Q1 If a line makes an angle of $\frac{\pi}{3}$ with each X and Y axis, then the acute angle made by Z-axis is

(A) p/3 (B) p/2

(C) p/3 (D) p/6

Q2 The length of perpendicular drawn from the point (3, -1, 11) to the line $\frac{x}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ is

(A) $\sqrt{29}$ (B) $\sqrt{33}$

(C) $\sqrt{53}$ (D) $\sqrt{66}$

Q3 The equation of the plane through the points (2, 1, 0), (3, 2, -2) and (3, 1, 7) is

(A) 2x - 3y + 4z - 27 = 0

(B) 6x - 3y + 2z - 7 = 0

(C) 7x - 9y - z - 5 = 0

(D) 3x - 2y + 6z - 27 = 0

Q4 The point of intersection of the line $x+1=\frac{y+3}{3}=\frac{-z+2}{2}$ with the plane 3x + 4y + 5z = 10 is

(A) (2, -6, -4) (B) (2, 6, -4)

(C) (2, 6, 4) (D) (-2, 6, -4)

Q5 If (2, 3, -1) is the foot of the perpendicular from (4, 2, 1) to a plane, then the equation of the plane is

(A) 2x + y + 2z - 1 = 0

(B) 2x - y + 2z = 0

(C) 2x + y + 2z - 5 = 0

(D) 2x - y + 2z + 1 = 0

Q6 $|a\times b|^2+|a\cdot b|^2=144$ and $|a|=4$, then |b| is equal to

(A) 3 (B) 8

(C) 4 (D) 12

Q7 If a + 2b + 3c = 0 and (a x b) + (b x c) + (c x a) = l(b x c), then the value of l is equal to

(A) 3 (B) 4

(C) 6 (D) 2

Q8 A bag contains 2n + 1 coins. It is known that n of these coins have head on both sides whereas, the other n + 1 coins are fair. One coin is selected at random and tossed. If the probability that toss results in heads is 31/42, then the value of n is

(A) 6 (B) 8

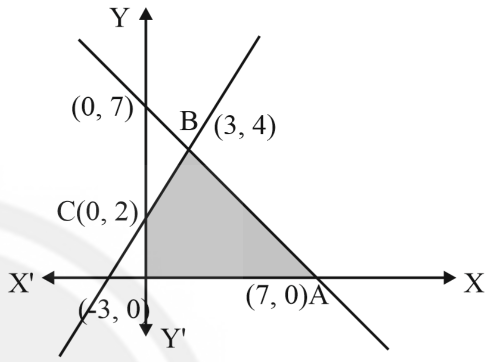
(C) 10 (D) 5

Q9 Let A = {x, y, z, u} and B = {a, b}. A function f : A → B is selected randomly. The probability that the function is an onto function is

(A) 1/8 (B) 5/8

(C) 1/35 (D) 7/8

Q10 The shaded region in the figure given is the solution of which of the inequations?



(A) $x+y\geq 7,\ 2x-3y+6\leq 0,\ x\geq 0,\ y\geq 0$

(B) $x+y\geq 7,\ 2x-3y+6\geq 0,\ x\geq 0,\ y\geq 0$

(C) $x+y\leq 7,\ 2x-3y+6\leq 0,\ x\geq 0,\ y\geq 0$

(D) $x+y\leq 7,\ 2x-3y+6\geq 0,\ x\geq 0,\ y\geq 0$

Q11 If A and B are events, such that $P(A)=\frac{1}{4}$, $P(A/B)=\frac{1}{2}$ and $P(B/A)=\frac{2}{3}$, then P(B) is

(A) 1/3 (B) 2/3

(C) 1/2 (D) 1/6

Q12 The value of $e^{\log_{10}\tan 1^{\circ}+\log_{10}\tan 2^{\circ}+\log_{10}\tan 3^{\circ}+....+\log_{10}\tan 89^{\circ}}$ is

(A) 3 (B) 1/e

(C) 1 (D) 0

Q13 The value of $\begin{vmatrix}\sin^2 14^{\circ} & \sin^2 66^{\circ} & \tan 135^{\circ}\\ \sin^2 66^{\circ} & \tan 135^{\circ} & \sin^2 14^{\circ}\\ \tan 135^{\circ} & \sin^2 14^{\circ} & \sin^2 66^{\circ}\end{vmatrix}$ is

(A) 0 (B) 1

(C) 2 (D) -1

Q14 The modulus of the complex number $\frac{(1+i)^2(1+3i)}{(2-6i)(2-2i)}$ is

(A) $\frac{2}{\sqrt{2}}$ (B) $\frac{1}{\sqrt{2}}$

(C) $\frac{\sqrt{2}}{4}$ (D) $\frac{4}{\sqrt{2}}$

Q15 Given that a, b and x are real numbers and a < b, x < 0, then

(A) $\frac{a}{x} \geq \frac{b}{x}$ (B) $\frac{a}{x} < \frac{b}{x}$

(C) $\frac{a}{x} \leq \frac{b}{x}$ (D) $\frac{a}{x} > \frac{b}{x}$

Q16 Ten chairs are numbered as 1 to 10. Three women and two men wish to occupy one chair each. First the women choose the chairs marked 1 to 6 , then the men choose the chairs from the remaining. The number of possible ways is

(A) ${}^6P_3 \times {}^4P_2$ (B) ${}^6C_3 \times {}^4P_2$

(C) ${}^6P_3 \times {}^4C_2$ (D) ${}^6C_3 \times {}^4C_2$

Q17 Which of the following is an empty set?

(A) $\{x : x^2 + 1 = 0,\ x \in R\}$

(B) $\{x : x^2 - 9 = 0, x \in R\}$

(C) $\{x : x^2 = x + 2,\ x \in R\}$

(D) $\{x : x^2 - 1 = 0,\ x \in R\}$

Q18 If f(x) = ax + b, where a and b are integers, f(-1) = -5 and f(3) = 3, then a and b are respectively.

(A) 2, -3 (B) 0, 2

(C) 2, 3 (D) -3, -1

Q19 If $p\left(\frac{1}{q} + \frac{1}{r}\right),\ q\left(\frac{1}{r} + \frac{1}{p}\right),\ r\left(\frac{1}{p} + \frac{1}{q}\right)$ are in AP, then p, q, r

(A) are in GP (B) are not in GP

(C) are in AP (D) are not in AP

Q20 A line passes through (2, 2) and is perpendicular to the line 3x + y = 3. Its y-intercept is

(A) 2/3 (B) 1

(C) 4/3 (D) 1/3

Q21 The distance between the foci of a hyperbola is 16 and its eccentricity is $\sqrt{2}$. Its equation is

(A) $\frac{x^2}{4} - \frac{y^2}{9} = 1$

(B) $2x^2 - 3y^2 = 7$

(C) $y^2 - x^2 = 32$

(D) $x^2 - y^2 = 32$

Q22 If $\lim_{x \to 0} \frac{\sin(2+x) - \sin(2-x)}{x}$ = A cos B, then the values of A and B respectively are

(A) 1, 2 (B) 2, 1

(C) 1, 1 (D) 2, 2

Q23 if n is even and the middle term in the expansion of $\left(x^2 + \frac{1}{x}\right)^n$ is $924x^6$, then n is equal to

(A) 14 (B) 12

(C) 8 (D) 10

Q24 nth term of the series $1 + \frac{3}{7} + \frac{5}{7^2} + \frac{1}{7^2} + \ldots$ is

(A) $\frac{2n+1}{7^n}$

(B) $\frac{2n-1}{7^n}$

(C) $\frac{2n+1}{7^n - 1}$

(D) $\frac{2n-1}{7^n - 1}$

Q25 $f : R \to R$ adn $g : [0,\ \infty)$.

$\to R$ defined by $f(x) = x^2$ and $g(x) = \sqrt{x}$

Which one of the following is not true?

