Subject : Mathematics
Class : XI

Kannada

Straight Line

- **Distance Formula:** $d=\sqrt{\left(x_1-x_2\right)^2+\left(y_1-y_2\right)^2}$.
- **Section Formula:** $x=\frac{mx_2 \pm nx_1}{m \pm n}; y=\frac{my_2 \pm ny_1}{m \pm n}$
- **Centroid, Incentre & Excentre:**

  Centroid $G\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$,

  Incentre $I\left(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}\right)$

  Excentre $I_1\left(\frac{-ax_1+bx_2+cx_3}{-a+b+c}, \frac{-ay_1+by_2+cy_3}{-a+b+c}\right)$

Remarks:

(i) If the triangle is equilateral, then centroid, incentre, orthocenter, circumcenter, coincides.

(ii) Orthocentre, centroid and circumcentre are always collinear and centroid divides the line joining. Orthocentre and circumcentre in the ratio $2:1$.

(ii) In a isosceles triangle centroid, orthocentre, incentre, circumcentre lies on the same line.

Area of Triangle

Let $A\left(x_1, y_1\right), B\left(x_2, y_2\right)$ and $C\left(x_3, y_3\right)$ are vertices of a triangle, then

Area of $\Delta ABC=\left|\frac{1}{2}\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix}\right|$

Equation of Straight Line

(a) Equation of a line parallel to x-axis at a distance a is $y=a$ or $y=-a$.

(b) Equation of x-axis is $y=0$.

(c) Equation of line parallel to y-axis at a distance b is $x=b$ or $x=-b$.

(d) Equation of y-axis is $x=0$.

If $A(x_1,y_1)$ and $B(x_2,y_2)\&x_1\neq x_2$ then slope of line $AB=\dfrac{y_2-y_1}{x_2-x_1}$

Standard Forms of Equations of a Straight Line

(a) **Slope Intercept form :** Let m be the slope of a line and c its intercept on y-axis, then the equation of this straight line is written as : $y=mx+c$.

(b) **Point Slope form :** If m be the slope of a line and it passes through a point (x_1,y_1), then its equation is written as : $y-y_1=m(x-x_1)$.

(c) **Two point form :** Equation of a line passing through two points (x_1,y_1) and (x_2,y_2) is written as :

$$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1) \text{ or } \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix}=0$$

(d) **Intercept form :** If a and b are the intercepts made by a line on the axes of x and y, its equation is written as : $\dfrac{x}{a}+\dfrac{y}{b}=1$.

(e) **Normal form :** If p is the length of perpendicular on a line from the origin and α the angle which this perpendicular makes with positive x-axis, then the equation of this line is written as :

$x\cos\alpha+y\sin\alpha=p$ (p is always positive), where $0\leq\alpha<2\pi$.

(f) **Parametric form:** $\dfrac{x-h}{\cos\theta}=\dfrac{y-k}{\sin\theta}=r$ is the equation.

(g) **General form:** We know that a first degree equation in x and $y, ax+by+c=0$ always represents a straight line. This form is known as general form of straight line.

(i) Slope of this line $=\dfrac{-a}{b}=\dfrac{\text{coefficient of } x}{\text{coefficient of } y}$

(ii) Intercept by this line on x-axis $=-\dfrac{c}{a}$ and intercept by this line on y-axis $=-\dfrac{c}{b}$.

(iii) To change the general form of a line to normal form, first take c to right hand side and make it positive, then divide the whole equation by $\sqrt{a^2+b^2}$.

Angle Between Two Lines

(a) If θ be the angle between two lines : $y=m_1x+c_1$ and $y=\ m_2x+c_2$, then $\tan\theta=\pm\left(\frac{m_1-m_2}{1+m_1m_2}\right)$.

(b) If equation of lines are $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2\ =0$, then these line are-

(i) Parallel $\Leftrightarrow\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$

(ii) Perpendicular $\Leftrightarrow a_1a_2+b_1b_2=0$

(iii) Coincident $\Leftrightarrow\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

(iv) Intersecting $\Leftrightarrow\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$

Length of Perpendicular from a Point on a Line

Lengthofperpendicularfroma point (x_1,y_1) on theline $ax+by+c=0$ is $=\left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$.

In particular the length of the perpendicular from the origin on the line $ax+by+c=0$ is $P=\frac{|c|}{\sqrt{a^2+b^2}}$.

