PYP - 2023

Mathematics

Q1 Given $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} x & y \\ 1 & 4 \end{bmatrix}$, If A=B, then x and y are:

(A) $x = 2, y = 3$ (B) $x = 1, y = 2$

(C) $x = 3, y = 2$ (D) $x = -2, y = -3$

Q2 If m and n are respectively the order and degree of the differential equation:

$$\left(\frac{d^2y}{dx^2}\right)^5 + 6\frac{\left(\frac{d^2y}{dx^2}\right)^3}{\frac{d^3y}{dx^3}} + \frac{d^3y}{dx^3} = x^2 + 5,$$ then:

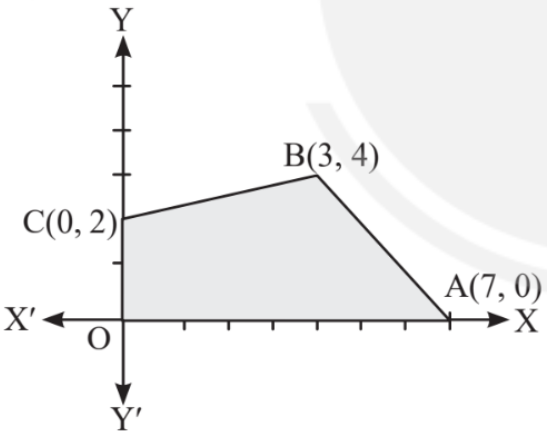
(A) m = 3, n = 3

(B) m = 2, n = 3

(C) m = 3, n = 2

(D) m = 3, n = 5

Q3 The feasible region for a LPP is shown in the given figure. The maximum value of z = 2x + 5y is:



(A) 14 (B) 10

(C) 36 (D) 26

Q4 If order of matrix A is m × p and order of matrix B is p × n, then what is the order of matrix AB?

(A) m × p

(B) m × n

(C) p × n

(D) m × 2

Q5 Match List-I with List-II. Match the integrating factors:

List - I	List - II
Differential Equation	Integrating factor
(A) $\frac{dy}{dx} + 3y = e^{-2x}$	(i) $\frac{1}{2}$
(B) $x\frac{dy}{dx} + y = 3x^2$	(ii) e^{-x}
(C) $x\frac{dy}{dx} - y = 3x^2$	(iii) x
(D) $\frac{dy}{dx} - y = x$	(iv) e^{3x}

Choose the correct answer from the options given below

(A) A-(iv), B-(iii), C-(i), D-(ii)

(B) A-(iii), B-(iv), C-(i), D-(ii)

(C) A-(ii), B-(iii), C-(iv), D-(i)

(D) A-(i), B-(ii), C-(iii), D-(iv)

Q6 The slope of the tangent to the curve $x = at^2, y = 2at$ at 't' is:

(A) $\frac{1}{t}$ (B) $\frac{1}{t^2}$

(C) $-\frac{1}{t}$ (D) $-\frac{1}{t^2}$

Q7 The area enclosed between $y^2 = 4x, x = 1, x = 4$ in first quadrant is:

(A) $\frac{28}{3}$ sq. unit

(B) $\frac{27}{2}$ sq. unit

(C) $\frac{25}{2}$ sq. unit

(D) $\frac{27}{5}$ sq. unit

Q8 In a Linear Programming problem, the objective function is always:

(A) Linear (B) Quadratic

(C) Cubic (D) Biquadratic

Q9 The mean number of heads in two tosses of a coin is:

(A) 2 (B) $\frac{1}{2}$

(C) 1 (D) $\frac{3}{2}$

Q10 If $\begin{vmatrix} 3x & 4 \\ 7 & x \end{vmatrix} = \begin{vmatrix} 6 & 3 \\ 2 & 1 \end{vmatrix}$ then:

(A) $x^2 = \frac{26}{3}$ (B) $x^2 = \frac{25}{3}$

(C) $x^2 = \frac{23}{3}$ (D) $x^2 = \frac{28}{3}$

Q11 $x = a\left(t - \frac{1}{t}\right), y = b\left(t + \frac{1}{t}\right)$,then $\frac{dy}{dx} =$

(A) $\frac{x}{y}$ (B) $\frac{b^2x}{a^2y}$

(C) $\frac{bx}{ay}$ (D) $\frac{a^2y}{b^2x}$

Q12 If the probability distribution of a random variable X is as given below:

X	-1	0	1	2	3
P(X)	K	$\frac{1}{5}$	$2K$	$\frac{3}{10}$	K

Then the value of K is:

(A) $\frac{3}{8}$ (B) $\frac{1}{4}$

(C) $\frac{5}{8}$ (D) $\frac{1}{8}$

Q13 If the function $f(x) = x^4 - 62x^2 + ax + 9$ attains its local maximum value at x = 1, then a is equal to:

(A) 120 (B) 110

(C) 100 (D) 90

Q14 The sum of the products of elements of any row with the cofactors of corresponding elements is equal to:

(A) The value of the determinant

(B) 0

(C) Sum of cofactors

(D) Adjoint of matrix

Q15 $\int \left(x + \frac{1}{x}\right)^2 dx$ equals:

(A) $\frac{x^3}{3} + \frac{1}{x} - 2x + c$ (B) $\frac{x^3}{3} - \frac{1}{x} + 2x + c$

(C) $\frac{x^3}{3} - \frac{1}{x} - 2x + c$ (D) $\frac{x^3}{3} + \frac{1}{x} + 2x + c$

Q16 The two curves $x^3 - 3xy^2 + 15 = 0$ and $3x^2y - y^3 + 17 = 0$:

(A) cut at right angles

(B) touch each other

(C) cut at angle $\frac{\pi}{4}$

(D) cut at angle $\frac{\pi}{3}$

Q17 Relation R on Real Numbers is defined as R = {(a, b) : a ≤ b}.The relation is:

(A) Reflexive and Symmetric but not Transitive

(B) Symmetric and Transitive but not Reflexive

(C) Reflexive and Transitive but not Symmetric

(D) Equivalence relation

Q18 If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, then $A^2 - 5A + 7I =$

(A) O (B) I

(C) $2I$ (D) $3I$

Q19 Let $a \leq \tan^{-1} x + \cot^{-1} x + \sin^{-1} x \leq b$. If α *and* β denote the minimum and maximum possible values of a and b respectively, then:

(A) $\alpha = 0, \beta = \pi$

(B) $\alpha = 0, \beta = \frac{\pi}{2}$

(C) $\alpha = \frac{\pi}{2}, \beta = \pi$

(D) $\alpha = -\frac{\pi}{2}, \beta = \frac{\pi}{2}$

Q20 Area of the region bounded by the curve y = cosx and x-axis between x = 0 and x = π is:

(A) 2 sq. units (B) 3 sq. units

(C) 4 sq. units (D) 1 sq. unit

Q21 The derivative of $\sec(\tan\sqrt{x})$ with respect to x is:

(A) $\frac{\sec(\tan\sqrt{x})\tan(\tan\sqrt{x})\sec^2\sqrt{x}}{2\sqrt{x}}$

(B) $\sec^2\left(\tan\sqrt{x}\right)$

(C) $\frac{\sec(\tan\sqrt{x})\tan(\tan\sqrt{x})\sec^2\sqrt{x}}{x}$

(D) $\sec^2\left(\tan x^{1/3}\right)$

Q22

The corner points of the feasible region determined by the following system of linear inequalities:
$2x + y \leq 10,\ x + 3y \leq 15,\ x, y \geq 0$ are (0, 0), (5, 0), (3, 4) and (0, 5). Let $z = px + qy$, where p, q > 0 condition on p and q so that maximum of z occurs at both (3, 4) and (0, 5) is:
(A) p = q (B) p = 2q
(C) p = 3q (D) q = 3p

Q23 The interval in which the function $f(x) = 2x^3 - 3x^2 - 36x + 7$ is strictly decreasing is:
(A) (–3, –2)
(B) (–2, 3)
(C) (2, 3)
(D) (2, –3)

Q24 The maximum value of (sin x)(cos x) is:
(A) 1 (B) $\frac{1}{2}$
(C) $\frac{1}{4}$ (D) $\sqrt{2}$

Q25 If $\left|\vec{a}\right| = 3\ and\ \left|\vec{b}\right| = 4$, then a value of λ for which $\vec{a} + \lambda\vec{b}$ and $\vec{a} - \lambda\vec{b}$ are perpendicular is:
(A) $\frac{9}{16}$ (B) $\frac{3}{4}$
(C) $\frac{3}{2}$ (D) $\frac{4}{3}$

Q26 $\int e^x \sec x(1 + \tan x)dx$ equals:
(A) $e^x \sec x + c$ (B) $e^x \tan x + c$
(C) $e^x \sin x + c$ (D) $e^x \cos x + c$