(A) $(f \circ g)(-4) = 4$

(B) $(f \circ g)(2) = 2$

(C) $(g \circ f)(-2) = 2$

(D) $(g \circ f)(4) = 4$

Q26 Let f : $f : R \to R$ be defined by $f(x) = 3x^2 - 5$ and $g : R \to R$ by $g(x)$ then $= \frac{x}{x^2+1}$,

g o f is

(A) $\frac{3x^2-5}{9x^4-6x^2+26}$

(B) $\frac{3x^2}{x^4+2x^2-4}$

(C) $\frac{3x^2}{9x^4+30x^2-2}$

(D) $\frac{3x^2-5}{9x^4-30x^2+26}$

Q27 Let the relation R be defined in N by aRb, if 3a + 2b = 27, then R is

(A) $\left\{\left(0,\ \frac{27}{2}\right),\ (1,\ 12),\ (3,\ 9),\ (5,\ 6),\ (7,\ 3)\right\}$

(B) {(1, 12), (3, 9), (5, 6) (7, 3), (9, 0)}

(C) {(2, 1), (9, 3), (6, 5), (3, 7)}

(D) {(1, 12), (3, 9), (5, 6), (7, 3)}

Q28 Let f(x) = sin 2x + cos 2x and g(x) = x^2 - 1 then g(f(x)) is invertible in the domain

(A) $x \in \left[\frac{-\pi}{8},\ \frac{\pi}{8}\right]$

(B) $x \in \left[\frac{-\pi}{2},\ \frac{\pi}{2}\right]$

(C) $x \in \left[0,\ \frac{\pi}{4}\right]$

(D) $x \in \left[\frac{-\pi}{4},\ \frac{\pi}{4}\right]$

Q29 The contrapositive of the statement.
"If two lines do not intersect in the same plane, then they are parallel." is
(A) If two lines are parallel, then they intersect in the same plane.
(B) B If two lines are not parallel, then they do not intersect in the same plane.
(C) If two lines are parallel, then they do not intersect in the same plane.
(D) If two lines are not parallel, then they intersect in the same plane.

Q30 The mean of 100 observations is 50 and their standard deviation is 5. Then, the sum of squares of all observations is
(A) 252500 (B) 250000
(C) 255000 (D) 50000

Q31 If $x\begin{bmatrix}3\\2\end{bmatrix}+y\begin{bmatrix}1\\-1\end{bmatrix}=\begin{bmatrix}15\\5\end{bmatrix}$, then the value of x and y are
(A) x = 4, y = -3 (B) x = -4, y = -3
(C) x = -4, y = 3 (D) x = 4, y = 3

Q32 If A and B are two matrices, such that AB = B and BA = A, then $A^2 + B^2$ equals to
(A) 2AB (B) AB
(C) 2BA (D) A + B

Q33 If $A=\begin{bmatrix}2-k & 2\\ 1 & 3-k\end{bmatrix}$ is singular matrix, then the value of 5k - k^2 is equal to
(A) -6 (B) -4
(C) 6 (D) 4

Q34 The area of a triangle with vertices (-3, 0), (3, 0) and (0, k) is 9 sq units, find the value of k is
(A) -9 (B) 6
(C) 3 (D) 9

Q35 If $\Delta=\begin{vmatrix}1 & a & a^2\\ 1 & b & b^2\\ 1 & c & c^2\end{vmatrix}$ and $\Delta_1=\begin{vmatrix}1 & 1 & 1\\ bc & ca & ab\\ a & b & c\end{vmatrix}$, then
(A) $\Delta_1 = 3\Delta$
(B) $\Delta_1 \neq \Delta$
(C) $\Delta_1 = -\Delta$
(D) $\Delta_1 = \Delta$

Q36 If
$\sin^{-1}\left(\frac{2a}{1+a^2}\right)+\cos^{-1}\left(\frac{1-a^2}{1+a^2}\right)=\tan^{-1}\left(\frac{2x}{1-x^2}\right)$
where a, $x \in (0,\ 1)$ then the value of x is
(A) a/2 (B) $\frac{2a}{1+a^2}$
(C) $\frac{2a}{1-a^2}$ (D) 0

Q37 The value of $\cot^{-1}\left[\frac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}-\sqrt{1+\sin x}}\right]$, where x is $\in\left(0,\ \frac{\pi}{4}\right)$
(A) $\frac{x}{2}-\pi$
(B) $\pi-\frac{x}{3}$
(C) $\pi-\frac{x}{2}$
(D) x/2

Q38 The function f(x) = cot x is discontinuous on every point of the set
(A) $\{x = 2n\pi;\ n \in Z\}$
(B) $\{x = (2n+1)\frac{\pi}{2};\ n \in Z\}$
(C) $\{x = \frac{n\pi}{2};\ n \in Z\}$
(D) $\{x = n\pi;\ n \in Z\}$

Q39 If the function is f(x) = 1/x+2, then the point of discontinuity of the composite function y = f(f(x)) is
(A) 5/2 (B) 2/5
(C) 1/2 (D) -5/2

Q40 If y = a sin x + b cos x, then $y^2+\left(\frac{dy}{dx}\right)^2$ is
(A) function of y
(B) function of x and y
(C) constant
(D) function of x

Q41 If
$f(x) = 1 + nx + \frac{n(n-1)}{2}x^2 + \frac{n(n-1)(n-2)}{6}x^3 + \ldots + x^n$, then $f^n(1)$
is equal to:
(A) $n(n-1)2^{n-2}$
(B) $n(n-1)2^n$
(C) 2^{n-1}
(D) $(n-1)2^{n-1}$

Q42 If $A = \begin{bmatrix} 1 & \tan\alpha/2 \\ -\tan\alpha/2 & 1 \end{bmatrix}$ and $AB = I$, then B is equal to

(A) $\cos^2\alpha/2 \cdot A$

(B) $\cos^2\alpha/2 \cdot I$

(C) $\sin^2\alpha/2 \cdot A$

(D) $\cos^2\alpha/2 \cdot A^T$

Q43 If $u = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ and $v = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, is then $\frac{du}{dv}$

(A) 2 (B) $\frac{1-x^2}{1+x^2}$

(C) 1 (D) 1/2

Q44 The distance 's' in meters travelled by a particle in 't' seconds is given by $s = \frac{2t^3}{3} - 18t + \frac{5}{3}$. The acceleration when the particle comes to rest is :

(A) 10 m^2/s (B) 12 m^2/s

(C) 18 m^2/s (D) 3 m^2/s

Q45 A particle moves along the curve $\frac{x^2}{16} + \frac{y^2}{4} = 1$. When the rate of change of abscissa is 4 times that of its ordinate, then the quadrant in which the particle lies is

(A) II or IV (B) III or IV

(C) II or III (D) I or III

Q46 An enemy fighter jet is flying along the curve, given by y = x^2 + 2. A soldier is placed at (3, 2) wants to shoot down the jet when it is nearest to him. Then, the nearest distance is

(A) $\sqrt{6}$ units (B) 2 units

(C) $\sqrt{5}\ units$ (D) $\sqrt{3}$ units

Q47 $\int_2^8 \frac{5^{\sqrt{10-x}}}{5^{\sqrt{x}}+5^{\sqrt{10-x}}}$ is equal to:

(A) 6 (B) 4

(C) 3 (D) 5

Q48 $\int \sqrt{\operatorname{cosec} x - \sin x}\ dx$ is equal to

(A) $\frac{\sqrt{\sin x}}{2} + C$ (B) $2\sqrt{\sin x} + C$

(C) $\frac{2}{\sqrt{\sin x}} + C$ (D) $\sqrt{\sin x} + C$

Q49 If f(x) and g(x) are two functions with g(x) = x - 1/x and f o g(x) $= x^3 - \frac{1}{x^3}$, then $f'(x)$ is equals to

(A) $3x^2 + \frac{3}{x^4}$ (B) $x^2 - \frac{1}{x^2}$

(C) $1 - \frac{1}{x^2}$ (D) $3x^2 + 3$

Q50 A circular plate of radius 5 cm is heated. Due to expansion, its radius increase at the rate of 0.05 cm/s. The rate at which its area is increasing when the radius is 5.2 cm is

(A) 27.4p cm^2/s (B) 5.05p cm^2/s

(C) 0.52p cm^2/s (D) 5.2p cm^2/s

Q51 $\int_{-2}^{0}\left(x^3 + 3x^2 + 3x + 3 + (x+1)\cos(x+1)\right)dx$ is equals to

(A) 3 (B) 4

(C) 1 (D) 0

Q52 $\int_0^\pi \frac{x \tan x}{\sec x \cdot \operatorname{cosec} x} dx$ is equals to