Distance Between two Parallel Lines

(i) The distance between two parallel lines $ax+by+c_1=0$ and $ax+by+c_2=0$ is $\frac{|c_1-c_2|}{\sqrt{a^2+b^2}}$.

(**Note:** The coefficients of $x\ \&\ y$ in both equations should be same).

(ii) The area of the parallelogram $=\frac{p_1p_2}{\sin\theta}$, where $p_1\ \&\ p_2$ are distance between two pairs of opposite sides $\&\theta$ is the angle between any two adjacent sides. Note that area of the parallelogram bounded by the lines $y=m_1x+c_1,y=m_1x+c_2$ and $y=m_2x+d_1,y=m_2x+d_2$ is given $\left|\frac{(c_1-c_2)(d_1-d_2)}{m_1-m_2}\right|$.

Equation of lines Parallel and Perpendicular to a Given Line

(i) Equation of line parallel to line $ax+by+c=0$. $ax+by+\lambda=0$

(ii) Equation of line perpendicular to line $ax+by+c=0$. $bx-ay+k=0$

Here λ, k , are parameters and their values are obtained with the help of additional information given in the problem.

Straight Line Making a given Angle with a Line

Equations of lines passing through a poing (x_1, y_1) and making an angle α , with the line $y = mx + c$ is written as:

$$y - y_1 = \frac{m \pm \tan\alpha}{1 \mp m\tan\alpha}(x - x_1)$$

Position of Two Points with Respect to a Given Line

Let the given line be $ax + by + c = 0$ and $P(x_1, y_1), Q(x_2, y_2)$ be two points. If the quantities $ax_1 + by_1 + c$ and $ax_2 + by_2 + c$ have the same signs, then both the points P and Q lie on the same side of the $ax + by + c = 0$. If the quantities $ax_1 + by_1 + c$ and $ax_2 + by_2 + c$ have opposite signs, then they lie on the opposite sides of the line.

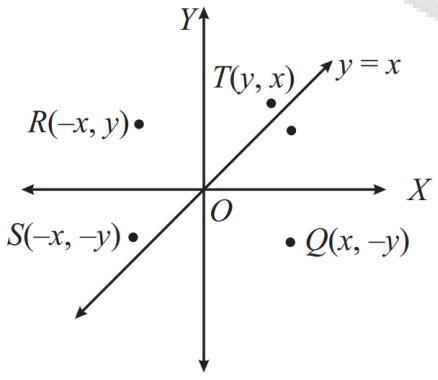
Concurrency of Lines

Three lines $a_1x + b_1y + c_1 = 0; a_2x + b_2y + c_2 = 0$ and $a_3x + b_3y + c_3 = 0$ are concurrent,

if $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$.

Reflection of a Point

Let $P(x, y)$ be any point, then its image with respect to



(i) x -axis is $Q(x, -y)$

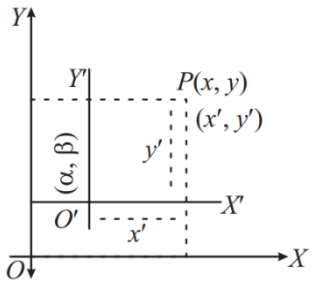
(ii) y -axis is $R(-x, y)$

(iii) origin is $S(-x, -y)$

(iv) line $y = x$ is $T(y, x)$

Transformation of Axes

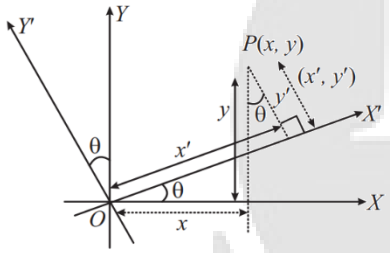
(a) **Shifting of origin without rotation of axes :** If coordinates of any point $P(x, y)$ with respect to new origin (a, b) will be (x', y')



then $x = x' + \alpha,\ y = y' + \beta$ or $x' = x - \alpha,\ y' = y - \beta$

Thus if origin is shifted to point (α, β) without rotation of axes, then new equation of curve can be obtained by putting $x + \alpha$ in place of x and $y + \beta$ in place of y.

