) $3x^3-90x^2+72x+18$

(D) $3x^3-45x^2+36x+9$

286. If $x=\sin t$ and $y=\sin pt$, then the value of $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+p^2y=$

(A) 0 (B) 1

(C) –1 (D) $\sqrt{2}$

287. $f(x)=(\cos x+i\sin x)\cdot(\cos 3x+i\sin 3x)\cdots$. $\left[\cos(2n-1)x+i\sin(2n-1)x\right], n\in\mathbb{N}$ Then $f''(x)=$ ____, (Where $i=\sqrt{-1}$)

(A) $n^2f(x)$ (B) $-n^4f(x)$

(C) $-n^2f(x)$ (D) $n^4f(x)$

288. The equation of the tangent to the curve $y=be^{-x/a}$ at the point where it crosses the Y axis is: **[2025]**

(A) $x+y=ab$ (B) $\frac{x}{a}+\frac{y}{b}=1$

(C) $ax+by=1$ (D) $x+y=a+b$

289. If the line $ax+by+c=0$ is normal to the curve $xy=1$, then **[2025, 2023]**

(A) $a>0,\ b>0$

(B) $a>0, b<0$

(C) $a<0,\ b\geqslant 0$

(D) $a<0,\ b<0$

290. The combined equation of the tangent and normal to the curve $xy=15$ at the point $(5,3)$ is: **[2025]**

(A) $15x^2-15y^2+16xy=480$

(B) $15x^2+16xy-198x+10y+480-15y^2=0$

(C) $15x^2-16xy+19x-10y-480+15y^2=0$

(D) $15x^2+16xy+198x-10y-480+15y^2=0$

291. The area of the triangle formed by the co-ordinate axes and a tangent to the curve $xy=a^2$ at the point (x_1,y_1) is ____ sq. units (where a, x_1 and y_1 are non-zero) **[2025]**

(A) $\frac{a^2x_1}{y_1}$ (B) $\frac{a^2y_1}{x_1}$

(C) $2a^2$ (D) $4a^2$

292. If the tangent and the normal at the point $(\sqrt{3},1)$ to the circle $x^2+y^2=4$, and the X -axis form a triangle, then the area (in sq.units) of this triangle is: **[2025]**

(A) $\frac{1}{\sqrt{2}}$

(B) $\frac{2}{\sqrt{3}}$

(C) $\frac{4}{\sqrt{3}}$

(D) $\frac{1}{3}$

293. If the curves $y^2 = 6x$ and $9x^2 + by^2 = 16$ intersect each other at right angles, then the value of b is **[2025, 2024, 2023, 2022]**

(A) 4 (B) $\frac{7}{2}$

(C) 6 (D) $\frac{9}{2}$

294. The equation of motion of the particle is $s = at^2 + bt + c$. If the displacement after 1 second is 20 m, velocity after 2 seconds is 30 m / seconds and the acceleration is 10 m / seconds ^{2}, then **[2025, 2024]**

(A) $a + c = 2$ b (B) $a + c = b$

(C) $a - c = b$ (D) $a + c = 3$ b

295. A particle moves along a curve $y = \frac{2x^3 - 1}{3}$. The points on the curve at which the y co-ordinate is changing 18 times the x co-ordinate are: **[2025]**

(A) $\left(-3, -\frac{55}{3}\right), \left(3, -\frac{53}{3}\right)$

(B) $\left(-3, \frac{53}{3}\right), \left(3, \frac{55}{3}\right)$

(C) $\left(-3, -\frac{53}{3}\right), \left(3, \frac{55}{3}\right)$

(D) $\left(-3, -\frac{55}{3}\right), \left(3, \frac{53}{3}\right)$

296. The length and breadth of a rectangle are x cm and y cm respectively. If the length decreases at the rate of 5 cm / minute and the breadth increases at the rate of 3 cm / minute, then the rates of change of the perimeter and area respectively when the length is 5 cm and breadth is 2 cm, are : **[2025]**

(A) -4 and 5

(B) -5 and 3

(C) 3 and 5

(D) 3 and -5

297. Let x be the length of each of the equal sides of an isosceles triangle and θ be the angle between these sides. If x is increasing at the rate $\frac{1}{12}$ m / hour and θ is increasing at the rate $\frac{\pi}{180}$ rad / hour, then the rate at which area of the triangle is increasing when $x = 12$ m and $\theta = \frac{\pi}{4}$ is: **[2025]**

(A) $\left(\frac{\pi}{5} + \frac{1}{2}\right)$ m^2 / hour

(B) $\sqrt{2}\left(\frac{\pi}{5} + \frac{1}{2}\right)$ m^2 /hour

(C) $2\left(\frac{\pi}{5} + \frac{1}{2}\right)$ m^2 / hour

(D) $\sqrt{3}\left(\frac{\pi}{5} + \frac{1}{2}\right)$ m^2 / hour

298. The radius of the base of a cone is increasing at the rate 3 cm / minute and the altitude is decreasing at the rate 4 cm / minute . The rate at which the lateral surface area is changing, when the radius is 7 cm and altitude is 24 cm is: **[2025]**

(A) 75π cm^2 / minute

(B) 25π cm^2 / minute

(C) 3π cm^2 / minute

(D) 54π cm^2 / minute

299. By dropping a stone in a quiet lake, a wave in the form of circle is generated. The radius of the circular wave increases at the rate of 2.1 cm / sec . Then the rate of increase of the enclosed circular region, when the radius of the circular wave is 10 cm, is ____. (Given $\pi = \frac{22}{7}$) **[2025]**

(A) 66 cm^2 / second

(B) 122 cm^2 / second

(C) 132 cm^2 / second

(D) 110 cm^2 / second

300. The approximate value of $\cos(59°30')$ is: (given $1° = 0.0175^c$, sin60°==0.8660) **[2025]**

(A) 0.5076 (B) 0.5176
(C) 0.5256 (D) 0.5150

301. If the function $f(x) = x(x+3)e^{-\frac{x}{2}}$ satisfies all the conditions of Rolle's theorem in $[-3,0]$, then c is

(A) 0 (B) -1
(C) -2 (D) -3

302. In the mean value theorem, $f'(c) = \frac{f(b)-f(a)}{b-a}$, if $a = 0$, $b = \frac{1}{2}$ and $f(x) = x(x-1)(x-2)$, then the value of c is:

(A) $1-\frac{\sqrt{15}}{6}$ (B) $1-\frac{\sqrt{13}}{6}$
(C) $1-\frac{\sqrt{21}}{6}$ (D) $1+\frac{\sqrt{21}}{6}$

303. The abscissae of the points of the curve $y = x^3$ are in the interval $[-2,2]$, where the slope of the tangents can be obtained by mean value theorem for the interval $[-2,2]$ are.

(A) 0 (B) $\pm\sqrt{3}$
(C) $\pm\frac{2}{\sqrt{3}}$ (D) $\frac{\sqrt{3}}{2}$

304. The function $f(x) = [x(x-2)]^2$ is increasing in the set:

(A) $(-\infty,0)\cup(2,\infty)$ (B) $(-\infty,1)$
(C) $(1,2)$ (D) $(0,1)\cup(2,\infty)$

305. If $f(x) = \frac{k\sin x + 2\cos x}{\sin x + \cos x}$ is strictly increasing for all real values of x, then

(A) $k = 1$
(B) $k > 1$
(C) $k < 2$
(D) $k > 2$

306. The function $f(x) = \sin^4 x + \cos^4 x$ increases if

(A) $0 < x < \frac{\pi}{8}$ (B) $\frac{\pi}{4} < x < \frac{\pi}{2}$
(C) $\frac{3\pi}{8} < x < \frac{5\pi}{8}$ (D) $\frac{5\pi}{8} < x < \frac{3\pi}{4}$

307. The function f defined by $f(x) = (x+2)e^{-x}$ is

(A) decreasing for all $x \in \mathbb{R}$
(B) decreasing in $(-\infty,-1)$ and increasing in $(-1,\infty)$
(C) decreasing in $(-1,\infty)$ and increasing in $(-\infty,-1)$
(D) increasing for all $x \in \mathbb{R}$

308. The function $x^5 - 5x^4 + 5x^3 - 10$ has a maximum, when x is equal to

(A) 0 (B) 1
(C) 2 (D) 3

309. A population $p(t)$ of 1000 bacteria introduced into a nutrient medium grows according to the relation $p(t) = 1000 + \frac{1000t}{100+t^2}$. The maximum size of this bacterial population is: **[2025]**

(A) 1100 (B) 1250
(C) 1050 (D) 950

310. A manufacturer sells x items at a price of ₹ $\left(6-\frac{x}{40}\right)$ each. The cost price of x items is ₹ $\left(\frac{x}{5}+193\right)$. The maximum profit in ₹ ____ is **[2025]**

(A) 134.4
(B) 144.3
(C) 143.4
(D) 133.4

311. If $y=\alpha\log x+\beta x^3-x$ has extreme values at $x=-1$ and $x=1$, then α and β are respectively: **[2025]**

(A) 0 and $\frac{1}{3}$ (B) 0 and $\frac{-1}{3}$

(C) $\frac{-1}{3}$ and $\frac{1}{3}$ (D) $\frac{1}{3}$ and $\frac{1}{3}$

312. The value of 'a' so that the sum of squares of the roots of the equation $x^2-(a-2)x-a+1=0$ assumes the least value is:

(A) 2 (B) 1

(C) 4 (D) 0

313. An open tank with a square bottom is to contain 4000 cubic cm. of liquid. The dimensions of the tank so that the surface area of the tank is minimum, is:

(A) side $=20$ cm, height $=10$ cm

(B) side $=10$ cm, height $=20$ cm

(C) side $=10$ cm, height $=40$ cm

(D) side $=20$ cm, height $=05$ cm

314. If x is real, then the difference between the greatest and least values of $\frac{x^2-x+1}{x^2+x+1}$ is:

(A) $\frac{10}{3}$ (B) $\frac{8}{3}$

(C) $\frac{5}{3}$ (D) $\frac{1}{3}$

315. Angle between the parabola $y^2=4(x-1)$ and $x^2+4(y-3)=0$ at the common end of their latus rectum is: **[2025]**

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{6}$ (D) $\frac{\pi}{3}$

316. The length of the perpendicular drawn from the origin on the normal to the curve $x^2+2xy-3y^2=0$ at the point $(2,2)$ is:

(A) $\sqrt{2}$ units (B) $3\sqrt{2}$ units

(C) $2\sqrt{2}$ units (D) $\frac{1}{\sqrt{2}}$ units

317. $\int\frac{e^{2030\log x}-e^{2029\log x}}{e^{2028\log x}-e^{2027\log x}}\,dx=\ldots.$ **[2025]**

(A) $\frac{x^2}{2}+c$, where c is the constant of integration

(B) $x+c$, where c is the constant of integration

(C) $\frac{x^3}{3}+c$, where c is the constant of integration

(D) $\frac{x}{3}+c$, where c is the constant of integration

318. $\int\cos\left(\frac{x}{16}\right)\cdot\cos\left(\frac{x}{8}\right)\cdot\cos\left(\frac{x}{4}\right)\cdot\sin\left(\frac{x}{16}\right)dx=$ **[2025]**

(A) $\frac{\cos 16x}{256}+c$, where c is the constant of integration

(B) $\frac{-\cos 16x}{256}+c$, where c is the constant of integration

(C) $\frac{\sin 16x}{256}+c$, where c is the constant of integration

(D) $\frac{-\cos\left(\frac{x}{2}\right)}{4}+c$, where c is the constant of integration

319. $\int\frac{dx}{\sin^2 x\cos^2 x}=$ **[2025, 2004]**

(A) $\tan x+\cot x+c$, where c is the constant of integration.

(B) $\tan x-\cot x+c$, where c is the constant of integration.

(C) $\tan x\cot x+c$, where c is the constant of integration.

(D) $\tan x-\cot 2x+c$, where c is the constant of integration.

320. If $\int \tan^4 x\, dx = a\tan^3 x + b\tan x + cx + k$ (where k is the constant of integration) then the value of $a - b + c =$

(A) $\frac{7}{3}$ (B) $\frac{5}{3}$

(C) $\frac{4}{3}$ (D) $\frac{1}{3}$

321. $\int \frac{dx}{2+\cos x} =$

(A) $2\tan^{-1}\left(\frac{1}{\sqrt{3}}\tan\frac{x}{2}\right)+c$, where c is the constant of integration

(B) $\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{1}{\sqrt{3}}\tan\frac{x}{2}\right)+c$, where c is the constant of integration

(C) $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{1}{\sqrt{3}}\tan\frac{x}{2}\right)+c$, where c is the constant of integration constant of integration

(D) $\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}\tan\frac{x}{2}\right)+c$, where c is the constant of integration

322. $\int \frac{\sin x}{\sqrt{5\sin^2 x + 6\cos^2 x}} dx$

(A) $\log\left(\cos x+\sqrt{\cos^2 x+5}\right)+c$, where c is the constant of integration

(B) $\log\left(\sin x+\sqrt{6\cos^2 x+5}\right)+c$, where c is the constant of integration

(C) $-\log\left(\cos x+\sqrt{\cos^2 x+6}\right)+c$, where c is the constant of integration

(D) $-\log\left(\cos x+\sqrt{\cos^2 x+5}\right)+c$, where c is the constant of integration

323. $\int \sec^{\frac{2}{3}} x \cdot cosec^{\frac{4}{3}} x dx =$

(A) $3\tan^{\frac{-1}{3}} x + c$, where c is the constant of integration

(B) $-3\tan^{\frac{-1}{3}} x + c$, where c is the constant of integration

(C) $-3\cot^{\frac{-1}{3}} x + c$, where c is the constant of integration

(D) $-\frac{3}{4}\tan^{\frac{-4}{3}} x + c$, where c is the constant of integration

324. $\int\left(\frac{x-3}{x^2+9}\right)^2 dx =$

(A) $\frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right)-\frac{3}{x^2+9}+c$, where c is the constant of integration.

(B) $\frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right)-\frac{1}{x^2+9}+c$, where c is the constant of integration.

(C) $\frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right)+\frac{3}{x^2+9}+c$, where c is the constant of integration.

(D) $\frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right)-\frac{1}{x^2+9}+c$, where c is the constant of integration.