(A) $p^2/4$ (B) p/2

(C) $p^2/2$ (D) p/4

Q53 $\int \sqrt{5 - 2x + x^2}\ dx$ is equals to

(A) $\frac{x}{2}\sqrt{5-2x+x^2} + 4\log\left|(x+1) + \sqrt{x^2-2x+5}\right| + C$

(B) $\frac{x-1}{2}\sqrt{5+2x+x^2} + 2\log\left|(x-1) + \sqrt{5+2x+x^2}\right| + C$

(C) $\frac{x-1}{2}\sqrt{5-2x+x^2} + 2\log\left|(x-1) + \sqrt{5-2x+x^2}\right| + C$

(D) $\frac{x-1}{2}\sqrt{5-2x+x^2} + 2\log\left|(x+1) + \sqrt{x^2+2x+5}\right| + C$

Q54 $\int \frac{1}{1+3\sin^2 x+8\cos^2 x} dx$ is equals to

(A) $\tan^{-1}\left(\frac{2\tan x}{3}\right) + C$

(B) $\frac{1}{6}\tan^{-1}\left(\frac{2\tan x}{3}\right) + C$

(C) $6\tan^{-1}\left(\frac{2\tan x}{3}\right) + C$

(D) $\frac{1}{6}\tan^{-1}(2\tan x) + C$

Q55 If a curve passes through the point (1, 1) and at any point (x, y) on the curve, the product of the slope of its tangent and x coordinate of the point is equal to the y coordinate of the point, then the curve also passes through the point

(A) (3, 0) (B) (-1, 2)

(C) ($\sqrt{3}$, 0) (D) (2, 2)

Q56 The degree of the differential equation $1 + \left(\frac{dy}{dx}\right)^2 + \left(\frac{d^2y}{dx^2}\right)^2 = \sqrt[3]{\frac{d^2y}{dx^2} + 1}$ is

(A) 3 (B) 1

(C) 2 (D) 6

Q57 If $\left|\vec{a} + \vec{b}\right| = \left|\vec{a} - \vec{b}\right|$, then

(A) $\vec{a}$ and $\vec{b}$ -are parallel

(B) $\vec{a}$ and $\vec{b}$ are coincident.

(C) Inclined to each other at 60^o

(D) $\vec{a}$ and $\vec{b}$ -are perpendicular

Q58 The component of $\hat{i}$ in the direction of the vector $\hat{i}+\hat{j}+2\hat{k}$ is

(A) 6 (B) $6\sqrt{6}$

(C) $\frac{\sqrt{6}}{6}$ (D) $\sqrt{6}$

Q59 In the interval (0, p/2) area lying between the curves y = tan x and y = cot x and the X-axis is

(A) 2 log 2 sq. units

(B) 4 log 2 sq. units

(C) log 2 sq. units

(D) 3 log 2 sq. units

Q60 The area of the region bounded by the line y = x + 1 and the lines x = 3 and x = 5 is

(A) 7/2 sq. units (B) 11/2 sq. units

(C) 7 sq. units (D) 10 sq. units

Answer Key

Q1	C
Q2	C
Q3	C
Q4	B
Q5	D
Q6	A
Q7	C
Q8	C
Q9	D
Q10	D
Q11	A
Q12	C
Q13	A
Q14	C
Q15	D
Q16	A
Q17	A
Q18	A
Q19	C
Q20	C
Q21	D
Q22	D
Q23	B
Q24	D
Q25	A
Q26	D
Q27	D
Q28	A
Q29	D
Q30	A
Q31	D
Q32	D
Q33	D
Q34	C
Q35	C
Q36	C
Q37	C
Q38	D
Q39	D
Q40	C
Q41	A
Q42	D
Q43	C
Q44	B
Q45	A
Q46	C
Q47	C
Q48	B
Q49	D
Q50	C
Q51	B
Q52	A
Q53	C
Q54	B
Q55	D
Q56	D
Q57	D
Q58	C
Q59	C
Q60	D

Hints & Solutions

Note: scan the QR code to watch video solution

Q1 Text Solution:

Given, $\alpha = \beta = \frac{\pi}{3}$

Let acute angle made by Z-axis be g.

We know that

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

$$\Rightarrow \left(\tfrac{1}{2}\right)^2 + \left(\tfrac{1}{2}\right)^2 + \cos^2 \gamma = 1$$

$$\Rightarrow \tfrac{1}{4} + \tfrac{1}{4} + \cos^2 \gamma = 1$$

$$\Rightarrow \cos^2 \gamma = 1 - \tfrac{1}{2} \Rightarrow \cos \gamma = \pm \tfrac{1}{\sqrt{2}}$$

$$\Rightarrow \gamma = \tfrac{\pi}{4} \quad [\because y \text{ is acute}]$$

Q2 Text Solution:

Let the foot point of the perpendicular drawn from the point P(3, -1, 11) on the straight line be L.

\ Hence, L lies on the straight line

Let $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4} = t$

\ L(2t, 2 + 3t, 3 + 4t) [where, t is arbitrary constant]

\ The direction ratios of PL are (2t - 3, 2 + 3t + 1, 3 + 4t - 11) or (2t - 3, 3t + 3, 4t - 8)

Again, the direction ratios of the straight line.

$\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ are $(2,\ 3,\ 4)$

Since, PL is perpendicular on the straight line.

Then;

$$(2t-3)\cdot 2 + (3t+3)\cdot 3 + (4t-8)\cdot 4 = 0$$

$$4t - 6 + 9t + 9 + 16t - 32 = 0$$

$$29t = 29$$

$$t = 1$$

Hence, $L(2, 5, 7)$

$$\therefore (PL)$$

$$= \sqrt{(2-3)^2 + (5+1)^2 + (7-11)^2} = \sqrt{53}$$

Q3 Text Solution:

We know that the equation of a plane passing through three non-collinear points.

$(x_1, y_1, z_1), (x_2, y_2, z_2)$ and (x_3, y_3, z_3)

$$\because \begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x - 2 & y - 1 & z - 0 \\ 3 - 2 & 2 - 1 & -2 - 0 \\ 3 - 2 & 1 - 1 & 7 - 0 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x - 2 & y - 1 & z \\ 1 & 1 & -2 \\ 1 & 0 & 7 \end{vmatrix} = 0$$

$$\Rightarrow (x-2)(7-0) - (y-1)(7+2) + z(0-1) = 0$$

$$\Rightarrow 7x - 14 - (y-1)9 + (-z) = 0$$

$$\Rightarrow 7x - 9y - z - 5 = 0$$

Q4 Text Solution:

Let $\frac{x+1}{1} = \frac{y+3}{3} = \frac{-z+2}{2} = k$

Thus, any point on this line will have co-ordinates.

x = k - 1, y = 3k - 3, z = -2k + 2

This line intersects the plane 3x + 4y + 5z = 10

Since, the value of x, y, z must satisfy equation of plane.

$$\Rightarrow 3(k-1) + 4(3k-3) + 5(-2k+2) = 10$$

$$\Rightarrow 3k - 3 + 12k - 12 + 10 - 10k = 10$$

$$\Rightarrow 5k - 5 = 10$$

$$\Rightarrow k = 3$$

$\therefore$ The point of intersection is (x, y, z)

$$\Rightarrow x = 3 - 1, y = 3 \times 3 - 3, z = -2 \times 3 + 2$$

$$\Rightarrow (x, y, z) = (2, 6, -4)$$

Q5 Text Solution:

Since, the line joining the two points is perpendicular to the plane, it's DR's will given the normal to the plane.