Q27 The area enclosed between the curve $y = x^2 + 2$ and x-axis between x = 0 and x = 3 is :
(A) 14 sq. units (B) 15 sq. units
(C) 16 sq. units (D) 18 sq. units

Q28 If $\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$, then the value of $\cos^{-1} x + \cos^{-1} y$ is:
(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$
(C) $\frac{2\pi}{3}$ (D) $\frac{\pi}{6}$

Q29 If A and B are invertible matrices of order 3, |A| = 2 and $\left|(AB)^{-1}\right| = -\frac{1}{6}$,then the value of |B| is:
(A) 3 (B) -3
(C) 2 (D) $\frac{1}{6}$

Q30 Which of the following statements are correct?
1. If f: R → R then f(x) = |x| is continuous everywhere.
2. If f: R → R then f(x) = |x| is continuous everywhere but not differentiable at x = 0.
3. Let f: R – {0} → R then f(x) = $\frac{1}{x}$ is continuous everywhere.
4. Let f: R → R then f(x) = |x – 1| + |x – 2| is continuous everywhere but not differentiable at exactly 2 points.
5. If f: R → R then f(x) = cot x is continuous everywhere.
Choose the correct answer from the options given below:
(A) 1 only
(B) 1, 3 only
(C) 1, 2, 3, 4 only
(D) 4, 5 only

Q31 If $f(x) = \begin{cases} \frac{k\cos x}{\pi - 2x}, & x \neq \frac{\pi}{2} \\ 3, & x = \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$, then k is:
(A) 6 (B) 4
(C) 3 (D) 2

Q32 $A = \begin{bmatrix} 1 & -2 & 3 \\ 1 & 2 & 1 \\ \lambda & 2 & -3 \end{bmatrix}$ .If A^{-1} does not exist, then λ =
(A) -2 (B) 2

(C) 1 (D) -1

Q33 Solution of differential equation xdy – ydx = 0 represents

(A) Family of straight lines passing through origin

(B) Family of parabolas whose vertex is at origin

(C) Family of circles whose centre is at origin

(D) Family of straight lines passing through (1, 1)

Q34 The variance of number of heads in three tosses of a coin is:

(A) $\frac{3}{2}$ (B) $\frac{3}{4}$

(C) 1 (D) 2

Q35 The corner points of the feasible region determined by the system of linear constraints are $(0,\ 0),\ (0,\ 40),\ (60,\ 20),\ (60,\ 0)$. The objective function is $z\ =\ 4x\ +\ 3y$. Compare the quantity in Column-A and Column-B.

Column-A	Column-B
Maximum value of z	350

(A) The quantity in column A is greater

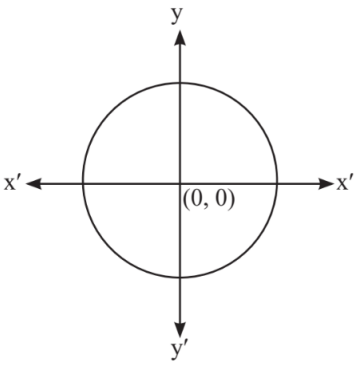
(B) The quantity in column B is greater

(C) The two quantities are equal

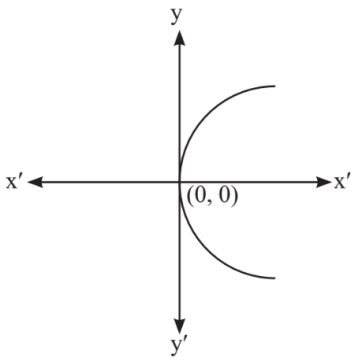
(D) The quantity in column B is greater than twice the quantity in column A

Q36 Which of the following graphs represent a function?

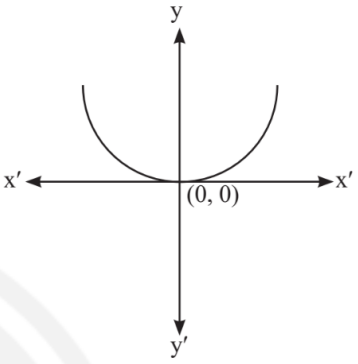
(A)



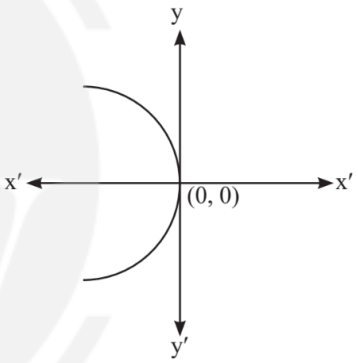
(B)



(C)



(D)



Q37 $\int \frac{\sqrt{\tan x}}{\sin x \cos x} dx$ equals:

(A) $2\sqrt{\tan x} + c$ (B) $2\sqrt{\cot x} + c$

(C) $\sqrt{\tan x} + c$ (D) $\frac{2}{\sqrt{\tan x}} + c$

Q38 If $A = \begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 \\ 3 & -4 \\ 2 & 4 \end{bmatrix}$ then product AB is:

(A) Not possible (B) $\begin{bmatrix} 1 & -6 & 6 \\ 8 & -8 & 16 \end{bmatrix}$

(C) $\begin{bmatrix} 9 \\ 19 \\ 15 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 18 \\ 10 & 16 \end{bmatrix}$

Q39 The degree of the differential equation $\left[1+\left(\frac{dy}{dx}\right)\right]^3=\left(\frac{d^2y}{dx^2}\right)^2$ is:

(A) 1 (B) 2

(C) 3 (D) 4

Q40 If a, b and c are all different from zero and $\begin{vmatrix}1+a & 1 & 1\\ 1 & 1+b & 1\\ 1 & 1 & 1+c\end{vmatrix}=0$, then the value of $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$ is:

(A) 0 (B) abc

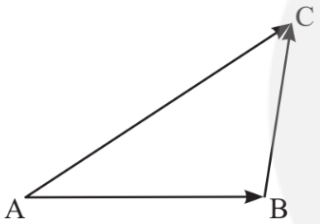
(C) -1 (D) $\frac{1}{abc}$

Q41 If matrix A = $\begin{bmatrix}3 & x\\ y & 0\end{bmatrix}$ and $A'=A$, then:

(A) x = y (B) x = 0, y = 3

(C) x = 3, y = 0 (D) x + y = 3

Q42 In ΔABC:



A. $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}=\overrightarrow{O}$

B. $\overrightarrow{AB}+\overrightarrow{BC}-\overrightarrow{AC}=\overrightarrow{O}$

C. $\overrightarrow{AB}+\overrightarrow{BC}-\overrightarrow{CA}=\overrightarrow{O}$

D. $\overrightarrow{AB}-\overrightarrow{CB}+\overrightarrow{CA}=\overrightarrow{O}$

E. $\overrightarrow{AB}-\overrightarrow{CB}-\overrightarrow{CA}=\overrightarrow{O}$

Choose the correct answer from the options given below:

(A) A, B, D only (B) A, B, E only

(C) B, E only (D) A, D, E only

Q43 Match List-I with List-II.

List – I	List – II
A. If A and B are mutually exclusive events, then P(A ∪ B) =	**(i)** $\frac{P(A\cap B)}{P(B)}$, $P(B)\neq 0$
B. If A and B are independent events, then P(A ∩ B) =	**(ii)** $\frac{P(A\cap B)}{P(A)}$, $P(A)\neq 0$
C. If A and B are two events of a sample space of an experiments, then P(A / B) =	**(iii)** $P(A)\cdot P(B)$
D. If A and B are two events of a sample space of an experiments, then P(B / A) =	**(iv)** $P(A)+P(B)$

Choose the correct answer from the options given below:

(A) A-(iv), B-(iii), C-(i), D-(ii)

(B) A-(iii), B-(iv), C-(i), D-(ii)

(C) A-(ii), B-(iii), C-(iv), D-(i)

(D) A-(i), B-(ii), C-(iii), D-(iv)

Q44 The vector equation of the line joining the points (–2, –3, –4) and (1, –2, 4) is:

(A) $\vec{r}=\left(-2\hat{i}-3\hat{j}-4\hat{k}\right)+\lambda\left(\hat{i}-2\hat{j}+4\hat{k}\right)$

(B) $\vec{r}=\left(2\hat{i}+3\hat{j}+4\hat{k}\right)+\lambda\left(3\hat{i}-\hat{j}+8\hat{k}\right)$

(C) $\vec{r}=\left(-2\hat{i}-3\hat{j}-4\hat{k}\right)+\lambda\left(3\hat{i}+\hat{j}+8\hat{k}\right)$

(D) $\vec{r}=\left(2\hat{i}+3\hat{j}+4\hat{k}\right)+\lambda\left(3\hat{i}+\hat{j}+8\hat{k}\right)$

Q45 The angle between the two planes $x+y-z=0$ and $3x+2y+z=5$ is:

(A) $\cos^{-1}4$ (B) $\cos^{-1}\frac{2\sqrt{42}}{21}$

(C) $\cos^{-1}\frac{1}{4}$ (D) $\cos^{-1}\frac{1}{\sqrt{42}}$

Q46 If a set P contains 5 elements and the set Q contains 8 elements, then the number of one-one functions from A to B is:

(A) ${}^8C_5$

(B) ${}^8C_5 \times 5$!