325. $\int\sqrt{x^2-6x-16}\, dx$ equals

(A) $\left(\frac{x-3}{2}\right)\sqrt{x^2-6x-16}$
$+\frac{5}{2}\log\left(x-3+\sqrt{x^2-6x-16}\right)+c$
where c is the constant of integration

(B) $\left(\frac{x-3}{2}\right)\sqrt{x^2-6x-16}$
$-\frac{25}{2}\log\left(x-3+\sqrt{x^2-6x-16}\right)+c$
where c is the constant of integration

(C) $\left(\frac{x-3}{2}\right)\sqrt{x^2-6x-16}$
$+\frac{25}{2}\log\left(x-3+\sqrt{x^2-6x-16}\right)+c$
where c is the constant of integration

(D) $\left(\frac{x-3}{2}\right)\sqrt{x^2-6x-16}$
$-\frac{5}{2}\log\left(x-3+\sqrt{x^2-6x-16}\right)+c$
where c is the constant of integration

326. $\int \frac{dx}{(x+a)^{\frac{9}{7}}(x-b)^{\frac{5}{7}}} =$

(A) $\frac{7}{a+b}\left(\frac{x-b}{x+a}\right)^{\frac{9}{7}}+c$, where c is the constant of integration.

(B) $\frac{7}{a+b}\left(\frac{x-b}{x+a}\right)^{\frac{5}{7}}+c$, where c is the constant of integration.

(C) $\frac{7}{2(a+b)}\left(\frac{x-b}{x+a}\right)^{\frac{2}{7}}+c$, where c is the constant of integration.

(D) $\frac{7}{a+b}\left(\frac{x-b}{x+a}\right)^{\frac{1}{7}}+c$, where c is the constant of integration.

327. If $\int \frac{\left(x^4+1\right)}{x\left(x^2+1\right)^2}\,dx = A\log|x| + \frac{B}{1+x^2}+c$, then $A-B$ is (where c is the constant of integration)

(A) 0 (B) 1
(C) 2 (D) -1

328. $\int \frac{\sqrt{\tan x}}{\sin x.\cos x}dx =$

(A) $2\sqrt{\sec x}+c$, where c is a constant of integration

(B) $2\sqrt{\tan x}+c$, where c is a constant of integration

(C) $\frac{2}{\sqrt{\tan x}}+c$, where c is a constant of integration

(D) $\frac{2}{\sqrt{\sec x}}+c$, where c is a constant of integration

329. $\int \frac{dx}{x^{\frac{1}{2}}+x^{\frac{1}{3}}} = Ax^{\frac{1}{2}}+Bx^{\frac{1}{3}}+Cx^{\frac{1}{6}}+D\log\left(x^{\frac{1}{6}}+1\right)+k$

(where k is the integeration constant) then values of A, B, C and D are respectively, **[2025]**

(A) $2,-3,6,-6$ (B) $2,3,-6,6$
(C) $2,-3,-6,6$ (D) $-2,-3,6,6$

330. $\int e^x \frac{(x-1)}{(x+1)^3}dx =$

(A) $e^x(x+1)^2+c$, where c is the constant of integration

(B) $e^x(x+1)^3+c$, where c is the constant of integration

(C) $\frac{e^x}{(x+1)^2}+c$, where c is the constant of integration

(D) $\frac{e^x}{(x+1)^3}+c$, where c is the constant of integration

331. $\int e^{2x}\frac{(\sin 2x\cos 2x-1)}{\sin^2 2x}dx =$ **[2025]**

(A) $e^{2x}\cot(2x)+c$, where c is the constant of integration

(B) $2e^{2x}\cot(2x)+c$, where c is the constant of integration

(C) $4e^{2x}\cot(2x)+c$, where c is the constant of integration

(D) $\frac{1}{2}e^{2x}\cot(2x)+c$, where c is the constant of integration

332. $\int x^2\cos x\,dx =$ **[2025]**

(A) $x^2\sin x+2x\cos x-2\sin x+c$, where c is the constant of integration,

(B) $x^2\sin x-2x\cos x-2\sin x+c$, where c is the constant of integration

(C) $x^2\sin x-2x\cos x+2\sin x+c$, where c is the constant of integration

(D) $x^2\sin x+2x\cos x+2\sin x+c$, where c is the constant of integration

333. $\int \frac{dx}{x\left(x^3+1\right)} =$ **[2025]**

(A) $\log\left(\frac{x^3}{x^3+1}\right)+c$, where c is the constant of integration

(B) $\frac{1}{3}\log\left(\sqrt[3]{\frac{x^3}{x^3+1}}\right)+c$, where c is the constant of integration

(C) $\log\left(\sqrt[3]{\frac{x^3}{x^3+1}}\right)+c$, where c is the constant of integration

(D) $\frac{1}{3}\log\left(\frac{x^3+1}{x^3}\right)+c$, where c is the constant of integration

334. If $\int \frac{2x+3}{(x-1)\left(x^2+1\right)}dx$

$= \log_e\left\{(x-1)^{\frac{5}{2}}\left(x^2+1\right)^a\right\} - \frac{1}{2}\tan^{-1}x + A$

where A is an arbitrary constant, then the value of a is:

[2025]

(A) $\frac{5}{4}$ (B) $-\frac{5}{4}$

(C) $-\frac{5}{3}$ (D) $-\frac{5}{6}$

335. If $\int \frac{2x^2+3}{(x^2-1)(x^2-4)}dx = \log\left[\left(\frac{x-2}{x+2}\right)^a \cdot \left(\frac{x+1}{x-1}\right)^b\right]+c$, (where c is the constant of integration) then the value of $a+b$ is equal to: **[2025]**

(A) $\frac{1}{12}$ (B) $\frac{21}{12}$

(C) $\frac{-1}{12}$ (D) $\frac{-21}{12}$

336. $\int \frac{dx}{\cos x(1+\cos x)} =$ **[2025]**

(A) $\log(\sec x+\tan x)+2\tan\left(\frac{x}{2}\right)+c$, where c is the constant of integration

(B) $\log(\sec x+\tan x)-2\tan\left(\frac{x}{2}\right)+c$, where c is the constant of integration

(C) $\log(\sec x+\tan x)+\tan\left(\frac{x}{2}\right)+c$, where c is the constant of integration

(D) $\log(\sec x+\tan x)-\tan\left(\frac{x}{2}\right)+c$, where c is the constant of integration

337. $\int \frac{dx}{2e^{2x}+3e^x+1} =$ **[2025]**

(A) $x+\log\left(e^x+1\right)-2\log\left(2e^x+1\right)+c$, where c is the constant of integration

(B) $x-\log\left(e^x+1\right)+4\log\left(e^x+1\right)+c$, where c is the constant of integration

(C) $x+\log\left(e^x+1\right)-4\log\left(2e^x+1\right)+c$, where c is the constant of integration

(D) $x-\log\left(e^x+1\right)+2\log\left(2e^x+1\right)+c$, where c is the constant of integration

338. The value of $\int_1^4 \log[x]dx$, where $[x]$ is the greatest integer function less than or equal to x is equal to

(A) log5

(B) log6

(C) log2

(D) log3

339. The value of $\int_{-3}^{3} \sin^7 x \cos^{16} x\, dx$ is

(A) 1 (B) 2

(C) 0 (D) –1

340. $\int_{-2}^{2}|x^2-x-2|dx$

(A) $\frac{17}{3}$ (B) $\frac{19}{3}$

(C) 19 (D) 17

341. $\int_0^3\frac{dx}{(x+2)\sqrt{x+1}}=$

(A) $\tan^{-1}\left(\frac{1}{3}\right)$ (B) $2\tan^{-1}\left(\frac{1}{3}\right)$

(C) $3\tan^{-1}\left(\frac{1}{3}\right)$ (D) $4\tan^{-1}\left(\frac{1}{3}\right)$

342. $\int_0^{\frac{\pi}{2}}\frac{dx}{1+(\cot x)^{101}}=$

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{8}$ (D) π

343. $\int_0^1\tan^{-1}xdx=$

(A) $\frac{\pi}{4}-\log 2$ (B) $\frac{\pi}{4}-\log\sqrt{2}$

(C) $\frac{\pi}{4}+\log 2$ (D) $\frac{\pi}{4}+\log\sqrt{2}$

344. $\int_0^{\frac{\pi}{4}}\frac{\cos^2 x\sin^2 x}{\cos^3 x+\sin^3 x}dx=$

(A) $\frac{1}{3}$ (B) $\frac{-1}{3}$

(C) $\frac{1}{6}$ (D) $\frac{-1}{6}$

345. $\int_0^2\frac{3x+1}{x^2+4}dx=$

(A) $\log\left(2\sqrt{2}\right)+\frac{\pi}{4}$ (B) $\log\left(2\sqrt{2}\right)+\frac{\pi}{6}$

(C) $\log\left(2\sqrt{2}\right)+\frac{\pi}{8}$ (D) $\log\left(2\sqrt{2}\right)+\frac{\pi}{12}$

346. The value of the integral $\int_1^2\frac{xdx}{(x+2)(x+3)}$ is

(A) $\log\left(\frac{125}{16}\right)$ (B) $\log\left(\frac{1024}{1125}\right)$

(C) $\log\left(\frac{16}{125}\right)$ (D) $\log\left(\frac{1125}{1024}\right)$

347. The value of $\int_0^1\tan^{-1}\left(1-x+x^2\right)dx$ is

(A) $\frac{\pi}{2}-\log 2$ (B) $\frac{\pi}{2}+\log 2$

(C) $\log 2$ (D) 0

348. $\int_0^1\log\left(\frac{1}{x}-1\right)dx$

(A) $\frac{1}{2}$ (B) 1

(C) 2 (D) 0

349. $\int_0^1\frac{1}{2+\sqrt{x}}dx=$

(A) $2\log\left(\frac{2e}{3}\right)$ (B) $2\log\left(\frac{4e}{9}\right)$

(C) $\log\left(\frac{2e}{3}\right)$ (D) $\log\left(\frac{4e}{9}\right)$

350. $\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}}\sin^{-4}xdx=$

(A) $\frac{8}{3}$ (B) $-\frac{8}{3}$

(C) $\frac{2}{3}$ (D) $-\frac{2}{3}$

351. $\int_{\frac{1}{2}}^{2}\frac{1}{x}\text{cosec}^{101}\left(x-\frac{1}{x}\right)dx$

(A) 0 (B) 1

(C) $\frac{1}{4}$ (D) $\frac{101}{2}$

352. $\int_0^{\frac{\pi}{2}} \frac{300\sin x + 100\cos x}{\sin x + \cos x} dx = \ldots$

(A) 100π (B) 300π

(C) 200π (D) 150π

353. $\int_0^1 \log(x+1)dx =$

(A) $\log 2 - 1$ (B) $\log 2 + 1$

(C) $2\log 2 + 1$ (D) $2\log 2 - 1$

354. The area bounded by the curve $y = |x-2|, x = 1$, $x = 3$ and X -axis is: **[2025, 2023]**

(A) 3 sq. units (B) 2 sq. units

(C) 1 sq. units (D) 4 sq. units

355. The area enclosed between the curves $y^2 = 4x$ and $y = |x|$ is:

(A) $\frac{8}{3}$ sq. units (B) $\frac{5}{3}$ sq. units

(C) $\frac{4}{3}$ sq. units (D) $\frac{2}{3}$ sq. units

356. AOB is the positive quadrant of the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$ in which $OA = 5, OB = 3$. The area between the arc AB and the chord AB of the ellipse in sq. units is

(A) $\frac{3}{5}(\pi - 2)$ (B) $\frac{15}{2}(\pi - 2)$

(C) $\frac{3}{10}(\pi - 2)$ (D) $\frac{15}{4}(\pi - 2)$

357. The area of the region bounded by the parabola $y^2 = 27x$ and the line $x = 1$ is _____ sq.units.

(A) $2\sqrt{3}$ (B) $3\sqrt{3}$

(C) $4\sqrt{3}$ (D) $7\sqrt{3}$

358. The area bounded by the curve $x = 2 - y - y^2$ and the Y -axis is: **[2025]**

(A) $\frac{7}{6}$ sq. units (B) $\frac{13}{2}$ sq. units

(C) $\frac{9}{2}$ sq. units (D) $\frac{27}{2}$ sq. units

359. The area bounded by the curve $y = x^2 + 3, y = x, x = 3$ and y -axis is: **[2025]**

(A) $\frac{9}{2}$ sq. units

(B) 18 sq. units

(C) $\frac{27}{2}$ sq. units

(D) $\frac{27}{3}$ sq. units

360. If the area bounded by the curve $x^2 = 4y, X$ -axis and the line $x = 4$ is divided into equal areas by the line $x = \alpha$, then the value of α is ... **[2025]**

(A) $\frac{1}{32}$ (B) 32

(C) $(32)^{\frac{1}{2}}$ (D) $(32)^{\frac{1}{3}}$

361. The degree of the differential equation $\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 = x^2 \log\left(\frac{d^2y}{dx^2}\right)$ is

(A) 1 (B) 2

(C) 3 (D) Not defined

362. The order and degree of the differential equation $\sqrt{\frac{dy}{dx}} - 4\frac{dy}{dx} - 7x = 0$ is respectively:

(A) 1, 2 (B) 2, 1

(C) 2, 2 (D) 3, 1

363. The differential equation of all straight lines passing through the point $(1, -1)$ is: **[2025]**

(A) $y = (x-1)\frac{dy}{dx} - 1$

(B) $x = (x-1)\frac{dy}{dx} + 1$

(C) $y = (x-1)\frac{dy}{dx}$

(D) $y = 2(x-1)\frac{dy}{dx}$

364. The differential equation satisfied by $y = X\sin(6t+5) + Y\cos(6t+5)$ is (where X and Y are constants)

(A) $\frac{d^2y}{dt^2} + 6y = 0$ (B) $\frac{d^2y}{dt^2} = 0$

(C) $\frac{d^2y}{dt^2} + 36y = 0$ (D) $\frac{d^2y}{dt^2} + 25y = 0$

365. The differential equation which represents the family of curves $y = c_1 e^{c_2 x}$, where c_1, c_2 are arbitrary constants is

(A) $y'' = y'y$ (B) $yy'' = y'$

(C) $yy'' = (y')^2$ (D) $y' = y^2$

366. The differential equation of all circles touching the Y -axis at the origin and centre on the X -axis is:

(A) $x^2 + y^2 + 2xy\frac{dy}{dx} = 0$

(B) $x^2 - y^2 + 2xy\frac{dy}{dx} = 0$

(C) $2x^2 + y^2 + xy\frac{dy}{dx} = 0$

(D) $x^2 - 2y^2 + 2xy\frac{dy}{dx} = 0$

367. The differential equation representing the family of parabolas having vertex at the origin and axis along the positive Y -axis is: **[2025]**