(b) **Rotation of axes without shifting the origin :** Let O be the origin. Let $P \equiv (x, y)$ with respect to axes OX and OY and let $P \equiv (x', y')$ with respect to axes OX' and OY', where $\angle X'OX = \angle YOY' = \theta$



then $x = x'\cos\theta - y'\sin\theta$

$y = x'\cos\theta - y'\cos\theta$

or $y' = x\cos\theta + y\sin\theta$

$y' = -x\sin\theta + y\cos\theta$

The above relation between (x, y) and (x', y') can be easily obtained with the help of following table

Old / New	$x\downarrow$	$y\downarrow$
$x' \rightarrow$	$\cos\theta$	$\sin\theta$
$y' \rightarrow$	$-\sin\theta$	$\cos\theta$

Equation of Bisectors of Angles between Two Lines

If equation of two intersecting lines are $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, then equation of bisectors of the angles between these lines are written are: $\dfrac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = \pm\dfrac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$ ... (1)

(a) **Equation of bisector of angle containing origin :** If the equation of the lines are written with constant terms c_1 and c_2 positive, then the equation of bisectors of the angle containing the origin is obtained by taking sign in (1).

(b) **Equation of bisector of acute/obtuse angles :** See whether the constant terms c_1 and c_2 in the two equation are +ve or not. If not then multiply both sides of given equation by -1 to make the constant terms positive.

Determinate the sign of $a_1a_2 + b_1b_2$

If sign of $a_1a_2 + b_1b_2$	For obtuse angle bisector	For acute angle bisector
+	use + sing in eq. (1)	use – sign in eq. (1)
–	use – sign in eq. (1)	use + sign in eq. (1)

i.e., if $a_1a_2 + b_1b_2 > 0$, then the bisector corresponding to + sign gives obtuse angle bisector

$$\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = \frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$$

Family of Lines

If equation of two lines be $P = a_1x + b_1y + c_1 = 0$ and $Q \equiv a_2x + b_2y + c_2 = 0$, then the equation of the lines passing through the point of intersection of these lines is : $P + \lambda Q = 0$ or $a_1x + b_1y + c_1 + \lambda\left(a_2x + b_2y + c_2\right) = 0$.

The value of λ is obtained with the help of the additional information given in the problem.

General Equation and Homogeneous Equation of Second Degree

(a) A general equation of second degree $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represent a pair of straight lines if $\Delta = abc + 2fgh - af^2 - bg^2 - ch^2 = 0$ or $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$.

(b) If θ be the angle between the lines, then $\tan\theta = \pm\frac{2\sqrt{h^2 - ab}}{a + b}$. Obviously these lines are

(i) Parallel, if $\Delta = 0, h^2 = ab$ or if $h^2 = ab$ and $bg^2 = af^2$.

(ii) Perpendicular, if $a + b = 0$ i.e. coeff. of x^2 + coeff. of $y^2 = 0$.

(c) Homogeneous equation of 2^{nd} degree $ax^2 + 2hxy + by^2 = 0$ always represent a pair of straight lines whose equations are

$$y = \left(\frac{-h \pm \sqrt{h^2 - ab}}{b}\right)x \equiv y = m_1x \text{ and } y = m_2x \text{ and } m_1 + m_2 = -\frac{2h}{b}; m_1m_2 = \frac{a}{b}$$

These straight lines passes through the origin and for finding the angle between these lines same formula as given for general equation is used.

The condition that these lines are :

(i) At right angles to each other is $a+b=0$. i.e. co-efficient of x^2+ co-efficient of $y^2=0$.

(ii) Coincident is $h^2=ab$.

(iii) Equally inclined to the axis of x is $h=0$. i.e. coefficient of $xy=0$.

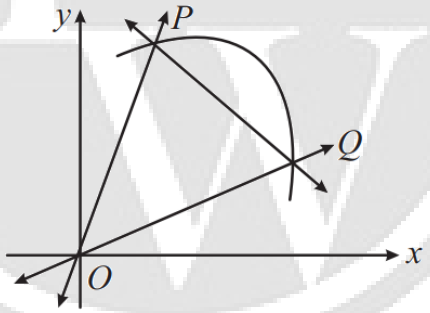
(d) The combined equation of angle bisectors between the lines represented by homogeneous equation of 2^{nd} degree is given by $\frac{x^2-y^2}{a-b}=\frac{xy}{h}, a\neq b, h\neq 0$.

(e) Pair of straight lines perpendicular to the lines $ax^2+2hxy+by^2=0$ and through origin are given by $bx^2-2hxy+ay^2=0$.