(C) 5^8

(D) 8^5

Q47 The equation of tangent to the curve given by $x = a\sin^3 t$, $y = b\cos^3 t$ at a point where $t = \frac{\pi}{2}$ is:

(A) $x = 0$ (B) $y = 0$

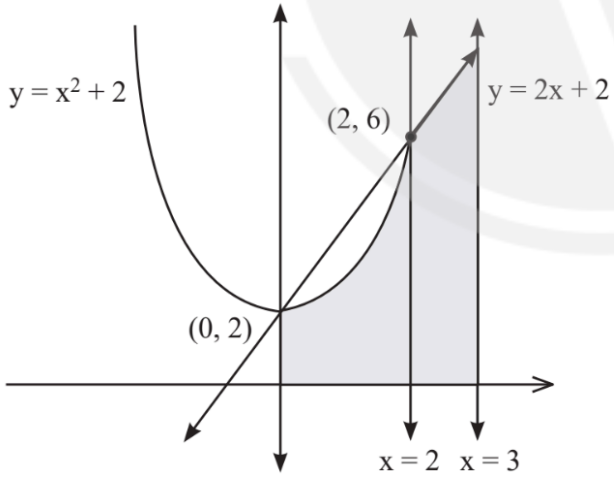
(C) $y = \frac{\pi}{2}$ (D) $x = \frac{\pi}{2}$

Q48 The rate of change in area of a triangle having sides 10 cm and 12 cm when the variable angle between them is $\theta = 60^\circ$, is:

(A) $30cm^2$/radian (B) $120cm^2$/radian

(C) $30\sqrt{3}cm^2$/radian (D) $60\sqrt{3}cm^2$/radian

Q49 Which of the following regions will represent the shaded area in the given figure?



(A) $\{(x, y) : 0 \le y \le x^2 + 2, 0 \le y \le 2x + 2, 0 \le x \le 3\}$

(B) $\{(x, y) : 0 \le y \le x^2 + 2, y \ge 2x + 2, x \le 3\}$

(C) $\{(x, y) : y \ge x^2 + 2, x \le 2, x \ge 3\}$

(D) $\{(x, y) : y \ge x^2 + 2, x \ge 0, x \ge 3\}$

Q50 If the equation of a floor of a room is given by $x + y - z + 4 = 0$ and the equation of roof is given by $x + y - z + 5 = 0$. Then, the height of the room is:

(A) $\frac{1}{6}$units (B) $\frac{1}{3}$units

(C) $\frac{1}{\sqrt{3}}$units (D) $\frac{1}{\sqrt{6}}$units

Answer Key

Q1	(A)
Q2	(C)
Q3	(D)
Q4	(B)
Q5	(A)
Q6	(A)
Q7	(A)
Q8	(A)
Q9	(C)
Q10	(D)
Q11	(B)
Q12	(D)
Q13	(A)
Q14	(A)
Q15	(B)
Q16	(A)
Q17	(C)
Q18	(A)
Q19	(A)
Q20	(A)
Q21	(A)
Q22	(D)
Q23	(B)
Q24	(B)
Q25	(B)
Q26	(A)
Q27	(B)
Q28	(B)
Q29	(B)
Q30	(C)
Q31	(A)
Q32	(D)
Q33	(A)
Q34	(B)
Q35	(B)
Q36	(C)
Q37	(A)
Q38	(D)
Q39	(B)
Q40	(C)
Q41	(A)
Q42	(A)
Q43	(A)
Q44	(C)
Q45	(B)
Q46	(B)
Q47	(B)
Q48	(A)
Q49	(A)
Q50	(C)

Hints & Solutions

Q1 Text Solution:

Given: $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} x & y \\ 1 & 4 \end{bmatrix}$ and A = B.

So, $\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} x & y \\ 1 & 4 \end{bmatrix}$

on comparing both sides we get x = 2 and y = 3

Hence, x = 2 and y = 3.

Q2 Text Solution:

Given:

$$\left(\frac{d^2y}{dx^2}\right)^5 + \frac{6\left(\frac{d^2y}{dx^2}\right)^3}{\left(\frac{d^3y}{dx^3}\right)} + \frac{d^3y}{dx^3} = x^2 + 5$$

$$\Rightarrow \frac{\left(\frac{d^2y}{dx^2}\right)^5\left(\frac{d^3y}{dx^3}\right) + 6\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{d^3y}{dx^3}\right)^2}{\left(\frac{d^3y}{dx^3}\right)} = x^2 + 5$$

$$\Rightarrow \left(\frac{d^2y}{dx^2}\right)^5\left(\frac{d^3y}{dx^3}\right) + 6\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{d^3y}{dx^3}\right)^2 = \left(x^2+5\right)\left(\frac{d^3y}{dx^3}\right)$$

So, order = m = 3 and degree = n = 2

Hence, m = 3 and n = 2.

Q3 Text Solution:

Given,

Objective function

z = 2x + 5y

Now,

$Z_{(0,2)} = 2 \times 0 + 5 \times 2$

$= 0 + 10 = 10$

$Z_{(3,4)} = 2 \times 3 + 5 \times 4 = 6 + 20 = 26$

$Z_{(7,0)} = 2 \times 7 + 5 \times 0 = 14 + 0 = 14$

Hence, it gives maximum value 26 at (3, 4)

Q4 Text Solution:

Given,

matrix A of order m × p and matrix B of order p × n

order of A × B = (m × p) × (p × n)

order of AB = m × n

Q5 Text Solution:

We know,

Linear differential equation

$\frac{dy}{dx} + py = q$.....(i)

and Integrating factor (I.F.)= $e^{\int pdx}$,

Now,

(A) $\frac{dy}{dx} + 3y = e^{-2x}$

On comparing with eqn (i), we get

p = 3, q= e^{-2x}

Now,

I.F.= $e^{\int pdx} = e^{\int 3dx} = e^{3x}$

(B) Given,

$x\frac{dy}{dx} + y = 3x^2$

$\frac{dy}{dx} + \frac{y}{x} = 3x$

on comparing with equation (i), we get

$p = \frac{1}{x}, q = 3x$

Now, I.F= $e^{\int pdx} = e^{\int 1/xdx} = e^{\log x} = x$

(C) Given,

$\frac{dy}{dx} - \frac{y}{x} = 3x$

on comparing with eqn. (i), we get

$p = \frac{-1}{x}, q = 3x$

$Now\ I.F = e^{\int pdx} = e^{-\int \frac{1}{x}dx} = e^{-\log x}$

$= e^{\log x^{-1}}$

$= x^{-1} = \frac{1}{x}$

(D) Given, $\frac{dy}{dx} - y = x$

on comparing, with eqn. (i), we get

$p = -1,\ q = x$

$Now.\ I.F = e^{\int pdx} = e^{\int -1dx} = e^{-x}$

Hence, (A) → (iv), (B) → (iii), (C) → (i), (D) → (ii)

Q6 Text Solution:

Given, $x = at^2, y = 2at$

We know,

Slope of tangent to curve $= \frac{dy}{dx}$

Now,

differentiating both sides, w.r.t. 't'.

$\frac{dx}{dt} = \frac{d}{dt}\left(at^2\right) = 2at$.....(i)

Now, differentiating y = 2at on both sides, w.r.t. 't'.

$\frac{dy}{dt} = \frac{d}{dt}\left(2at\right) = 2a$.....(ii)

Now, $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2a}{2at} = \frac{1}{t}$

Hence the slope of tangent to the curve is $\frac{1}{t}$

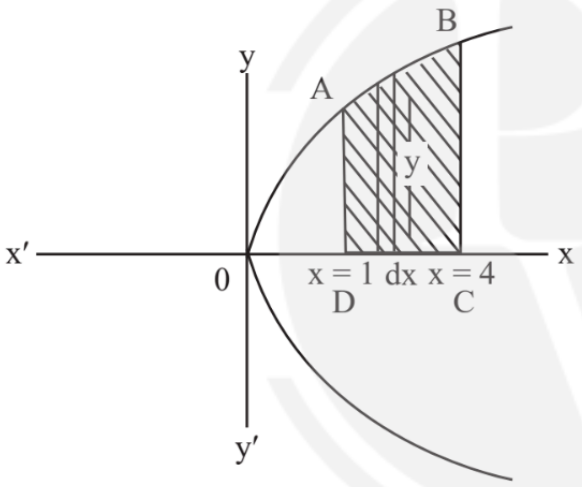
Therefore, option (A) is the correct option.