(A) $x\frac{dy}{dx} - 2y = 0$ (B) $\frac{dy}{dx} + xy = 0$

(C) $x\frac{dy}{dx} + y = 0$ (D) $x^2\frac{dy}{dx} + y = 0$

368. The differential equation whose solution is $Ax^2 + By^2 = 1$, where A and B are arbitrary constants is of: **[2025]**

(A) degree 1 and order 2

(B) degree 2 and order 1

(C) degree 3 and order 2

(D) degree 1 and order 3

369. The general solution of the differential equation $\frac{dy}{dx} + \sin\left(\frac{x+y}{2}\right) = \sin\left(\frac{x-y}{2}\right)$ is: **[2025, 2024]**

(A) $\log\tan\left(\frac{y}{2}\right) = c - 2\sin\frac{x}{2}$, where c is the constant of integration

(B) $\log\tan\left(\frac{y}{4}\right) = c - 2\sin\left(\frac{x}{2}\right)$, where c is the constant of integration

(C) $\log\left[\tan\left(\frac{y}{2} + \frac{\pi}{4}\right)\right] = c - 2\sin x$, where c is the constant of integration

(D) $\log\left[\tan\left(\frac{y}{4} + \frac{\pi}{4}\right)\right] = c - 2\sin\frac{x}{2}$, where c is the constant of integration

370. The solution of $\frac{dy}{dx} = (x+y)^2$ is [2025]

(A) $\tan^{-1}(x+y) = x + c$, where c is the constant of integration

(B) $x + y = \tan x + c$, where c is the constant of integration

(C) $x + y = \cot^{-1} x + c$, where c is the constant of integration

(D) $x + y = \sin^{-1}(x+y) + c$, where c is the constant of integration

371. A particular solution of $\frac{dy}{dx} = (x+9y)^2$, when $x = 0, y = \frac{1}{27}$ is: **[2025]**

(A) $3x + 27y = \tan\left[3\left(x + \frac{\pi}{12}\right)\right]$

(B) $3x + 27y = \tan\left(x + \frac{\pi}{4}\right)$

(C) $3x + 27y = \tan\left(x + \frac{\pi}{12}\right)$

(D) $3x + 27y = \tan\left[3\left(x + \frac{\pi}{4}\right)\right]$

372. A particular solution of $3e^x \tan y\,dx + (1-e^x)\sec^2 y\,dy = 0$ with $y(1) = \frac{\pi}{4}$ is **[2025]**

(A) $\tan y = \left(\frac{1-e^3}{1-e^x}\right)^3$ (B) $\tan y = \left(\frac{1-e^2}{1-e^x}\right)^3$

(C) $\tan y = \left(\frac{1-e}{1-e^x}\right)^3$ (D) $\tan y = \left(\frac{1-e^x}{1-e}\right)^3$

373. If $y = y(x)$ and $\left(\frac{2+\sin x}{y+1}\right)\frac{dy}{dx} = -\cos x, y(0) = 1$, then $y\left(\frac{\pi}{2}\right) =$

[2025, 2024, 2023, 2022]

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$

(C) $-\frac{1}{3}$ (D) 1

374. The solution of $\log\left(\frac{dy}{dx}\right) = 2x - 5y, y(0) = 0$ is **[2025]**

(A) $2e^{2x} + 5e^{5y} = 6$ (B) $5e^{2x} - 2e^{5y} = 3$

(C) $2e^{2x} - 5e^{5y} = 6$ (D) $5e^{2x} + 2e^{5y} = 3$

375. Which of the following is not a homogeneous function? **[2025]**

(A) $y^2 + 2xy$ (B) $2x - 3y$

(C) $\sin\left(\frac{y}{x}\right)$ (D) $\cos x + \sin y$

376. If $x\frac{dy}{dx} = y(\log y - \log x + 1)$, then the solution of the equation is: **[2025, 2024]**

(A) $\log\frac{x}{y} = cy$, where c is the constant of integration

(B) $\log\frac{y}{x} = cy$, where c is the constant of integration

(C) $\log\frac{x}{y} = cx$, where c is the constant of integration

(D) $\log\frac{y}{x} = cx$, where c is the constant of integration

377. The integrating factor of the differential equation $x\frac{dy}{dx} + y\log x = x \cdot e^x x^{-\frac{1}{2}}\log x (x > 0)$ is: **[2025]**

(A) $(\log x)^x$ (B) $x^{\log x}$

(C) $\left(\sqrt{x}\right)^{\log x}$ (D) $e^{\sqrt{x}\log x}$

378. The solution of $(1+y^2) + \left(x - e^{\tan^{-1} y}\right)\frac{dy}{dx} = 0$ is

(A) $2xe^{\tan^{-1} y} = e^{2\tan^{-1} y} + k$, where k is the constant of integration

(B) $x \cdot e^{\tan^{-1} y} = e^{\tan^{-1} y} + k$, where k is the constant of integration

(C) $x \cdot e^{2\tan^{-1} y} = e^{\tan^{-1} y} + k$, where k is the constant of integration

(D) $x = 2 + k \cdot e^{-\tan^{-1} y}$, where k is the constant of integration

379. If $y + \frac{d}{dx}(xy) = x(\sin x + \log x)$ then

(A) $y = \cos x + \frac{2\sin x}{x} + \frac{2}{x^2}\cos x + \frac{x}{3}\log x - \frac{x}{9} + \frac{c}{x^2}$, where c is the constant of integration.

(B) $y = -\cos x - \frac{2}{x}\sin x + \frac{2}{x^2}\cos x + \frac{x}{3}\log x - \frac{x}{9} + \frac{c}{x^2}$ where c is the constant of integration.

(C) $y = -\cos x + \frac{2}{x}\sin x + \frac{2}{x^2}\cos x + \frac{x}{3}\log x - \frac{x}{9} + \frac{c}{x^2}$, where c is the constant of integration.

(D) $y = \cos x - \frac{2}{x}\sin x + \frac{2}{x^3}\cos x + \frac{x}{3}\log x - \frac{x}{9} + \frac{c}{x^2}$ where c is the constant of integration.

380. The equation of the curve passing through the point $(0,2)$ given that the sum of the ordinate and abscissa of any point exceeds the slope of the tangent to the curve at that point by 5 is:
[2025]

(A) $y = x - 4 - 2e^x$

(B) $y = 4 - x - 2e^x$

(C) $y = 4 + x - 2e^x$

(D) $y = 4 - x + 2e^x$

381. The money invested in a company is compounded continuously. ₹ 400 invested today becomes ₹800 in 6 years, then at the end of 33 years, it will become .. $\left(\sqrt{2}=1.4142\right)$ **[2025]**

(A) 9050.88 (B) 18101.76
(C) 6788.16 (D) 12067.84

382. The assets of a person reduced in his business such that the rate of reduction is proportional to the square root of the existing assets. If the assets were initially ₹ 10 lakhs and due to loss they reduce to ₹ 10000 after 3 years, then the number of years required for the person to be bankrupt will be:
[2025, 2024]

(A) $\frac{20}{3}$ years (B) $\frac{10}{3}$ years
(C) $\frac{10}{9}$ years (D) $\frac{20}{9}$ years

383. In a culture bacteria count is $1,00,000$ initially. The number increases by 10% in first 2 hours. In how many hours will the count reach $2,00,000$, if the rate of growth of bacteria is proportional to the number present?

(A) $\frac{2\log\left(\frac{11}{10}\right)}{\log 2}$ (B) $\frac{\log\left(\frac{11}{10}\right)}{\log 2}$

(C) $\frac{2\log 2}{\log\left(\frac{11}{10}\right)}$ (D) $\frac{\log(2)}{\log\left(\frac{11}{10}\right)}$

384. A wet substance in the open air loses its moisture at a rate proportional to the moisture content. If a sheet, hung in the open air, loses half its moisture during the first hour, then 90% of the moisture will be lost in _____ hours.

(A) $2\log_2 10$ (B) $\frac{4\log 10}{\log 2}$

(C) $\log_2 10$ (D) $\frac{3\log 10}{\log 2}$

385. The rate at which a substance cools in moving air, is proportional to the difference between the temperature of the substance and that of air. The temperature of air is 290 K and the substance cools from 370 K to 330 K in 10 minutes. Then the time to cool the substance upto 295 K is: **[2025, 2020]**

(A) 40 min (B) 95 min
(C) 50 min (D) 60 min

386. The rate of increase of the population of a city is proportional to the population present at that instant. In the period of 40 years the population increased from 30,000 to 40,000. At any time t the population is $(a)(b)^{\frac{t}{40}}$. Then the values of a and b are respectively **[2025]**

(A) $30,000, \frac{2}{3}$ (B) $30,000, \frac{4}{3}$

(C) $40,000, \frac{2}{3}$ (D) $40,000, \frac{3}{4}$

387. The population of towns A and B increase at the rate proportional to their population present at that time. At the end of the year 1984, the population of both the towns was 20,000. At the end of the year 1989, the population of town A was 25,000 and that of town B was 28,000. The difference of populations of towns A and B at the end of 1994 was

(A) 5950
(B) 8000
(C) 7950
(D) 6950

388. The differential equation $x\frac{dy}{dx}=2y$ represents …….. [2025]

(A) a family of circles with radius *c*.

(B) a family of parabolas with vertex at the origin and axis along the positive Y -axis

(C) a family of parabolas with vertex at origin and axis along the positive X -axis

(D) a family of ellipses

389. A coin is tossed until one head appears or a tail appears 4 times in succession. The probability distribution of the number of tosses is:

(A)

X	1	2	3	4
P(X = x)	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{2}$	$\frac{1}{4}$

(B)

X	1	2	3	4
P(X = x)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{8}$	$\frac{1}{8}$

(C)

X	1	2	3	4
P(X = x)	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{2}$

(D)

X	1	2	3	4
P(X = x)	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$

390. A random variable X takes the values $0,1,2,3,$ ____ with probability

$P(X=x)=k(x+1)\left(\frac{1}{5}\right)^x$, where k is a constant.

Then $P(X=0)$ is:

(A) $\frac{16}{25}$ (B) $\frac{7}{25}$

(C) $\frac{19}{25}$ (D) $\frac{18}{25}$

391. If a random variable X has the following probability distribution of X.

$X = x$	0	1	2	3	4	5	6	7
$P(X=x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

Then $P(X\geq 6)=$

(A) $\frac{19}{100}$ (B) $\frac{81}{100}$

(C) $\frac{9}{100}$ (D) $\frac{91}{100}$

392. The probability distribution of a discrete random variable X is:

X	0	1	2	3	4
P(X = x)	2k	k	2k	4k	k

If $a=P(x<3)$ and $b=P(2\leq X<4)$, then

(A) $a=b$ (B) $a>b$

(C) $a<b$ (D) $a=\frac{1}{2}$ b

393. A student studies for X number of hours during a randomly selected school day. The probability that X can take the values, has the following form, where k is some constant.

$$P(X=x)=\begin{cases}0.2, & \text{if } x=0\\ kx, & \text{if } x=1 \text{ or } 2\\ k(6-x), & \text{if } x=3 \text{ or } 4\\ 0, & \text{otherwise}\end{cases}$$

The probability that the student studies for at most two hours is:

(A) 0.1 (B) 0.5

(C) 0.3 (D) 0.7

394. The c.d.f. of a discrete random variable X is

X	–3	–1	0	1	3	5	7	9
F (X =x)	0.1	0.3	0.5	0.65	0.75	0.85	0.90	1

Then $\frac{P[X=-3]}{P[X<0]}=$

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$

(C) $\frac{1}{6}$ (D) $\frac{1}{7}$

395. A random variable X has the following probability distribution

X:	0	1	2	3	4
P(X):	k	2k	4k	2k	k

then the value of $P(1 \leqslant X < 4 \mid X \leqslant 2) =$

(A) $\frac{5}{6}$ (B) $\frac{6}{7}$

(C) $\frac{7}{8}$ (D) $\frac{8}{9}$

396. The cumulative distribution function of a discrete random variable X is

$X=x$	-4	-2	0	2	4	6	8	10
$F(X-x)$	0.1	0.3	0.5	0.65	0.75	0.85	0.90	1

then $\frac{P(X \leqslant 0)}{P(X > 0)} =$

(A) $\frac{1}{2}$ (B) 1

(C) $\frac{1}{3}$ (D) $\frac{1}{5}$

397. In a game, 3 coins are tossed. A person is paid ₹ 150 if he gets all heads or all tails and he is supposed to pay ₹50 if he gets one head or two heads. The amount he can expect to win / lose on an average per game in ₹ is:

(A) 100
(B) 0
(C) 200
(D) –100

398. Two numbers are selected at random, without replacement from the first 6 positive integers. Let X denote the larger of the two numbers. Then $E(X) =$

(A) $\frac{14}{3}$ (B) $\frac{3}{14}$

(C) $\frac{14}{5}$ (D) $\frac{15}{41}$

399. In a game a man wins ₹40 if he gets 5 or 6 on a throw of a fair die and loses ₹ 20 for getting any other number on the die. If he decides to throw the die either till he gets a five or six or to a maximum of three throws, then his expected gain/loss (in rupees) is

(A) –10
(B) 10
(C) 0
(D) 1

400. The following is the probability distribution of X.

[2025]

X	0	1	2	3
P(X=x)	$\frac{1+p}{5}$	$\frac{2-2p}{5}$	$\frac{2-p}{5}$	$\frac{2p}{5}$

For a minimum value of p, the value of $5E(X)$ is

(A) 5 (B) 6
(C) 7 (D) 8

401. Let X be a discrete random variable. The probability distribution of X is given below

X	30	10	–10
P(X)	$\frac{1}{5}$	A	B

and $E(X) = 4$, then the value of AB is equal to:

[2025]

(A) $\frac{3}{10}$ (B) $\frac{2}{15}$

(C) $\frac{1}{15}$ (D) $\frac{3}{20}$

402. Two cards are drawn simultaneously from a well shuffled pack of 52 cards. If X is the random variable of getting queens, then the value of $2E(X) + 3E(X^2)$ for the number of queens is

[2025]