(f) If lines $ax^2+2hxy+by^2+2gx+2fy+c=0$ are parallel then distance between them is $=2\sqrt{\frac{g^2-ac}{a(a+b)}}$.

Equations of Lines Joining the Points of Intersection of a Line and a Curve to the Origin

Let the equation of curve be:



$ax^2+2hxy+by^2+2gx+2fy+c=0$ ... (i)

and straight line be

$lx+my+n=0$ ... (ii)

$$ax^2+2hxy+by^2+2(gx+fy)\left(\frac{lx+my}{-n}\right)+c\left(\frac{lx+my}{-n}\right)^2=0$$

STANDARD RESULTS

(i) Area of rhombus formed by lines $a|x|+b|y|+c=0$ or $\pm ax\pm by+c=0$ is $\frac{2c^2}{|ab|}$.

(ii) Area of triangle formed by line $ax+by+c=0$ and axes is $\frac{c^2}{2|ab|}$.

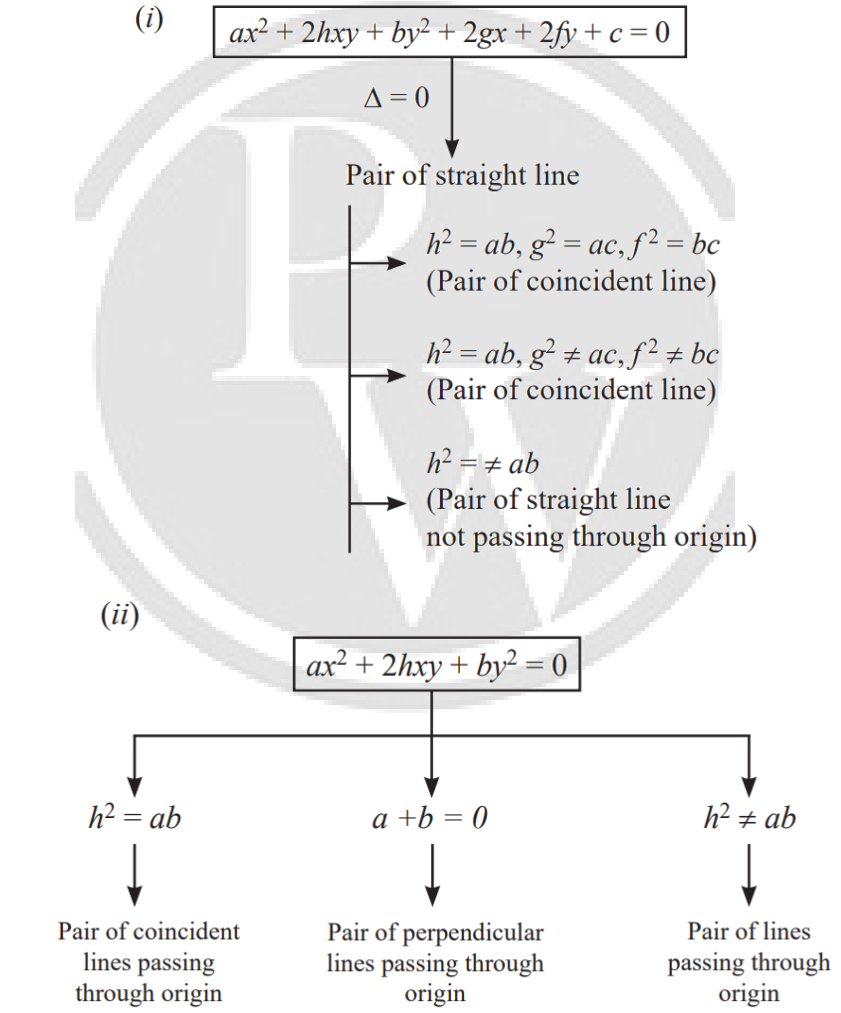
(iii) Co-ordinate of foot of perpendicular (h,k) from (x_1,y_1) to the line $ax+by+c=0$ is given by

$$\frac{h-x_1}{a}=\frac{k-y_1}{b}=\frac{-(ax_1+by_1+c)}{a^2+b^2}.$$

(iv) Image of point (x_1,y_1) w.r.t to the line $ax+by+c=0$ is given by

$$\frac{h-x_1}{a}=\frac{k-y_1}{b}=\frac{-2(ax_1+by_1+c)}{a^2+b^2}.$$

Chart





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