Q7 Text Solution:

Given:

$y^2 = 4x, x = 1, x = 4$ in I quadrant.

$y = 2\sqrt{x}$



So, the required area = Area of ABCD

$= \int_1^4 ydx = \int_1^4 2\sqrt{x}dx$

$= 2\left(\frac{(x)^{3/2}}{3/2}\right)_1^4 = \frac{4}{3}\left(4^{\frac{3}{2}} - 1^{\frac{3}{2}}\right)$

$= \frac{4}{3}(8-1) = \frac{28}{3}$ sq. units

Q8 Text Solution:

In linear programming, objective function are always linear.

Therefore, option(A) is the correct option.

Q9 Text Solution:

Given, If a single coin is tossed twice then

Sample space S = {HH, HT, TH, TT} we know.

$Probability = \frac{Favourable\ outcomes}{Total\ outcomes}$

Now, considering Head as a random variable then

$P(x=0) = P(TT) = \frac{1}{4}$

$P(x=1) = P(HT, TH) = \frac{2}{4} = \frac{1}{2}$

$P(x=2) = P(HH) = \frac{1}{4}$

Therefore the probability distribution is

x	0	1	2
P(x)	1/4	1/2	1/4

Now, mean E(x)

$= \sum x_i P(x_i) = 0 \times \frac{1}{4} + 1 \times \frac{1}{2} + 2 \times \frac{1}{4}$

$= 0 + \frac{1}{2} + \frac{1}{2} = 1$

Hence, the mean is 1.

Q10 Text Solution:

Given,

$\begin{vmatrix} 3x & 4 \\ 7 & x \end{vmatrix} = \begin{vmatrix} 6 & 3 \\ 2 & 1 \end{vmatrix}$

Now,

$\Rightarrow 3x^2 - 28 = 6 - 6 \Rightarrow 3x^2 - 28 = 0 \Rightarrow x^2 = \frac{28}{3}$

Q11 Text Solution:

Given,

$x = a\left(t - \frac{1}{t}\right), y = b\left(t + \frac{1}{t}\right)$

Now, we know. $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$.....(i)

So, $\frac{dx}{dt} = \frac{d}{dt}a\left(t - \frac{1}{t}\right)$

$= a\left(1 - \left(\frac{-1}{t^2}\right)\right) \Rightarrow \frac{dx}{dt} = a\left(\frac{t^2+1}{t^2}\right)$.....(ii)

Now,

$\frac{dy}{dt} = \frac{d}{dt}b\left(t + \frac{1}{t}\right)$

$\Rightarrow \frac{dy}{dt} = b\left(\frac{t^2-1}{t^2}\right)$.....(iii)

put values of eqn. (ii) and (iii) in eqn. (i), we get

$\frac{dy}{dx} = \frac{b\left(\frac{t^2-1}{t^2}\right)}{a\left(\frac{t^2+1}{t^2}\right)} = \frac{b}{a}\left(\frac{t^2-1}{t^2+1}\right)$

on dividing numerator and denominator by t, we get

$$\frac{dy}{dx} = \frac{b}{a} \cdot \frac{\left(\frac{t^2-1}{t}\right)}{\left(\frac{t^2+1}{t}\right)} = \frac{b\left(t-\frac{1}{t}\right)}{a\left(t+\frac{1}{t}\right)}$$

$$\frac{dy}{dx} = \frac{b \cdot x/a}{a \cdot y/b} = \frac{b^2 x}{a^2 y}$$

Q12 **Text Solution:**

Given,

x	–1	0	1	2	3
P(x)	k	1/5	2k	3/10	k

We know that;

$\Sigma P\left(x\right) = 1$

$Now,$

$\Rightarrow P\left(-1\right) + P\left(0\right) + P\left(1\right) + P\left(2\right)$

$+ P\left(3\right) = 1$

$\Rightarrow k + \frac{1}{5} + 2k + \frac{3}{10} + k = 1$

$\Rightarrow \frac{10k+2+20k+3+10k}{10} = 1$

$\Rightarrow 40k + 5 = 10 \Rightarrow k = \frac{5}{40}$

$\Rightarrow k = \frac{1}{8}$

Q13 **Text Solution:**

Given,

$f\left(x\right) = x^4 - 62x^2 + ax + 9$

We know that for critical point $\frac{dy}{dx} = 0.$

differentiating both sides w.r. to x, we get

$f'\left(x\right) = 4x^3 - 124x + a$

$\text{Now, } f'\left(1\right) = 0$

$\Rightarrow 4\left(1\right)^3 - 124\left(1\right) + a = 0$

$\Rightarrow a - 120 = 0$

$\Rightarrow a = 120$

Therefore, option(A) is the correct option.

Q14 **Text Solution:**

Let,

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{32} \end{vmatrix}$$

Expanding along R_1,we get

$$\Delta = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{32} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{32} \end{vmatrix}$$

$$+ a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$= a_{11}M_{11} + a_{12}M_{12} + a_{13}M_{13}$$

$\left(where, M_{11}, M_{12} \ \& \ M_{13} \ are \ cofactors\right)$

Hence, the sum of the products of element of any row with the co-factors of corresponding elements is equal to the value of the determinant of the given matrix.

Q15 **Text Solution:**

Let, $I = \int \left(x + \frac{1}{x}\right)^2 dx$ Now,

$\Rightarrow I = \int \left(x^2 + \frac{1}{x^2} + 2x \cdot \frac{1}{x}\right) dx$

$\Rightarrow I = \int \left(x^2 + x^{-2} + 2\right) dx$

$\Rightarrow I = \frac{x^3}{3} + \frac{x^{-1}}{-1} + 2x + c$

$\Rightarrow I = \frac{x^3}{3} - \frac{1}{x} + 2x + c$

Q16 **Text Solution:**

Given,

$c_1 : x^3 - 3y^2x + 15 = 0$

$c_2 : 3x^2y - y^3 + 17 = 0$

We know that,

$m = \frac{dy}{dx}$

$Now, \ For \ c_1, x^3 - 3xy^2 + 15 = 0$

On diff. both sides, we get

$\frac{d}{dx}\left(x^3 - 3xy^2 + 15\right) = 0$

$\Rightarrow 3x^2 - 3\left(x \cdot 2y\frac{dy}{dx} + y^2 \cdot 1\right) + 0 = 0$

$\Rightarrow 3x^2 - 6xy\frac{dy}{dx} - 3y^2 = 0$

$\Rightarrow -6xy\frac{dy}{dx} = 3y^2 - 3x^2$

$\therefore m_1 = \frac{dy}{dx} = \frac{y^2-x^2}{-2xy} \ldots\ldots\ldots\ldots (i)$

$Now.\ for\ c_2$

$3x^2y - y^3 + 17 = 0$

$on\ differentiating\ both\ side\ w.r.t.\ x,$

$\frac{d}{dx}\left(3x^2y - y^3 + 17\right)$

$\Rightarrow 3\left(x^2\frac{dy}{dx} + y \cdot 2x\right) - 3y^2\frac{dy}{dx} + 0 = 0$

$\Rightarrow 3x^2\frac{dy}{dx} + 6xy - 3y^2\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx}\left(3y^2 - 3x^2\right) = 6xy$

$\therefore m_2 = \frac{dy}{dx} = \frac{2xy}{y^2-x^2} \ldots\ldots\ldots\ldots\ldots (ii)$

From Eqn. (i) and (ii), on multiply, we get

$\Rightarrow m_1 \times m_2 = \frac{y^2-x^2}{-2xy} \times \frac{2xy}{y^2-x^2}$

$\therefore m_1 \times m_2 = -1$

Hence, both curve cut each other at right angles.

Q17 Text Solution:

Given,

R = {(a, b); a ≤ b}

we know, if (a, a) ∈ R, ∀a ∈ R then R is Reflexive.

If (a, b) ∈ R then (b, a) ∈ R, ∀a, b ∈ R then R is symmetric.

If (a, b) ∈ R and (b, c) ∈ R then, (a, c) ∈ R, ∀abc ∈ R then R is transitive.

For Reflexive,

(a, a) ∈ R

then, a ≤ a is true ∀a∈A

So, R is Reflexive

For symmetric,

let (a, b) ∈ R

(3, 4) ∈ R ⇒ 3 ≤ 4

but $4 \text{ is not } \leq 3$, So, (4, 3) ∉ R .