DR's of the normal = (4 - 2, 2 - 3, 1 + 1) = (2, -1, 2)

Hence, a = 2, b = -1 and c = 2

Since, the plane passes through (2, 3, -1)

d = 2(2) + 3(-1) - 1(2)

d = -1

Hence, the required equation of plane is 2x - y + 2z = -1

Q6 Text Solution:

Given, $|a \times b|^2 + |a \cdot b|^2 = 144$ and $|a| = 4$

$\Rightarrow |a|^2|b|^2 \sin^2\theta + |a||b|^2 \cos^2\theta = 144$

$\Rightarrow |a|^2|b|^2 [\sin^2\theta + \cos^2\theta] = 144$

$\Rightarrow (4)^2 \times |b|^2 = 144$

$\Rightarrow |b|^2 = \frac{144}{16} = 9$

$|b| = 3$

Q7 Text Solution:

Given, $a + 2b + 3c = 0$

$(a + 2b + 3c) \times c = 0 \Rightarrow c \times a = 2(b \times c)$

$(a + 2b + 3c) \times b = 0$

$\Rightarrow a \times b = 3(b \times c)$

So, $(b \times c) + (c \times a) + (a \times b) = \lambda(b \times c)$

$(b \times c) + 2(b \times c) + 3(b \times c) = \lambda(b \times c)$

$6(b + c) = \lambda(b \times c)$

On comparing with coefficient of (b x c), we get

l = 6

Q8 Text Solution:

Given, n coins have head on both the sides and (n + 1) coins are fair coins.

Total coins = 2n + 1

Let evetns E_1, E_2 be the following

E_1 = Event that an unfair coin is selected

E_2 = Event that a fair coin is selected

$\therefore P(E_1) = \frac{n}{2n+1}$ and $P(E_2) = \frac{n+1}{2n+1}$

From the law of total probability,

$\therefore P(E) = P(E_1) \times P\left(\frac{E}{E_1}\right) + P(E_2) \times P\left(\frac{E}{E_2}\right)$

$\Rightarrow \frac{31}{42} = \frac{n}{2n+1} \times 1 + \frac{n+1}{2n+1} \times \frac{1}{2}$

$\Rightarrow \frac{31}{42} = \frac{2n+n+1}{2(2n+1)}$

$\Rightarrow 31 \times 2(2n + 1) = 42(3n + 1)$

$\Rightarrow 124n + 62 = 126n + 42$

$\Rightarrow 2n = 20$

$\Rightarrow n = 10$

Q9 Text Solution:

Total cases $= 2 \times 2 \times 2 \times 2 = 16$

We know that

Probability of onto = 1 - (Probability of not onto)

Probability $= \frac{\text{Number of favorable event}}{\text{Total event}}$

Probability of onto $= 1 - \frac{2}{16} = \frac{14}{16} = \frac{7}{8}$

Q10 Text Solution:

The line joining A(7, 0) and (0, 7) is

x + y = 7 ...(i)

And the line joing C(0, 2) and B(3, 4) is

2x - 3y + 6 = 0

$x,\ y \geq 0$ ...(ii)

The shaded region is bounded by $x + y \leq 7,\ 2x - 3y + 6 \geq 0,\ x \geq 0,\ y \geq 0$

Hence, option (d) is correct option.

Q11 Text Solution:

Given, $P(A) = \frac{1}{4}, P\left(\frac{A}{B}\right) = \frac{1}{2}$ and $P\left(\frac{B}{A}\right) = \frac{2}{3}$

We know that, $P\left(\frac{E_1}{E_2}\right) = \frac{P(E_1 \cap E_2)}{P(E_2)}$

$\therefore P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)}$

$\Rightarrow \frac{2}{3} = \frac{P(A \cap B)}{1/4} \quad [\because A \cap B = B \cap A]$

$\Rightarrow P(A \cap B) = \frac{2}{3} \times \frac{1}{4} = \frac{1}{6}$

$\because P\left(\frac{A}{B}\right) = \frac{1}{2}$

$\therefore \frac{P(A \cap B)}{P(B)} = \frac{1}{2}$

$\Rightarrow P(B) = 2P(A \cap B) = 2 \times \frac{1}{6} = \frac{1}{3}$

Q12 Text Solution:

We know that

$\tan\theta \tan(90^\circ - \theta) = \tan\theta \cdot \cot\theta = 1$

$\tan 1^\circ \tan 89^\circ = \tan 2^\circ \tan 88^\circ = \tan 3^\circ \tan 87^\circ = \tan 44^\circ \tan 46^\circ = 1$

So, $\log_{10} \tan 1^\circ + \log_{10} \tan 2^\circ \ldots \log_{10} \tan 89^\circ$

$= (\log_{10} \tan 1^\circ + \log_{10} \tan 89^\circ)$

$+ (\log_{10} \tan 2^\circ + \log_{10} \tan 88^\circ) + \ldots$

$+ (\log_{10} \tan 44^\circ + \log_{10} \tan 46^\circ) + \log_{10} \tan 45^\circ$

$= \log_{10}(\tan 1^\circ + \tan 89^\circ) + \log_{10} (\tan 2^\circ + \tan 88^\circ) + \ldots + \log_{10} (\tan 44^\circ + \tan 46^\circ) + \log_{10} \tan 45^\circ = \log_{10} 1 + \ldots + \log_{10} 1 + \log_{10} 1$

$= 0$

Thus, $e^{e^{\log_{10} \tan 1^\circ + \log_{10} \tan 2^\circ + \ldots + \log_{10} \tan 89^\circ}})$

$= e^0 = 1$

Q13 Text Solution:

Here, $\begin{vmatrix} \sin^2 14^\circ & \sin^2 66^\circ & \tan 135^\circ \\ \sin^2 66^\circ & \tan 135^\circ & \sin^2 14^\circ \\ \tan 135^\circ & \sin^2 14^\circ & \sin^2 66^\circ \end{vmatrix}$

Applying $C_1 \rightarrow C_1 + C_2 + C_3$

$$= \begin{vmatrix} \sin^2 14^\circ + \sin^2 66^\circ + \tan 135^\circ & \sin^2 66^\circ & \tan 135^\circ \\ \sin^2 14^\circ + \sin^2 66^\circ + \tan 135^\circ & \tan 135^\circ & \sin^2 14^\circ \\ \sin^2 14^\circ + \sin^2 66^\circ + \tan 135^\circ & \sin^2 14^\circ & \sin^2 66^\circ \end{vmatrix}$$

$$= \sin^2 14^\circ + \sin^2 66^\circ + \tan 135^\circ \begin{vmatrix} 1 & \sin^2 66^\circ & \tan 135^\circ \\ 1 & \tan 135^\circ & \sin^2 14^\circ \\ 1 & \sin^2 14^\circ & \sin^2 66^\circ \end{vmatrix}$$

$$= \sin^2 14^\circ + \sin^2 66^\circ - 1 \begin{vmatrix} 1 & \sin^2 66 & -1 \\ 1 & -1 & \sin^2 14^\circ \\ 1 & \sin^2 14^\circ & \sin^2 66^\circ \end{vmatrix}.$$

Applying $R_1 \rightarrow R_1 - R_2,\ R_2 \rightarrow R_2 - R_3$

$$\begin{vmatrix} 0 & 1 + \sin^6 66^\mathrm{o} & -1 - \sin^2 14^\mathrm{o} \\ 0 & -1 - \sin^2 14^\mathrm{o} & \sin^2 14^\mathrm{o} - \sin^2 66^\mathrm{o} \\ 1 & \sin^2 14^\mathrm{o} & \sin^2 66^\mathrm{o} \end{vmatrix}$$

$$\left(\cos^2 66^\mathrm{o} + \sin^2 66^\mathrm{o} - 1\right) = 0$$

Q14 Text Solution:

Here $\frac{(1+i)^2(1+3i)}{(2-6i)(2-2i)} = \frac{(1+i^2+2i)(1+3i)}{4-4i-12i+12i^2}$

$= \frac{2i+6i^2}{-8-16i} = \frac{2i-6}{-8-16i}$

$= \frac{i-3}{-4-8i}$

$= \frac{3-i}{(4+8i)} \times \frac{(4-8i)}{(4-8i)}$

$= \frac{12-28i-8}{16+64}$

$= \frac{4-28i}{80} = \frac{1-7i}{20}$

$= \frac{1}{20} - \frac{7i}{20}$

Let $z = x + iy$

$\because \left|z\right| = \sqrt{x^2 + y^2}$

$\Rightarrow \left|z\right| = \sqrt{\left(\frac{1}{20}\right)^2 + \left(\frac{7}{20}\right)^2}$

$\Rightarrow |z| = \frac{1}{20}\sqrt{50} = \frac{5}{20}\sqrt{2}$

$\Rightarrow\ |z| = \frac{\sqrt{2}}{4}$

Q15 Text Solution:

Given, a < b

Dividing the above in equation by "x"

$\Rightarrow\ \frac{a}{x} > \frac{b}{x}\quad [\because x < a]$

[Sign of inequality changes when its both sides are divided by a negative number]

Q16 Text Solution:

Since the three women choose their chairs first and their choices are limited to the chairs numbered 1 to 6, we must calculate the number of possible ways they can select these chairs. Order matters in this selection because each woman will sit in a unique chair; therefore, permutations should be used.