(A) $\frac{132}{221}$ (B) $\frac{108}{221}$

(C) $\frac{176}{221}$ (D) $\frac{68}{221}$

403. A fair n faced die is rolled repeatedly until a number less than n appears. If the mean of the number of tosses required is $\frac{n}{9}$, then $n=$ (where $n\in\mathbb{N}$) **[2025]**

(A) 4 (B) 6

(C) 8 (D) 10

404. Let mean and standard deviation of probability distribution

X=x	–3	0	1	α
P(X=x)	$\frac{1}{4}$	K	$\frac{1}{4}$	$\frac{1}{3}$

be μ and σ respectively and if $\sigma-\mu=2$ then $\sigma=$ **[2025]**

(A) $\frac{3}{2}$ (B) $\frac{5}{2}$

(C) $\frac{7}{2}$ (D) $\frac{9}{2}$

405. A random variable X has p.m.f. $P(X=x)=\frac{{}^4C_x}{2^4}, x=0,1,2,3,4$ and μ and σ^2 are mean and variance respectively of random variable X, then: **[2025]**

(A) $\mu=2,\sigma^2=4$ (B) $\mu=2,\sigma^2=1$

(C) $\mu=3,\sigma^2=4$ (D) $\mu=2,\sigma^2=5$

406. If a random variable X has the p.d.f. $f(x)=\begin{cases}\frac{k}{x^2+1} & ,\ \text{if}\ 0<x<\infty\\ 0 & ,\ \text{otherwise}\end{cases}$ then c.d.f. of X is

(A) $2\tan^{-1}x$ (B) $\frac{\pi}{2}\tan^{-1}x$

(C) $\frac{2}{\pi}\tan^{-1}x$ (D) $\tan^{-1}x$

407. If a random variable X has p.d.f. $f(x)=\begin{cases}\frac{ax^2}{2}+bx & ,\ \text{if}\ 1\leqslant x\leqslant 3\\ 0 & ,\ \text{otherwise}\end{cases}$ and $f(2)=2$, then the values of a and b are, respectively:

(A) $11,-10$ (B) $-9,10$

(C) $\frac{1}{6},\frac{5}{6}$ (D) $9,-8$

408. The p.d.f. of a continuous random variable X is $f(x)=\begin{cases}\frac{x^2}{18} & \text{if}\ -3<x<3\\ 0 & \text{otherwise}\end{cases}$ Then $P[|X|<2]=$

(A) $\frac{1}{27}$ (B) $\frac{2}{13}$

(C) $\frac{8}{27}$ (D) $\frac{4}{27}$

409. A fair coin is tossed 99 times. If X is the number of times head occur then $P[X=r]$ is maximum when $r=$

(A) 48 (B) 49

(C) 51 (D) 52

410. In a box containing 100 apples, 10 are defective. The probability that in a sample of 6 apples, 3 are defective is

(A) 0.1548 (B) 0.1458

(C) 0.01854 (D) 0.01458

411. A fair coin is tossed 100 times. The chance of getting a head even number of times is:

(A) $\frac{1}{8}$

(B) $\frac{3}{8}$

(C) $\frac{1}{2}$

(D) $\frac{3}{4}$

412. A pair of fair dice is thrown 4 times. If getting the same number on both dice is considered as a success, then the probability of two successes are

(A) $\frac{25}{216}$

(B) $\frac{25}{36}$

(C) $\frac{25}{108}$

(D) $\frac{25}{104}$

413. The probability that a certain kind of component will survive a given test is $\frac{2}{3}$. The probability that at most 2 components out of 4 tested, will survive is :

(A) $\frac{31}{3^4}$ (B) $\frac{32}{3^4}$

(C) $\frac{33}{3^4}$ (D) $\frac{35}{3^4}$

414. If $X \sim B(35, p)$ such that $7P(X=0)=P(X=1)$ then the value of $\frac{P(X=15)}{P(X=20)}$ is equal to

(A) $\frac{3125}{7776}$ (B) 3125

(C) 7776 (D) $\frac{625}{1296}$

415. If a random variable X follows the Binomial distribution $B(33, p)$ such that $3P(X=0)=P(X=1)$, then the variance of X is

(A) $\frac{11}{144}$ (B) $\frac{35}{48}$

(C) $\frac{121}{48}$ (D) $\frac{33}{144}$

416. Numbers are selected at random, one at a time from the two-digit numbers $00, 01, 02, \ldots\ldots, 99$ with replacement. An event E occurs only if the product of the two digits of a selected number is 24. If four numbers are selected, then probability, that the event E occurs at least 3 times, is

(A) $\frac{24}{(25)^4}$ (B) $\frac{4}{(25)^4}$

(C) $\frac{97}{(25)^4}$ (D) $\frac{96}{(25)^4}$

MHTCET PYQ T(2025)

Marathi

Mathematics

Hint & Solution

1. (A)

Using $\tan(A+B)=\dfrac{\tan A+\tan B}{1-\tan A\tan B}$ gives

$\tan(A+B)=\sqrt{\dfrac{x^2+x+1}{x^3}}=\tan C$; hence

$A+B=C$.

2. (A)

Direct simplification gives

$\sqrt{3}\cot 20^\circ-4\cos 20^\circ=1$.

3. (D)

After dividing by $\sin\theta$, $6\cos\theta=6-8\sin^2\theta$; hence

$\cos\theta=-\dfrac{1}{4}$ and $\sin\theta=\dfrac{\sqrt{15}}{4}$.

4. (B)

With $t=\tan 10^\circ$, the standard cubic identity gives

$3t^6-27t^4+33t^2=1$.

5. (D)

Writing $X=A+B$ in $\sin(X-B)=n\sin(X+B)$

gives $\tan X=\dfrac{1+n}{1-n}\tan B$.

6. (A)

Since $\dfrac{x}{y}=\tan\dfrac{A+B}{2}$, $\sin(A+B)=\dfrac{2xy}{x^2+y^2}$.

7. (B)

$\sec x+\tan x=2$ gives $\sin x=\dfrac{3}{5}$;

successive half-angle formulae yield

$\sin\dfrac{x}{4}=\dfrac{1}{\sqrt{2\left(10+3\sqrt{10}\right)}}$.

8. (D)

The equation gives $\sin\theta+\cos\theta=\dfrac{1}{2}$; therefore

$\sin\left(\dfrac{\pi}{4}+\theta\right)=\dfrac{1}{2\sqrt{2}}$.

9. (A)

$\alpha+\beta=\dfrac{\pi}{2}$ and $\alpha-\beta=\dfrac{\pi}{6}$, so the required product

is $=1$.

10. (D)

Given:

$3\sin\alpha=5\sin\beta\Rightarrow\dfrac{\sin\alpha}{\sin\beta}=\dfrac{5}{3}$

Identity: $\dfrac{\sin\alpha+\sin\beta}{\sin\alpha-\sin\beta}=\dfrac{\tan\left(\dfrac{\alpha+\beta}{2}\right)}{\tan\left(\dfrac{\alpha-\beta}{2}\right)}$

Substitute: $\dfrac{\sin\alpha+\sin\beta}{\sin\alpha-\sin\beta}=\dfrac{5+3}{5-3}=\dfrac{8}{2}=4$

11. (A)

$\tan 20^\circ\tan 80^\circ\cot 50^\circ=\sqrt{3}$.

12. (A)

If $s=\frac{1}{2}\left(x+x^{-1}\right)=\sin\theta$, then

$\frac{1}{2}\left(x^3+x^{-3}\right)=4s^3-3s=-\sin3\theta$; the sum is 0.

13. (A)

$\tan\frac{P}{2}+\tan\frac{Q}{2}+\tan\frac{P}{2}\tan\frac{Q}{2}=1;$

Vieta gives $a+b=c$.

14. (B)

The centre of the rectangle is

$\left(\frac{8+10}{2},\frac{11+12}{2}\right)=\left(9,\frac{23}{2}\right).$

15. (B)

The median from vertex A goes to the midpoint of side BC.

First, find the midpoint M of $B(2,1)$ and $C(-4,5)$:

$M=\left(\frac{2+(-4)}{2},\frac{1+5}{2}\right)=\left(\frac{-2}{2},\frac{6}{2}\right)=(-1,3)$

Since the median passes through $A(3,k)$ and $M(-1,3)$, its equation is given as:

$x+4y=p$

Both A and M satisfy this equation.

For $M(-1,3)$:

$-1+4(3)=-1+12=11$

so $p=11$.

For $A(3,k)$:

$3+4k=11$

$4k=8$

$k=2$

16. (B)

The $3:2$ division point is $\left(\frac{8}{5},\frac{14}{5}\right)$, so $k=6$ and $(k+1):(k-1)=7:5$.

17. (D)

The original line has inclination 45°.; after a 45° anticlockwise rotation about $(2,0)$ it is $x=2$.

18. (C)

For diagonals $(0,4)$ and $(4,-2)$, $\tan\theta=2$; hence $\theta=\tan^{-1}2$.

19. (A)

Concurrency gives $-ab+2ac-bc=0$, or $\frac{2}{b}=\frac{1}{a}+\frac{1}{c}$; hence a,b,c are in H.P.

20. (B)

Along direction $(1,3)$, $(1+t,2+3t)$ meets $x+y=0$ at $t=-\frac{3}{4}$; distance = $\frac{3\sqrt{10}}{4}$.

21. (B)

The slopes are 2 and -1, so $\tan\theta=\left|\frac{2+1}{1-2}\right|=3$.

22. (A)

The parallel lines have constants 10 and $-\frac{5}{2}$ on $4x+3y$; hence $OA:OB=4:1$.

23. (D)

The distances are $\frac{4}{5},\frac{5}{\sqrt{13}},\frac{12}{5},\frac{3}{\sqrt{29}}$; the last is least.

24. (C)

The pair factors as $(x-y+1)(x+y-3)=0$; centre $(1,2)$ and radius 1 give $(x-1)^2+(y-2)^2=1$.

25. **(A)**

The radius condition gives:

$\left(\frac{k}{2}\right)^2+\left(\frac{1-k}{2}\right)^2-5\le 25\,\$$

$2k^2-2k-119\le 0$

Roots: $k=\frac{1\pm\sqrt{239}}{2}\approx -7.23, 8.23$

Integral values: -7 to $8\to 8-(-7)+1=16$

26. **(A)**

Centre $(7,5)$, radius 15, and $CP=13$ give minimum and maximum distances $2, 28$.

27. **(B)**

The circumradius is $2\sqrt{5}$; an equilateral-triangle median is $\frac{3}{2}R=3\sqrt{5}$.

28. **(C)**

Centre $(2,1)$, radius 5, and $AC=10$ give $AM=5$, $AM'=15$.

29. **(B)**

A perpendicular tangent has form $x-5y+c=0$; $|c|/\sqrt{26}=6$ gives $c=\pm 6\sqrt{26}$.

30. **(D)**

The two right triangles have total area 25 and the 90° sector has area $25\pi/4$; required area = $25(4-\pi)/4$.

31. **(B)**

$\sin 30^\circ = 4/OP$, so $OP=8$ and the locus is $x^2+y^2=64$.

32. **(D)**

For $x^2=20y$, $a=5$; the triangle has base 20 and height 5, hence area 50.

33. **(D)**

$(y+2)^2=-4\left(x-\frac{1}{2}\right)$, so the directrix is $x=\frac{3}{2}$.

34. **(C)**

The tangent slopes satisfy $m^2-4m+1=0$; $\tan\theta=\sqrt{3}$, so $\theta=\pi/3$.

35. **(B)**

For $y=mx+c$ tangent to $y^2=4x$, $c=1/m$; $c=3$ gives $m=1/3$.

36. **(D)**

$e_1=1/\sqrt{2}$ and $e_1e_2=\sqrt{2}/3$, so $e_2=2/3$, $b^2=20$; product of axes = $12\cdot 4\sqrt{5}=48\sqrt{5}$.

37. **(B)**

For equal intercepts $x+y=c$, tangency gives $c^2=32+18=50$; area = $c^2/2=25$.

38. **(C)**

$-6=4\sqrt{3}\cos\phi$, $2=4\sin\phi$ gives $\phi=150^\circ$.

39. **(B)**

The conic is $(x-3)^2/16+y^2/25=1$; $c=3$, so foci $(3,\pm 3)$.

40. **(B)**

The common focal distance is 4; with $e=2$, $a=2$, $b^2=12$, giving $x^2/4-y^2/12=1$.

41. **(C)**

The tangent slope is $3/(2\sin\theta)$; equating it to 3 gives $\theta=30^\circ$.

42. **(B)**

A perpendicular tangent has slope $3/4$ and intercept $c=\pm 5/2$; the listed intercept pair is $(10/3, -5/2)$.

43. **(B)**
Valid ordered pairs total $1+2+4=7$ out of 16; probability $7/16$.

44. **(C)**
$P(\text{I's together}) = 9!/(10!/2!) = 1/5$; hence not together = $4/5$.

45. **(B)**
$$\frac{3\cdot3\cdot1+2\cdot2\cdot1+2\cdot3\cdot4}{125}=\frac{37}{125}.$$

46. **(D)**
$$P(OEO)+P(EOE)=\frac{5}{9}\frac{4}{8}\frac{4}{7}+\frac{4}{9}\frac{5}{8}\frac{3}{7}=\frac{5}{18}.$$

47. **(A)**
Independence gives $P(A)=1/4$ and $P(A'|B)=3/4$, a root of $4x^2-7x+3=0$.

48. **(C)**
Of the seven outcomes containing at least one girl, only GGG qualifies; probability $1/7$.

49. **(A)**
Let $P(A)=a$; then $a+\frac{3}{2}a+\frac{3}{4}a=1$, so $a=4/13$

50. **(C)**
Solving $a(1-b)=3/25$ and $(1-a)b=8/25$ gives the listed admissible value $P(A)=1/5$.

51. **(B)**
Bayes: $\dfrac{(1/3)(2/5)}{(1/2)(1/3)+(1/3)(2/5)+(1/6)(3/7)}$.
$=14/39$

52. **(A)**
Posterior for d_2 is $0.5/(0.7+0.5+0.8)=1/4$.

53. **(A)**
Event probability is $2/6$; odds against are $4:2=2:1$.

54. **(D)**
Solving $(2z-n)/(2z+n)=2i-1$ with $\Im z=10$ gives $z=-10+10i$, $n=40$.

55. **(B)**
The real-part condition gives
$x^2+y^2-\frac{1}{2}x-\frac{1}{2}=0$, radius $3/4$.