Hence, it is not symmetric.

For transitive,

(a, b) ∈ R (3, 4) ∈ R 3 ≤ 4

(b, c) ∈ R (4, 5) ∈ R 4 ≤ 5

(a, c) ∈ R (3, 5) ∈ R 3 ≤ 5

So, it is true.

Hence, R is Reflexive and transitive but not symmetric.

Q18 Text Solution:

Given,

$A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$Now,\ A^2 = A \times A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$

$A^2 - 5A + 7I = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$+ 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$= \begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$

$A^2 - 5A + 7I = O$

Therefore, option(A) is the correct option.

Q19 Text Solution:

Given:

$a \leq \tan^{-1} x + \cot^{-1} x + \sin^{-1} x \leq b$

also, given, $a = \alpha$ and $b = \beta$

So, $\alpha \leq \tan^{-1} x + \cot^{-1} x + \sin^{-1} x \leq \beta$

we know that,

$\sin^{-1} x + \cos^{-1} x = \pi/2$

$\tan^{-1} x + \cot^{-1} x = \pi/2$

$Range\ of\ \sin^{-1} x = \left[-\pi/2, \pi/2\right]$

Now,

$\alpha \leq \tan^{-1} x + \cot^{-1} x + \sin^{-1} x \leq \beta$

$\alpha \leq \pi/2 + \sin^{-1} x \leq \beta \ldots\ldots\ldots (i)$

$-\frac{\pi}{2} \leq \sin^{-1} x \leq \frac{\pi}{2}\ (we\ know)$

$adding\ \pi/2\ in\ all\ sides,\ we\ get.$

$-\frac{\pi}{2} + \frac{\pi}{2} \leq \frac{\pi}{2} + \sin^{-1} x \leq \frac{\pi}{2} + \frac{\pi}{2}$

$0 \leq \frac{\pi}{2} + \sin^{-1} x \leq \pi$

on comparing with Eqn.(i) we get

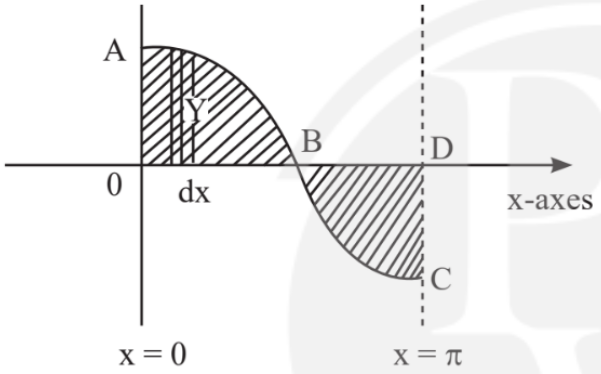
$\alpha = 0, \beta = \pi$

Q20 Text Solution:

Given:

y = cosx and x-axis

x = 0 and x = π



we know,

cosine function is symmetric from

$0\ to\ \pi/2\ and\ \pi/2\ to\ \pi$. So, their area is equal.

Required Area = Area of OABCD

= 2 Area of OAB

$= 2\int_0^{\pi/2} y dx$

$= 2\int_0^{\pi/2} \cos x dx$

$= 2\left(\sin x\right)_0^{\pi/2}$

= 2(1 – 0) = 2 sq. units

Q21 Text Solution:

Given,

$y = \sec(\tan\sqrt{x})$

on differentiating both side w.r to x,

$\frac{dy}{dx} = \frac{d}{dx}\left[\sec\left(\tan\sqrt{x}\right)\right]$.....(i)

By chain rule, we get

$\frac{dy}{dx} = \sec\left(\tan\sqrt{x}\right)\tan\left(\tan\sqrt{x}\right)\frac{d(\tan\sqrt{x})}{dx}$

$\frac{dy}{dx} = \sec\left(\tan\sqrt{x}\right)\tan\left(\tan\sqrt{x}\right)$

$\cdot \sec^2\sqrt{x}\frac{d}{dx}\sqrt{x}$

$\frac{dy}{dx} = \sec\left(\tan\sqrt{x}\right)\tan\left(\tan\sqrt{x}\right)\sec^2\sqrt{x}$

$\cdot \frac{1}{2\sqrt{x}}$

$\frac{dy}{dx} = \frac{\sec(\tan\sqrt{x})\tan(\tan\sqrt{x})\sec^2\sqrt{x}}{2\sqrt{x}}$

Q22 Text Solution:

Given,

Objective function. $z = px + qy$

Subject to constraints,

$2x + y \leq 10$

$x + 3y \leq 15$

$x, y \geq 0$

Corner points of feasible region are (0, 0), (5, 0), (3, 4) and (0, 5)

Maximum of z occurs at (3, 4) & (0, 5)

$So,\ z_{(3,4)} = z_{(0,5)}$

$(px + qy)_{(3,4)} = (px + qy)_{(0,5)}$

$\Rightarrow 3p + 4q = p \times 0 + 5q$

$\Rightarrow 3p + 4q = 5q$

$\Rightarrow q = 3p$

Q23 Text Solution:

Given,

$f(x) = 2x^3 - 3x^2 - 36x + 7$

we know that.

If f'(x) > 0 then f(x) is increasing. and if f'(x) < 0 then f(x) is decreasing.

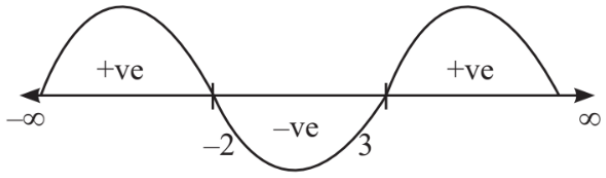
Now,

on differentiating both side, w.r.t we get

$f'(x) = 6x^2 - 6x - 36$

$\Rightarrow f'(x) = 6(x^2 - x - 6)$

$\Rightarrow f'(x) = 6(x - 3)(x + 2)$



f(x) is decreasing from (–2, 3)

Hence, f(x) is strictly decreasing in (–2, 3)

Therefore, option(B) is the correct option.

Q24 **Text Solution:**

f(x) = (sin x) (cos x)

we know, if $f''(x) > 0$ then f(x) is minimum at that value of x.

if $f''(x) < 0$ then f(x) is maximum at that value of x.

Now, $f(x) = \sin x \,.\, \cos x$

$\Rightarrow f(x) = \frac{\sin 2x}{2}$

on differentiating both side w.r.t. x, we get

$f'(x) = \frac{1}{2}\cos 2x \cdot 2$

$\Rightarrow f'(x) = \cos 2x \ldots\ldots\ldots (i)$

$\Rightarrow 0 = \cos 2x$

$\Rightarrow \cos 2x = \cos\pi/2 \Rightarrow x = \pi/4$

again differentiating both side w.r to x we get

$f''(x) = -\sin 2x \,.\, 2$

$f''(\pi/4) = -\sin\left(2 \times \frac{\pi}{4}\right) \cdot 2 = -2 < 0$

So, f(x) is maximum at $x = \frac{\pi}{4}$

Now, maximum value:

$\Rightarrow f\left(\pi/4\right) = \sin\pi/4 \,.\, \cos\, \pi/4$

$= \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}}$

$\Rightarrow f\left(\pi/4\right) = \frac{1}{2}$

Q25 **Text Solution:**

Given, $\left|\overrightarrow{a}\right| = 3, \left|\overrightarrow{b}\right| = 4$

we know that, If two vectors are perpendicular then their dot product is zero.So,

$\left(\overrightarrow{a} + \lambda\overrightarrow{b}\right) \cdot \left(\overrightarrow{a} - \lambda\overrightarrow{b}\right) = \overrightarrow{a} \cdot \overrightarrow{a} - \lambda\overrightarrow{a} \cdot \overrightarrow{b} + \lambda\overrightarrow{b} \cdot \overrightarrow{a} - \lambda^2\overrightarrow{b} \cdot \overrightarrow{b}$

$\Rightarrow 0 = \left|\overrightarrow{a}\right|^2 - \lambda^2\left|\overrightarrow{b} \cdot \overrightarrow{b}\right|^2$

$\Rightarrow 0 = 3^2 - \lambda^2\left(4\right)^2 \Rightarrow \lambda^2 \cdot 16 = 9$

$\Rightarrow \lambda^2 = \frac{9}{16}$

$\Rightarrow \lambda = \frac{3}{4}$

Q26 **Text Solution:**

Given, $I = \int e^x \sec x(1 + \tan x)dx$

$I = \int e^x \sec x dx + \int e^x \sec x \tan x dx$

$I = \sec x \int e^x dx - \int \left\{\frac{d}{dx}\sec x \int e^x dx\right\}dx + \int e^x \sec x \tan x dx$

$I = \sec x \cdot e^x - \int \sec x \cdot \tan x \cdot e^x dx + \int e^x \sec x \tan x dx + c$

$I = e^x secx + c$

$\therefore I = \int (e^x \sec x + e^x \sec \cdot \tan x) \cdot dx$

$= e^x \sec x + c$

Therefore,option(A) is the correct option.