The number of different ways that the first woman can choose a chair is 6. After she has chosen, there are 5 chairs left for the second woman. The third woman can then choose from the remaining 4 chairs. So, the number of ways the three women can select the chairs is a permutation of 6 items taken 3 at a time, which is denoted by $^6P_3$

fter the three women have taken their seats, there are 10 - 3 = 7 chairs left, but the men can only choose from the remaining chairs numbered 7 to 10, so they have 4 chairs to choose from. The number of different ways the first man can choose a chair is 4. After he has chosen, there are 3 chairs left for the second man. So, the number of ways the two men can select from the 4 chairs is a permutation of 4 items taken 2 at a time, which is denoted by $^4P_2$.

Now we need to multiply the number of ways the women can sit by the number of ways the men can sit, as these are independent events:

Number of ways women can choose = $^6P_3$

Number of ways men can choose = $^4P_2$

Total number of ways = $^6P_3 \times {}^4P_2$

So the correct answer is option A: $^6P_3 \times {}^4P_2$

Q17 Text Solution:

Here, $x^2 + 1 = 0$

$\Rightarrow\ x^2 = -1$

$\Rightarrow\ \dot{x} = \pm i$

It is an imaginary number.

Hence, {x : x is a real number of x^2 + 1 = 0} is an empty set.

Q18 Text Solution:

Given, f(x) = ax + b, where a and b are integers,
f(-1) = -5 and f(3) = 3
We have, f(x) = ax + b
Put, x = -1 in above equation, we get
f(-1) = a(-1) + b
-5 = -a + b ...(i)
Now, f(3) = 3
3 = 3a + b ...(ii)
From Eqs. (i) and (ii) we get
a = 2, b = a - 5 = 2 - 5 = -3

Q19 Text Solution:

$p\left(\frac{1}{q}+\frac{1}{r}\right), q\left(\frac{1}{r}+\frac{1}{p}\right), r\left(\frac{1}{p}+\frac{1}{q}\right)$

$= p\left(\frac{q+r}{qr}\right),\ q\left(\frac{p+r}{pr}\right), r\left(\frac{p+q}{pq}\right)$

$= \frac{p(q+r)}{qr}+1, \frac{q(p+r)}{pr}+1, r\left(\frac{p+q}{pq}\right)$

$+1$ (On adding 1 each term)

$= \frac{pq+pr+qr}{qr}, \frac{qp+qr+pr}{pr}, \frac{rp+rq+pq}{pq}$

On dividing each term by pq + pr + qr we get

$= \frac{1}{qr}, \frac{1}{pr}, \frac{1}{pq}$ in AP

$= \frac{pqr}{qr}, \frac{pqr}{pr},$

$\frac{pqr}{pq}$ in APs (on multiply pqr both order)

$\Rightarrow p,\ q,\ r$ also in AP.

Q20 Text Solution:

Equation of the line passing through (2, 2) and perpendicular to 3x + y = 3

$(x-2) - 3(y-2) = 0$

$\Rightarrow\ x - 3y = -4$

$\therefore$ y intercept of this line is $\frac{4}{3}$.

Q21 Text Solution:

Given, distance between the foci = 16 and

$e = \sqrt{2}$

$2ae = 16$

$ae = 8$

$\Rightarrow a = \frac{8}{\sqrt{2}} = 4\sqrt{2}$

So, $b^2 = a^2\left(e^2 - 1\right)$

$b^2 = 32(2-1) = 32$

Equation of hyperbola is

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\frac{x^2}{32} - \frac{y^2}{32} = 1$

$\Rightarrow x^2 - y^2 = 32$

Q22 Text Solution:

We have, $\lim\limits_{x\to 0} \frac{\sin(2+x)-\sin(2-x)}{x} = A\cos B$

$\lim\limits_{x\to 0} \frac{2\cos\left(\frac{2+x+2-x}{2}\right)\sin\left(\frac{2+x-2+x}{2}\right)}{x}$

$= \lim\limits_{x\to 0} 2\cos 2\frac{\sin x}{x}$

$= 2\cos 2 \lim\limits_{x\to 0}\frac{\sin x}{x} = 2\cos 2$

Thus, $A = 2$ and $B = 2$

Q23 Text Solution:

For $\left(x^2+\frac{1}{x}\right)^n$ have middle term n is even

$\Rightarrow n = 2a; a \in N$

Middle term ${}^{2a}C_a\left(x^2\right)^a \cdot \frac{1}{x^a} = {}^{2a}C_a x^a$

$= 924x^6$

$\Rightarrow a = 6$

Also, for $a = 6, {}^{2a}C_a = {}^{12}C_6 = 924$

So, $n = 2a = 12$

Q24 Text Solution:

We have,

$1 + \frac{3}{7} + \frac{5}{72} + \frac{7}{73} + \ldots$

$\frac{1}{70} + \frac{3}{7} + \frac{5}{72} + \frac{7}{73} + \ldots$

Numerator are in AP

So, T_n of $AP = 1 + (n-1)^2 = 2n - 1$

Denominator are in GP

So, T_n of $GP = 7^{n-1}$

T_n of given series $= \frac{2n-1}{7^{n-1}}$

Q25 Text Solution:

$f \circ g(x) = f(g(x))$

As domain of $g(x) \to [0,\ \infty)$ so domain of f o $g(\lambda)$ also $[0,\ \infty)$

$\therefore f\left(g\left(x\right)\right) = \left(\sqrt{x}\right)^2$

$g \circ f(x) = g(f(x))$

$g \circ f\left(x\right) = \sqrt{x^2} = \left|x\right|$

Hence, $f \circ g\left(-4\right)$

$= \left(\sqrt{-4}\right)^2$ is not define.

Q26 Text Solution:

Given that $f(x) = 3x^2 - 5$ and

$g(x) = \frac{x}{x^2+1}$

$g \circ t = g\{f(x)\} = g(3x^2 - 5)$

$= \frac{3x^2-5}{(3x^2-5)^2+1} = \frac{3x^2-5}{9x^4-30x^2+25+1}$

$= \frac{3x^2-5}{9x^4-30x^2+26}$

Q27 Text Solution:

Given that $3a + 2b = 27$

$\Rightarrow 2b = 27 - 3a$

$\Rightarrow b = \frac{27-3a}{2}$

For $a = 1,\ b = 12$

$a = 3,\ b = 9$

$a = 5,\ b = 6$

$a = 7,\ b = 3$

$R = \{(1, 12), (3, 9), (5, 6), (7, 3)\}$

Q28 Text Solution:

$f(x) = \sin 2x + \cos 2x$ and $g(x) = x^2 - 1$

$g[f(x)] = [(\sin 2x + \cos 2x)^2 - 1]$

$g[f(x)] = \sin^2 2x + \cos^2 2x + \sin 4x - 1$

$g[f(x)] = \sin 4x$

Let $gf(x) = y$

$y = \sin 4x \quad [\because \text{Domain of } \sin x \text{is} - \frac{\pi}{2}, \frac{\pi}{2}]$

$\therefore$ Domain of $\sin 4x = [-\frac{\pi}{8}, \frac{\pi}{8}]$

Q29 Text Solution:

Given, statement

"If two lines do not intersect in the same plane, then they are parallel.

Contrapositive It two lines intersect in the same plane, then they are not parallel.

Converse If two lines do not intersect in the same plane, then they are parallel.

Q30 Text Solution:

We know, standard deviation,

$P^2 = \frac{\Sigma x^2}{n} - (\overline{x})^2$

$P = 5,\ \Sigma x = 50,\ n = 100$

$\Rightarrow 25 = \frac{\Sigma x^2}{100} - (50)^2$

$\Rightarrow \Sigma x^2 = 252500$

Thus, sum of squares of all observation in 252500.