56. **(A)**
The locus is the perpendicular bisector of $(-1,1)$ and $(1,-1)$, namely $y=x$.

57. **(C)**
The successive positions are $6+5i$, $7+6i$, then multiplication by i gives $-6+7i$.

58. **(B)**
$y=1$ and $\sqrt{x^2+1}+x=3$ give $x=4/3$ and $|z|=5/3$.

59. **(D)**
The conjugate has modulus 25; a square root therefore has modulus $\sqrt{25}=5$.

60. **(B)**
$|z+i|=\sqrt{3}|z-i|$ gives $x^2+(y-2)^2=3$.

61. **(D)**
The base $\omega\omega^2$ is $\sqrt{3}$ and the perpendicular height from i is $1/2$; area $\sqrt{3}/4$.

62. **(B)**
Counting three-digit n with $\gcd(n,36)=2$ gives 150.

63. **(B)**

Endings are 25 or 75; each permits $P(5,2)=5\cdot 4=20$, total 40.

64. **(D)**

Under the paper's circular/reflection convention the alternating arrangement count is 576.

65. **(B)**

The number of 3-subsets of a 20-cycle with no adjacent vertices is $\frac{20}{17}\binom{17}{3}=800$.

66. **(A)**

$\binom{n}{4}+\binom{n}{5}>\binom{n+1}{4}$ reduces to $\binom{n}{5}>\binom{n}{3}$, hence $n>8$.

67. **(B)**

$\binom{8}{2}+2\binom{4}{2}+2(8)(4)=28+12+64=104$.

68. **(C)**

$\sum_{r=0}^{n}\binom{n}{r}/(r+1)=\left(2^{n+1}-1\right)/(n+1)=1023/10$, so $n=9$.

69. **(A)**

$\binom{11}{4}-\binom{5}{3}\binom{6}{1}-\binom{5}{4}=330-60-5=265$.

70. **(A)**

After fixed inclusions/exclusions, choose 5 from 14: $\binom{14}{5}=2002$.

71. **(C)**

$\binom{21}{12}11!8!=\frac{35}{9}19!$.

72. **(D)**

Pascal's identity gives $\binom{n+3}{2}=15(n+2)$, hence $n=27$.

73. **(C)**

For $P(7-x,x-1)$, $1\le x\le 4$ and x is integral; domain $\{1,2,3,4\}$.

74. **(C)**

The fractional-linear map is one-one and its range excludes $2/3$.

75. **(C)**

$f(2)=f(3)=0$ gives $m=-5,n=30$; $4m+5n=130$.

76. **(D)**

$\cos 2a\cos 2b=1/2\left[\cos 2(a-b)+\cos 2(a+b)\right]$; the expression is 0.

77. **(D)**

If the common value is K, the logarithm is $7(1/3+1/4+1/6+1/2)=35/4$.

78. **(A)**

Let $u=\log_3 x$; $4\sqrt{u}+1-u=5$ gives $u=4$, so $x=81$.

79. **(D)**

Exactly the terms with $k=50,\ldots,99$ have floor 1; the sum is 50.

80. **(B)**

$(f\circ g)(x)=3x^2+7$, hence $(f\circ g)^{-1}(x)=\sqrt{(x-7)/3}$.

81. (B)

$\frac{1+g(x)}{1-g(x)}=\left(\frac{1+x}{1-x}\right)^3$; therefore

$(f\circ g)(x)=3f(x)$.

82. (A)

$f^{-1}(x)=(2x-3)/(x-1)$ and

$g^{-1}(x)=(x+2)/3$; roots are $-1/2,3$, sum $5/2$.

83. (C)

$[x]=2$ or 3, giving $x\in[2,4)$.

84. (C)

$$f(x)=\begin{cases}3-x, & -1\le x<0\\ 1+\frac{5x}{3}, & -3\le x\le 2\end{cases}$$

$$g(x)=\begin{cases}-x, & -2\le x\le 3\\ x, & 0\le x\le 1\end{cases}$$

For $f\circ g$ to exist, $g(x)$ must lie in the domain of $f=[-3,2]$.

From $g(x)=-x$, $-2\le x\le 3$,

$g(x)\in[-3,2]$.

So, every value in $[-3,2]$ is attained.

Hence the range of $f\circ g$ is simply the range of f on $[-3,2]$.

For $-3\le t\le 2$,

$f(t)=1+\frac{5t}{3}$

gives range

$\left[-4,\frac{13}{3}\right]$.

For $-1\le t<0$,

$f(t)=3-t$

gives range

$(3,4]$.

But $(3,4]\subset\left[-4,\frac{13}{3}\right]$ since $\frac{13}{3}=4.\overline{3}$.

Therefore,

$\text{Range}(f\circ g)=\left[-4,\frac{13}{3}\right]$

85. (B)

For the limit to exist at $x=2$:

Left-hand limit (LHL):

$\lim\limits_{x\to 2} f(x)=b-2a$

Right – handlimit (RHL): $\lim\limits_{x\to 2} f(x)=a+4b$

For limit to exist:

$b-2a=a+4b$

$b-4b=a+2a$

$-3b=3a$

$\frac{a}{b}=-1$

86. (B)

Let $t=x-5$ Then

$x^2-12x+35=(x-5)(x-7)=t(t-2)$

$\lim\limits_{t\to 0}\frac{\sqrt{2-2\cos(t(t-2))}}{t}$

Using $2-2\cos\theta=4\sin^2(\theta/2)$:

$=\lim\limits_{t\to 0}\frac{2|\sin(t(t-2)/2)|}{t}=\lim\limits_{t\to 0}\frac{|t(t-2)|}{t}$

For $t\to 0^-$, $|t|=-t$:

$\frac{-t(2-t)}{t}=-(2-t)\to -2$

So the limit is -2.

87 (A)

The expression is $(84-x)^{1/4}$; differentiation at $x=3$ gives $-1/(4\cdot 27)=-1/108$.

88. (A)

The limit is $3k(\log 7)^4/4=3(\log 7)^3$, so $k=4/\log 7$.

89. (A)

The highest exponential dominates, giving limit 1.

90. **(B)**

Each of the 100 numerator terms is asymptotic to $(2x)^{50}$, as is the denominator; limit 100.

91. **(D)**

$\left(1+\frac{7}{x+1}\right)^{x+5} \to e^7$.

92. **(D)**

$\sum_{k=1}^{n} k^3 = \left[n(n+1)/2\right]^2$; division by $1-n^4$ gives $-1/4$.

93. **(B)**

$\sin\left(\pi\cos^2 x\right) \sim \pi x^2$, so the limit is $\pi/3$.

94. **(B)**

Continuity requires $\left(a^2-b^2\right)/\left(c^2-b^2\right) = -1$, or $a^2+c^2 = 2b^2$; A.P.

95. **(C)**

The right limit is $2(\log 2)^2$ and the left value is $2\lambda\log 2$; $\lambda = \log 2$, hence $1000e^{\lambda} = 2000$.

96. **(A)**

The limit is $\frac{d}{dx}\left[f(x)^3\right]_{x=3} = 3f(3)^2 f'(3) = 1$.

97. **(A)**

The limit is $(\log 3)^2/6 = (1/6)(\log 3)^2$;
$a+b+c = 1/6+3+2 = 31/6$.

98. **(A)**

The printed absolute-value expression is continuous at $x=1$.

99. **(A)**

Continuity gives $a-b = \pi/4$ (and $a+2b=0$); required value $\pi/4$.

100. **(C)**

An implication is false only when its antecedent is true and consequent false, giving $(p,q,r) = (T,F,T)$.

101. **(D)**

$q = F$ and the biconditional force $r = F$;
$(p \wedge r) \to (p \vee r)$ is always true.

102. **(D)**

False implication gives $p = q = T$, $r = F$, $s = T$.

103. **(D)**

The given false implication gives $(p,q,r) = (T,F,F)$; only option (D) is false.

104. **(C)**

The first implication and its contrapositive are false; the second and its contrapositive are true: (F,T).

105. **(D)**

The statement is $p \to (q \wedge \neg r)$; contrapositive $(r \vee \neg q) \to \neg p$.

106. **(D)**

The equivalent contrapositive is: not equilateral $\Rightarrow$ vertices cannot be cube roots of unity.

107. **(C)**

A truth-table check gives $TTTT$.

108. **(A)**

$[\sim(\sim p \vee q) \vee (p \wedge r)] \wedge (\sim q \wedge r)$

1. $\sim(\sim p \vee q) = p \wedge \sim q$
2. So: $[(p \wedge \sim q) \vee (p \wedge r)] \wedge (\sim q \wedge r)$
3. Factor: $[p \wedge (\sim q \vee r)] \wedge (\sim q \wedge r)$
4. $(\sim q \vee r) \wedge \sim q = \sim q$
5. So: $p \wedge \sim q \wedge r = (p \wedge r) \wedge \sim q$

109. (D)
The antecedent is $\neg p \wedge \neg q$; choosing $r \equiv \neg q$ makes the implication a tautology.

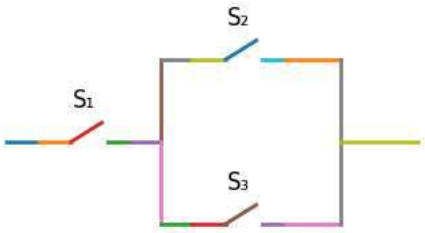
110. (C)
The original implication is a tautology; its negation is a contradiction.

111. (D)
Option (D) contains $p \vee \neg p$ and is therefore a tautology.

112. (A)
Direct substitution gives T; the dual also gives T

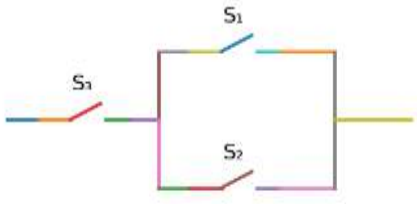
113. (C)
Negating yields $\neg(E \vee I) \vee I \vee \neg R$, equivalent to option (C).

114. (D)
Circuits (A) and (C) both represent $S_1(S_2 \vee S_3)$.



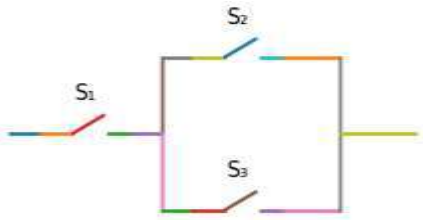
Equivalent: S₁ in series with (S₂ ‖ S₃)

115. (B)
The expression simplifies to $r(p \vee q)$, so S_3 is in series with $S_1 \| S_2$.



Simplified: S₃ in series with (S₁ ‖ S₂)

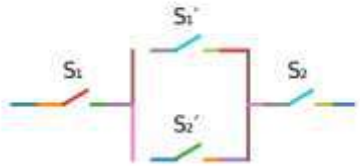
116. (A)
The lower series path is absorbed; the circuit is $p(q \vee r)$.



Circuit for p(q v r)

117. (B)
From the diagram:
Parallel branches: $(S_2' \cdot S_3') + S_1 + (S_2 \cdot S_3)$
This parallel combination is in series with another S_1 (near lamp)
So:
$L = S_1 \cdot [(S_2' \cdot S_3') + S_1 + (S_2 \cdot S_3)]$
Simplification
$L = S_1 \cdot (S_2' \cdot S_3') + S_1 \cdot S_1 + S_1 \cdot (S_2 \cdot S_3)$
$L = S_1 \cdot (S_2' \cdot S_3') + S_1 + S_1 \cdot (S_2 \cdot S_3)$
$L = S_1 \cdot [(S_2' \cdot S_3') + 1 + (S_2 \cdot S_3)] = S_1$
Equivalent Circuit just 1 switch (S_1)
Answer: 1

118. (B)
The series condition $p(\neg p \vee \neg q)q$ is identically false; the lamp is always off.



Series condition is impossible; lamp remains OFF

119. (C)
The truth values are $p = F, q = F, r = T$; only $(\neg p \vee q) \wedge r$ is true.

120. (D)
Only statements C and D are true.

121. (D)
$(p,q,r,s)=(F,T,F,T)$; only option (D) evaluates true.

122. (C)
The required adjoint entry is cofactor $C_{32}=4$.

123. (D)
$AA^T=|A|I$ gives $a=2/5, b=3$; $5a+b=5$.

124. (C)
$A(I+\text{adj}A)=A+|A|I$ and $|A|=8$, yielding option (C).

125. (D)
$A^{-1}=\sin^2(\theta/2)A^T=\dfrac{1-\cos\theta}{2}A^T$.

126. (B)
$A^TA^{-1}=\begin{bmatrix}\cos 2x & -\sin 2x\\ \sin 2x & \cos 2x\end{bmatrix}$.

127. (C)
Multiplying the stated matrices and equating to I gives $(a,b,c)=(-3,0,1)$.

128. (C)
$B^{-1}=(AB)^{-1}A=\dfrac{1}{2}\begin{bmatrix}-1 & 3\\ 1 & 2\end{bmatrix}$.

129. (C)
Using $A^2=4A-3I$, $(A+3I)^{-1}=\dfrac{7}{24}I-\dfrac{1}{24}A$.

130. (A)
$\text{adj}A=kA^{-1}$, hence $(\text{adj}A)^{-1}=A/k$.

131. (C)
$|A|=5$; $|\text{adj}B|/|5A|=|A|^4/(125|A|)=1$.

132. (D)
The system gives $(x,y,z)=(3,2,1)$; sum of squares 14.

133. (D)
$\sin 4\theta(2\cos 3\theta+1)=0$ gives the six values listed in option (D).

134. (A)
Let $u=16^{\sin^2 x}$; $u+16/u=10$ gives exponents $1/4, 3/4$, producing 8 solutions.

135. (C)
$5+3\sin\theta\neq 0$; $2\cos\theta+1=0$ gives $2\pi/3, 4\pi/3$.

136. (C)
The sides are $5k, 12k, 13k$ and $30k^2=270$; $k=3$.

137. (D)
$\tan(A/2)\tan(C/2)=(s-b)/s=1/3$.

138. (D)
Expanding and using $a\sin B=b\sin A$ gives $c\cos\theta$.

139. (A)
$\cos A=3/5$, so $\sin A=4/5$.

140. (B)
The product is $4\cos^2(A/2)=2+\sqrt{3}$ for $A=30^\circ$.