Q27 **Text Solution:**

Given,

$y = x^2 + 2$

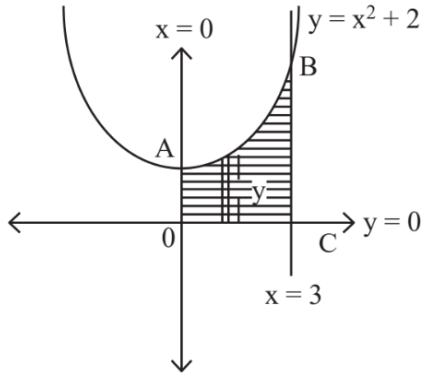
$\Rightarrow (y - 2) = x^2$

$x - axis \Rightarrow y = 0$

$x = 0\, and\, x = 3$

Here, $(y - 2) = x^2$ is a top sided parabola whose vertex is at (0, 2) x-axis means y = 0.

Now plot the arc an on xy-axis we get



Required area = area of OABC

$= \int_0^3 y dx = \int_0^3 (x^2 + 2) dx$

$= \left(\frac{x^3}{3} + 2x\right)_0^3 = \left(\frac{3^3}{3} + 2 \times 3\right)$

$- \left(\frac{0^3}{3} + 2 \times 0\right)$

= (9 + 6) – (0) = 15 sq. units

Q28 Text Solution:

Given, $\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$

We know,

$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2} \Rightarrow \sin^{-1} x = \frac{\pi}{2}$

$- \cos^{-1} x$

Now, $\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$

$\Rightarrow \frac{\pi}{2} - \cos^{-1} x + \frac{\pi}{2} - \cos^{-1} y = \frac{2\pi}{3}$

$\Rightarrow \pi - \left(\cos^{-1} x + \cos^{-1} y\right) = \frac{2\pi}{3}$

$\Rightarrow \cos^{-1} x + \cos^{-1} y = \pi - \frac{2\pi}{3}$

$\therefore \cos^{-1} x + \cos^{-1} y = \frac{\pi}{3}$

Q29 Text Solution:

Given, A and B are invertible of order 3.

$|A| = 2 \; and \; |(AB)^{-1}| = \frac{-1}{6}$

We know, $|A^{-1}| = \frac{1}{|A|}$

Now, $|(AB)^{-1}| = \frac{1}{|AB|} = \frac{-1}{6}$

$\Rightarrow |AB| = -6$

$\Rightarrow |A||B| = -6$

$\Rightarrow 2|B| = -6$

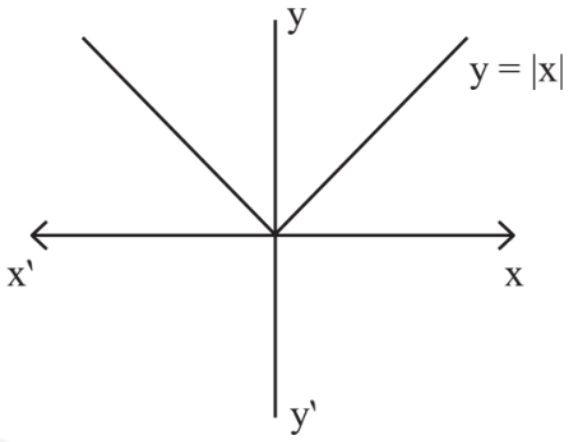
$\Rightarrow |B| = \frac{-6}{2} = -3$

Therefore, option(B) is the correct option.

Q30 Text Solution:

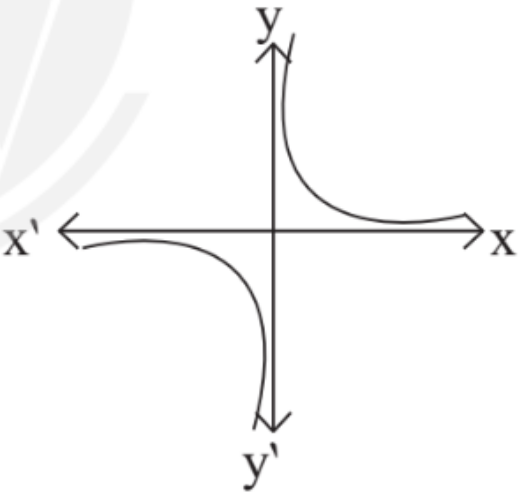
(1) for f(x) = |x| on the basis of graph, we can say that f(x) is continuous ∀x ∈ R

So, Statement (A) is correct.



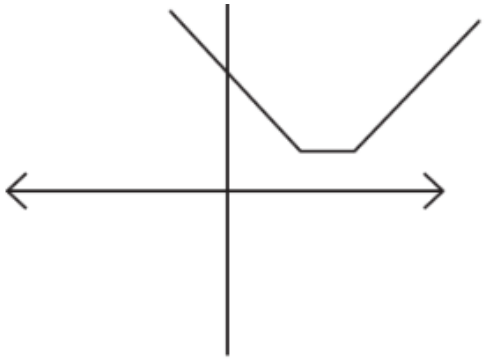
(2) f : R → R f(x) = |x| from above graph, we can conclude the f(x) is continuous for all value of x ∈ R but f(x) in not differentiable at its corner point that is at x = 0. So, statement (B) is also correct.

(3) f : R – {0} → R, f(x) = 1/x from the graph we can say that f(x) is continuous for all real number except 0.

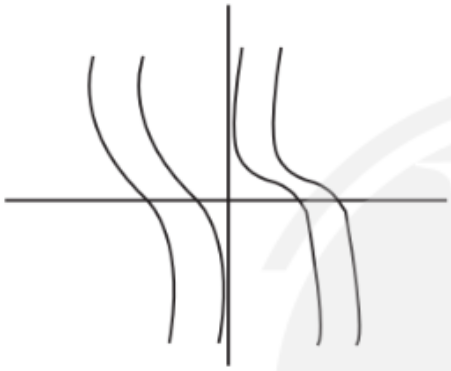


So, statement (c) is also correct.

(4) f : R → R, f(x) = |x – 1| + |x – 2| from the graph of given function we can observe that f(x) is continuous for all real number but not differentiable at its corner point i.e at x=1 and x=2 . So, Statement (4) is correct.



(5) f : R → R, f(x) = cot x from the graph we can clearly observe that f(x) is discontinuous at x=0, x=π or at $x = n\pi$



Therefore, option(C) is the correct option.

Q31 Text Solution:

Given,

$$f(x) = \begin{cases} \frac{k\cos x}{\pi - 2x}, & x \neq \pi/2 \\ 3, & x = \pi/2 \end{cases}$$

we know that, If f(x) is continuous then

LHL = RHL= f(a) ...(i)

Now,

$$\text{LHL} = \lim_{x \to \frac{\pi}{2}^-} \frac{k\cos x}{\pi - 2x}$$

$$= \lim_{h \to 0} \frac{k\cos\left(\frac{\pi}{2} - h\right)}{\pi - 2\left(\frac{\pi}{2} - h\right)}$$

$$= \lim_{h \to 0} \frac{k\sin h}{\pi - \pi + 2h}$$

$$= \lim_{h \to 0} \frac{k\sin h}{2h} = \frac{k}{2} \ldots\ldots\ldots (ii)$$

Now,

$$f\left(\frac{\pi}{2}\right) = 3 \ldots\ldots (iii)$$

From Eqn. (i), (ii) and (iii), we get

$$\Rightarrow \frac{k}{2} = 3$$

$$\Rightarrow k = 6$$

Therefore, option(A) is the correct option.

Q32 Text Solution:

Given,

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 1 & 2 & 1 \\ \lambda & 2 & -3 \end{bmatrix}$$

we know, A^{-1} does not exist for singular matrix.

$$\Rightarrow |A| = 0$$

Now,

$$\Rightarrow |A| = \begin{vmatrix} 1 & -2 & 3 \\ 1 & 2 & 1 \\ \lambda & 2 & -3 \end{vmatrix}$$

$$\Rightarrow 0 = 1(-6-2) + 2(-3-\lambda) + 3(2-2\lambda)$$

$$\Rightarrow 8\lambda = -8 \Rightarrow \lambda = -1$$

Q33 Text Solution:

Given,

$$xdy - ydx = 0$$

$$\Rightarrow \frac{dy}{y} = \frac{dx}{x}$$

on integrating both sides, we get

$$\Rightarrow \int \frac{dy}{y} = \int \frac{dx}{x}$$

$$\Rightarrow \log y = \log x + \log c$$

$$\Rightarrow \log y = \log(xc)$$

$$\Rightarrow y = xc$$

Hence, given differential equation represents family of straight line passing through origin.