Q31 Text Solution:

We have,

$x\begin{bmatrix} 3 \\ 2 \end{bmatrix} + y\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 15 \\ 5 \end{bmatrix}$

$\begin{bmatrix} 3x \\ 2x \end{bmatrix} + \begin{bmatrix} y \\ -y \end{bmatrix} = \begin{bmatrix} 15 \\ 5 \end{bmatrix}$

$\begin{bmatrix} 3x + y \\ 2x - y \end{bmatrix} = [\frac{15}{5}]$

$3x + y = 15 \quad ...(i)$

$2x - y = 5 \quad ...(ii)$

Adding Eqs. (i) and (ii), we get

$5x = 20 \quad \Rightarrow x = 4$

So, $y = 15 - 3x$ [From Eq. (i)]

$y = 15 - 3 \times 4 = 3$

Q32 Text Solution:

Given, $AB = B$ and $BA = A$

$A^2 + B^2 = AA + BB = A(BA) + B(AB)$

$= (AB)(A) + (BA)B = BA + AB = A + B$

Q33 Text Solution:

$A = \begin{bmatrix} 2-k & 2 \\ 1 & 3-k \end{bmatrix}$ is singular matrix we know that A is singular matrix, then |A| = 0

$|A| = (2-k)(3-k) - 2$

$\Rightarrow 0 = 6 - 2k - 3k + k^2 - 2$

$\Rightarrow k^2 - 5k + 4 = 0 \Rightarrow k^2 - 5k = -4$

$\Rightarrow 5k - k^2 = 4$

Q34 Text Solution:

We know that areas of a triangle with vertices $(x_1,\ y_1),\ (x_2,\ y_2)$ and $(x_3,\ y_3)$ is given by

$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$\therefore \Delta = \frac{1}{2}\begin{vmatrix} -3 & 0 & 1 \\ 3 & 0 & 1 \\ 0 & k & 1 \end{vmatrix}$

Expanding along C_2

$9 = \frac{1}{2}[-k(-3-3)]$

$\Rightarrow 18 = 3k + 3k = 6k$

$\therefore k = \frac{18}{6} = 3$

Q35 Text Solution:

Given,

$$\Delta = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

and $$\Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ bc & ca & ab \\ a & b & c \end{vmatrix}$$

$$\Delta = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

$R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$

$$\Delta = \begin{vmatrix} 0 & a-b & a^2-b^2 \\ 0 & b-c & b^2-c^2 \\ 1 & c & c^2 \end{vmatrix}$$

$$\Delta = \left|a-b\right|\left|b-c\right| \begin{vmatrix} 0 & 1 & a+b \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix}$$

$\Delta = (a-b)(b-c)(c-a) \quad ...(i)$

Now, $$\Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ bc & ca & ab \\ a & b & c \end{vmatrix}$$

$C_1 \to C_1 - C_2, C_2 \to C_2 - C_3$

$$\Delta_1 = \begin{vmatrix} 0 & 0 & 1 \\ c(b-a) & a(c-b) & ab \\ a-b & b-c & c \end{vmatrix}$$

$$\Delta_1 = \left|a-b\right|\left|b-c\right| \begin{vmatrix} 0 & 0 & 1 \\ -c & -a & ab \\ 1 & 1 & c \end{vmatrix}$$

$\Delta_1 = (a-b)(b-c)(a-c)$

$\Delta_1 = -(a-b)(b-c)(c-a)$

From Eqs. (i) and (ii), we get

Q36 Text Solution:

Here, $\sin^{-1}\left(\frac{2a}{1+a^2}\right)+\cos^{-1}\left(\frac{1-a^2}{1+a^2}\right)$

$= \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

We know that $2\tan^{-1} x = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

and $2\tan^{-1} x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

$\because 2\tan^{-1} a + 2\tan^{-1} a = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

$2\left[\tan^{-1}\left(\frac{a+a}{1-a\times a}\right)\right] = 2\tan^{-1} x$

$\Rightarrow \frac{2a}{1-a^2} = x$

Q37 Text Solution:

Here, $\cot^{-1}\left[\frac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}-\sqrt{1-\sin x}}\right]$

$= \cot^{-1}\left[\frac{\left(\sqrt{1-\sin x}+\sqrt{1+\sin x}\right)^2}{(1-\sin x)-(1+\sin x)}\right]$

$= \cot^{-1}\left(\frac{1-\sin x+1+\sin x+2\sqrt{1-\sin^2 x}}{1-\sin x-1-\sin x}\right)$

$\Rightarrow \cot^{-1}\left(\frac{2+2\sqrt{\cos^2 x}}{-2\sin x}\right)$

$\Rightarrow \cot^{1}\left(\frac{1+\cos x}{-\sin x}\right)$

$\Rightarrow \cot^{-1}\left(\frac{1+2\cos^2\frac{x}{2}-1}{-2\sin\frac{x}{2}\cos\frac{x}{2}}\right)$

$\Rightarrow \cot^{1}\left(\frac{-\cos\frac{x}{2}}{\sin\frac{x}{2}}\right)$

$\Rightarrow \cot^{-1}\left(-\cot\frac{x}{2}\right) = \cot^{-1}\left(\cot\left(\pi-\frac{x}{2}\right)\right)$

$= \pi - \frac{x}{2}$

Q38 Text Solution:

Given, $f(x) = \cot x$

$\Rightarrow f(x) = \frac{\cos x}{\sin x}$

We know that sin x = 0, if f(x) is discontinuous

$\therefore$ If $\sin x = 0$

$\therefore x = n\pi,\ n \in n\pi$

So, the given function f(x) is discontinuous on the set $\{x = n\pi,\ n \in Z\}$

Q39 Text Solution:

$y = f(f(x)) = f\left(\frac{1}{2+x}\right)$

$= \frac{1}{2+\frac{1}{2+x}} = \frac{2+x}{4+2x+1} = \frac{2+x}{5+2x}$

This function is not continuous at x = -2 (because f(x) is not defined at this point) and x = -5/2 because f(f(x)) is not defined at this point.

Q40 Text Solution:

Given, $y = a\sin x + b\cos x$

$\frac{dy}{dy} = a\cos x - b\sin x$

$y^2 + \left(\frac{dy}{dx}\right)^2 = (a\sin x + b\cos x)^2$

$+ (a\cos x - b\sin x)^2$

$= a^2\sin^2 x + b^2\cos^2 x + 2ab\sin x\cdot\cos x$

$+ a^2\cos^2 x + b^2\sin^2 x - 2ab\sin x\cdot\cos x$

$= a^2(\sin^2 x + \cos^2 x) + b^2(\sin^2 x + \cos^2 x)$

$= a^2 + b^2 \quad [\because \sin^2 x + \cos^2 x = 1]$

$=$ constant

Q41 Text Solution:

Given, $f\left(x\right) = 1 + nx + \frac{n(n-1)}{2}x^2 + \frac{n(n-1)(n-2)}{6}x^3 + \ldots + x^n$.

The given equation can be written as f(x) = $(1+x)^n$

$\because (1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots x^n$

Now, $f'(x) = n(1+x)^{n-1}$ and $f''(x) = n(n-1)(1+x)^{n-2}$

Put $x = 1$

$\therefore f''(1) = n(n-1)2^{n-2}$

Q42 Text Solution:

Given, $A = \begin{bmatrix} 1 & \tan\frac{\alpha}{2} \\ -\tan\frac{\alpha}{2} & 1 \end{bmatrix}$ and $AB = I$

$\Rightarrow B = A^{-1}$

Now, $A = \begin{bmatrix} 1 & \tan\frac{\alpha}{2} \\ -\tan\frac{\alpha}{2} & 1 \end{bmatrix}$

$\left|A\right| = \begin{bmatrix} 1 & \tan\frac{\alpha}{2} \\ -\tan\frac{\alpha}{2} & 1 \end{bmatrix} = 1 + an^2\frac{\alpha}{2} = \sec^2\frac{\alpha}{2}$

$\text{adj } A = \begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix}$

$A^{-1} = \frac{1}{\sec^2\frac{\alpha}{2}}\begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix}$

$= \cos^2\frac{\alpha}{2}\begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix} = \cos^2\frac{\alpha}{2}AT$