141. (B)
$a\cos B=b\cos A$ gives $a=b$; area is $\dfrac{c}{4}\sqrt{4a^2-c^2}$

142. (B)
The sine rule gives $B=\pi/2$ and $\cos A:\cos C=\sqrt{3}:1$.

143. (C)
Computing the angles from the half-angle tangents gives $a+c=2b$; a,b,c are in A.P.

144. (A)

The equation becomes $\left(a^2+b^2-c^2\right)^2=2a^2b^2$, hence $\cos C=-1/\sqrt{2}$ and $C=135^\circ$.

145. (C)

Squaring the half-angle relation gives a Pythagorean relation; the triangle is right-angled.

146. (C)

The half-angle identity reduces to $a^2=b^2+c^2$.

147. (A)

$\sum\cot(A/2)=\sum(s-a)/r=s/r=s^2/\Delta$.

148. (B)

Since $B=60^\circ$, the third side is $5+\sqrt{6}$; perimeter $24+\sqrt{6}$.

149. (A)

Testing the cosine law in $\cos(A-B)=31/32$ gives $c=6$.

150. (B)

$b\cos C+c\cos B=a$, so the given expression is $ab\sin C=2\Delta$; area 21.

151. (A)

The A.P. sides are $3,5,7$; area = $\frac{1}{2}(3)(5)\sin 120^\circ=15\sqrt{3}/4$.

152. (B)

The altitude $b\sin A=21/4$ exceeds $a=5$; no triangle is possible.

153. (C)

The relation gives $2ab(1+\cos C)=3ab$, so $C=\pi/3$.

154. (C)

With $\alpha=\sin^{-1}x$, $4\alpha+(\pi/2-\alpha)=\pi$; $x=1/2$.

155. (D)

$x=\pi/4$ satisfies both principal inverse-tangent sides.

156. (D)

$x=\cos\left(\sin^{-1}(1/3)+\sin^{-1}(3/5)\right)$

$=\left(8\sqrt{2}-3\right)/15$

157. (D)

The tangent of the sum is $41/12$; the value is $\tan^{-1}(41/12)$.

158. (C)

The cotangent condition gives $x=y=1/2$; the expression is $1/2$.

159. (B)

The equation has the two real solutions $x=\pm\sqrt{2}$

160. (D)

The inequality reduces to $\sin^{-1}x<\pi/4$ on $[-1,1]$; hence $-1\le x<1/\sqrt{2}$.

161. (B)

The tangent-sum equation gives $(x+1)\left(x^2-10x+6\right)=0$; with $x<0$, $x=-1$, and the value is 1767.

162. (B)

The equation becomes $(xy+1)/(y-x)=3$; positive integer pairs are $(1,2),(2,7)$, so two solutions.

163. (D)

$\sin x=\cos\left(2\tan^{-1}\sqrt{\cos\alpha}\right)=\tan^2(\alpha/2)$.

164. (C)

The real part is $3-4\sin^2\theta$; zero gives $\theta=n\pi\pm\pi/3$.

165. (C)
The roots are $2,4,5$ and the sum equals $\left(a^2+b^2+c^2\right)/(2abc)=9/16$.

166. (A)
The nth term is $\tan^{-1}\left(2^n\right)-\tan^{-1}\left(2^{n-1}\right)$; the sum is $\pi/4$.

167. (C)
$\cot^{-1}\left(2n^2\right)=\tan^{-1}\left(1/(2n-1)\right)$
$-\tan^{-1}\left(1/(2n+1)\right)$; sum $\pi/4$.

168. (C)
The argument is $\cos(23\pi/20)$; principal arccosine is $17\pi/20$.

169. (A)
$(y-3)^2-(x-5)^2=0$ gives option (A).

170. (C)
$(y-3)^2-(x+2)^2=0$ gives $x^2-y^2+4x+6y-5=0$.

171. (B)
The slope ratio satisfies $r+1/r=17/4$, so one slope is four times the other.

172. (A)
The parallel lines are $u=0, u=-k$ with normal length $\sqrt{5}$; $|k|/\sqrt{5}=\sqrt{5}$, so $k=5$.

173. (A)
Angle bisectors satisfy $11\left(x^2-y^2\right)+2xy=0$.

174. (C)
The degeneracy determinant is $-4k(k-5)$; $k=0$ or 5.

175. (A)
The pair is $(x+1)(y-1)=0$; concurrency at $(-1,1)$ gives $k=4$.

176. (A)
Let $u=4x-3y$; the equation is $(u+5)(u+7)=0$ distance $2/5$.

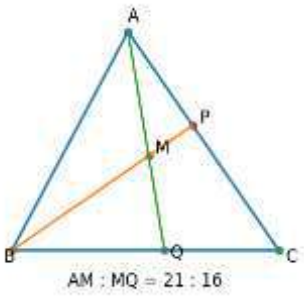
177. (C)
The lines form a right triangle with hypotenuse $2\sqrt{2}$; circumradius $\sqrt{2}$.

178. (B)
The determinant is $2(p-1)\left(p^2+7p+13\right)$; only one integral value.

179. (D)
The four points form a square of side 2; area 4.

180. (A)
An internal angle-bisector vector is proportional to $\vec{a}/|a|+\vec{b}/|b|$, so $x=y$.

181. (C)
Coordinate intersection gives $AM:MQ=21:16$.



182. (C)
$|a-b|^2+|b-c|^2+|c-a|^2$
$=3\sum|a|^2-|a+b+c|^2\le 249$.

183. (C)
Projection = $10\cos A=10(85/140)=85/14$.

184. (C)
The projection relation gives $|a| = 2|b+c|$;
$|b+c|^2 = 40$, so $|a| = 4\sqrt{10}$.

185. (B)
$3 = |a+b|^2 = 2 + 2\cos\theta$, so $\theta = \pi/3$.

186. (A)
Writing $d = a + (1+2y)b - yc$ gives
$|d|^2 = 13y^2 + 14y + 5$.

187. (B)
The hypothesis gives $a\cdot b + a\cdot c = 2$; expansion yields 32.

188. (C)
The dot product is $ct^2 + 6ct - 12 < 0$ for every real t iff $-4/3 < c < 0$.

189. (C)
$|b\times c| = 1/2$; the unit normal is $\pm 2(b\times c)$.

190. (D)
Point-to-plane distance is 11 units.

191. (B)
Area = $\frac{1}{2}|(2a-b)\times(4a-5b)| = 3/\sqrt{2}$.

192. (C)
$m = r - 5q = (-1,-8,2)$.

193. (A)
$|(1,-1,2)\times(2,3,\alpha)|^2 = 93$ gives $\alpha = -4, 2$.

194. (C)
The vector relation makes $a \parallel (2b+3c)$;
substitution gives the required ratio 3.

195. (C)
$b\times c = (2,-1,2)$, $d = 2(b\times c)$; $|a\times d|^2 = 720$.

196. (A)
With A as origin, $(B-A)\times(C-A) = (-5,2,1)$.

197. (B)
Altitude = $|a\times b + b\times c + c\times a| / |c-b|$.

198. (C)
A normal is $(2,1,1)$; unit normals are
$\pm(2i + j + k)/\sqrt{6}$.

199. (B)
The two triangle areas total $(3/2+1)|a\times b| = 5/2$ times the parallelogram area.

200. (D)
Using the vector triple-product identity and $a\cdot b = 0$ gives -5.

201. (D)
Multilinearity gives
$3[p,r,s] + 5[q,r,s] + 2[p,q,s]$.

202. (A)
$|[abc]| = 72$ and its scalar projection is 4, so $|b\times c| = 18$.

203. (A)
The intercept-plane condition at $(1,1,1)$ is
$1/x + 1/y + 1/z = 1$.

204. (D)
$b\cdot c = -48$ and $|c| = 8\sqrt{3}$, so $\cos\theta = -\sqrt{3}/2$ and $\theta = 5\pi/6$.

205. (C)
The vector equals $(a\times b)\times(c\times d)$; magnitude = $\left(\sqrt{3}/2\right)^2\sin(\pi/6)=3/8$.

206. (B)
$a\cdot b=0$, $|a|^2=26$; four cross products give $26^2 b=676b$.

207. (D)
The scalar triple product of the three edge vectors is identically 1, independent of both parameters.

208. (B)
The intersection condition reduces to $(c-a)\cdot b=0$, or $a\cdot b=b\cdot c$.

209. (C)
The determinant condition is $(a\times b)\cdot(c\times d)=0$; hence $12-\lambda=0$.

210. (B)
The printed condition is inconsistent; under the intended $a\cdot b=1$, $3l^2=2$.

211. (D)
$p-c$ is parallel to b and $p\cdot a=0$, giving $p=(1,-1,2)$.

212. (C)
The triple-product identity gives $a\cdot c=0$ and $b\cdot c=-1/2$; angles $90^\circ, 120^\circ$.

213. (A)
Coplanarity makes $a\perp(b\times c)$ and $|b\times c|=8$; required magnitude 8.

214. (A)
The coefficient determinant is $-\lambda$; nonzero (x,y,z) requires $\lambda=0$.

215. (B)
The determinant factors as $-(a-b)(a-c)(b-c)(abc+1)$; noncoplanarity gives $abc=-1$.

216. (B)
The direction-cosine dot product is $-1/2$, so the directed angle is 120°.

217. (A)
The two direction triples are proportional to $(-1,-2,1)$ and $(-2,1,1)$; $\sin\theta=\sqrt{35}/6$.

218. (B)
The determinant is x^3-x+1; local extreme values are $1\pm 2/\left(3\sqrt{3}\right)$.

219. (B)
$\cos\angle(AB,AD)=8/9$; the rotation to perpendicular has $\cos\alpha=\sqrt{17}/9$.

220. (C)
The equal projections cancel the cross terms and $v\perp w$; $|u-v+w|^2=14$.

221. (C)
$\sin\theta=2\sqrt{\lambda}/\left[3\sqrt{5+\lambda}\right]=1/3$ gives $\lambda=5/3$.

222. (D)
A plane parallel to the XY-plane has constant z; here $z=6$.

223. (B)
Solving the two line parameters gives $k=9/2$.

224. (A)
The direction-vector cosine is $1/\sqrt{6}$, hence $\cos^{-1}\left(1/\sqrt{6}\right)$.

225. (C)
The lines meet at $(-1,-1,-1)$; joining to $(2,1,-2)$ gives direction $(3,2,-1)$.

226. (B)
Directions $(1,1,0)$ and $(1,0,0)$ make 45°.

227. (A)
The internal-bisector direction is $AB/7 + AC/14 \propto (-2,2,3)$.

228. (D)
The foot is $(8/3, 4/3, 1/3)$; reflection has coordinate sum $23/3$.

229. (A)
The parameters $\lambda = 1, \mu = 0$ give an intersection, and the direction dot product is nonzero.

230. (C)
Distance = $|(P-A)\times d| / |d| = 2\sqrt{19}$.

231. (B)
The shortest distance is $(2\alpha + 15)/3 = 9$; $\alpha = 6$.

232. (A)
The lines intersect at $(3,6,2)$; its distance from $(2,4,0)$ is 3.

233. (D)
The constant-distance line must be parallel to $(3,-2,1)$; only $(-5,6,2)$ is not on it.

234. (C)
The simultaneous parameter equations are inconsistent; the lines do not intersect.

235. (A)
The feet are $(a,a,1)$ and $(0,0,-1)$; perpendicularity gives $a^2 = 1$.

236. (C)
The plane is $2x - 3y - 5z + 9 = 0$, which contains $(-2,0,1)$.

237. (A)
$5(x+y+z-1) - (3x+4y+5z-2) = 0$ gives $2x + y - 3 = 0$.

238. (A)
The intersection direction is $(-3,5,4)$; divide by $5\sqrt{2}$.

239. (B)
The required plane is $11x - 3y - 2z - 54 = 0$; coefficient sum -48.

240. (D)
The line-plane parameter is -1, giving $(1,-2,7)$.

241. (A)
The plane is $x - y + z = 0$; the paper records intercept sum 0.

242. (C)
$(x-2)(x-6) = 0$ represents two planes parallel to the YZ-plane.

243. (C)
The normal is $(5,-3,-1)$; plane $5x - 3y - z = 19$

244. (A)
The intercept points give line through $(1,-2,0)$ in direction $(-1,1/2,-1/2)$.

245. (C)
A normal is $(-4,-9,2)$; divide by $\sqrt{101}$.

246. (B)
The two values are 3 and $227/55$; their difference is $62/55$.

247. (A)

The plane is $x+y+cz-1=0$; the 45° condition gives $c=\pm\sqrt{2}$.

248. (A)

The direction vectors have zero dot product; angle $\pi/2$.

249. (A)

Point on line: $A(3,-2,1)$

Plane: $2x+y+z=4$

Distance:

$$d=\frac{|2(3)+(-2)+1-4|}{\sqrt{2^2+1^2+1^2}}=\frac{|6-2+1-4|}{\sqrt{6}}=\frac{1}{\sqrt{6}}$$

250. (A)

Imposing that the joining vector lies parallel to the plane gives parameter $t=2$ and distance 7.

251. (B)

Coplanarity gives $k=\pm 2$ and the planes $y\pm z+1=0$.

252. (A)

The normal is $(5,-2,-3)$; distance from origin $7/\sqrt{38}$.

253. (A)

The line meets the plane at $(1,-2,7)$; distance from P is 7.

254. (D)

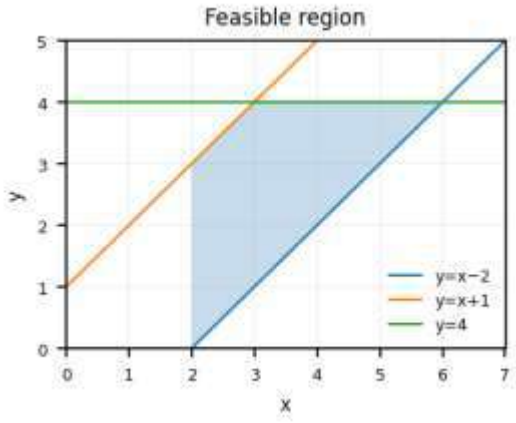
The coplanarity scalar triple product is $3(b-1)/(ab)$; hence $b=1$, $a\neq 0$ arbitrary.

255. (A)

The plane normal is $(7,-7,-7)$; distance $2/\sqrt{3}$.