Q34 Text Solution:

Given,

A coin is tossed thrice the total outcomes S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

we know,

variance = $E(x^2) - (E(x))^2$

$Probability = \frac{Favourable\ outcome}{Total\ outcomes}$

Now, considering head as random variable.

$P(X=0)=P(TTT)=\frac{1}{8}$

$P(X=1)=P(HTT,THT,TTH)=\frac{3}{8}$

$P(X=2)=P(HHT,HTH,THH)=\frac{3}{8}$

$P(X=3)=P(HHH)=\frac{1}{8}$

So,

x	0	1	2	3
P(x)	1/8	3/8	3/8	1/8
x^2	0	1	4	9
xP(x)	0	3/8	6/8	3/8
$x^2P(x)$	0	3/8	12/8	9/8

Now,

$\text{var}(x)=\Sigma x^2P(x)-\left(\Sigma xP(x)\right)^2$

$=\left[0+\frac{3}{8}+\frac{12}{8}+\frac{9}{8}\right]-\left[0+\frac{3}{8}+\frac{6}{8}+\frac{3}{8}\right]^2$

$=\frac{24}{8}-\left(\frac{12}{8}\right)^2=3-\left(1.5\right)^2=\frac{3}{4}$

$Hence,\ var(x)=\frac{3}{4}$

Q35 Text Solution:

Given,

Objective Function

$z=4x+3y$

Also, corner points of feasible region are,

$(0,\ 0),\ (0,\ 40),\ (20,\ 40),\ (60,\ 20),\ (60,\ 0)$

Now, put the corner points in objective junction

$Z(0,\ 0)=4\times 0+3\times 0=0$

$Z(0,\ 40)=4\times 0+3\times 40=120$

$Z(20,\ 40)=4\times 20+3\times 40=200$

$Z(60,\ 20)=4\times 60+3\times 20=300$

From the above data we can say that. z is maximum at (60, 20) i.e., 300.

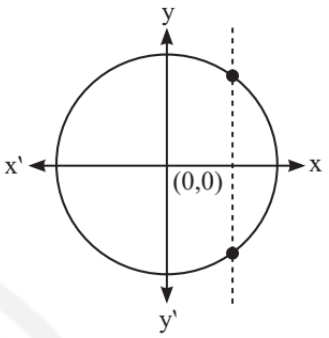
So, value of column A is 300 value of column B is 350 (given)

Hence, on comparing both the columns and we can conclude that column B is greater than column A.

Q36 Text Solution:

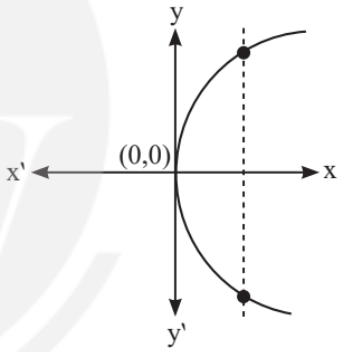
We know, function: It is a special type of relation in which each and every element of its domain is the preimage of unique value in its co-domain.

Graphical way to Identify: Draw a line parallel to y-axis if it cuts at one point then that graph represents a function and if it cuts at more than one point then it is not a function.



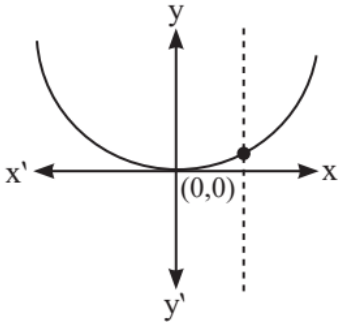
(i)

The line cuts the graph at two points, hence it is not a function.



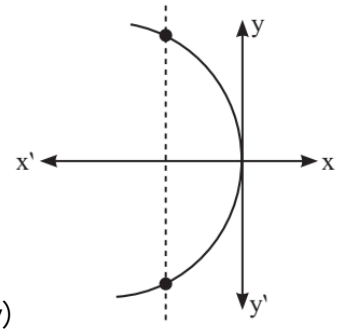
(ii)

The line cuts the graph at two points, hence it is not a function.



(iii)

The line cuts the graph at only one point, hence it is a function.



(iv)

The line cuts the graph at two points, hence it is not a function.

Q37 Text Solution:

Let $I = \int \frac{\sqrt{\tan x}}{\sin x \cdot \cos x} dx$

$\Rightarrow I = \int \frac{\sqrt{\tan x}}{\sin x \cdot \cos x \cdot \frac{\cos x}{\cos x}} dx$

$\Rightarrow I = \int \frac{\sqrt{\tan x}}{\frac{\sin x}{\cos x} \cdot \cos^2 x} dx$

$\Rightarrow I = \int \frac{\sec^2 x}{\sqrt{\tan x}} dx$

Let $\tan x = t \Rightarrow \sec^2 x dx = dt$

$\Rightarrow I = \int \frac{dt}{\sqrt{t}} = \int t^{-1/2} dt$

$\Rightarrow I = \frac{t^{1/2}}{1/2} + c = 2\sqrt{t} + c = 2\sqrt{\tan x} + c$

Therefore, option(A) is the correct option.

Q38 Text Solution:

Given, $A = \begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -2 \\ 3 & -4 \\ 2 & 4 \end{bmatrix}$

Now, $AB = \begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 4 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 3 & -4 \\ 2 & 4 \end{bmatrix}$

$= \begin{bmatrix} 1 \times 1 + (-2) \times 3 + 3 \times 2 & 1 \times (-2) + (-2) \times (-4) + 3 \times 4 \\ -4 \times 1 + 2 \times 3 + 4 \times 2 & (-4) \times (-2) + (2) \times (-4) + 4 \times 4 \end{bmatrix}$

$= \begin{bmatrix} 1 - 6 + 6 & -2 + 8 + 12 \\ -4 + 6 + 8 & 8 - 8 + 16 \end{bmatrix}$

$= \begin{bmatrix} 1 & 18 \\ 10 & 16 \end{bmatrix}$

Hence, the value of AB is $\begin{bmatrix} 1 & 18 \\ 10 & 16 \end{bmatrix}$

Q39 Text Solution:

Given,

$\left[1 + \left(\frac{dy}{dx}\right)\right]^3 = \left(\frac{d^2y}{dx^2}\right)^2$

We know, Degree of the differential equation represents the highest power of highest derivative.

Now, Here highest derivative is 2 and its power is 2. Hence, the degree is 2.

Therefore,option(B) is the correct option.

Q40 Text Solution:

Given, $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = 0$

$R_1 \to R_1 - R_2$; $R_3 \to R_3 - R_2$

$\begin{vmatrix} a & -b & 0 \\ 1 & 1+b & 1 \\ 0 & -b & c \end{vmatrix} = 0$

Expanding and along R_1,we get

$\Rightarrow a\begin{vmatrix} 1+b & 1 \\ -b & c \end{vmatrix} - \left(-b\right)\begin{vmatrix} 1 & 1 \\ 0 & c \end{vmatrix} + 0 = 0$

$\Rightarrow a\Big(\Big(1+b\Big)c - \Big(-b\Big) \times 1\Big)$

$+ b\Big(c - 0\Big) + 0 = 0$

$\Rightarrow a\Big(c + bc + b\Big) + bc = 0$

$\Rightarrow ac + ab + bc = -abc$

$\Rightarrow \frac{ac}{abc} + \frac{ab}{abc} + \frac{bc}{abc} = -\frac{abc}{abc}$

$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = -1$

Q41 Text Solution:

Given, $A = \begin{bmatrix} 3 & x \\ y & 0 \end{bmatrix}$ and $A' = A$

Now, we know. If two matrices are equal then their corresponding elements are also equal.

So,

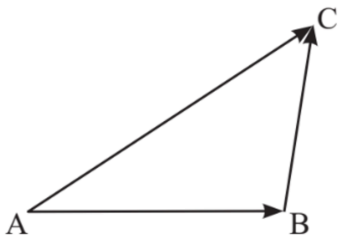
$A = \begin{bmatrix} 3 & x \\ y & 0 \end{bmatrix}$ and $A' = \begin{bmatrix} 3 & y \\ x & 0 \end{bmatrix}$

$$A' = \begin{bmatrix} 3 & y \\ x & 0 \end{bmatrix} = \begin{bmatrix} 3 & x \\ y & 0 \end{bmatrix}$$

on comparing, we get

x = y

Q42 Text Solution:



Given: In ΔABC,

By triangle law of addition

$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

$\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{AC} = \overrightarrow{0}$ $\left(B\right)$ $is\ true.$

$\overrightarrow{AB} + \overrightarrow{BC} - \left(-\overrightarrow{CA}\right) = \overrightarrow{0}$

$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \overrightarrow{0}$ $\left(A\right)$ $is\ true.$

$\overrightarrow{AB} + \left(-\overrightarrow{CB}\right) + \overrightarrow{CA} = \overrightarrow{0}$

$\overrightarrow{AB} - \overrightarrow{CB} + \overrightarrow{CA} = \overrightarrow{0}$ $\left(D\right)$ $is\ true$

Hence, (A), (B) and (D) only are correct.