$B = \left[\cos^2\frac{\alpha}{2}\right]AT$

Q43 Text Solution:

Here, $u = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

$u = 2\tan^{-1}x\left[\because 2\tan^{-1}x = \sin^{-1}\left(\frac{2x}{1+x^2}\right)\right]$

$\Rightarrow \frac{du}{dx} = 2 \times \frac{1}{1+x^2} \quad \ldots(\text{i})$

and $v = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

$y = 2\tan^{-1}x\left[\because 2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)\right]$

$\Rightarrow \frac{dv}{dx} = \frac{2}{1+x^2} \quad \ldots(\text{ii})$

From Eqs. (i) and (ii), we get

$\frac{dv}{dx} = 1$

Q44 Text Solution:

To find the acceleration when the particle comes to rest, we first have to determine at what time 't' the particle comes to rest. A particle comes to rest when its velocity is zero. The velocity 'v' of a particle is given by the derivative of the position function with respect to time, which in this case is

$v(t) = \frac{ds}{dt}$

Let us find the velocity function by differentiating the given position function:

$v\left(t\right) = \frac{d}{dt}\left(\frac{2t^3}{3} - 18t + \frac{5}{3}\right) = \frac{d}{dt}\left(\frac{2t^3}{3}\right) - \frac{d}{dt}\left(18t\right) + \frac{d}{dt}\left(\frac{5}{3}\right)$

Applying the differentiation rules, we get:

$v\left(t\right) = \frac{2}{3}\cdot 3t^2 - 18 = 2t^2 - 18$

Now, we will set the velocity to zero to find the time 't' when the particle comes to rest:

$0 = v\left(t\right) = 2t^2 - 18$

Solve for 't' gives:

$2t^2 = 18$

$t^2 = \frac{18}{2}$

$t^2 = 9$

$t = \pm 3$

Since time cannot be negative, we consider 't - 3' seconds.

Now we need to find the acceleration at t=3 seconds. Acceleration 'a' is the derivative of the velocity function, i.e., $a(t) = \frac{dv}{dt}$

Let us differentiate the velocity function to find the acceleration function:

$a(t) = \frac{d}{dt}\left(2t^2 - 18\right) = \frac{d}{dt}\left(2t^2\right) - \frac{d}{dt}(18)$

Applying the differentiation rules, we get:

a(t) = 2 . 2t - 0 = 4t

Finally, we find the acceleration when the particle comes to rest, i.e. at t = 3 seconds:

a(3) = 4.3 = 12 m/s^2

Therefore, the acceleration when the particle comes to rest is 12 m/s^2 which corresonds to Option B.

Q45 Text Solution:

Given, particles moves along the curve

$given\ \frac{dx}{dt} = 4\frac{dy}{dt}$

$\frac{x^2}{16} + \frac{y^2}{4} = 1$

$\Rightarrow \frac{2x}{16}\left(4\frac{dy}{dt}\right) + \frac{2y}{4}\left(\frac{dy}{dt}\right) = 0$

$\frac{x}{2} + \frac{y}{2} = 0$

$y = -x$

since y and x are opposite sign

particle must lie in II and IV quadrant

Q46 Text Solution:

Let P(x, y) be the position of the jet and the solider is placed at A(3, 2)

$\Rightarrow AP = \sqrt{(x-3)^2 + (y-2)^2}$ ....(i)

As, $y = (x^2+2) \Rightarrow x^2 = y - 2$ ....(ii)

$\Rightarrow (AP)^2 = (x-3)^2 + x^4 = z$ (say)

$\Rightarrow \frac{dz}{dx} = 2\left(x-3\right) + 4x^3$ and $\frac{d^2z}{dx^2} = 2 + 12x^2$

For local points of maxima and minima

$\frac{dz}{dx} = 0 \Rightarrow 2\left(x-3\right) + 4x^3 = 0$

$x = 1$ and $\frac{d^2z}{dx^2}\left(\text{at } x = 1\right) = 14 > 0$

\ z is minimum, where x = 1, y = 1 + 2 = 3

Also, minimum distance

$= \sqrt{(1-3)^2 + (3-2)^2} = \sqrt{5}$ units

Q47 Text Solution:

Let $I = \int_2^8 \frac{5^{\sqrt{10-x}}}{5^{\sqrt{x}} + 5^{\sqrt{10-x}}} dx$ ..(i)

Using property

$\int_0^a f(x)dx = \int_0^a f(0+a-x)dx$

$I = \int_2^8 5^{\sqrt{10-(2+8-x)}}$

$I = \int_2^8 \frac{5^{\sqrt{10-10+x}}}{5^{\sqrt{10-x}} + 5^{\sqrt{10-10+x}}}$

$I = \int_2^8 \frac{5^{\sqrt{x}}}{5^{\sqrt{10-x}} + 5^{\sqrt{x}}}$ ...(ii)

Adding Eq.s (i) and (ii), we get

$2I = \int_2^8 \frac{5^{\sqrt{10-x}} + 5^{\sqrt{x}}}{5^{\sqrt{x}} + 5^{\sqrt{10-x}}} dx$

$2I = \int_2^8 dx$

$I = \frac{1}{2}[x]_2^8 = \frac{1}{2} \times 6 = 3$

Q48 Text Solution:

Let $I = \int \sqrt{\text{cosec } x - \sin x} dx$

$I = \int \frac{\sqrt{1-\sin^2 x}}{\sqrt{\sin x}} dx$

$= \int \frac{\cos x}{\sqrt{\sin x}} dx$

Put, sin x - k

cos x dx = dk

$I = \int \frac{dk}{\sqrt{k}} = \frac{k^{\frac{-1}{2}+1}}{\frac{-1}{2}+1} + C$

$= \frac{k^{\frac{1}{2}}}{\frac{1}{2}} + C = 2\sqrt{k} + C$

$I = 2\sqrt{\sin x} + C \quad \left[\because k = \sin x\right]$

Q49 Text Solution:

Here, $g(x) = x - \frac{1}{x}$

$f \circ g(x) = x^3 - \frac{1}{x^3} = \left(x - \frac{1}{x}\right)^3 + 3x \cdot \frac{1}{x}\left(x - \frac{1}{x}\right)$

$f \circ g(x) = \left(x - \frac{1}{x}\right)^3 + 3\left(x - \frac{1}{x}\right)$

$\therefore f(x) = x^3 + 3x$

$f'(x) = 3x^2 + 3$

Q50 Text Solution:

Let r be the radius of the circular disc and A be the area of the circular disc at any instant of time. we know that

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

According to the question, the circular disc expands on heating with the rate of change of radius is 0.05 cm/s

$\frac{dr}{dt} = 0.05$ cm/s

$\frac{dA}{dt} = 2 \times \pi \times 5.2 \times 0.05$

$\Rightarrow \frac{dA}{dt} = 0.52\pi \text{ cm}^2/\text{s}$

So, the rate at which its area is increasing, when radius is 5.2 cm is $0.52\pi \text{ cm}^2$/s.

Q51 Text Solution:

Let $I = \int_{-2}^{2} x^3 + 3x^2 + 3x + 3 + (x+1)\cos(x+1)\, dx$

$I = \int_{-2}^{0} \left[(x+1)^3 + 2 + (x+1)\cos(x+1)\right] dx$

Putting, x + 1 = t

Then dx = dt, we get

$I = \int_{-1}^{1} \left[t^3 + 2 + t\cos t\right] dt$

$I = \int_{-1}^{1} (2)u = [2t]_{-1}^{1} = 2[1 - (-1)] = 4$

At t^3 + t cos t is an odd function.