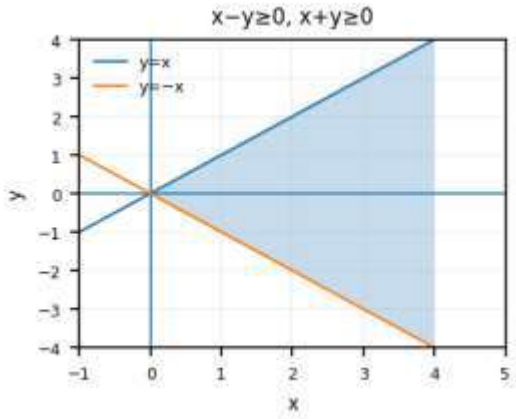
256. (B)

The feasible polygon has vertices $(2,0),(2,3),(3,4),(6,4)$, matching option (B).



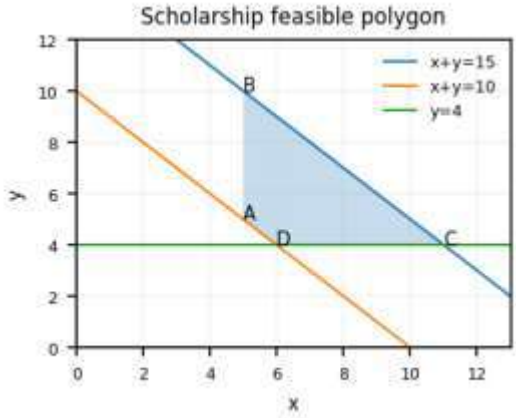
257. (A)

The shaded wedge satisfies $y\le x$ and $y\ge -x$.



258. (A)

At A,B,C,D, the objective values are $4250,5750,7250,4500$; maximum 7250.



259. (B)

The strip $|x-y|\le 1$ in the first quadrant is unbounded.

260. (B)

At the feasible vertices the maximum value is 22 at $(7/3, 8/3)$.

261. (A)

Constraints: $2x + 3y \geq 12 \rightarrow$ above the line

$-x + y \leq 3 \rightarrow$ below the line $y = x + 3$

* $x \leq 4 \rightarrow$ left of $x = 4$
* $y \geq 3 \rightarrow$ above $y = 3$

The region satisfying all four is S_1.

262. (C)

The extreme objective values are 65 and -160; difference $225 = 5k + 200$, so $k = 5$.

263. (C)

* $x - y \geq 0 \Rightarrow y \leq x \rightarrow$ below line $y = x$
* $x - 5y \leq -5 \Rightarrow y \geq \dfrac{x+5}{5} \rightarrow$ above line $x - 5y = -5$
* $x \geq 0, y \geq 0 \rightarrow$ first quadrant

The feasible region is between the two lines in the first quadrant, starting from their intersection at $x = 5/4$.

264. (B)

$z = 10(3x + 2y)$ and the active boundary $3x + 2y = 6$ contains a segment; infinitely many minimizers.

265. (C)

The minimum $x + y = 2$ occurs along a feasible line segment; infinitely many bounded minimizers.

266. (D)

Chain rule gives $3^3 + 2(1)(3) = 33$.

267. (C)

$f(x) = 1 - \dfrac{1}{2}\sin 2x$, so

$f'(\pi/6) = -\cos(\pi/3) = -1/2$.

268. (B)

Given:

$y = \tan^{-1}(\sqrt{1+x^2} - 1)$

Step 1: Use formula $\dfrac{d}{dx}\tan^{-1} u = \dfrac{1}{1+u^2} \cdot \dfrac{du}{dx}$

Here:

$u = \sqrt{1+x^2} - 1$

Step 2: Find $\dfrac{du}{dx}$

$\dfrac{du}{dx} = \dfrac{x}{\sqrt{1+x^2}}$

Step 3: Compute $1 + u^2$

$1 + u^2 = 1 + (\sqrt{1+x^2} - 1)^2$

$= 1 + (1 + x^2 - 2\sqrt{1+x^2} + 1)$

$= x^2 - 2\sqrt{1+x^2} + 3$

Step 4: Put together

$\dfrac{dy}{dx} = \dfrac{1}{x^2 - 2\sqrt{1+x^2} + 3} \cdot \dfrac{x}{\sqrt{1+x^2}}$

$\dfrac{dy}{dx} = \dfrac{x}{\sqrt{1+x^2}(x^2 - 2\sqrt{1+x^2} + 3)}$

269. (B)

$y'(1/2) = 6 + 2\sqrt{3} + k = 2\sqrt{3}$, so $k = -6$.

270. (C)

The three derivatives at 0 are $-1/2, -3/10, -1/10$; sum $-9/10$.

271. (C)

The inner quotient has derivative 8 at 0 and value 0; hence $dy/dx = 8$.

272. (A)

Direct differentiation simplifies to $5/(1 + 25x^2)$.

273. (A)

Logarithmic differentiation gives $\sum \tan\theta_i = -f'(\theta)/f(\theta)$.

274. (D)

$\frac{d}{dx}x^{x^x} = x^{x^x}\left[x^{x-1} + x^x \log x(1+\log x)\right]$.

275. (A)

Implicit differentiation at $(1,2)$ gives

$dy/dx = -2(1+\log 2)$.

276. (C)

Putting both radicals equal to t gives $y = t^2 + t$,

$x = t^2 - t$; $dy/dx = (y-x+1)/(y-x-1)$.

277. (B)

At $x = e$, $y = \sqrt{4+e^2}$ and

$dy/dx = (2e-1)/\left(2\sqrt{4+e^2}\right)$.

278. (B)

With $t = \tan^{-1}x$, $du/dv = \left(1+t^2\right)/(1+t)^2$.

279. (C)

$dy/dx = -\tan\theta$, hence $\sqrt{1+(dy/dx)^2} = \sec\theta$.

280. (B)

The derivatives at 0 are $1/2$ and 2; the derivative with respect to the second is $1/4$.

281. (C)

The parameters reduce to $x = 1/2\tan^{-1}t$,

$y = 2\tan^{-1}t$; $dy/dx = 4$.

282. (C)

Twice differentiating gives $x^2y'' = n(n+1)y$.

283. (B)

With $u = y/x$, the relation gives

$x^3y'' = (xy' - y)^2$.

284. (D)

$d^n(\log x)/dx^n = (-1)^{n-1}(n-1)!/x^n$.

285. (B)

Coefficient comparison yields

$f(x) = \frac{1}{7}\left(21x^3 - 90x^2 + 72x + 126\right)$.

286. (A)

The parametric curve satisfies

$\left(1-x^2\right)y'' - xy' + p^2y = 0$.

287. (B)

The product is e^{in^2x}; therefore $f'' = -n^4 f$.

288. (B)

At $(0,b)$ the slope is $-b/a$, giving

$x/a + y/b = 1$.

289. (B)

The normal slope is positive; in the standard normal equation $a > 0, b < 0$.

290. (B)

Multiplying $3x + 5y - 30 = 0$ and $5x - 3y - 16 = 0$ gives option (B).

291. (C)

The tangent intercepts are $2x_1, 2y_1$; area

$2x_1y_1 = 2a^2$.

292. (B)

The tangent intercept on the x-axis is $4/\sqrt{3}$ and the height is 1; area $2/\sqrt{3}$.

293. (D)

Orthogonality of slopes gives $by^2 = 27x$; using

$y^2 = 6x$ gives $b = 9/2$.

294. (B)

$2a=10$, $4a+b=30$, and $a+b+c=20$ give $a+c=b$.

295. (D)

$dy/dx=2x^2=18$ gives $x=\pm 3$ and the points in option (D).

296. (A)

$dP/dt=2(-5+3)=-4$ and

$dA/dt=5(3)+2(-5)=5$.

297. (B)

Step 1: Area of isosceles triangle Area A =

$\frac{1}{2}x^2\sin\theta$

Step 2: Differentiate with respect to time t

$$\frac{dA}{dt}=\frac{1}{2}\left(2x\sin\theta\frac{dx}{dt}+x^2\cos\theta\frac{d\theta}{dt}\right)$$

$$\frac{dA}{dt}=x\sin\theta\frac{dx}{dt}+\frac{1}{2}x^2\cos\theta\frac{d\theta}{dt}$$

Step 3: Substitute given values

$$x=12\theta=\frac{\pi}{4}\frac{dx}{dt}=\frac{1}{12}\frac{d\theta}{dt}=\frac{\pi}{180}$$

$$\sin\frac{\pi}{4}=\cos\frac{\pi}{4}=\frac{1}{\sqrt{2}}$$

$$\frac{dA}{dt}=12\cdot\frac{1}{\sqrt{2}}\cdot\frac{1}{12}+\frac{1}{2}\cdot(12)^2\cdot\frac{1}{\sqrt{2}}\cdot\frac{\pi}{180}$$

$$=\frac{1}{\sqrt{2}}+\frac{1}{2}\cdot 144\cdot\frac{1}{\sqrt{2}}\cdot\frac{\pi}{180}$$

$$=\frac{1}{\sqrt{2}}+\frac{72\pi}{180\sqrt{2}}$$

$$=\frac{1}{\sqrt{2}}+\frac{2\pi}{5\sqrt{2}}$$

$$=\frac{1}{\sqrt{2}}\left(1+\frac{2\pi}{5}\right)$$

$$=\frac{1}{\sqrt{2}}\left(\frac{2\pi}{5}+1\right)$$

$$=\sqrt{2}\left(\frac{\pi}{5}+\frac{1}{2}\right)$$

298. (D)

$l=25$, $\dot{l}=-3$;

$d(\pi rl)/dt=\pi(25\cdot 3+7\cdot -3)=54\pi$.

299. (C)

For the intended radius 10 cm,

$dA/dt=2\pi r\dot{r}=132\,\text{cm}^2/\text{s}$.

300. (A)

$\cos(60^\circ-0.5^\circ)\approx 0.5+(0.00875)(0.8660)$.

$=0.5076$

301. (C)

$f'(x)\propto -(x-3)(x+2)$; the Rolle point in $(-3,0)$ is $c=-2$.

302. (C)

The secant slope is $3/4$; solving

$3c^2-6c+2=3/4$ gives $c=1-\sqrt{21}/6$.

303. (C)

The MVT slope is 4; $3x^2=4$ gives $x=\pm 2/\sqrt{3}$

304. (D)

$f'(x)=4x(x-1)(x-2)>0$ on $(0,1)\cup(2,\infty)$.

305. (D)

$f'(x)=(k-2)/(\sin x+\cos x)^2$; strict increase requires $k>2$.

306. (B)

$f'(x)=-\sin 4x$; it is positive on $\pi/4<x<\pi/2$.

307. (C)

$f(x)=(x+2)e^{-z}$

Differentiate using product rule:

$f'(x)=1.e^{-x}+(x+2)(-e^{-x})=e^{-x}(1-x-2)$

$=-(x+1)e^{-x}$

Since $e^{-x}>0$ for all x:

- $f'(x)>0$ when $-(x+1)>0 \Rightarrow x-1\rightarrow$ increasing
- $f'(x)<0$ when $x>-1\rightarrow$ decreasing

So: increasing in $(-\infty,-1)$, decreasing in $(-1,\infty)$

Answer: (C)

308. (B)

$f'(x)=5x^2(x-1)(x-3)$ changes $+\rightarrow-$ at $x=1$

309. (C)

$t/\left(100+t^2\right)$ is maximal at $t=10$; population 1050.

310. (C)

Profit is $5.8x-x^2/40-193$, maximized at $x=116$ with value 143.4.

311. (A)

The two derivative equations give $\alpha=0$, $\beta=1/3$

312. (B)

The sum of root squares is $(a-1)^2+1$, least at $a=1$.

313. (A)

For an open square tank the optimum satisfies $h=s/2$; option (A).

314. (B)

The extrema occur at $x=\pm1$ and are $3,1/3$; difference $8/3$.

315. (A)

At $(2,2)$ the two tangent slopes are $1,-1$; angle $\pi/2$.

316. (C)

The normal is $x+y-4=0$; distance from origin $2\sqrt{2}$.

317. (C)

The integrand reduces to x^2; integral $x^3/3+C$.

318. (D)

The product is $\frac{1}{8}\sin(x/2)$; integral $-\frac{1}{4}\cos(x/2)+C$.

319. (B)

$\sec^2x+\csc^2x=1/\left(\sin^2x\cos^2x\right)$; integral $\tan x-\cot x+C$.

320. (A)

$\int\tan^4x\,dx=\frac{1}{3}\tan^3x-\tan x+x+C$;

$a-b+c=7/3$.

321. (B)

With $t=\tan(x/2)$, the integral is $\frac{2}{\sqrt{3}}\tan^{-1}\left(t/\sqrt{3}\right)+C$.

322. (D)

Let $u=\cos x$; the integral is $-\log\left(u+\sqrt{u^2+5}\right)+C$.

323. (B)

The integrand is $-3d\left(\tan^{-1/3}x\right)$; result $-3\tan^{-1/3}x+C$.

324. (C)

Integration gives $\frac{1}{3}\tan^{-1}(x/3)+\frac{3}{x^2+9}+C$.

325. (B)

Evaluate

$I=\int\sqrt{x^2-6x-16}dx$

Step 1: Complete square

$x^2-6x-16=(x-3)^2-25$

S,

$I=\int\sqrt{(x-3)^2-5^2}\,dx$

Step 2 : Use the standard formula

$\int\sqrt{u^2-a^2}\,du=\frac{u}{2}\sqrt{u^2-a^2}-\frac{a^2}{2}\ln\left|u+\sqrt{u^2-a^2}+C\right|$

Here,

- $u=x-3$
- $a=5$

Hence,

$I=\frac{x-3}{2}\sqrt{x^2-6x-16}-\frac{25}{2}\ln\left|x-3+\sqrt{x^2-6x-16}\right|+C$

This matches option (B):

$\left(\frac{x-3}{2}\right)\sqrt{x^2-6x-16}-\frac{25}{2}\log\left(x-3+\sqrt{x^2-6x-16}\right)+C$

Correct Answer : (B)

326. (C)

With $r=(x-b)/(x+a)$, the integral is

$\frac{7}{2(a+b)}r^{2/7}+C$.

327. (A)

Partial fractions give

$x^2/2+\log|x|-\log\left(x^2+1\right)+C$; $A-B=0$.

328. (B)

With $t=\tan x$, the integral becomes

$\int t^{-1/2}dt=2\sqrt{\tan x}+C$.