Q43 Text Solution:

we know,

Conditional Probability

$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}, P\Big(B\Big) \neq 0$

$and\ P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}, P\Big(A\Big) \neq 0$

Also, $P(A \cup B) = P(A) + P(B) - P.$ $(A \cap B)$

Mutual Exclusive Events

P (A ∩ B) = 0

Independent Events

$P\left(A \cap B\right) = P(A)\ .\ P(B)$

Now,

(A) $P(A \cup B) = P(A) + P(B) -$ P if A $(A \cap B)$

and

B are mutually exclusive then,

$P\left(A \cap B\right) = 0$

So, $P(A \cup B) = P(A) + P(B)$

(B) If A and B independent events

P(A ∩ B) = P(A) . P(B)

(C) Conditional probability

$P\Big(A/B\Big) = \frac{P(A \cap B)}{P(B)}, P\Big(B\Big) \neq 0$

(D) Conditional probability

$P\Big(B/A\Big) = \frac{P(A \cap B)}{P(A)}, P\Big(A\Big) \neq 0$

Hence, From Equation (I), (II), (III) and (IV) we can conclude that

A → (IV), (B)→ (III), (C) → (I), (D)→ (II).

Q44 Text Solution:

Given, points (–2, –3, –4) and (1, –2, 4) we know that,

Equation line passing through two points (in vector form)

$\overrightarrow{r} = \overrightarrow{a} + \lambda\left(\overrightarrow{b} - \overrightarrow{a}\right)$.....(i)

Now,

$\overrightarrow{a} = -2\hat{i} - 3\hat{j} - 4\hat{k}$

$\overrightarrow{b} = \hat{i} - 2\hat{j} + 4\hat{k}$

put in Eqn. (i), we get

$\overrightarrow{r} = \left(-2\hat{i} - 3\hat{j} - 4\hat{k}\right) + \lambda\Big[\left(\hat{i} - 2\hat{j} + 4\hat{k}\right) - \left(-2\hat{i} - 3\hat{j} - 4\hat{k}\right)\Big]$

$\overrightarrow{r} = \left(-2\hat{i} - 3\hat{j} - 4\hat{k}\right) + \lambda\left(3\hat{i} + \hat{j} + 8\hat{k}\right)$

hence, the equation of straight line in vector form is

$\overrightarrow{r} = \left(-2\hat{i} - 3\hat{j} - 4\hat{k}\right) + \lambda\left(3\hat{i} + \hat{j} + 8\hat{k}\right)$

Q45 Text Solution:

Given, Equation of planes,

$P_1 : x + y - z$

$P_2 : 3x + 2y + z$

So, $\vec{n}_1 = \hat{i} + \hat{j} - \hat{k}\ and\ \vec{n}_2 = 3\hat{i} + 2\hat{j} + \hat{k}$

we know that,

Angle between two planes is $\cos\theta = \frac{\vec{n}_1 \cdot \vec{n}_2}{\left|\vec{n}_1\right|\left|\vec{n}_2\right|}$

Now,

$\cos\theta = \frac{\left(\hat{i}+\hat{j}-\hat{k}\right)\cdot\left(3\hat{i}+2\hat{j}+\hat{k}\right)}{\sqrt{1^2+1^2+(-1)^2}\sqrt{3^2+2^2+1^2}}$

$\cos\theta = \frac{3+2-1}{\sqrt{3}\sqrt{9+4+1}} = \frac{4}{\sqrt{42}} \times \frac{\sqrt{42}}{\sqrt{42}}$

$\cos\theta = \frac{4\sqrt{42}}{42} = \frac{2\sqrt{42}}{21}$

$\Rightarrow \theta = \cos^{-1}\left(\frac{2\sqrt{42}}{21}\right)$

Q46 Text Solution:

Given, n(P) = 5, n(Q) = 8

For one-one mapping 5 Elements can be selected out of 8 elements of set Q in ${}^8C_5$ ways. and those 5 elements can be mapped with 5 elements of set P in 5 ! ways. So, the number of one-one mapping P to Q

$= {}^8C_5 \times 5!$

Hence the number one-one functions from A to Q $= {}^8C_5 \times 5!$

Q47 Text Solution:

$Given,\ x = a\sin^3 t$

$y = b\cos^3 t\ and\ t = \pi/2$

we know,

Now. Eqn. of tangent is

$y - y_1 = \frac{dy}{dx}(x - x_1)$.....(i)

$\text{Now, } x_1 = a\sin^3\left(\frac{\pi}{2}\right) = a$

$y_1 = b\cos^3\left(\pi/2\right) = 0$

$\Rightarrow \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\frac{d}{dt}b\cos^3 t}{\frac{d}{dt}a\sin^3 t}$

$\Rightarrow \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\frac{d}{dt}b\cos^3 t}{\frac{d}{dt}a\sin^3 t}$

$\Rightarrow \frac{dy}{dx} = \frac{b}{a}\left(\frac{3\cos^2 t\cdot(-\sin t)}{3\sin^2 t(\cos t)}\right)$

$\Rightarrow \frac{dy}{dx} = \frac{-b}{a}\frac{\cos t}{\sin t} = \frac{-b}{a}\cot t$

$\Rightarrow \frac{dy}{dx}\Big|_{t=\pi/2} = \frac{-b}{a}\cot\frac{\pi}{2} = 0$

Now, put the values of $x_1, y_1\ and\ \frac{dy}{dx}$ in eqn. (i), we get

$\Rightarrow$ y – 0 = 0 (x – a) $\Rightarrow$ y = 0

Hence, the equation of tangent is y = 0.

Q48 Text Solution:

Given, one side of triangle = 10 cm another side=12 cm,

Now, we know the area of triangle

$A = \frac{1}{2}ab\sin\theta$

On differentiating both side w.r.t. θ, we get

$\frac{d}{d\theta}A = \frac{1}{2}ab\frac{d}{d\theta}\sin\theta$

$\Rightarrow \frac{dA}{d\theta} = \frac{1}{2}ab\left(\cos\theta\right)$

$\text{Put } a = 10\ cm,\ b = 12\ cm,\ \theta = \pi/3$

$\Rightarrow \frac{dA}{d\theta} = \frac{1}{2} \times 10 \times 12 \times \left(\cos\pi/3\right)$

$\Rightarrow \frac{dA}{d\theta} = \frac{1}{2} \times 10 \times 12 \times \frac{1}{2}$

$\Rightarrow \frac{dA}{d\theta} = 30\ cm^2/radian$

Hence, area of triangle is decreasing at the rate of $30cm^2/radian.$

Q49 Text Solution:

Given, From graph.

Equation of parabola, $\Rightarrow y = x^2 + 2$

$x = 0, y = 0, x = 3, y = 2x + 2$

Now, on the basis of shaping from due given graph

⇒ Shading is below the parabola and lies in first quadrant only.

So,

$0 \leq y \leq x^2 + 2$.....(i)

⇒ Line which passes through both x and y axis,

y= 2x + 2.

The shading is also done below the line So,

$0 \leq y \leq 2x + 2$ ...(ii)

⇒ From the remaining lines x = 0 and x = 3, the shading is done between the mu. So, we can say that x lies between 0 and 3

$0 \leq x \leq 3$.....(iii)

Hence,

$\{(x, y); 0 \leq y \leq x^2 + 2, 0 \leq y \leq 2x + 2, 0 \leq x \leq 3\}$

represents the shaded area correctly.

Q50 Text Solution:

Given,

$\text{Equation of floor } P_1 \Rightarrow x + y - z + 4 = 0$

$\text{Equation of roof } P_2 \Rightarrow x + y - z + 5 = 0$

We know, Distance between two parallel planes,

$d = \left|\frac{d_2 - d_1}{\sqrt{a^2+b^2+c^2}}\right|$

$So, \ d_1 = 4, d_2 = 5, a = 1, b = 1, c = -1$

$D = \left|\frac{5-4}{\sqrt{1^2+1^2+(-1)^2}}\right| = \frac{1}{\sqrt{3}}$

Hence, Height of the room is $\frac{1}{\sqrt{3}}$.