Q52 Text Solution:

Let $I = \int_0^\pi \frac{x \tan x}{\sec x \cdot \text{cosec } x} dx$ ...(i)

Using property $I = \int_0^\pi f(x)dx$

$= \int_0^\pi f(0 + \pi - x) dx$

Then, $I = \int_0^\pi \frac{(\pi - x)\tan(\pi - x)}{\sec(\pi - x) cosec(\pi - x)} dx$

$I = \int_0^\pi \frac{(\pi - x)\tan x}{\sec x \text{ cosec } x} dx$

On Adding Eqs. (i) and (ii), we get

$2I=\int_0^{\pi}\frac{\tan x(x+\pi-x)}{\sec x\cdot\operatorname{cosec} x}dx$

$2I=\int_0^{\pi}\frac{\pi\tan x}{\sec x\operatorname{cosec} x}dx$

$2I=\pi\int_0^{\pi}\sin^2 xdx$

$2I=\pi\int_0^{\pi}\frac{1-\cos 2x}{2}dx$

$2I=\pi\left[\frac{x}{2}-\frac{\sin 2x}{4}\right]_0^{\pi}$

$2I=\pi\left[\frac{\pi}{2}-0\right]$

$I=\frac{\pi^2}{4}$

Q53 Text Solution:

Let $I=\int\sqrt{5-2x+x^2}dx$

$I=\int\sqrt{(x-1)^2+(2)^2}$

We know that

$\left[\because\int\sqrt{x^2+a^2}=\frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right|\right]$

$=\frac{x-1}{2}\sqrt{5-2x+x^2}$

$+2\log\left|(x-1)+\sqrt{5-2x+x^2}\right|+C$

Q54 Text Solution:

$I=\int\frac{1}{1+3\sin^2 x+8\cos^2 x}dx$

Dividing the numerator and denominator by $\cos^2 x$, we get

$\Rightarrow I=\int\frac{\sec^2 x}{\sec^2 x+3\tan^2 x+8}dx$

$\Rightarrow I=\int\frac{\sec^2 x}{1+\tan^2 x+3\tan^2 x+8}dx$

$\Rightarrow I=\int\frac{\sec^2 x}{4\tan^2 x+9}dx$

Putting, $\tan x=t\Rightarrow\sec^2 xdx=dt$ we get

$I=\int\frac{dt}{4t^2+9}=\frac{1}{4}\int\frac{dt}{t^2+\left(\frac{3}{2}\right)^2}+C$

$I=\frac{1}{4}\times\frac{1}{3/2}\tan^{-1}\left(\frac{t}{3/2}\right)+C$

$\Rightarrow I=\frac{1}{6}\tan^{-1}\left(\frac{2t}{3}\right)+C=\frac{1}{6}\tan^{-1}\left(\frac{2\tan x}{3}\right)+C$

Q55 Text Solution:

Let the equation of the curve by y = f(x) and curve passes through the point (1, 1), So, f(1) = 1. Also, we know that at any point (x, y) on the curve, the product of the slope of its tangent and x co-ordinate of the point is equal to the y co-ordinate of the point.

$\frac{y}{x}=\frac{dy}{dx}$

$xy'=y$

$\frac{y'}{y}=\frac{1}{x}$

On integrating both sides w.r.t. x, we get

$\ln\ |y|=|In|x\ |+e^c$

$y=kx$, where $k=e^c$

Now, we can use the fact that the curve passes through the point (1, 1) to find the value of k.

$\Rightarrow\ 1=k(1)\quad k=1$

Therefore, the equation of the curve is y = x

Substituting the value of x and y from the given option, we find that the curve passes through the point (2, 2)

Q56 Text Solution:

Given, $1+\left(\frac{dy}{dx}\right)^2+\left(\frac{d^2y}{dx^2}\right)^2=\sqrt[3]{\frac{d^2y}{dx^2}+1}$

On cubic both sides, we get

$\left[1+\left(\frac{dy}{dx}\right)^2+\left(\frac{d^2y}{dx^2}\right)^2\right]^3=\left(\frac{d^2y}{dx^2}+1\right)$

$\left[1+\left(\frac{dy}{dx}\right)^2\right]^3+\left(\frac{d^2y}{dx^2}\right)^6+3\left[1+\left(\frac{dy}{dx}\right)^2\right]$

$\cdot\frac{d^2y}{dx^2}$

$\left[1+\left(\frac{dy}{dx}\right)^2+\left(\frac{d^2y}{dx^2}\right)^2\right]=\frac{d^2y}{dx^2}+1$

So, degree of the differential equation is 6 .

Q57 Text Solution:

To determine which option is correct, we can analyze the given equation using the properties of vector norms. The condition $\left|\vec{a}+\vec{b}\right|=\left|\vec{a}-\vec{b}\right|$ implies that the vector sum and difference of vectors $\vec{a}$ and $\vec{b}$ have the same magnitude. Let's expand both sides using the formula for the magnitude of a vector sum and difference:

$\left|\vec{a}+\vec{b}\right|^2=\left(\vec{a}+\vec{b}\right)\cdot\left(\vec{a}+\vec{b}\right)=\left|\vec{a}\right|^2$

$+2\vec{a}\cdot\vec{b}+\left|\vec{b}\right|^2$

And for the difference, we have:

$$|\vec{a}-\vec{b}|^2 = (\vec{a}-\vec{b})\cdot(\vec{a}-\vec{b}) = |\vec{a}|^2 - 2\vec{a}\cdot\vec{b} + |\vec{b}|^2$$

Given that $\left|\vec{a}+\vec{b}\right|^2 = \left|\vec{a}-\vec{b}\right|^2$, we can equate the right-hand sides of the above equations:

$$\left|\vec{a}\right|^2 + 2\vec{a}\cdot\vec{b} + \left|\vec{b}\right|^2 = \left|\vec{a}\right|^2 - 2\vec{a}\cdot\vec{b} + \left|\vec{b}\right|^2$$

By simplifying this equation, we can solve for the dot product $\vec{a}\cdot\vec{b}$:

$$2\vec{a}\cdot\vec{b} = -2\vec{a}\cdot\vec{b}$$

$$4\vec{a}\cdot\vec{b} = 0$$

$$\vec{a}\cdot\vec{b} = 0$$

The dot product of two vectors is zero if and only if the vectors are orthogonal (perpendicular) to each other. Therefore, we can conclude that vectors $\vec{a}$ and $\vec{b}$ are perpendicular:

Hence, the correct answer is:

Option D

$\vec{a}$ and $\vec{b}$ are perpendicular.

Q58 Text Solution:

We know that component of a in the direction of b is

$\frac{|a\cdot b|}{|b|^2}$

Let $b = \hat{i} + \hat{j} + 2\hat{k}$ and $a = \hat{i}$

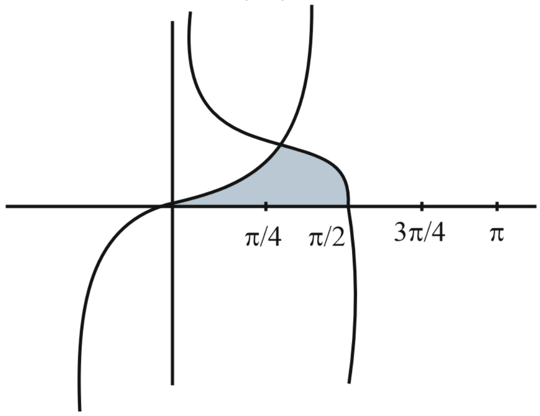
Then, we have $\left|a\cdot b\right| = 1$ and $|b| = \sqrt{6}$

Now, the component of i in the direction $\hat{i} + \hat{j} + 2\hat{k}$ is $\frac{1}{\sqrt{6}} = \frac{\sqrt{6}}{6}$

Q59 Text Solution:

Given, $c_1 \to y = \tan x, x \in \left[-\frac{\pi}{3}, \frac{\pi}{3}\right]$

$c_2 \to y = \cot x \in \left[\frac{\pi}{6}, \frac{\pi}{6}\right]$



The region is symmetric about lines $x = \frac{\pi}{4}$ and its first portion is bounded between the lines x = 0 and $x = \frac{\pi}{4}$

$\therefore$ Required Area $= 2\int_0^{\pi/4} \tan x \, dy$

$= 2\left[\log(\sec x)\right]_0^{\pi/4}$

$= 2\left[\log\left(\sec\frac{\pi}{4}\right) - \log\left(\sec 0\right)\right]$

$= 2\log\sqrt{2} = \log 2$ sq units.

Q60 Text Solution:

$\therefore$ Required area, $A = \int_3^5 \left(x+1\right)dx$

$= \left[\frac{x^2}{2} + x\right]_3^5 = \left[\frac{25}{2} + 5 - \frac{9}{2} - 3\right]$

$= \left[\frac{25+10-9-6}{2}\right] = \frac{20}{2}$

$= 10$ sq. units