329. (A)

Let $u=x^{1/6}$; integration gives coefficients

$(2,-3,6,-6)$.

330. (C)

$\frac{d}{dx}\left[e^x/(x+1)^2\right]=e^x(x-1)/(x+1)^3$.

331. (D)

The integrand is $\frac{1}{2}\frac{d}{dx}\left[e^{2x}\cot 2x\right]$.

332. (A)

Two integrations by parts give

$x^2\sin x+2x\cos x-2\sin x+C$.

333. (C)

$1/\left[x\left(x^3+1\right)\right]=1/x-x^2/\left(x^3+1\right)$; integral

$\frac{1}{3}\log\frac{x^3}{x^3+1}+C$.

334. (B)

Differentiating the proposed form and matching coefficients gives $a=-5/4$.

335. (B)

Matching logarithmic derivatives gives

$a=11/12, b=5/6$; $a+b=21/12$.

336. (D)

$1/\left[\cos x(1+\cos x)\right]=\sec x-1/(1+\cos x)$; integral

$\log(\sec x+\tan x)-\tan(x/2)+C$.

337. (A)

With $t=e^x$, partial fractions yield

$x+\log\left(e^x+1\right)-2\log\left(2e^x+1\right)+C$.

338. (B)

The integral is $\log 2+\log 3=\log 6$.

339. (C)

The integrand is odd on $[-3,3]$; value 0.

340. (B)

Roots: $x=-1,2$.

$\int_{-2}^{2}|x^2-x-2|\,dx=\int_{-2}^{-1}(x^2-x-2)dx-\int_{-1}^{2}(x^2-x-2)dx$

$$=\left[\frac{x^3}{3}-\frac{x^2}{2}-2x\right]_{-2}^{-1}-\left[\frac{x^3}{3}-\frac{x^2}{2}-2x\right]_{-1}^{2}$$

$$=\frac{11}{6}+\frac{27}{6}=\frac{38}{6}=\frac{19}{3}$$

341. (B)

With $u=\sqrt{x+1}$, the value is

$2\left(\tan^{-1}2-\pi/4\right)=2\tan^{-1}(1/3)$.

342. (B)

Pairing x with $\pi/2-x$ makes the two integrands sum to 1; integral $\pi/4$.

343. (B)

$\left[x\tan^{-1}x-1/2\log\left(1+x^2\right)\right]_0^1=\pi/4-\log\sqrt{2}$.

344. (C)

Given, $I=\int_0^{\pi/4}\frac{\cos^2 x\sin^2 x}{\cos^3 x+\sin^3 x}dx$

Trick: Use the property

Let $f(x)=\frac{\cos^2 x\sin^2 x}{\cos^3 x+\sin^3 x}$.

Using the substitution $x=\frac{\pi}{2}-t$,

$$f\left(\frac{\pi}{2}-x\right)=\frac{\sin^2 x\cos^2 x}{\sin^3 x+\cos^3 x}=f(x).$$

Hence,

$$2I=\int_0^{\pi/4}\left[f(x)+f\left(\frac{\pi}{2}-x\right)\right]dx=2\int_0^{\pi/4}f(x)dx.$$

Now simplify the denominator:

$\cos^3 x+\sin^3 x=(\sin x+\cos x)(1-\sin x\cos x)$.

Also,

$\sin^2 x\cos^2 x=(\sin x\cos x)^2$.

Let

$t=\sin x-\cos x,\quad dt=(\sin x+\cos x)dx.$

Since

$(\sin x-\cos x)^2=1-2\sin x\cos x,$

we get

$\sin x\cos x=\frac{1-t^2}{2}$.

Substituting,

$$I=\frac{1}{2}\int_{-1}^{0}\frac{1-t^2}{3+t^2}dt=\frac{1}{2}\left[t-2\tan^{-1}\left(\frac{t}{\sqrt{3}}\right)\right]_{-1}^{0}=\frac{1}{6}$$

345. (C)

The value is $\frac{3}{2}\log 2+\pi/8=\log\left(2\sqrt{2}\right)+\pi/8$.

346. (D)

Partial fractions give $\log(1125/1024)$.

347. (C)

Symmetry/evaluation gives $\log 2$.

348. (D)

$$I=\int_0^1\log\left(\frac{1}{x}-1\right)dx=\int_0^1\log(1-x)dx-\int_0^1\log x\,dx$$

Use standard results:

$\int_0^1\log x\,dx=-1,\quad\int_0^1\log(1-x)dx=-1$

So,

$I=(-1)-(-1)=0$

349. (B)

Let $u=\sqrt{x}$; the value is

$2-4\log(3/2)=2\log(4e/9)$.

350. (B)

$\csc^4 x$ has a non-integrable singularity at 0; the printed definite integral diverges.

351. (A)

The substitution $x\mapsto 1/x$ changes the integral to its negative; value 0.

352. (A)

Pairing x with $\pi/2-x$ gives $2I=400(\pi/2)$;

$I=100\pi$.

353. (D)

$\left[(1+x)\log(1+x)-(1+x)\right]_0^1 = 2\log 2 - 1.$

354. (C)

Two congruent triangles each have area $1/2$; total 1.

355. (A)

$\int_0^4 \left(2\sqrt{x}-x\right)dx = 8/3.$

356. (D)

Quarter-ellipse area minus triangle area is $15\pi/4 - 15/2 = 15(\pi-2)/4.$

357. (C)

$\int_{-3\sqrt{3}}^{3\sqrt{3}} \left(1-y^2/27\right)dy = 4\sqrt{3}.$

358. (C)

$\int_{-2}^{1} \left(2-y-y^2\right)dy = 9/2.$

359. (C)

$\int_0^3 \left(x^2-x+3\right)dx = 27/2.$

360. (D)

$\alpha^3/12$ is half of $16/3$; $\alpha^3 = 32$, so $\alpha = \sqrt[3]{32}$.

361. (D)

Because a highest derivative occurs inside a logarithm, the equation is not polynomial in derivatives; degree is undefined.

362. (A)

After removing the square root, the equation is first order and degree two.

363. (A)

$y+1 = m(x-1)$ and $m = y'$ give $y = (x-1)y' - 1.$

364. (C)

A linear combination of $\sin(6t+5)$ and $\cos(6t+5)$ satisfies $y'' + 36y = 0.$

365. (C)

For $y = c_1 e^{c_2 x}$, $yy'' = (y')^2.$

366. (B)

Eliminating the centre from $x^2 + y^2 - 2ax = 0$ gives $x^2 - y^2 + 2xyy' = 0.$

367. (A)

From $x^2 = 4ay$, eliminate a to get $xy' - 2y = 0.$

368. (A)

Eliminating the two constants requires a second derivative and the resulting equation is degree one.

369. (B)

Separation gives $\log\tan(y/4) = C - 2\sin(x/2).$

370. (A)

Let $v = x + y$; $dv/dx = 1 + v^2$, so $\tan^{-1}(x+y) = x + C.$

371. (A)

Let $t = x + 9y$

Then

$$\frac{dt}{dx} = 1 + 9\frac{dy}{dx} = 1 + 9t^2$$

So,

$$\frac{dt}{1+9t^2} = dx$$

Integrate:

$$\frac{1}{3}\tan^{-1}(3t) = x + C$$

$$3x + C' = \tan^{-1}(3t)$$

$$3t = \tan(3x + C)$$

Given $x=0, y=\frac{1}{27} \Rightarrow t=0+9\cdot\frac{1}{27}=\frac{1}{3}$.

So,

$3\cdot\frac{1}{3}=1=\tan(C)\Rightarrow C=\frac{\pi}{4}$

Thus,

$3(x+9y)=\tan\left(3x+\frac{\pi}{4}\right)$

$3x+27y=\tan\left(3x+\frac{\pi}{4}\right)=\tan\left[3\left(x+\frac{\pi}{12}\right)\right]$

372. (D)

With $u=\tan y$, $\left(1-e^x\right)du+3e^x u\,dx=0$; the initial condition gives option (D).

373. (A)

Integration gives $(y+1)(2+\sin x)=4$; hence $y(\pi/2)=1/3$.

374. (B)

Integration gives $5e^{2x}-2e^{5y}=3$.

375. (D)

$\cos x+\sin y$ is not homogeneous under common scaling.

376. (D)

With $v=y/x$, integration gives $\log(y/x)=cx$.

377. (C)

$IF=\exp\int(\log x)/x\,dx=e^{(\log x)^2/2}=\left(\sqrt{x}\right)^{\log x}$.

378. (A)

Given: $(1+y^2)+(x-e^{\tan^{-1}y})\frac{dy}{dx}=0$

Rewrite as:

$\frac{dx}{dy}=\frac{e^{\tan^{-1}y}-x}{1+y^2}$

$\frac{dx}{dy}+\frac{x}{1+y^2}=\frac{e^{\tan^{-1}y}}{1+y^2}$

This is linear in x. Integrating factor:

$I.F.=e^{\int\frac{1}{1+y^2}dy}=e^{\tan^{-1}y}$

Multiply:

$e^{\tan^{-1}y}\frac{dx}{dy}+\frac{e^{\tan^{-1}y}}{1+y^2}x=\frac{e^{2\tan^{-1}y}}{1+y^2}$

Left side is derivative of $xe^{\tan^{-1}y}$:

$\frac{d}{dy}\left(xe^{\tan^{-1}y}\right)=\frac{e^{2\tan^{-1}y}}{1+y^2}$

Integrate:

$xe^{\tan^{-1}y}=\int\frac{e^{2\tan^{-1}y}}{1+y^2}dy$

Let $t=\tan^{-1}y$, then $dy=(1+y^2)dt$:

$xe^t=\int e^{2t}dt=\frac{e^{2t}}{2}+C$

So,

$xe^{\tan^{-1}y}=\frac{e^{2\tan^{-1}y}}{2}+C$

Multiply by 2:

$2xe^{\tan^{-1}y}=e^{2\tan^{-1}y}+k$

379. (C)

The linear equation has IF x^2; integration gives option (C).

380. (B)

$y'-y=x-5$ and $y(0)=2$ give $y=4-x-2e^x$.

381. (B)

$e^{6k}=2$; amount after 33 years is $400\,2^{11/2}=18101.76$.

382. (B)

$\sqrt{A}=1000-300t$; bankruptcy occurs at $t=10/3$ years.

383. (C)

$e^{2k}=11/10$ and $e^{kt}=2$; $t=2\log 2/\log(11/10)$

384. (C)

Remaining fraction is $(1/2)^t=0.1$; $t=\log_2 10$.

385. (A)

The excess temperature halves every 10 min; $80\to 5$ needs four half-lives, 40 min.

386. (B)

$P(t)=30000(4/3)^{t/40}$; $(a,b)=(30000,4/3)$.

387. (C)

1994 populations are 31250 and 39200; difference 7950.

388. (B)

$xy'=2y$ integrates to $y=Cx^2$, a family of parabolas with vertex at the origin and axis along Y.

389. (D)

Probabilities for $X=1,2,3,4$ are $1/2,1/4,1/8,1/8$.

390. (A)

$\sum_{x\ge 0}(x+1)(1/5)^x=25/16$, so $k=P(X=0)=16/25$.

391. (A)

Normalization gives $k=1/10$; $P(X\ge 6)=9k^2+k=19/100$.

392. (C)

$k=1/10$, $a=5k$, $b=6k$; therefore $a<b$.

393. (B)

Normalization gives $k=0.1$; $P(X\le 2)=0.2+0.1+0.2=0.5$.

394. (B)

$P(X=-3)=0.1$, $P(X<0)=0.3$; ratio $1/3$.

395. (B)

The conditional probability is $(2k+4k)/(k+2k+4k)=6/7$.

396. (B)

$P(X\le 0)=0.5$ and $P(X>0)=0.5$; ratio 1.

397. (B)

Expected payoff = $150(1/4)-50(3/4)=0$.

398. (A)

$E(X)=\sum_{m=2}^{6} m(m-1)/15=14/3$.

399. (C)

Accounting for stopping and all losses gives expected gain 0.

400. (B)

Nonnegativity gives minimum $p=0$; then $5E(X)=6$.

401. (D)

$A+B=4/5$, $30/5+10A-10B=4$; $AB=3/20$.

402. (C)

$E(X)=2/13$, $E(X^2)=36/221$; $2E(X)+3E(X^2)=176/221$.

403. (D)

Geometric mean $n/(n-1)=n/9$ gives $n=10$.

404. (C)

Normalization gives $K=1/6$; the nonduplicate solution is $\alpha=6$, yielding $\sigma=7/2$.

405. (B)

This is $B(4,1/2)$; mean 2, variance 1.

406. (C)

$k = 2/\pi$ and $F(x) = \frac{2}{\pi}\tan^{-1}x$ for $x > 0$.

407. (B)

$a + b = 1$ and normalization give $(a,b) = (-9,10)$

408. (C)

$P(|X| < 2) = \int_{-2}^{2} x^2/18\,dx = 8/27$.

409. (B)

For $B(99,1/2)$ the modes are 49 and 50; the listed one is 49.

410. (D)

Using the binomial model intended by the options:

$\binom{6}{3}(0.1)^3(0.9)^3 = 0.01458$.

411. (C)

For a fair coin the probability of an even number of heads is $1/2$.

412. (A)

$\binom{4}{2}(1/6)^2(5/6)^2 = 25/216$.

413. (C)

$\sum_{r=0}^{2}\binom{4}{r}(2/3)^r(1/3)^{4-r} = 33/3^4$.

414. (B)

$7q^{35} = 35pq^{34}$ gives $q/p = 5$; the probability ratio is $(q/p)^5 = 3125$.

415. (C)

$3q^{33} = 33pq^{32}$ gives $p = 1/12$; variance $33pq = 121/48$.

416. (C)

The intended event has single-trial probability $1/25$;

$P(Y \geq 3) = 4(1/25)^3(24/25) + (1/25)^4 = 97/25^4$

MISSION MHT-CET 2027 | PYQ BOOKLET

This Booklet Brings Together 5 Years Of Previous MHT-CET Questions From Physics, Chemistry, Mathematics, Botany And Zoology, Giving Students The Opportunity To Practice Questions Based On The Actual Exam Pattern. It Will Help Strengthen Concepts, Improve Problem-Solving Skills, Understand The Types Of Questions Asked In MHT-CET And Boost Overall Exam Preparation And Confidence.



PW MAHARASHTRA YOUTUBE

PW MHTCET YOUTUBE